Pockels effect or the magneto-optic Faraday effect. Any orthonormal set of unit polarization vectors $\{\hat{e}_1, \hat{e}_2\}$ on the plane that is normal to the wave propagation direction $\hat{k}$ can be used to expand the unit polarization vector $\hat{e}$ on this plane as a linear superposition of two orthogonal polarizations:

$$\hat{e} = c_1\hat{e}_1 + c_2\hat{e}_2, \tag{10.9}$$

where c_1 and c_2 are two complex constants subject to the normalization condition of $c_1 \cdot c_1^* + c_2 \cdot c_2^* = 1$. On the $\{\hat{e}_1, \hat{e}_2\}$ basis, the unit polarization vector $\hat{e}_\perp$ that is orthogonal to the unit polarization vector $\hat{e}$ can be expressed as

$$\hat{e}_\perp = c_2^*\hat{e}_1 - c_1^*\hat{e}_2. \tag{10.10}$$

It is clear that $\{\hat{e}, \hat{e}_\perp\}$ is also an orthonormal basis because $\hat{e} \cdot \hat{e}^* = \hat{e}_\perp \cdot \hat{e}_\perp^* = 1$ and $\hat{e} \cdot \hat{e}_\perp^* = \hat{e}_\perp \cdot \hat{e}^* = 0$. Therefore, the two unit polarization vectors $\hat{e}_1$ and $\hat{e}_2$ can be expressed in terms of the $\{\hat{e}, \hat{e}_\perp\}$ basis as

$$\hat{e}_1 = c_1^*\hat{e} + c_2\hat{e}_\perp, \quad \hat{e}_2 = c_2^*\hat{e} - c_1\hat{e}_\perp. \tag{10.11}$$

As an example, any polarization state on the xy plane can be represented by the unit vector $\hat{e} = \hat{x}\cos\alpha + \hat{y}\mathrm{e}^{\mathrm{i}\varphi}\sin\alpha$ given in (1.65), which is the linear superposition of the two orthonormal linear polarization unit vectors $\hat{x}$ and $\hat{y}$ with $c_1 = \cos\alpha$ and $c_2 = \mathrm{e}^{\mathrm{i}\varphi}\sin\alpha$. In this case, $\hat{e}_1 = \hat{x}$, $\hat{e}_2 = \hat{y}$, and $\hat{e}_\perp = \hat{x}\mathrm{e}^{-\mathrm{i}\varphi}\sin\alpha - \hat{y}\cos\alpha$. As another example, the linear polarization unit vector $\hat{x}$ can be expressed as $\hat{e} = \hat{x} = (\hat{e}_+ + \hat{e}_-)/\sqrt{2}$ in terms of the linear superposition of the orthonormal circular polarization unit vectors with $c_1 = c_2 = 1/\sqrt{2}$. In this case, $\hat{e}_1 = \hat{e}_+$, $\hat{e}_2 = \hat{e}_-$, and $\hat{e}_\perp = \mathrm{i}\hat{y} = (\hat{e}_+ - \hat{e}_-)/\sqrt{2}$.

When the phases of the two orthogonally polarized field components are differentially modulated, the polarization vector of the modulated optical wave becomes a function of time:

$$\hat{e}_\mathrm{m}(t) = c_1\mathrm{e}^{\mathrm{i}\varphi_1(t)}\hat{e}_1 + c_2\mathrm{e}^{\mathrm{i}\varphi_2(t)}\hat{e}_2 = \left[c_1\hat{e}_1 + c_2\mathrm{e}^{\mathrm{i}\Delta\varphi(t)}\hat{e}_2\right]\mathrm{e}^{\mathrm{i}\varphi_1(t)}, \tag{10.12}$$

where

$$\Delta\varphi(t) = \varphi_2(t) - \varphi_1(t) \tag{10.13}$$

is the time-varying phase difference due to differential phase modulation between the $\hat{e}_1$ and $\hat{e}_2$ components of the optical field. By substituting $\hat{e}_1$ and $\hat{e}_2$ of (10.11) into (10.12), we can express the modulated time-varying unit polarization vector $\hat{e}_\mathrm{m}(t)$ in terms of $\hat{e}$ and $\hat{e}_\perp$ as

$$\begin{aligned}\hat{e}_\mathrm{m}(t) &= \left[c_1c_1^*\mathrm{e}^{\mathrm{i}\varphi_1(t)} + c_2c_2^*\mathrm{e}^{\mathrm{i}\varphi_2(t)}\right]\hat{e} + \left[c_1c_2\mathrm{e}^{\mathrm{i}\varphi_1(t)} - c_1c_2\mathrm{e}^{\mathrm{i}\varphi_2(t)}\right]\hat{e}_\perp \\ &= \left[c_1c_1^* + c_2c_2^*\mathrm{e}^{\mathrm{i}\Delta\varphi_1(t)}\right]\mathrm{e}^{\mathrm{i}\varphi_1(t)}\hat{e} + c_1c_2\left[1 - \mathrm{e}^{\mathrm{i}\Delta\varphi(t)}\right]\mathrm{e}^{\mathrm{i}\varphi_1(t)}\hat{e}_\perp.\end{aligned} \tag{10.14}$$

It is clear from (10.14) that $\hat{e}_\mathrm{m}(t) \cdot \hat{e}_\perp \neq 0$ and $\hat{e}_\mathrm{m}(t) \neq \hat{e}$ when $c_1c_2 \neq 0$ and $\Delta\varphi(t) \neq 2m\pi$, resulting in a polarization change caused by differential phase modulation.

As discussed in Section 1.6, the polarization state of a wave depends only on the phase difference and the magnitude ratio of the two orthogonally polarized field components. Therefore, the polarization state defined by $\hat{e}_\mathrm{m}(t)$ is determined by the phase difference $\Delta\varphi(t)$ and the magnitude ratio $|c_1/c_2|$ of the $\hat{e}_1$ and $\hat{e}_2$ components, and is independent of the common