Coupled line components: directional couplers and power dividers

Microwave Electronics

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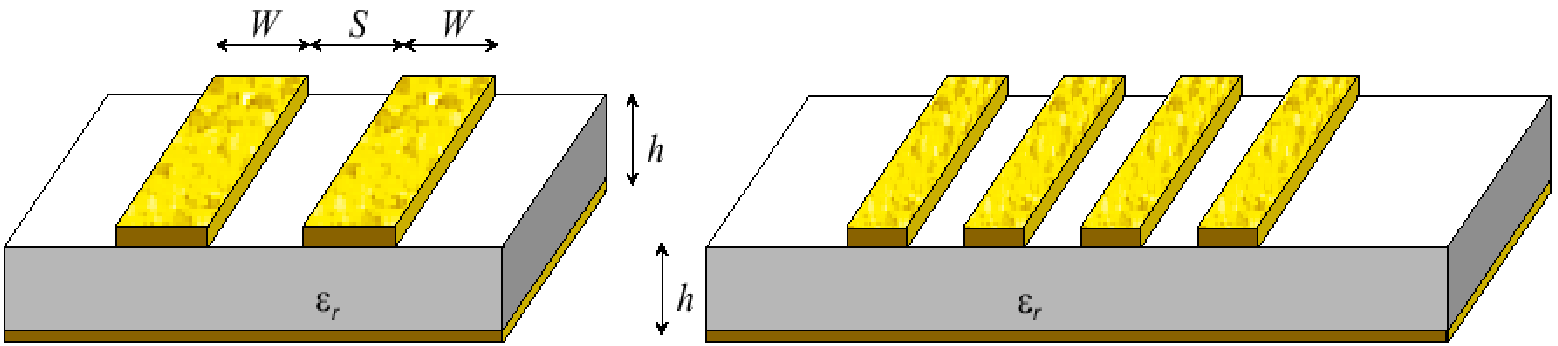
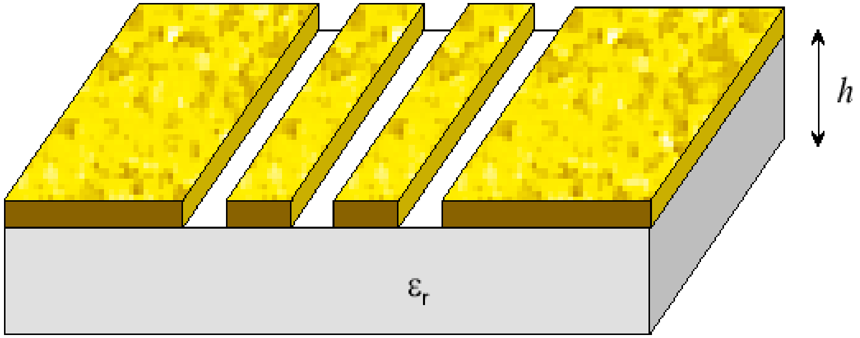
Outline

- **Coupled lines**
- Directional couplers
- Power dividers and combiners

Coupled lines

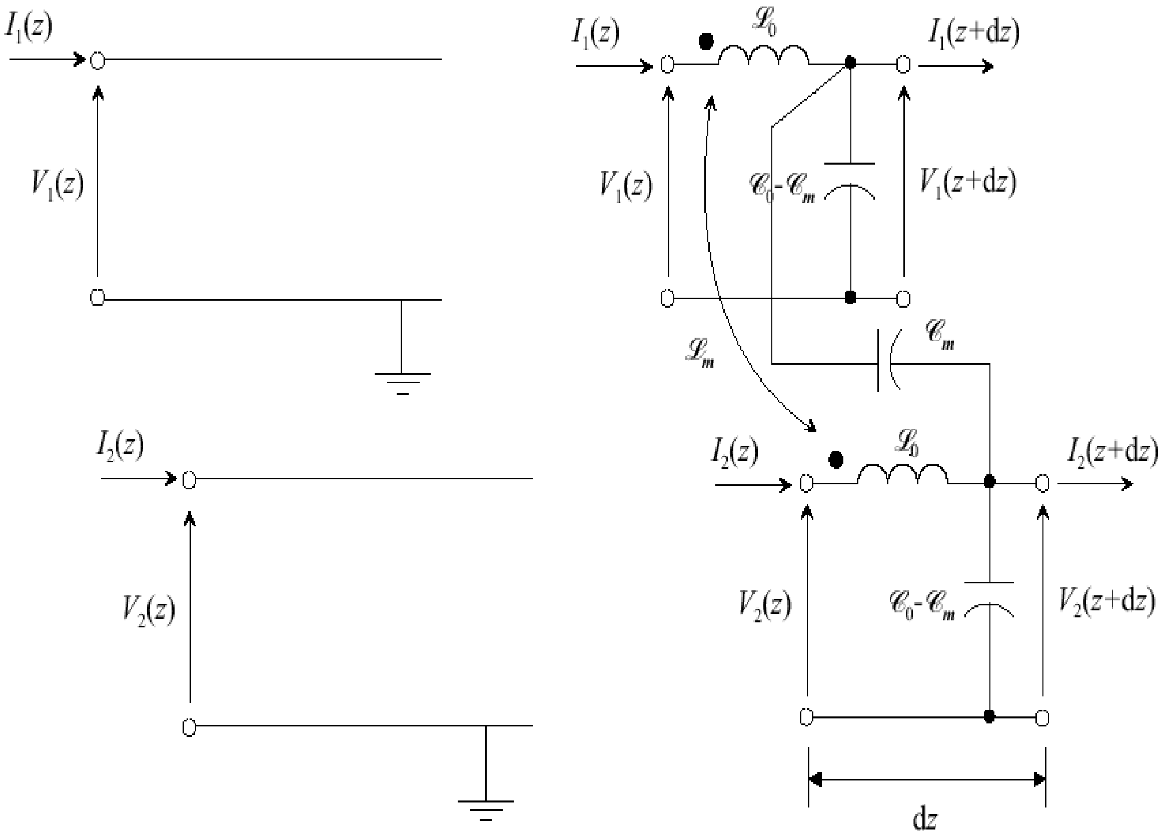


Two and four-conductor microstrip coupled lines



Coupled two-conductor coplanar lines

Equivalent circuit of lossless symmetric coupled lines



- Capacitance matrix

$$\mathbf{C} = \begin{pmatrix} \mathcal{C}_0 & -\mathcal{C}_m \\ -\mathcal{C}_m & \mathcal{C}_0 \end{pmatrix}$$

- Inductance matrix

$$\mathbf{L} = \begin{pmatrix} \mathcal{L}_0 & \mathcal{L}_m \\ \mathcal{L}_m & \mathcal{L}_0 \end{pmatrix}.$$

- A line with *N* conductors + ground carries *N* quasi-TEM propagation modes

Generalized telegraphers' equation and modal solution

- Voltage and current vectors on the two lines

$$\underline{V} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \quad \underline{I} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

- Generalized telegraphers' equations

$$\frac{\mathrm{d}\underline{V}(z)}{\mathrm{d}z} = \mathrm{j}\omega \mathbf{L}\underline{I}$$
$$\frac{\mathrm{d}\underline{I}(z)}{\mathrm{d}z} = \mathrm{j}\omega \mathbf{C}\underline{V}$$

- Second-order system

$$\frac{\mathrm{d}^2\underline{V}(z)}{\mathrm{d}z^2} = -\omega^2 \mathbf{LC}\underline{V}$$

- Trial solutions

$$\underline{V}(z) = \underline{V}_0 \exp(-\mathrm{j}\beta z)$$

Solutions in terms of even (p) and odd (d) modes

- Two modes, even and odd (p, d) with different propagation constants

$$\left(\beta^2 \mathbf{I} - \omega^2 \mathbf{LC}\right) \underline{V}_0 = 0$$

$$\beta_1 = \beta_e = \frac{\omega}{c_0}\sqrt{\frac{\mathcal{C}_e}{\mathcal{C}_{ea}}}$$

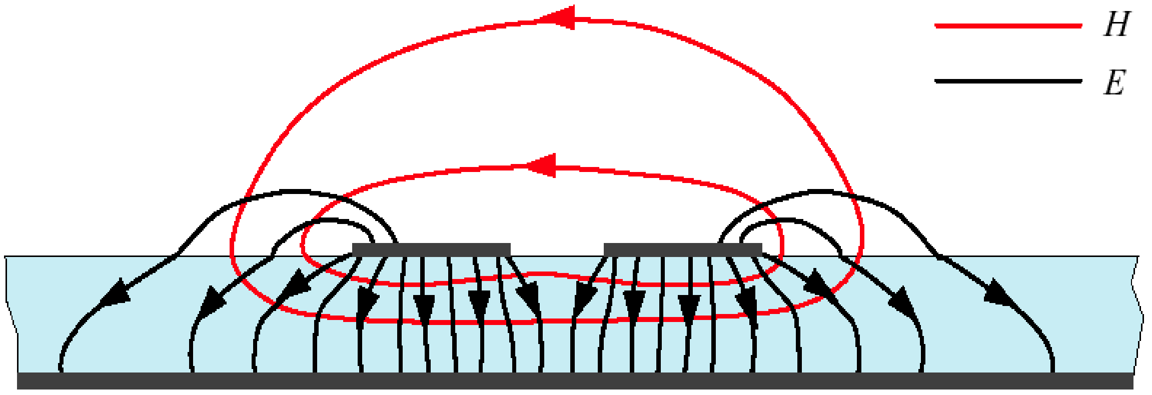
$$\beta_2 = \beta_o = \frac{\omega}{c_0}\sqrt{\frac{\mathcal{C}_o}{\mathcal{C}_{oa}}}$$

$$\mathcal{C}_e = \mathcal{C}_0 - \mathcal{C}_m$$
$$\mathcal{C}_o = \mathcal{C}_0 + \mathcal{C}_m$$

