Chapter 12

```
> with(plots):
Warning, the name changecoords has been redefined
```

Question 1

```
> solve(y=110+0.75*(y1+80-0.2*y1)+320+0.1*y1+330+440-10-0.2*y1,
  y);
```

$$1250. + .5000000000\, y1$$

```
> sol0:=rsolve({y(t)=1320+0.5*y(t-1),y(0)=2500},y(t));
```

$$sol0 := -140\left(\frac{1}{2}\right)^t + 2640$$

The difference equation which results is:

$Y_t = 1320 + .5\ Y_{t-2}$

In order to solve this second order equation we require to make assumptions about the level of income in period 0 and period 1. We assume that in period 0 income is 2500 and in period 1 it is 2570.

```
> sol1:=rsolve({y(t)=1320+0.5*y(t-2),y(0)=2500,y(1)=2570},y(t))
  ;
```

$$sol1 := -\frac{1}{2}(-2570 - 1250\sqrt{2})\left(\frac{1}{2}\sqrt{2}\right)^t\sqrt{2} + \frac{1}{2}(-2570 + 1250\sqrt{2})\left(-\frac{1}{2}\sqrt{2}\right)^t\sqrt{2}$$
$$+ 2640 - \frac{1}{2}(1320\sqrt{2} + 2640)\left(\frac{1}{2}\sqrt{2}\right)^t\sqrt{2} + \frac{1}{2}(-1320\sqrt{2} + 2640)\left(-\frac{1}{2}\sqrt{2}\right)^t\sqrt{2}$$

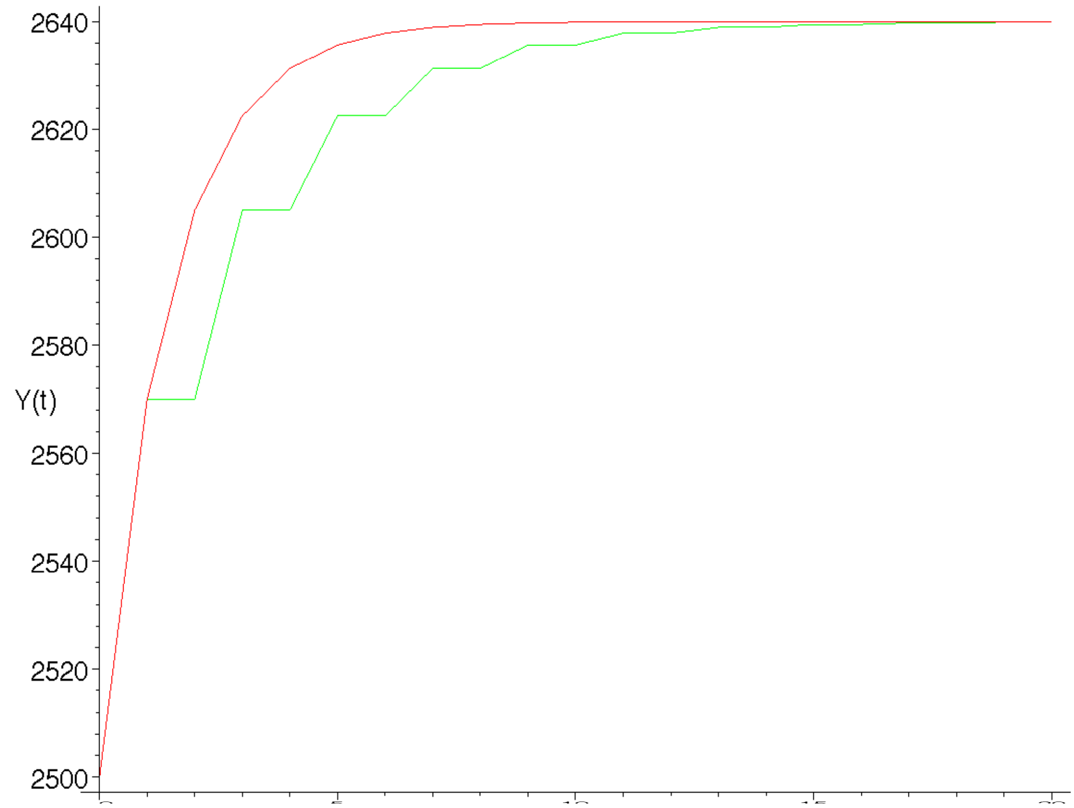
```
> evalf(sol1);
```

$$-119.497476\ .7071067810^t - 20.5025255\ (-.7071067810)^t + 2640.$$

```
>
```

t	y0(t)	y1(t)
0	2500.00	2500.00
1	2570.00	2570.00
2	2605.00	2570.00
3	2622.50	2605.00
4	2631.25	2605.00
5	2635.62	2622.50
6	2637.81	2622.50
7	2638.90	2631.25
8	2639.45	2631.25
9	2639.72	2635.62
10	2639.86	2635.62
11	2639.93	2637.81
12	2639.96	2637.81
13	2639.98	2638.90
14	2639.99	2638.90
15	2639.99	2639.45
16	2639.99	2639.45
17	2639.99	2639.72
18	2639.99	2639.72
19	2639.99	2639.86
20	2639.99	2639.86

```
> points0:=seq([t,sol0],t=0..20):
> points1:=seq([t,evalf(sol1)],t=0..20):
> plot([[points0],[points1]],labels=["t","Y(t)"]);
```