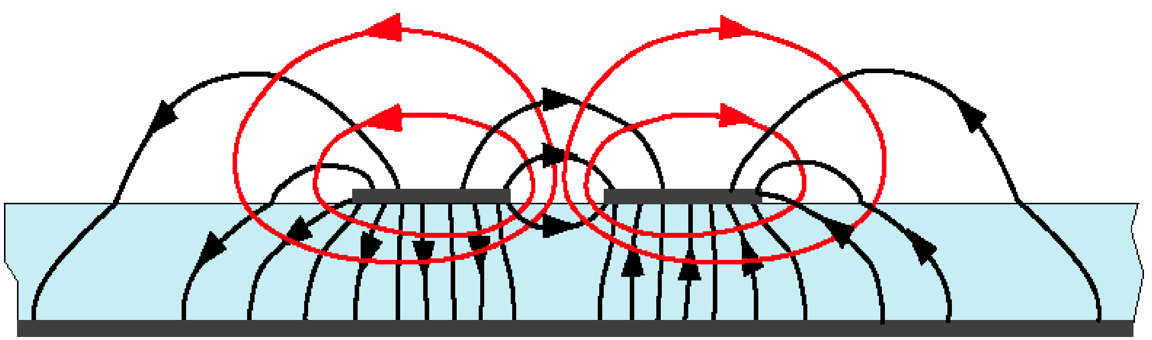
- Each mode has a progressive and regressive wave

- Even mode: both line have the same voltage
- Odd mode: the line voltages are equal but opposite

Even and odd mode topology in coupled lines



Even mode



Odd mode

Even (p) and odd (d) mode parameters

- Effective permittivity:

$$\epsilon_{\text{eff}\,e} = \frac{\mathcal{C}_e}{\mathcal{C}_{ea}}$$
$$\epsilon_{\text{eff}\,o} = \frac{\mathcal{C}_o}{\mathcal{C}_{oa}}$$

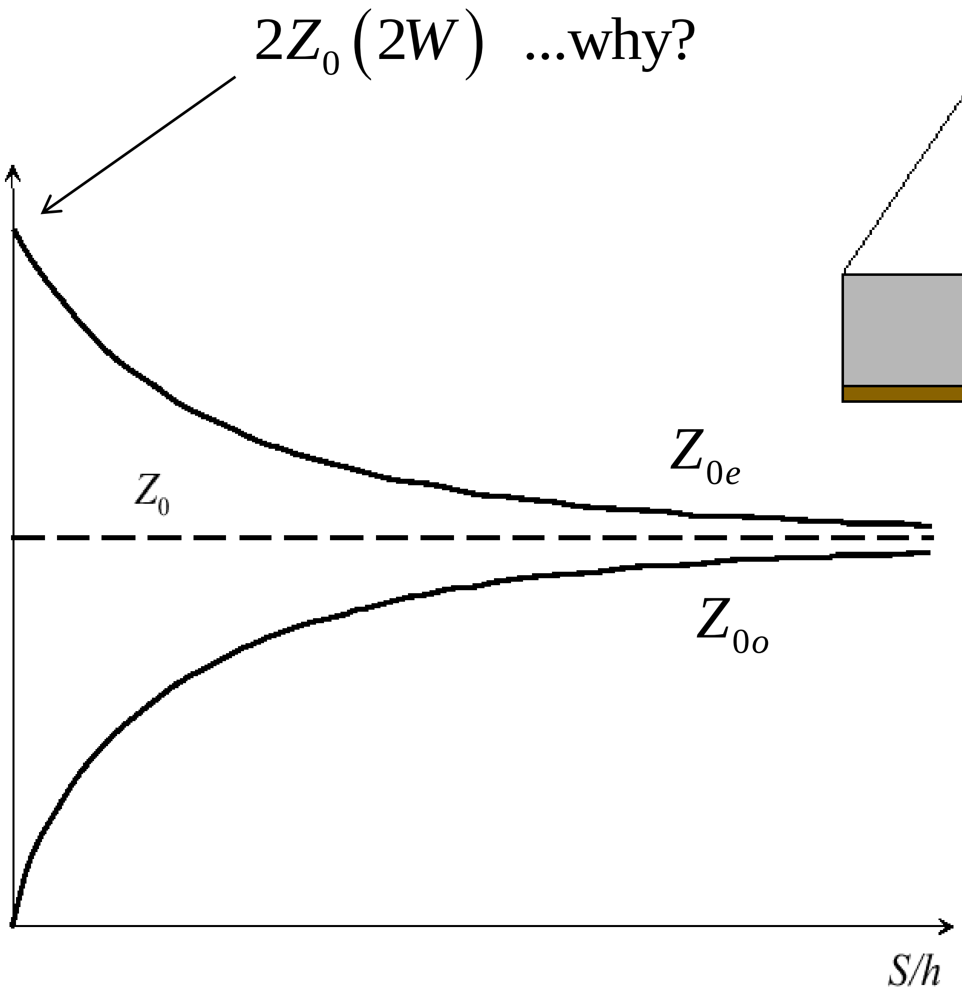
- Characteristic impedance:

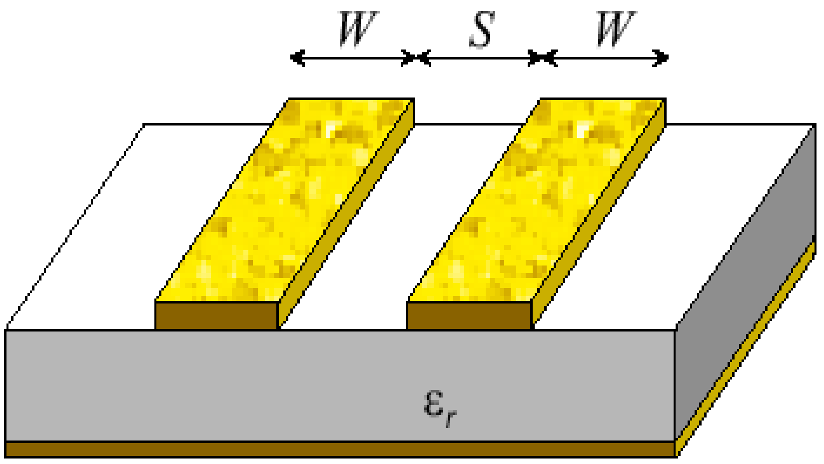
$$Z_{0e} = \frac{1}{c_0\sqrt{\mathcal{C}_e\mathcal{C}_{ea}}} = \frac{Z_{0ea}}{\sqrt{\epsilon_{\text{eff}\,e}}}$$
$$Z_{0o} = \frac{1}{c_0\sqrt{\mathcal{C}_o\mathcal{C}_{oa}}} = \frac{Z_{0oa}}{\sqrt{\epsilon_{\text{eff}\,o}}}$$

- Since the odd mode capacitance > than the even mode capacitance the odd mode impedance < than the even mode impedance (Z_0 is the uncoupled line impedance):

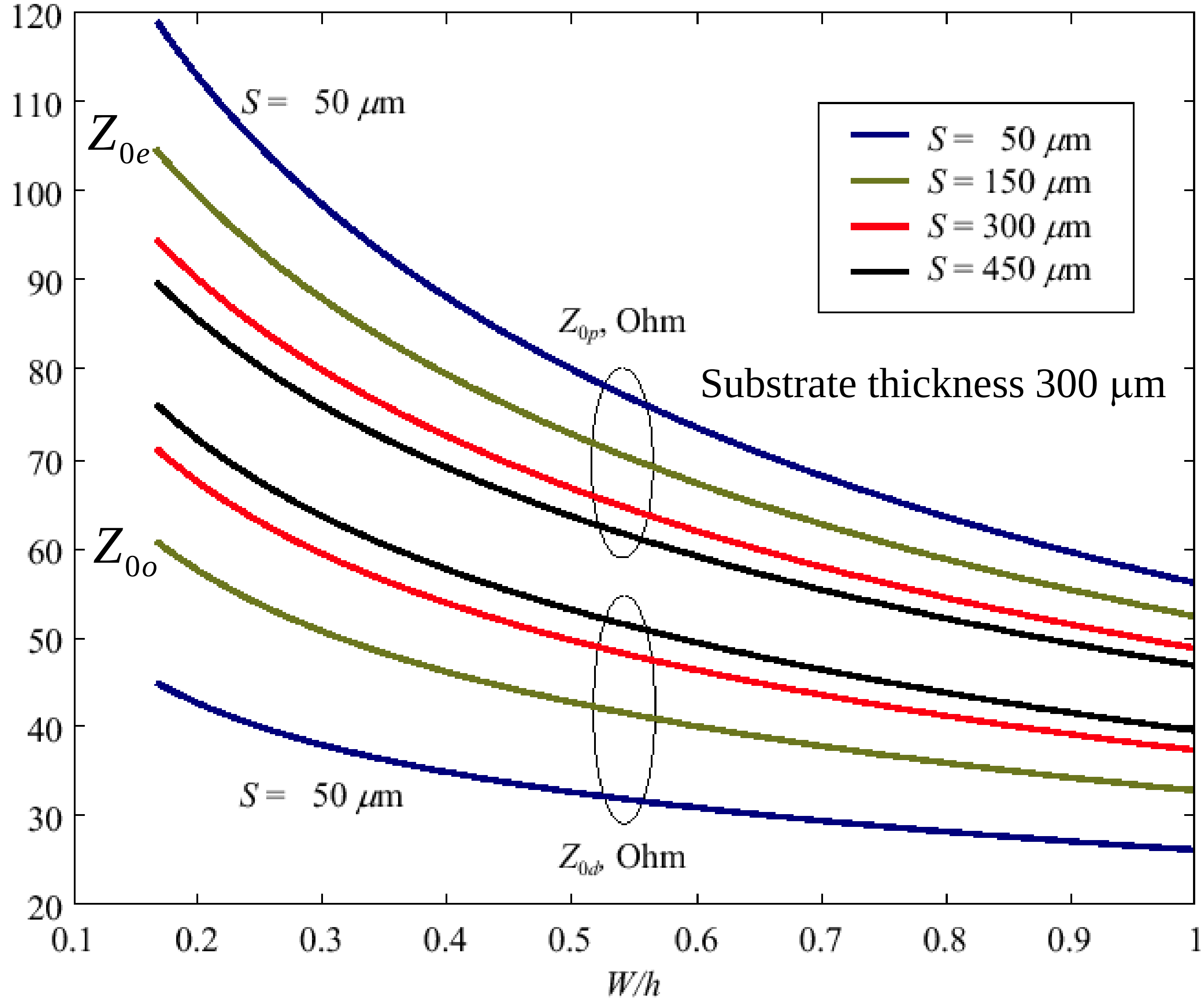
$$Z_{0e} \geq Z_0 \geq Z_{0o}$$

Even and odd mode parameters vs. line spacing *S*

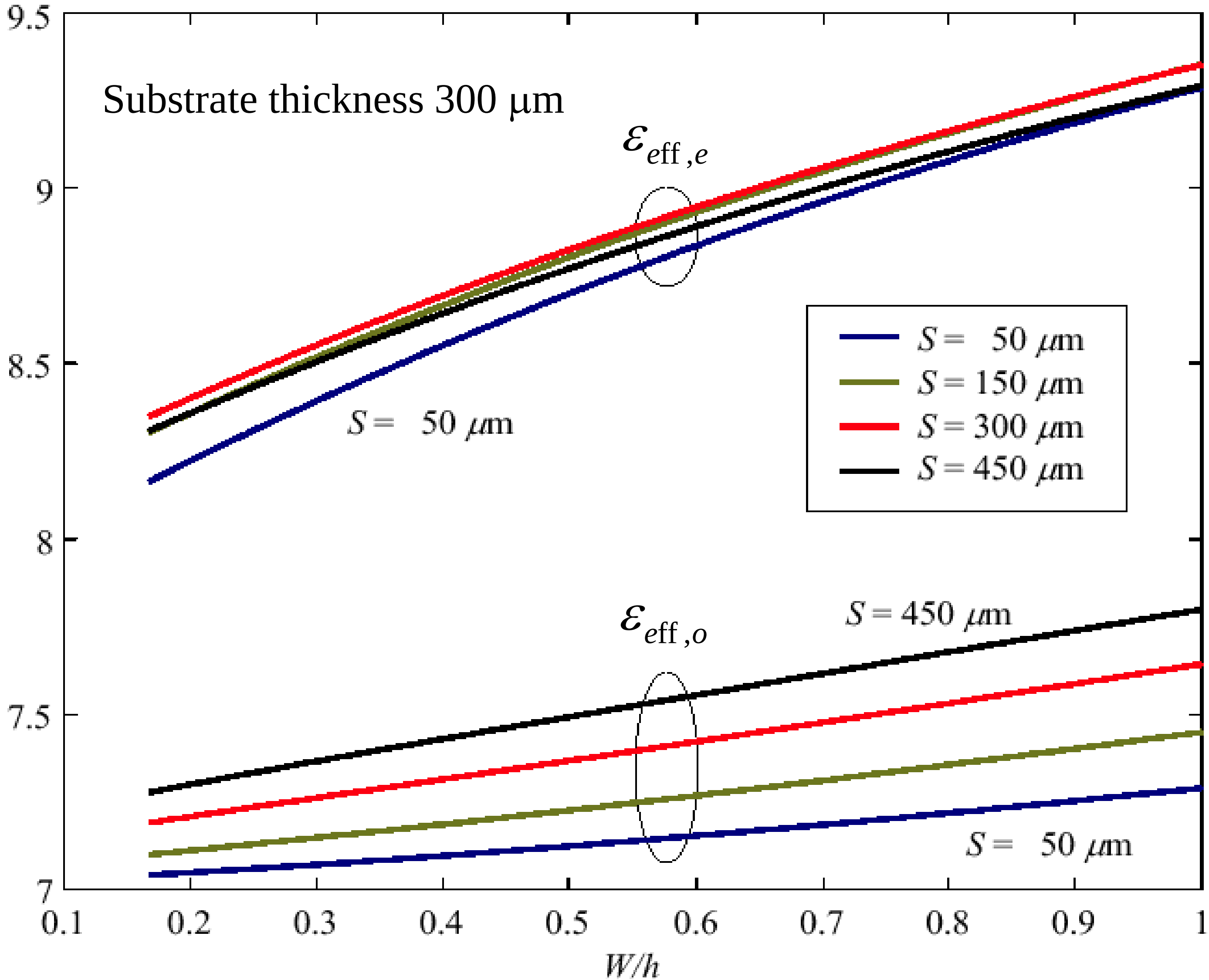




Coupled microstrip impedances (on GaAs, ε=13)



Effective permittivities (on GaAs, ε=13)

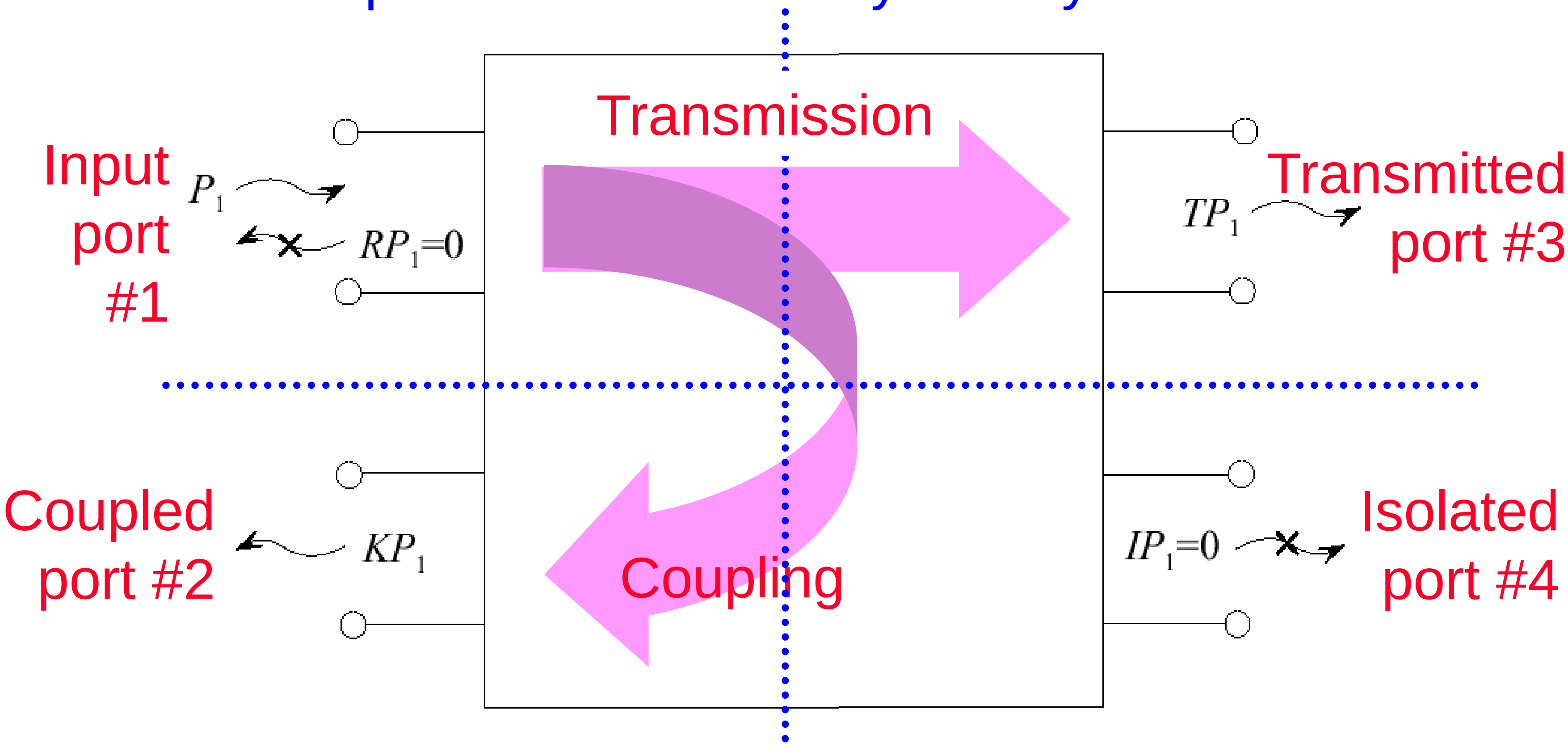


Outline

- Coupled lines
- **Directional couplers**
- Power dividers and combiners

Directional coupler

- Is a four-port dividing the input power at port 1 into two parts (generally not the same → 3dB coupler if same) (port 3 and 2) *with 90 or 180 phase difference (90 and 180 hybrids) between the two ports*; port 4 is isolated and all ports are matched. Couplers have twofold symmetry!



Coupler parameters

$$R\big|_{\mathrm{dB}} = 20\log_{10}\left|S_{11}\right| \text{ reflection coefficent } \approx 0$$

$$K\big|_{\mathrm{dB}} = -10\log_{10}\left(\frac{P_2}{P_1}\right) \equiv \left|C\right|_{\mathrm{dB}} = -20\log_{10}\left|\frac{b_2}{a_1}\right| \text{ coupling}$$

$$T\big|_{\mathrm{dB}} = -10\log_{10}\left(\frac{P_3}{P_1}\right) \text{ transmission coefficent}$$

$$I\big|_{\mathrm{dB}} = -10\log_{10}\left(\frac{P_4}{P_1}\right) \text{ isolation}$$

$$D\big|_{\mathrm{dB}} = 10\log_{10}\left(\frac{P_2}{P_4}\right) \text{ directivity}$$

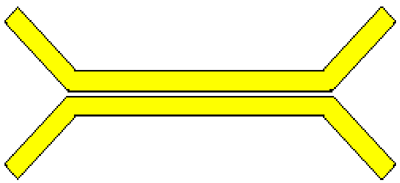
Couplers are reactive!

$$P_1 = P_2 + P_3 = KP_1 + TP_1$$

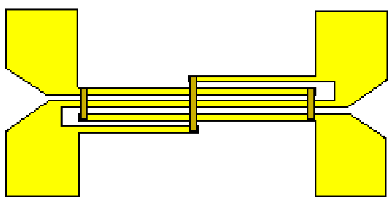
$$\rightarrow K + T = 1 \rightarrow T = 1 - \left|C\right|^2$$

(often C is taken as real, e.g. coupling $= jC$)

Directional coupler implementations



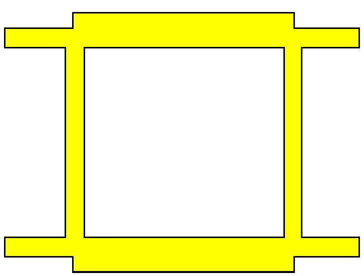
uniform coupled line coupler



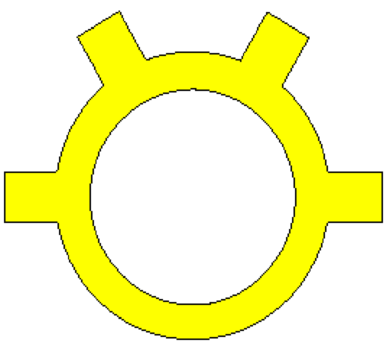
Lange coupler

- Coupled line couplers:
  - simple (two coupled lines), achieving 3 dB coupling is difficult (equal power division)
  - interdigitated (Lange), technologically more difficult, 3 dB couplers can be obtained
  - Coupled line couplers can easily obtain low coupling (what for?)