Question 2

(i)

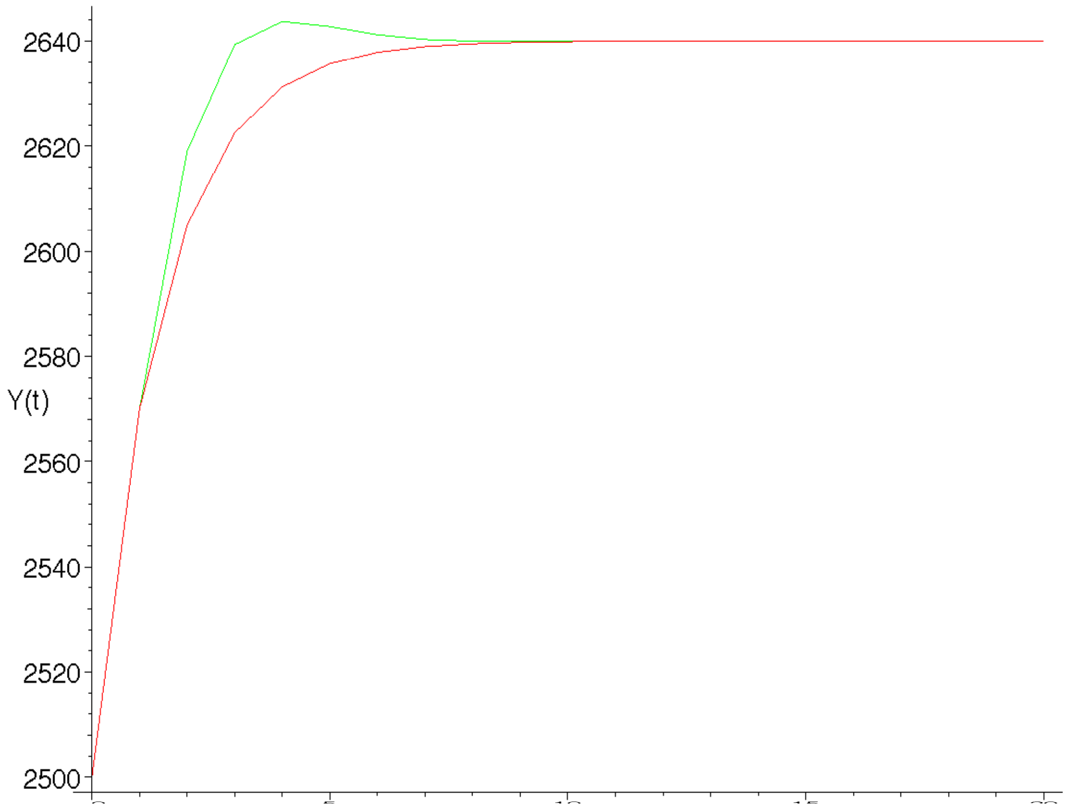
The new resulting difference equation is

$$Y_t = 1320 + .7\ Y_{t-1} - .2\ Y_{t-2}$$

```
> soly21:=evalf(rsolve({y(t)=1320+0.7*y(t-1)-0.2*y(t-2),y(0)
=2500,y(1)=2570},y(t)));
```

$$soly21 := -(70.000000 + 37.717114\ I)\ (.3500000000 - .2783882182\ I)^t - (70.000000 - 37.717114\ I)\ (.3500000000 + .2783882182\ I)^t + 2640.$$

```
> points21:=seq([t,evalf(soly21)],t=0..20):
> plot([[points0],[points21]],labels=["t","Y(t)"]);
```



(ii)

The resulting difference equation is

$$Y_t = 1320 + .5\ Y_{t-1}$$

which is the same equation as in Question 1, and hence the same time path for income.

Question 3

Since $NX_t = X_t - M_t$ and $X_t = X_0$ while $M_t = M_0 + m\ Y_t$ then

$$NX_t = X_0 - M_0 - m\ Y_t$$

$$\Delta\ NX_t = -m\ \Delta\ Y_t$$

But

$$k_t = \frac{\Delta\ Y_t}{\Delta\ G}$$

Therefore

$$\Delta\ NX_t = -m\ k_t\ \Delta\ G$$

Also

$$\lim_{t \to \infty} m\ k_t = m\ (\lim_{t \to \infty} k_t) = m\ k$$

Question 4

(i)-(ii)

```
> solve(y=110+0.75*(y1+80-0.2*y1)+320+0.1*y1+330+440-10-0.2*y1,
  y);
```

$$1250. + .5000000000\ y1$$

```
> solve(ystar1=1250+0.5*ystar1,ystar1);
```

$$2500.$$

```
> solve(y=110+0.75*(y1+80-0.2*y1)+320+0.1*y1+330+440-10-0.3*y1,
  y);
```