- Interference couplers:
  - Easy 3 dB couplers
  - Difficult low-coupling couplers
  - Large area



Branch line coupler



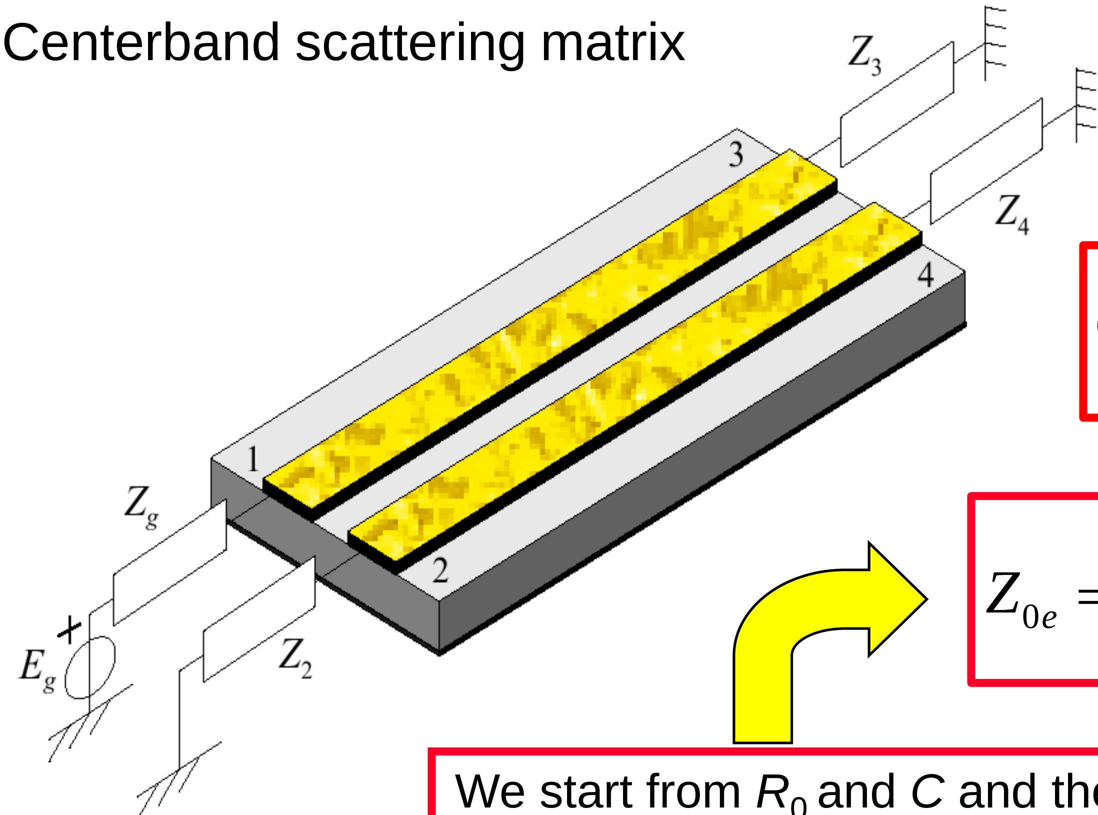
Rat race coupler

Size: quarter wavelength at centerband

Coupled line couplers

$$\mathbf{S}(\pi/2) = \begin{pmatrix} 0 & C & -\mathrm{j}\sqrt{1-C^2} & 0 \\ C & 0 & 0 & -\mathrm{j}\sqrt{1-C^2} \\ -\mathrm{j}\sqrt{1-C^2} & 0 & 0 & C \\ 0 & -\mathrm{j}\sqrt{1-C^2} & C & 0 \end{pmatrix}$$

Centerband scattering matrix



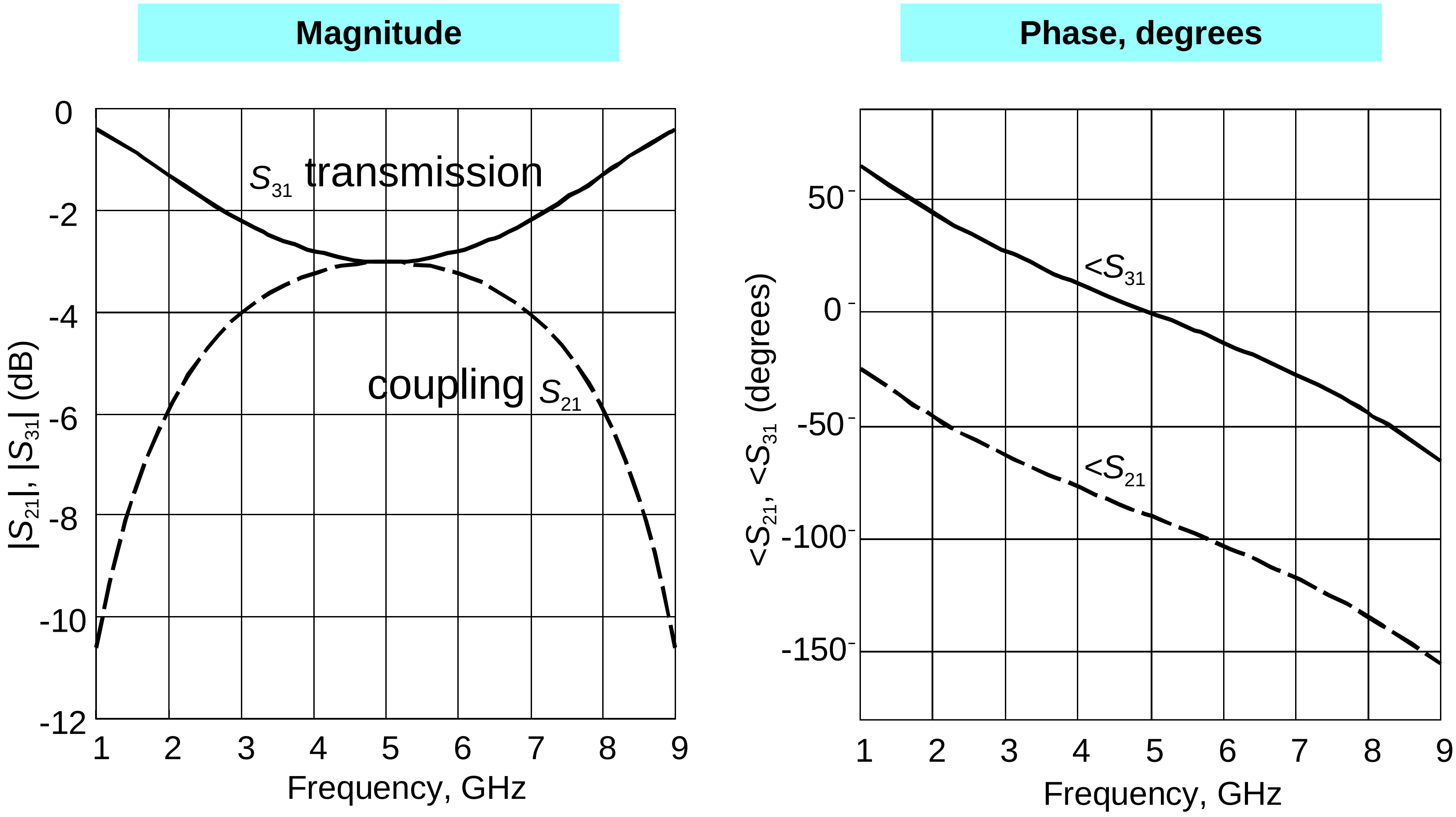
- The centerband coupler length is **lambda_g/4**
- To have matching (port 1), isolation (port 4), coupling *C* (port 2) the following conditions should be met:

$$C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \qquad R_0 = \sqrt{Z_{0e} Z_{0o}}$$

$$Z_{0e} = R_0 \sqrt{\frac{1+C}{1-C}} \qquad Z_{0o} = R_0 \sqrt{\frac{1-C}{1+C}}$$

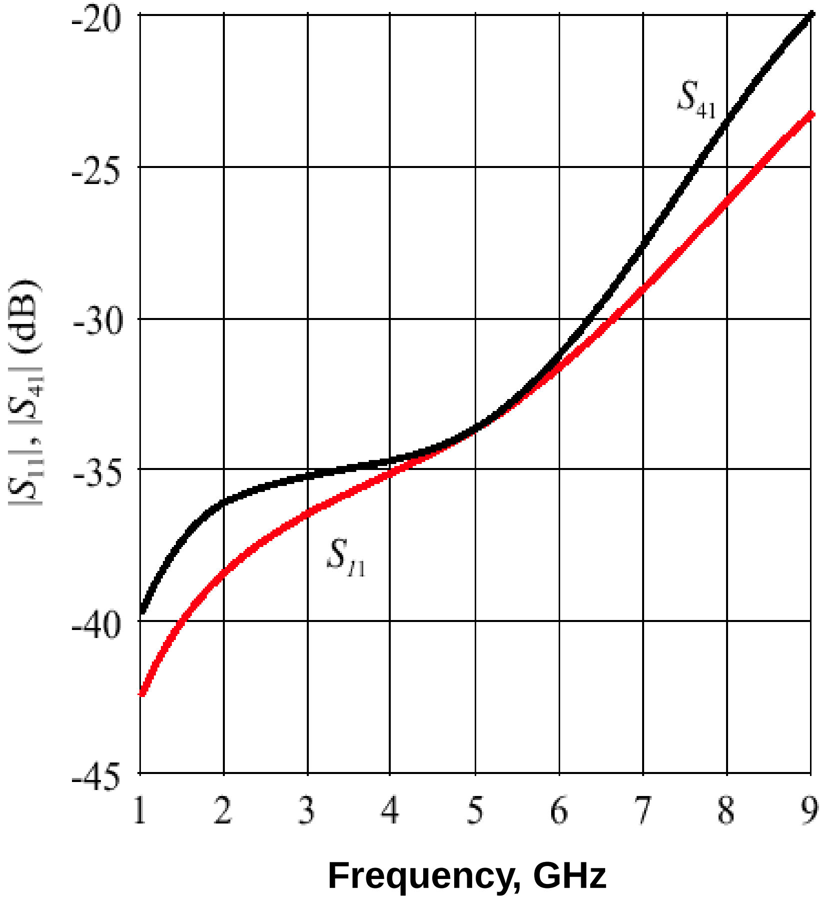
We start from R_0 and C and the design of coupled lines follows