$$1250. + .4000000000\, y1$$

```
> solve(ystar2=1250+0.4*ystar2,ystar2);
```

$$2083.333333$$

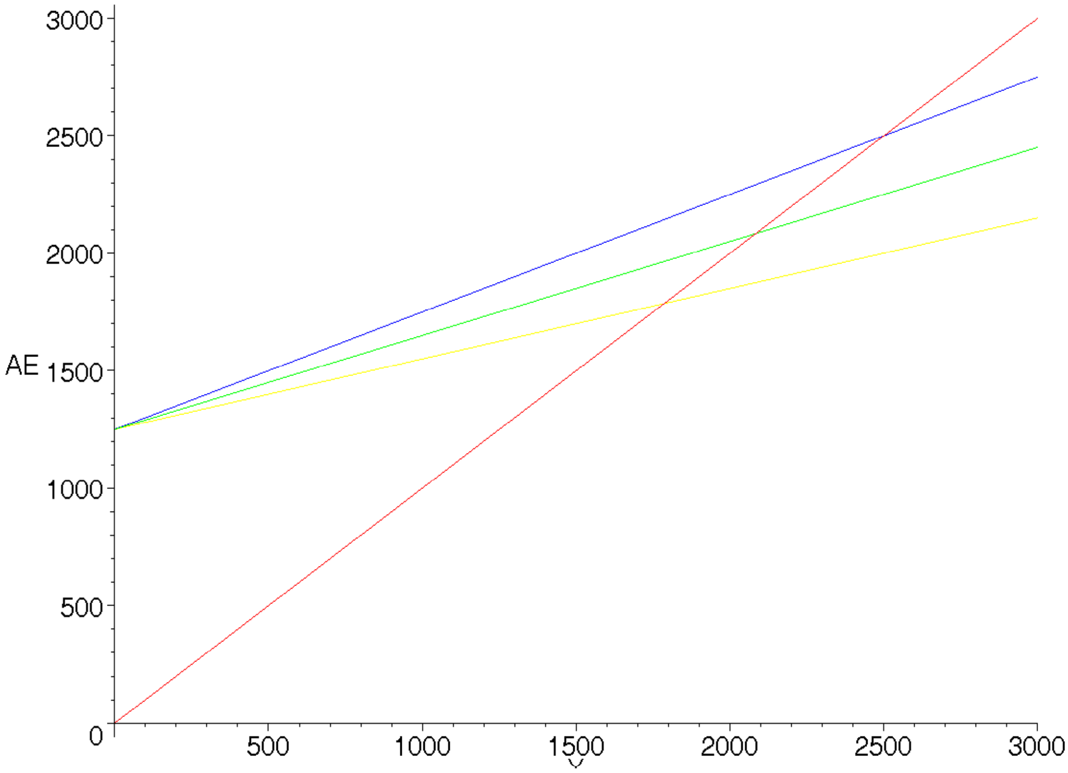
```
> solve(y=110+0.75*(y1+80-0.2*y1)+320+0.1*y1+330+440-10-0.4*y1,
  y);
```

$$1250. + .3000000000\, y1$$

```
> solve(ystar3=1250+0.3*ystar3,ystar3);
```

$$1785.714286$$

```
> plot({y,1250+0.5*y,1250+0.4*y,1250+0.3*y},
  y=0..3000,labels=["Y","AE"]);
```



(iii)

```
> sol1:=rsolve({y(t)=2500+0.5*y(t-1),y(0)=2000},y(t));
```

$$sol1 := -3000\left(\frac{1}{2}\right)^t + 5000$$

```
> sol2:=rsolve({y(t)=2500+0.4*y(t-1),y(0)=2000},y(t));
```

$$sol2 := -\frac{6500}{3}\left(\frac{2}{5}\right)^t + \frac{12500}{3}$$

```
> sol3:=rsolve({y(t)=2500+0.3*y(t-1),y(0)=2000},y(t));
```

$$sol3 := -\frac{11000}{7}\left(\frac{3}{10}\right)^t + \frac{25000}{7}$$

t	$m = .20$	$m = .30$	$m = .40$
0	2000.00	2000.00	2000.00
1	3500.00	3300.00	3100.00
2	4250.00	3820.00	3430.00
3	4625.00	4028.00	3529.00
4	4812.50	4111.20	3558.70
5	4906.25	4144.48	3567.61
6	4953.12	4157.79	3570.28
7	4976.56	4163.11	3571.08
8	4988.28	4165.24	3571.32
9	4994.14	4166.09	3571.39
10	4997.07	4166.43	3571.41
11	4998.53	4166.57	3571.42
12	4999.26	4166.63	3571.42
13	4999.63	4166.65	3571.42
14	4999.81	4166.66	3571.42
15	4999.90	4166.66	3571.42
16	4999.95	4166.66	3571.42
17	4999.97	4166.66	3571.42
18	4999.98	4166.66	3571.42
19	4999.99	4166.66	3571.42
20	4999.99	4166.66	3571.42

(iv)

Thus, the higher the marginal propensity to import, the lower the equilibrium value of income and the sooner the economy reaches equilibrium for a given level of income.