Ideal 3dB coupler response (synchronous even and odd modes)



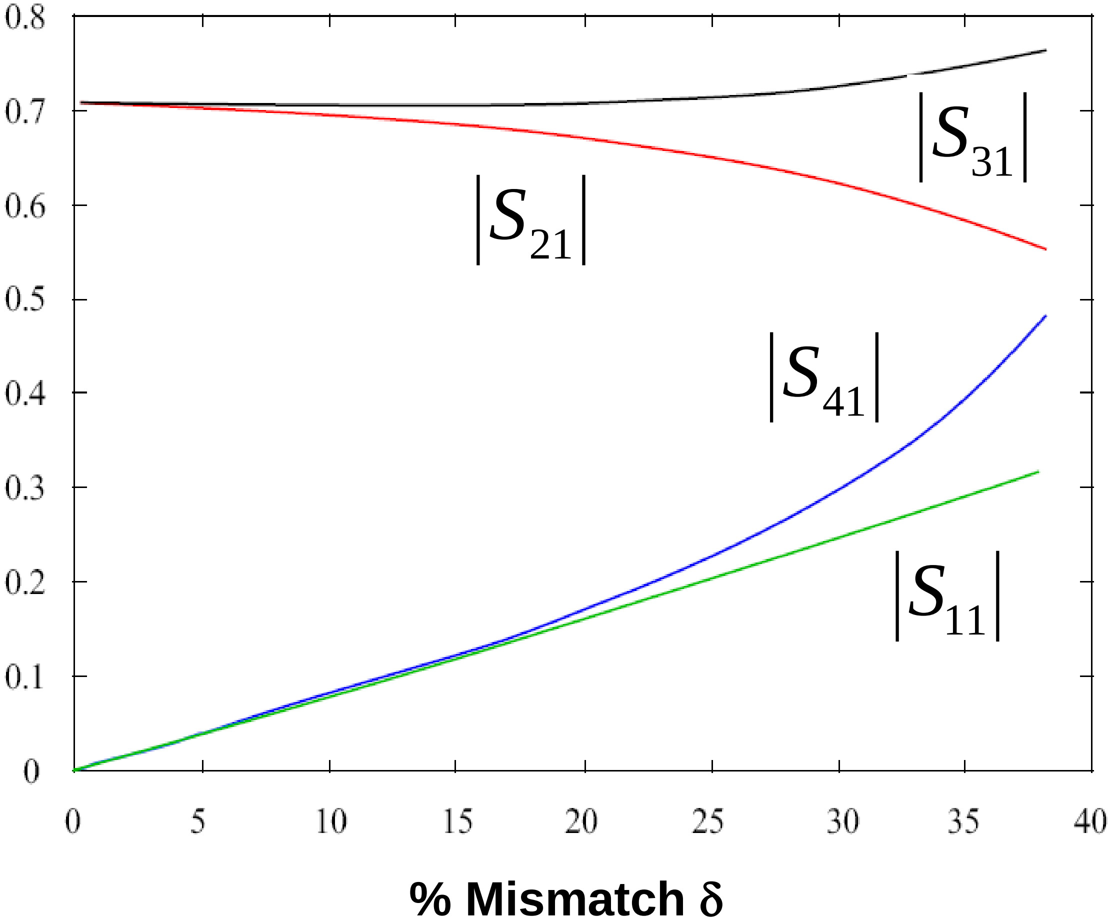
Ideal matching and isolation achieved on the whole band!

Isolation and matching in a real coupler



- In real couplers the even and odd modes do not have the same propagation velocity
- As a result, matching and isolation deteriorate, coupling and transmission are almost unchanged

Centerband S parameters as a function of velocity mismatch

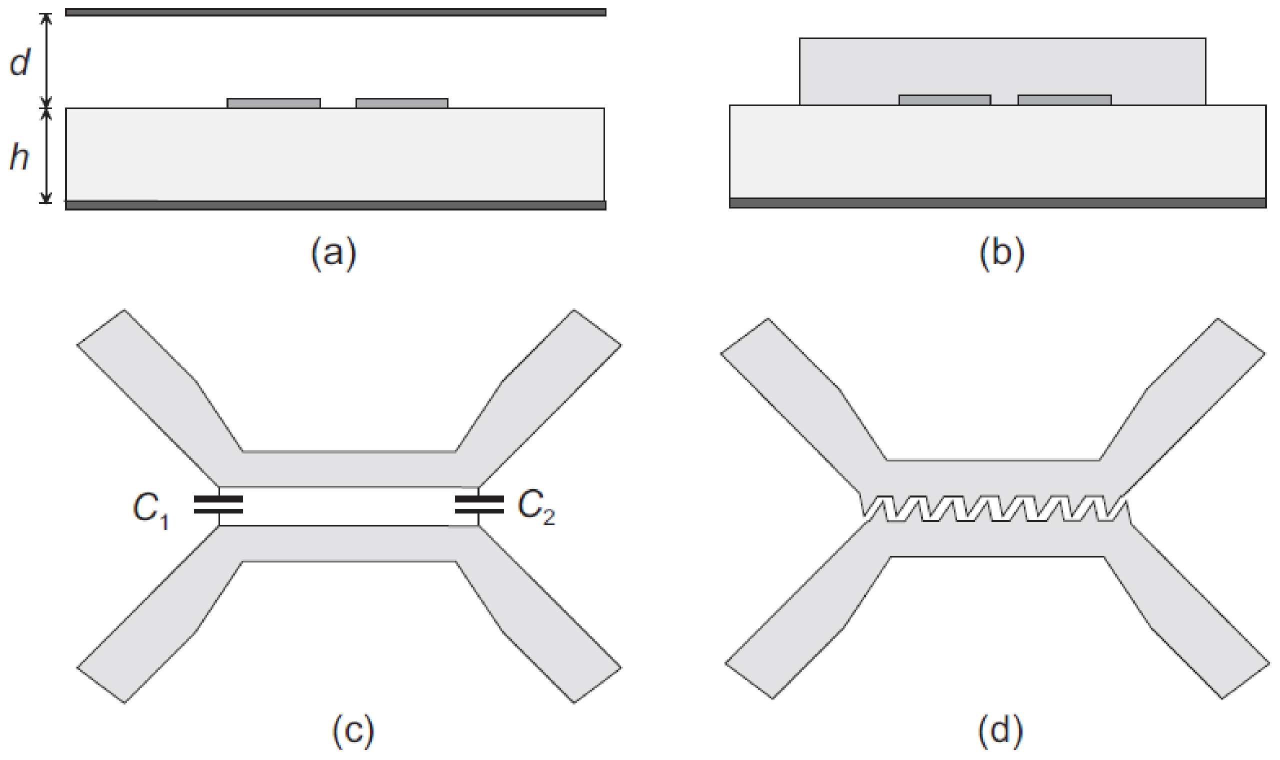


- In several applications the loss in isolation can be a serious problem
- Approximately:

$$\boxed{S_{41} \approx \delta(1 - C^2)}$$

$$\boxed{\delta = \frac{|v_e - v_o|}{v_e + v_o}}$$

Compensated couplers



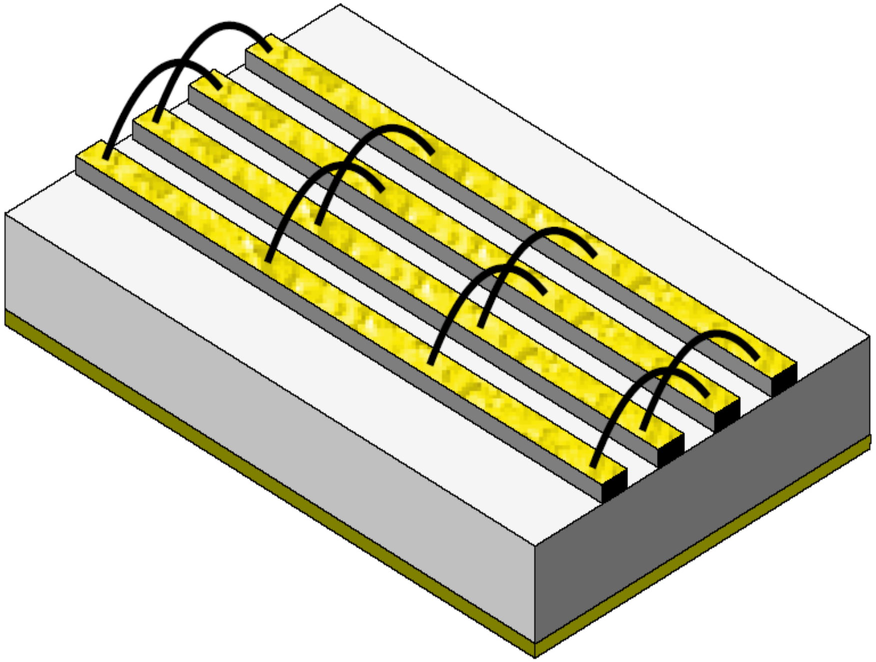
Velocity compensation techniques: shielded directional coupler (a) and with dielectric overlay (b); compensation through concentrated capacitances (c) e and line wiggle (d).

Integrated 3dB couplers

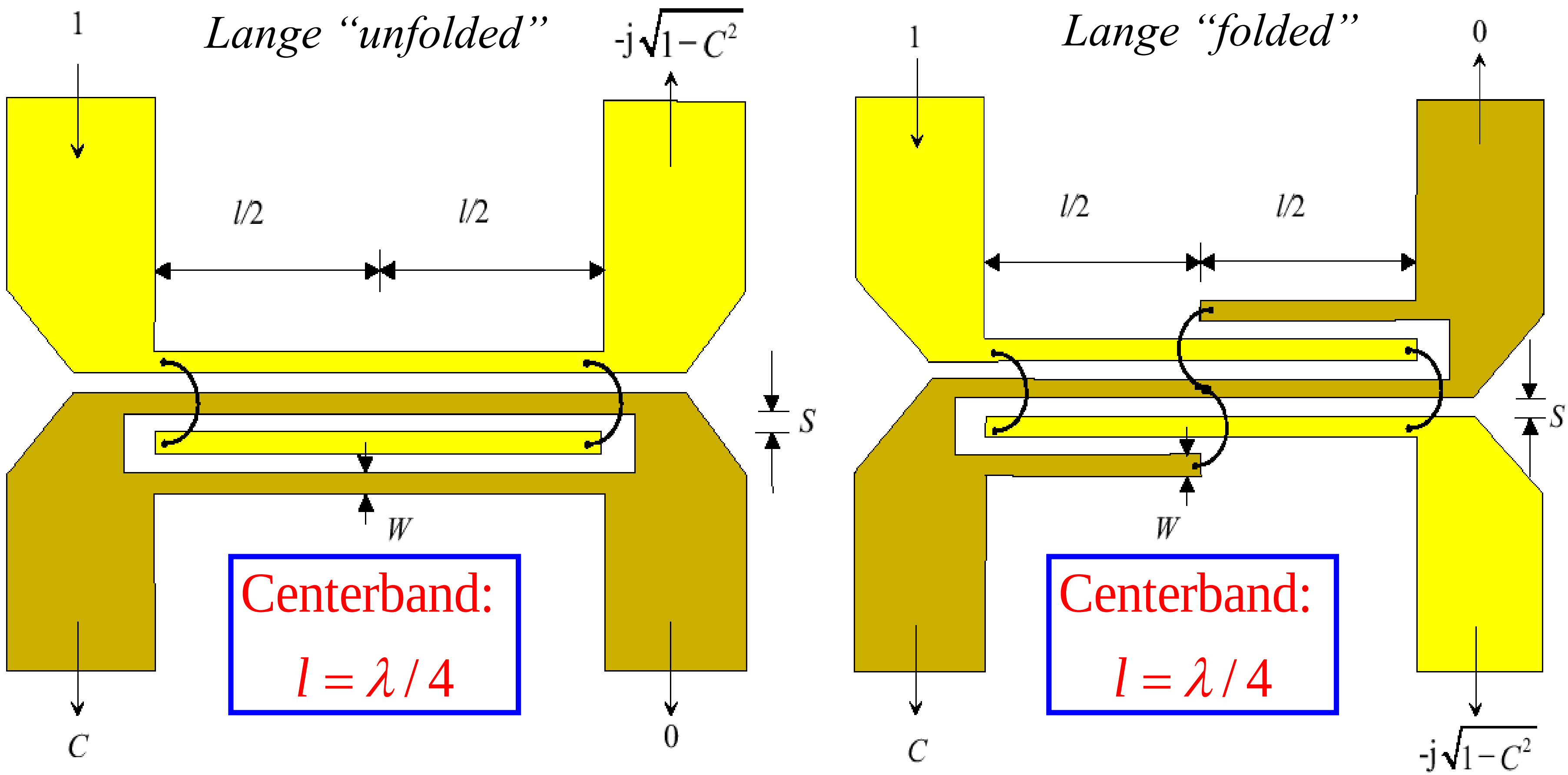
- 3dB couplers can be in principle implemented with two coupled microstrips; however, the slot width in a 50 Ohm system turns out to be technologically impossible to realize (a few microns)
- Unfortunately 3dB couplers with 90 degrees phase shift are required in many applications (e.g. balanced amplifiers)
- A solution that is technologically manageable is given by the **multiconductor couplers** → Lange couplers

Multiconductor microstrip and equivalent two-conductor line

- A multiconductor line is equivalent to a two-conductor line if the conductors are periodically connected
- The mutual capacitance of the equivalent two lines increases → coupling increases also for widely spaced lines
- 3 dB couplers can be implemented with 4, 6, 8 conductors (typically 4)
- Increasing the number of conductors the closing impedance decreases



Interdigitated four-conductor 3dB Lange couplers



- In principle directional couplers can also be made with two-conductor coupled microstrips, but for technological limits 3dB couplers can only be made interdigitated

Designing a multiconductor coupler

- Assume we know the centerband coupling C, the closing impedance R_0 (e.g. 50 Ohm) and the number of lines k.
- It can be shown that:

$$C(k) = \frac{(k-1)(1-R^2)}{(k-1)(1+R^2)+2R}, \quad R_0^2 = r\frac{(1+R)^2}{\left[(k-1)R+1\right]\left[(k-1)+R\right]}$$

- where $R = Z_{0o}(2)/Z_{0e}(2)$, $r = Z_{0o}(2)\,Z_{0e}(2)$; $Z_{0o}(2)$ and $Z_{0d}(2)$ are the odd and even mode impedances of a two-conductor line with the **same** geometrical parameters w and G, substrate etc.
- The two previous equations can be solved in order to find
  - the impedance ratio $R = Z_{0o}(2)/Z_{0e}(2)$
  - the impedance product $r = Z_{0o}(2)\,Z_{0e}(2)$
- From these we obtain $Z_{0o}(2)$ and $Z_{0e}(2)$, synthesis formulae exist giving the slot and line width needed to implement the two impedances.
- Of course the same formulae can be exploited for analysis

Example: 3dB 50 Ohm Lange coupler on GaAs

- With *k*=4 and *C*=1/sqrt(2) we obtain (from first equation):

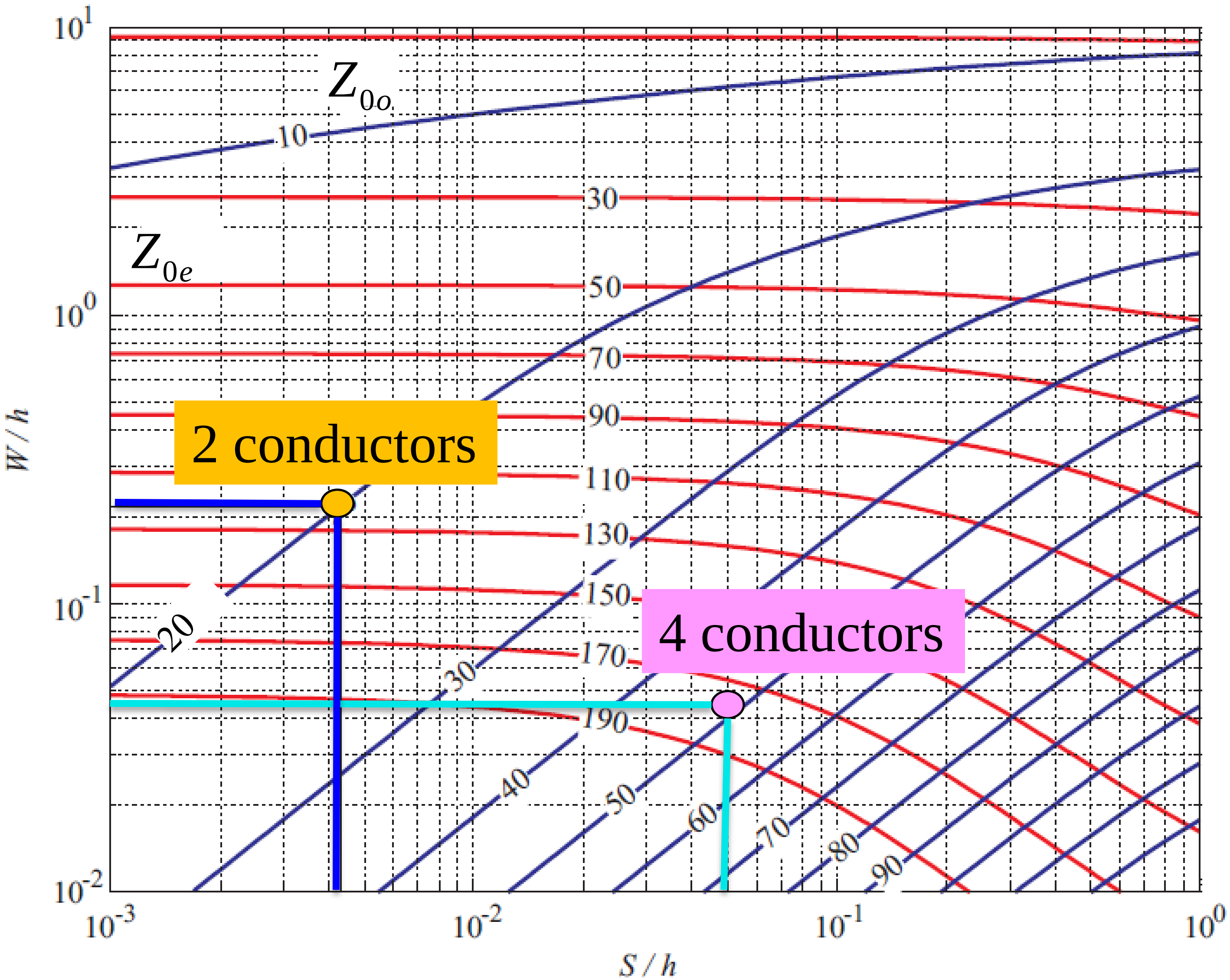
$$3\left(1+\frac{1}{\sqrt{2}}\right)R^2+2\frac{1}{\sqrt{2}}R-3\left(1-\frac{1}{\sqrt{2}}\right)=0$$