Question 5

The IS curve is given by equation (12.13)

$$r = \frac{a + nx_0 + (f + g)\,R}{h} - \frac{[1 - c\,(1 - t) - j + m]\,y}{h}$$

while the LM curve is given by equation (12.19)

$$r = \frac{m_0}{u} - \frac{k\,y}{u}$$

Note, however, that in Table 12.2 nx_0 is contained in the value of the parameter a. The BP curve is given by

$$r = rstar - \frac{bp_0 + (f+g)\,R}{v} + \frac{m\,y}{v}$$

The initial values are given by

```
> para0:={a=43.5,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=1.764,h=2,m0=3,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$$para0 := \{a = 43.5, c = .75, t = .3, j = 0, m = .2, f = 5, g = 2, R = 1.764, h = 2, m0 = 3, u = .5, k = .25, rstar = 15, bp0 = -3.5, v = 1\}$$

```
> intIS:=(a+(f+g)*R)/h;
```

$$intIS := \frac{a + (f+g)\,R}{h}$$

```
> slopeIS:=-(1-c*(1-t)-j+m)/h;
```

$$slopeIS := -\frac{1 - c\,(1-t) - j + m}{h}$$

```
> intIS0:=subs(para0,intIS);
```

$$intIS0 := 27.92400000$$

```
> slopeIS0:=subs(para0,slopeIS);
```

$$slopeIS0 := -.3375000000$$

```
> intLM:=-m0/u;
```

$$intLM := -\frac{m0}{u}$$

```
> slopeLM:=k/u;
```

$$slopeLM := \frac{k}{u}$$

```
> intLM0:=subs(para0,intLM);
```

$$intLM0 := -6.000000000$$

```
> slopeLM0:=subs(para0,slopeLM);
```

$$slopeLM0 := .5000000000$$

```
> intBP:=rstar-(bp0+(f+g)*R)/v;
```

$$intBP := rstar - \frac{bp0 + (f+g)\,R}{v}$$

```
> slopeBP:=m/v;
```

$$slopeBP := \frac{m}{v}$$

```
> intBP0:=subs(para0,intBP);
```

$$intBP0 := 6.152$$

```
> slopeBP0:=subs(para0,slopeBP);
```

$$slopeBP0 := .2$$

Initial equilibrium values are given by

```
> solve({r=intIS0+slopeIS0*y,r=intLM0+slopeLM0*y},{y,r});
```

$$\{y = 40.50626866, r = 14.25313433\}$$

The BP curve also intersects at the same values since

```
> solve({r=intIS0+slopeIS0*y,r=intBP0+slopeBP0*y},{y,r});
```

$$\{y = 40.50604651, r = 14.25320930\}$$

Also note that

```
> bp:=(x0-z0)+(f+g)*R-m*y+cf0+v*(r-rstar);
```

$$bp := x0 - z0 + (f + g)\, R - m\, y + cf0 + v\,(r - rstar)$$

```
> initbp:=subs({x0=0,z0=24,cf0=20.5,r=14.25313433,y=40.50626866
  },subs(para0,bp));
```

$$initbp := -.00011940$$

or initially $bp = 0$ taken to two decimal places.

(i) Rise in *a* from 43.5 to 50

The change in the parameter *a* affects only the IS curve, shifting it to the right. The IS cure will intersect the LM curve at a higher income level and a higher interest rate. With the higher interest rate and the real exchange rate constant, there will be a capital inflow and the balance of payments will go into surplus. To verify these statements:

```
> para1:={a=50,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=1.764,h=2,m0
  =3,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$$para1 := \{c = .75, t = .3, j = 0, m = .2, f = 5, g = 2, R = 1.764, h = 2, m0 = 3, u = .5, k = .25, rstar = 15, bp0 = -3.5, v = 1, a = 50\}$$

```
> intIS1:=subs(para1,intIS);
```

$$intIS1 := 31.17400000$$

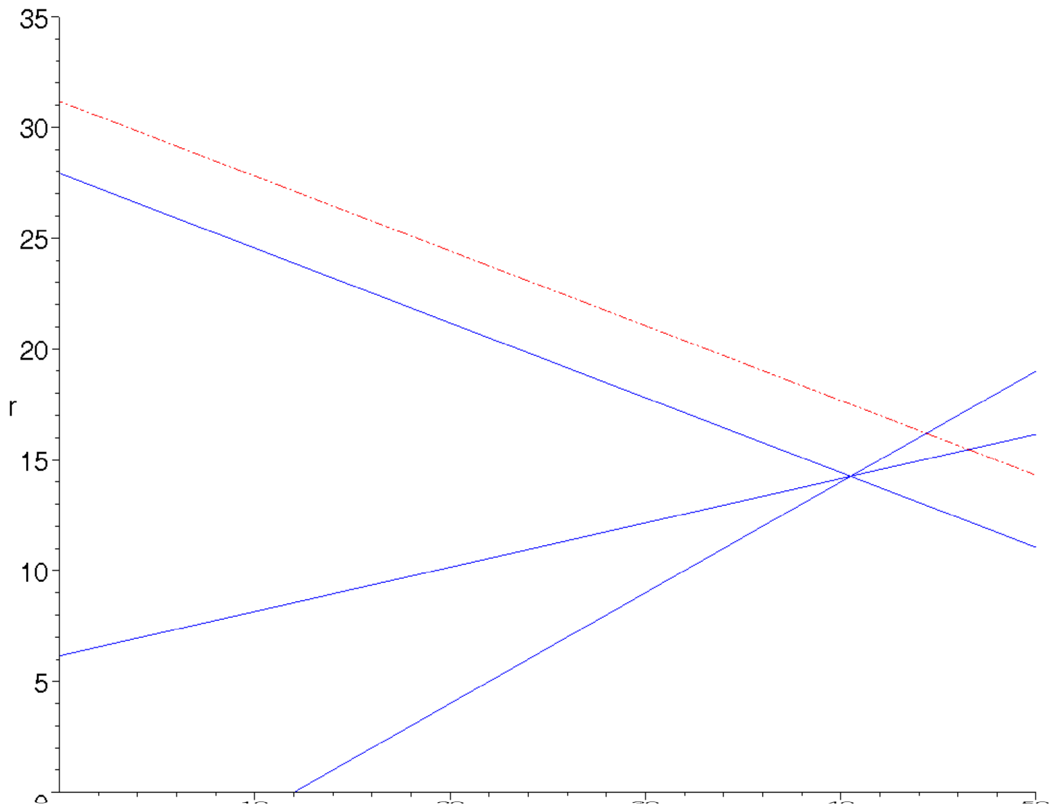
```
> solve({r=intIS1+slopeIS0*y,r=intLM0+slopeLM0*y},{y,r});
```