- i.e. solving for *R* and substituting (second equation, closing impedance 50 Ohm):

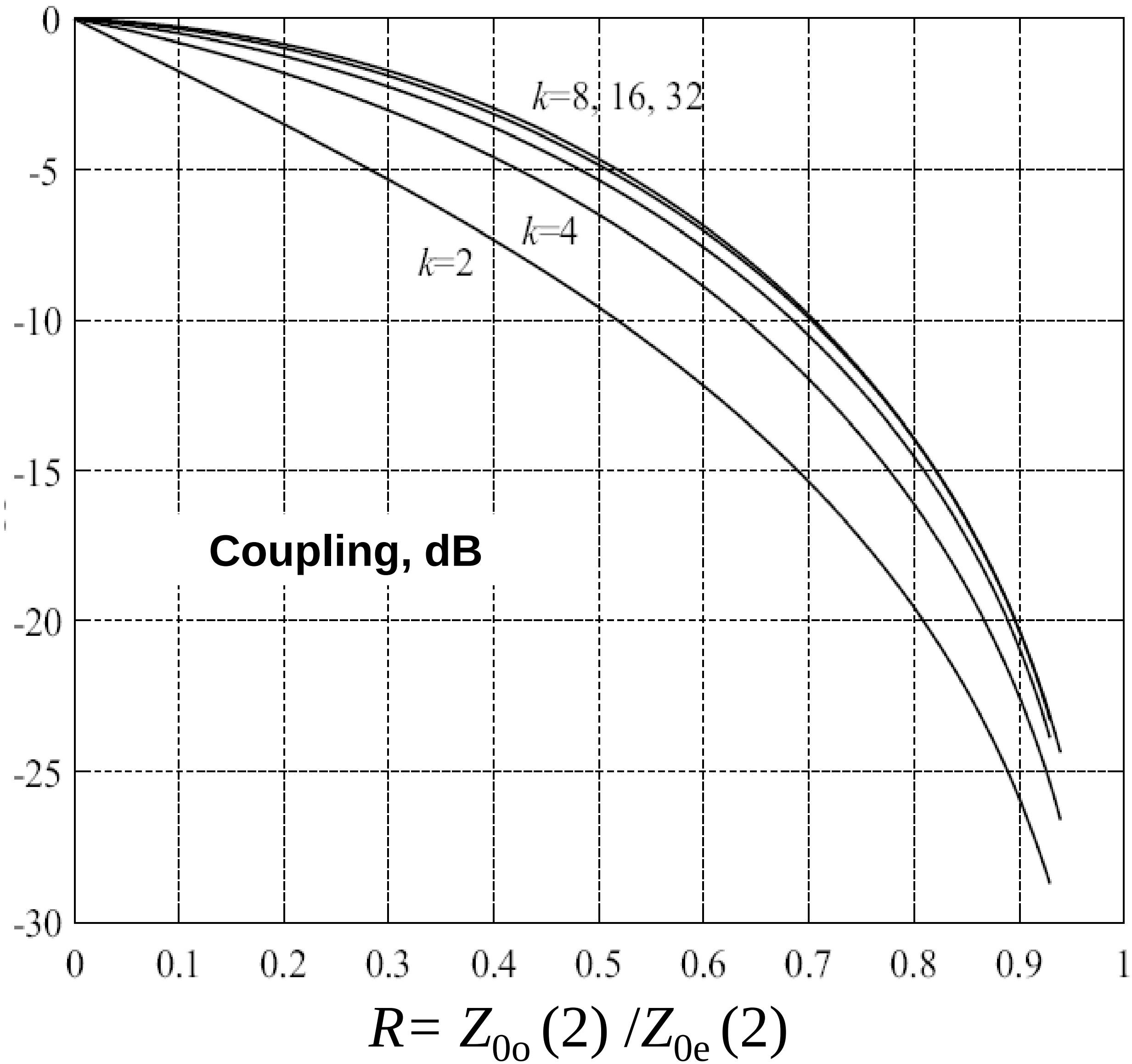
$$R=\frac{Z_o}{Z_e}=0.3; \quad r=Z_eZ_o=9275$$

- The two-strip impedance is:
  - Odd mode 53 Ohm
  - Even mode 177 Ohm
- An idea of the required spacing and strip thickness can be obtained from the graph in the next slide; for the 2 conductor plain coupler the odd and even mode impedances were around 120 and 20 Ohm, much less favourable

Design chart for the coupler (epsilon_r=13)