$$\{y = 44.38686567, r = 16.19343284\}$$

```
> bp1:=subs({x0=0,z0=24,cf0=20.5,y=44.38686567,r=16.19343284
  },subs(para1,bp));
```

$$bp1 := 1.16405971$$

```
> plot({intIS0+slopeIS0*y,intLM0+slopeLM0*y,intBP0+slopeBP0*
  y,intIS1+slopeIS0*y},y=0..50,0..35,colour=[blue,blue,blue,
  red],linestyle=[1,1,1,4],labels=["y","r"]);
```



(ii) A fall in *m*0 from 3 to 2

A fall in the money supply shifts the LM curve left, leaving the IS curve ane the BP curve unaffected. There is a resulting rise in the rate of interest and a fall in the level of income. The new LM curve intersects the IS curve above the BP curve. The increased interest rate leads to a capital inflow and an improvement in the balance of payments.

```
> para2:={a=43.5,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=1.764,h=2,
  m0=2,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$para2 := \{m0 = 2, a = 43.5, c = .75, t = .3, j = 0, m = .2, f = 5, g = 2, R = 1.764, h = 2, u = .5, k = .25, rstar = 15, bp0 = -3.5, v = 1\}$

```
> intLM2:=subs(para2,intLM);
```

$$intLM2 := -4.000000000$$

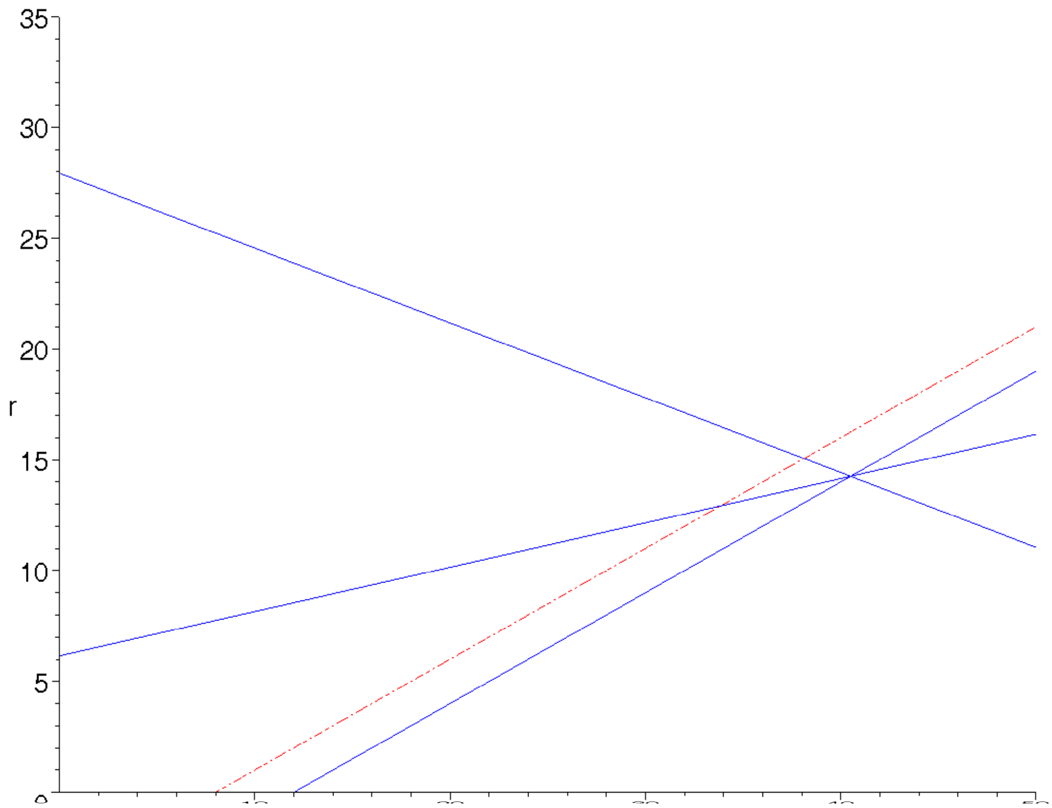
```
> solve({r=intIS0+slopeIS0*y,r=intLM2+slopeLM0*y},{y,r});
```

$$\{r = 15.05910448, y = 38.11820896\}$$

```
> bp2:=subs({x0=0,z0=24,cf0=20.5,y=38.11820896,r=15.05910448
  },subs(para2,bp));
```

$$bp2 := 1.28346269$$

```
> plot({intIS0+slopeIS0*y,intLM0+slopeLM0*y,intBP0+slopeBP0*
  y,intLM2+slopeLM0*y},y=0..50,0..35,colour=[blue,blue,blue,
  red],linestyle=[1,1,1,4],labels=["y","r"]);
```



(iii) Devaluation: rise in *R* from 1.764 to 2

A devaluation, rise in *R*, shifts the IS curve right and the BP curve down, leaving the LM curve unaffected. There results a rise in income and a rise in the rate of interest. The rise in the interest rate leads to a capital inflow and an improvement in the balance of payments. In the case of all shifts only the intercepts change.

```
> para3:={a=43.5,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=2,h=2,m0=3
  ,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$$para3 := \{R=2, a=43.5, c=.75, t=.3, j=0, m=.2, f=5, g=2, h=2, m0=3, u=.5, k=.25, rstar=15, bp0=-3.5, v=1\}$$

```
> intIS3:=subs(para3,intIS);
```

$$intIS3 := 28.75000000$$

```
> intBP3:=subs(para3,intBP);
```

$$intBP3 := 4.5$$

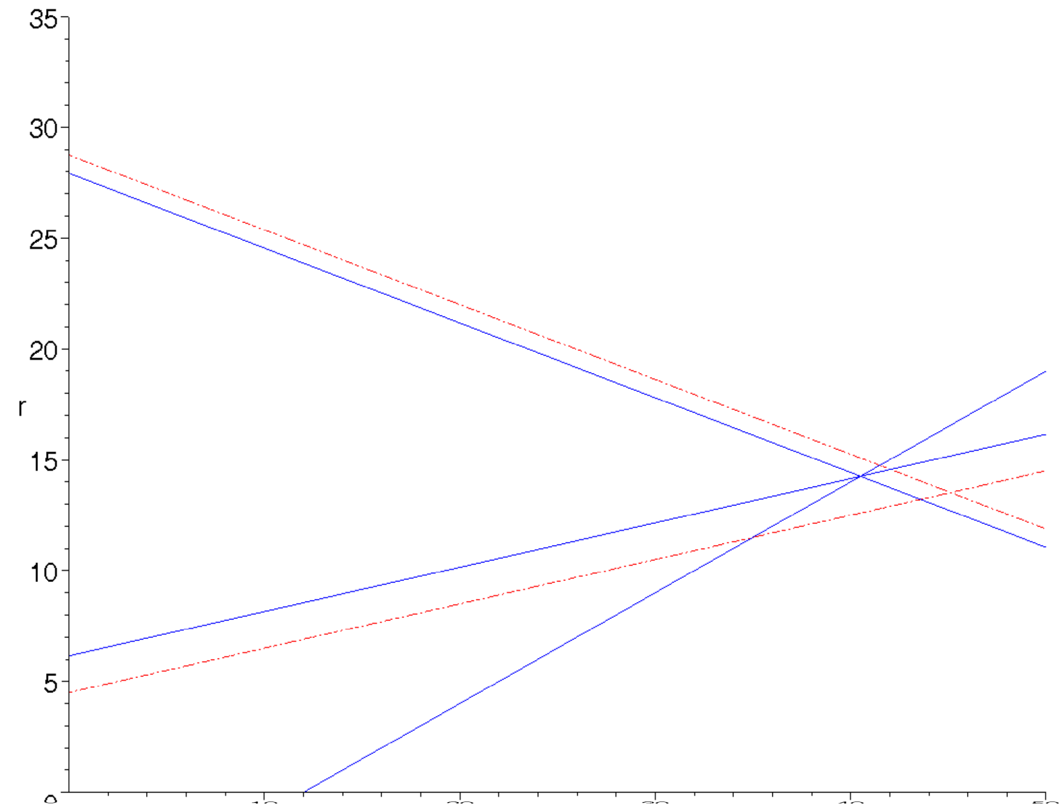
```
> solve({r=intIS3+slopeIS0*y,r=intLM0+slopeLM0*y},{y,r});
```

$$\{r=14.74626866, y=41.49253731\}$$

```
> bp3:=subs({x0=0,z0=24,cf0=20.5,y=41.49253731,r=14.74626866
  },subs(para3,bp));
```

$$bp3 := 1.94776120$$

```
> plot({intIS0+slopeIS0*y,intLM0+slopeLM0*y,intBP0+slopeBP0*
  y,intIS3+slopeIS0*y,intBP3+slopeBP0*y},y=0..50,0..35,colou
  r=[blue,blue,blue,red,red],linestyle=[1,1,1,4,4],labels=["
  y","r"]);
```



Question 6

(i) Rise in autonomous spending by 20

With a rise in autonomous spending of 10, the new interept for the IS curve rises by $\frac{\Delta a}{h} = \frac{20}{2}$ = 10, to the value 37.924

```
> solve({r=37.924-0.3375*y,r=-6+0.5*y},{y,r});
```

$\{y = 52.44656716, r = 20.22328358\}$

To solve for *bp* we use the information in Table 12.2 and the solution values just derived.

```
> newbp:=subs({x0=0, z0=24,
  cf0=20.5,y=52.44656716,r=20.22328358},subs(para0,bp));
```

$newbp := 3.58197015$

Under a floating exchange rate, the resulting surplus on the balance of payments leads to an appreciation of the exchange rate (*S* appreciates, and with *P* and *P** constant, *R* appreciates by the same amount). The value of *R* falls to 1.33.

```
> para6:={a=63.5,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=1.33,h=2,m0=3
  ,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$para6 := \{R = 1.33, a = 63.5, c = .75, t = .3, j = 0, m = .2, f = 5, g = 2, h = 2, m0 = 3, u = .5, k = .25, rstar = 15, bp0 = -3.5, v = 1\}$

```
> intIS6:=subs(para6,intIS);
```

$intIS6 := 36.40500000$

```
> intBP6:=subs(para6,intBP);
```

$intBP6 := 9.19$

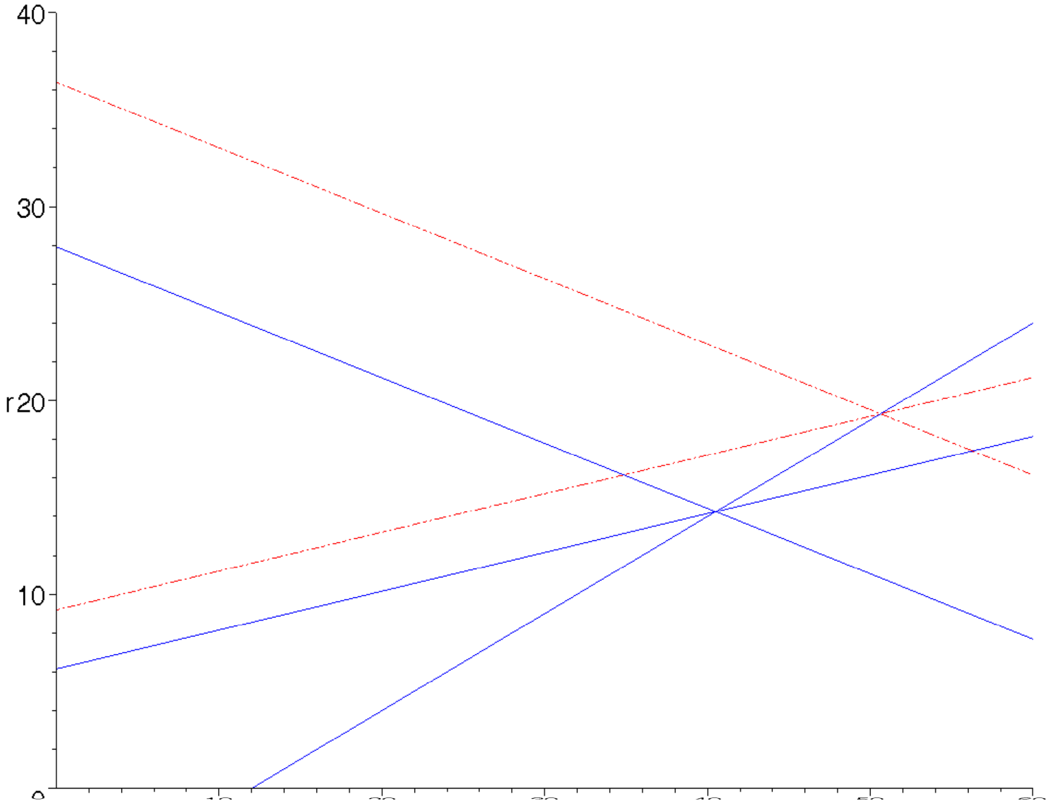
```
> solve({r=intIS6+slopeIS0*y,r=intLM0+slopeLM0*y},{y,r});
```

$\{y = 50.63283582, r = 19.31641791\}$

```
> bp6:=subs({y=50.63283582,r=19.31641791,x0=0,z0=24,cf0=20.5},subs(para6,bp));
```

$bp6 := -.00014925$

```
> plot([intIS0+slopeIS0*y, intLM0+slopeLM0*y, intBP0+slopeBP0*y, intIS6+slopeIS0*y, intBP6+slopeBP0*y],y=0..60,0..40,colour=[blue,blue,blue,red,red],linestyle=[1,1,1,4,4],labels=["y","r"]);
```



(ii) A rise in the money supply from 3 to 4

```
> solve({r=intIS0+slopeIS0*y,r=-8+0.5*y},{y,r});
```

$\{y = 42.89432836, r = 13.44716418\}$

To solve for *bp* we use the information in Table 12.2 and the solution values just derived.

```
> newbp:=subs({x0=0, z0=24, cf0=20.5,y=42.89432836,r=13.44716418},subs(para0,bp));
```

$newbp := -1.28370149$

Under a floating rate, the resulting deficit on the balance of payments leads to a depreciation of the exchange rate (*S* depreciates, and with *P* and *P** constant, *R* depreciates by the same amount). The value of *R* rises to 1.92.

```
> para62:={a=43.5,c=0.75,t=0.3,j=0,m=0.2,f=5,g=2,R=1.92,h=2,m0=3,u=0.5,k=0.25,rstar=15,bp0=-3.5,v=1};
```

$para62 := \{R = 1.92, a = 43.5, c = .75, t = .3, j = 0, m = .2, f = 5, g = 2, h = 2, m0 = 3, u = .5, k = .25, rstar = 15, bp0 = -3.5, v = 1\}$

```
> intIS62:=subs(para62,intIS);
```

$intIS62 := 28.47000000$

```
> intBP62:=subs(para62,intBP);
```

$intBP62 := 5.06$

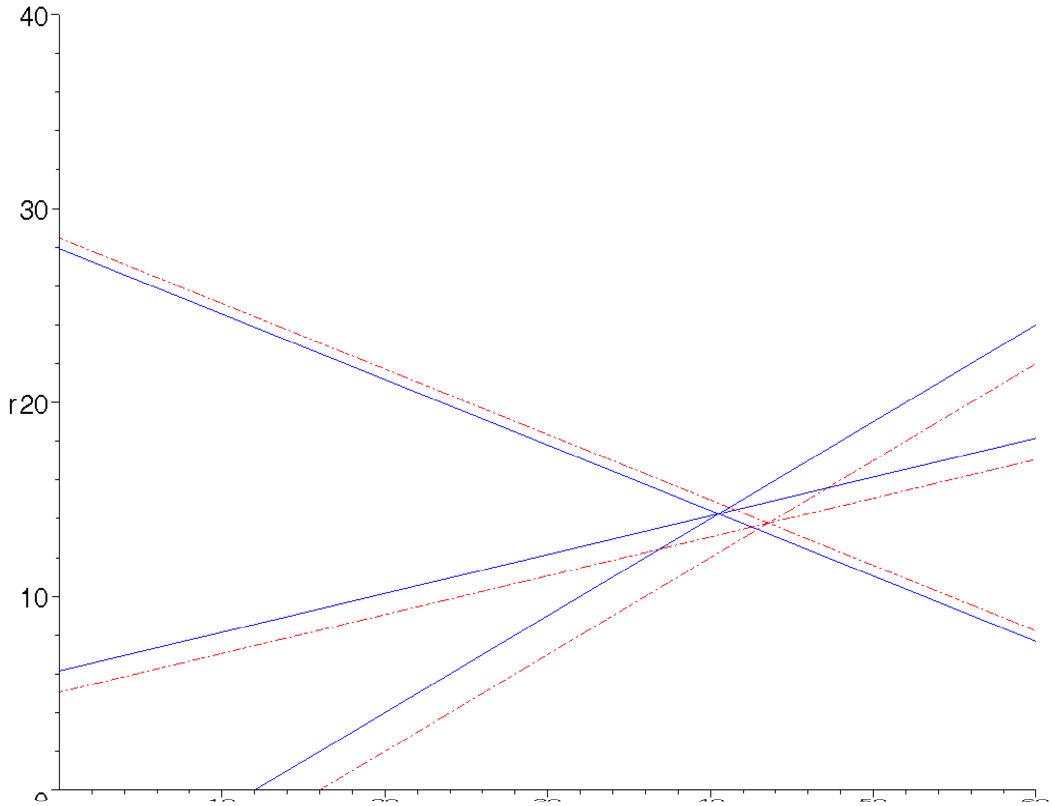
```
> solve({r=intIS61+slopeIS0*y,r=-8+0.5*y},{y,r});
```

$\{y = 1.194029851\ intIS61 + 9.552238806, r = .5970149254\ intIS61 - 3.223880597\}$

```
> bp62:=subs({x0=0, z0=24,
cf0=20.5,y=43.54626866,r=13.77313433},subs(para62,bp));
```

$bp62 := .00388060$

```
> plot([intIS0+slopeIS0*y,intLM0+slopeLM0*y,intBP0+slopeBP0*y,
-8+0.5*y,intIS62+slopeIS0*y,intBP62+slopeBP0*y],y=0..60,0..40,
colour=[blue,blue,blue,red,red,red],linestyle=[1,1,1,4,4,4],labels=["y","r"]);
```



Question 7

```
> solve({0=2.675-0.03375*y-0.1*r+0.35*s,0=-2.4+0.2*y-0.4*r,0=-0.00185-0.00002*y+0.0001*r+0.0007*s},{y,r,s});
```

$\{r = 16.78481013, s = 1.547016275, y = 45.56962025\}$

Although *Maple* will solve the difference equations, the solutions are rather meaningless.

Question 8

```
> solve({0=2.675-0.03375*y-0.1*r+0.35*s,0=-2.4+0.2*y-0.4*r,0=-0.925-0.01*y+0.05*r+0.35*s},{y,r,s});
```

$\{r = 16.78481013, s = 1.547016275, y = 45.56962025\}$

Again, *Maple* will solve the difference equations, but the solutions are meaningless.