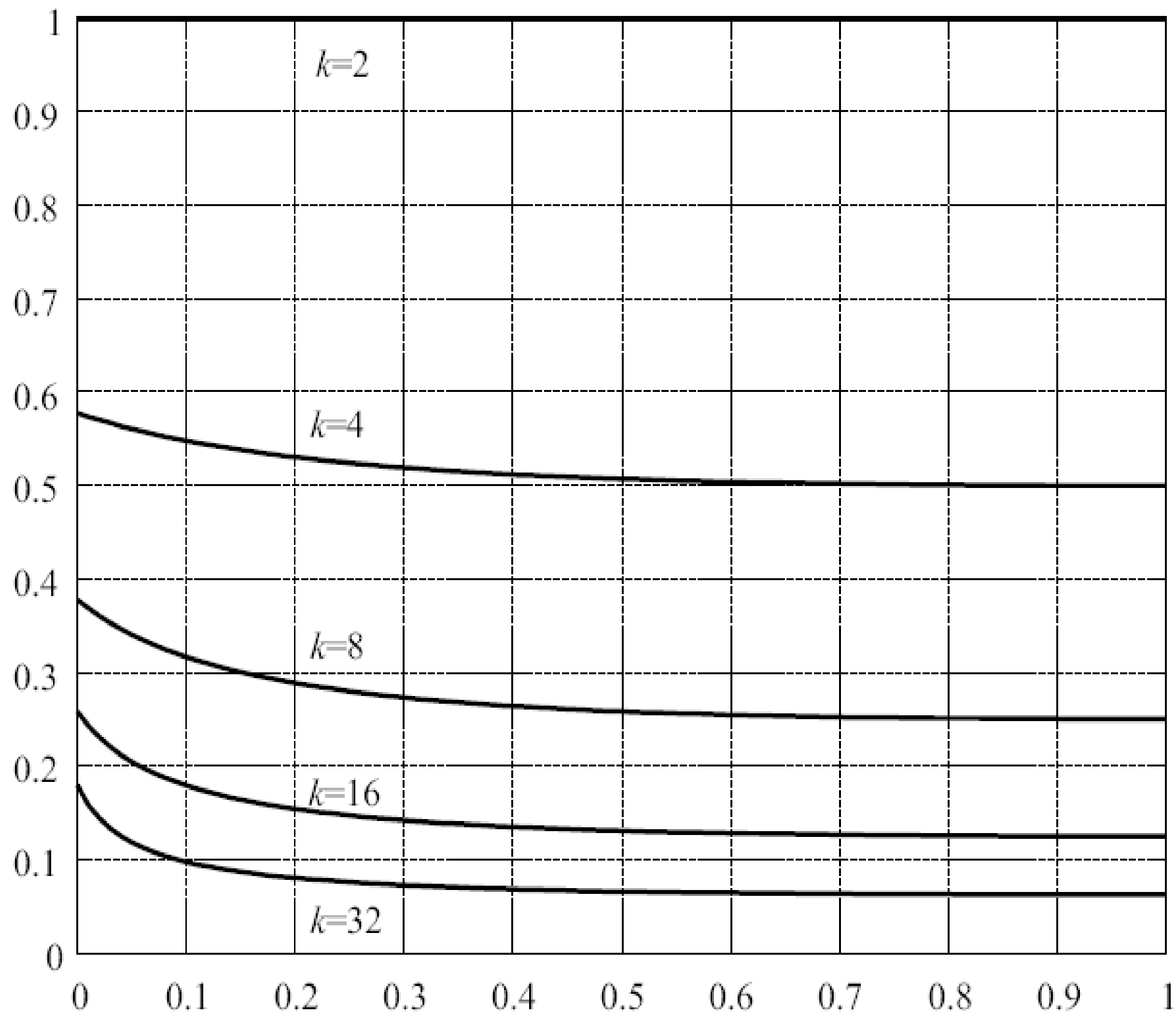


Coupling vs. *R=Z_odd/Z_even*



Matching impedance vs. *R*

$$\frac{R_0}{\sqrt{Z_{0e}(2)Z_{0o}(2)}}$$



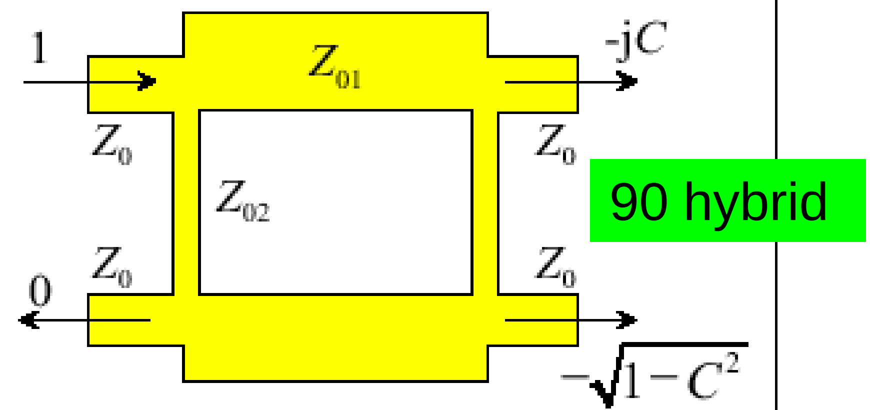
$$R = Z_{0o}(2) / Z_{0e}(2)$$

Interference couplers

Branch line coupler

$$Z_{02} = \frac{CZ_0}{\sqrt{1-C^2}}$$
$$Z_{01} = CZ_0$$

$$Z_{02} = Z_0$$
$$Z_{01} = \frac{Z_0}{\sqrt{2}}$$

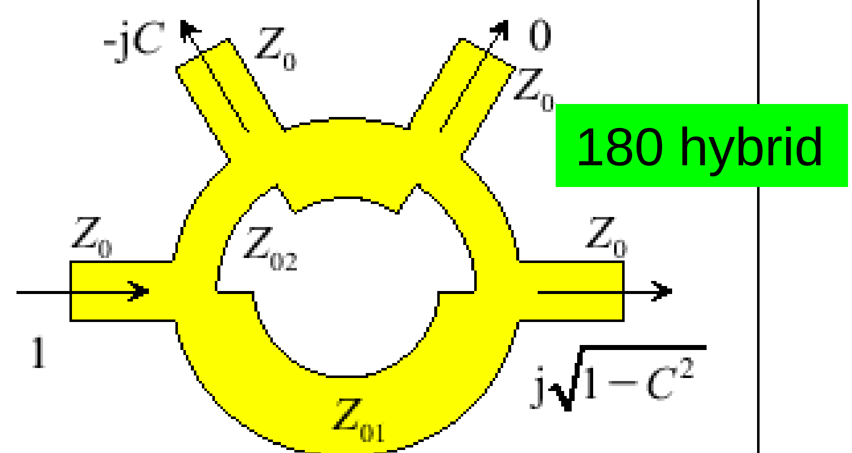


Rat race or hybrid ring coupler

$$Z_{01} = \frac{Z_0}{\sqrt{1-C^2}}$$
$$Z_{02} = \frac{Z_0}{C}$$

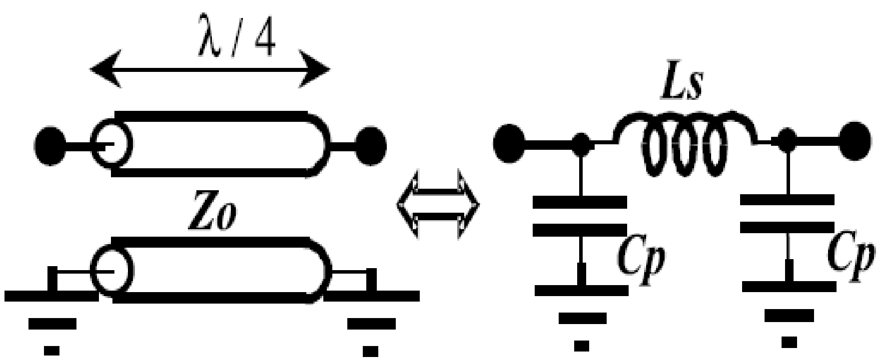
$$Z_{01} = Z_0\sqrt{2}$$
$$Z_{02} = Z_0\sqrt{2}$$

Generic coupling | ***3dB***

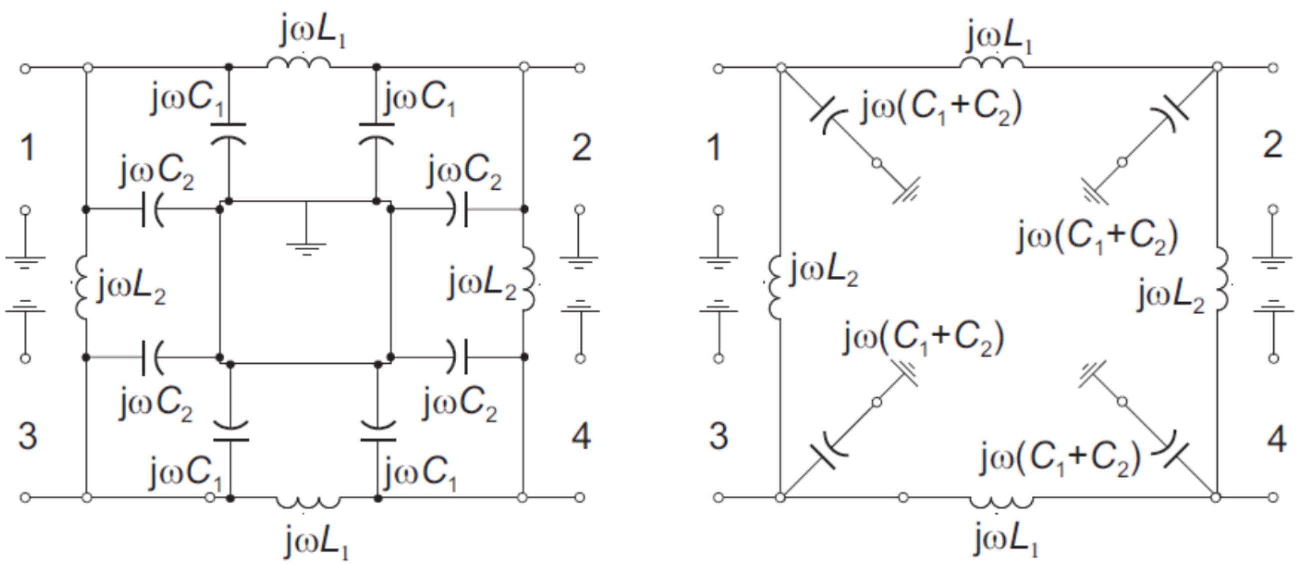


Centerband: branch coupler side $\lambda/4$, hybrid ring periphery $3\lambda/2$

Lumped interference couplers

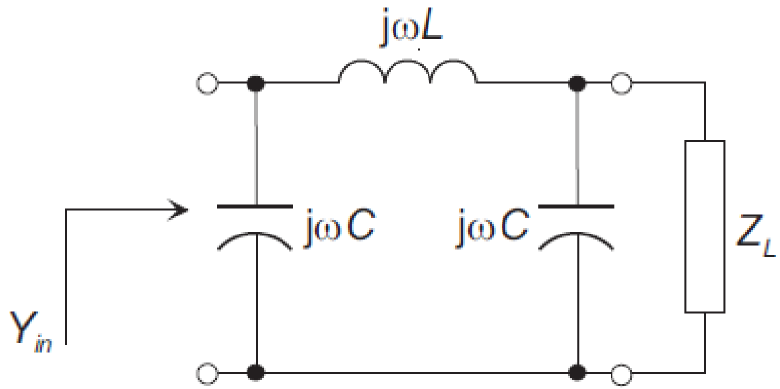


- Idea: replace at centerband a lambda/4 section with a pi lumped section
- ~Same behavior around centerband



Lumped parameter branch-line coupler: left, implementation from π sections; right, practical implementation.

Example: lumped section as quarter-wave transformer



At centerband suppose resonance $j\omega_0 L = -\dfrac{1}{j\omega_0 C} = jX$;

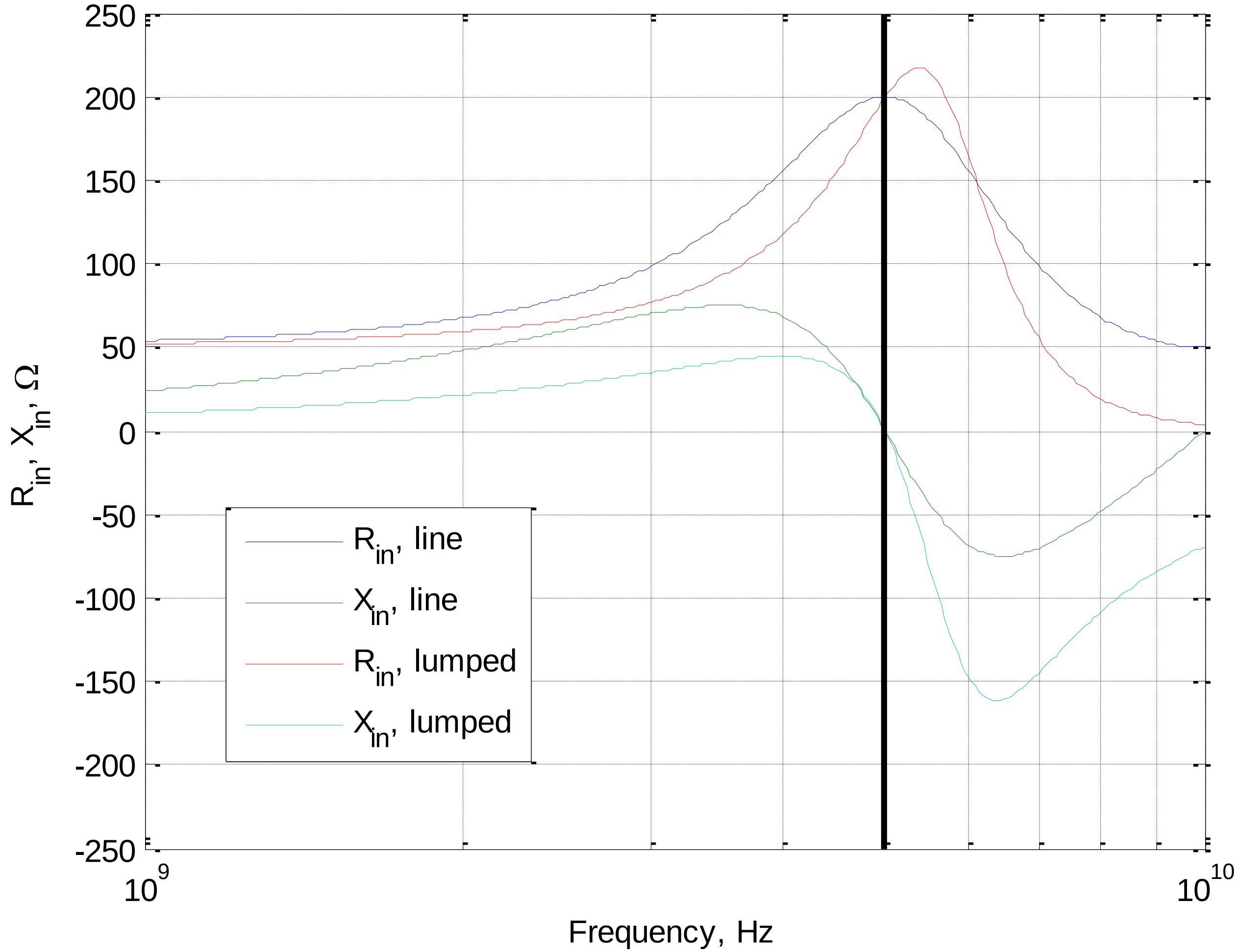
$$Z_1 = \frac{-jX \times Z_L}{-jX + Z_L} \qquad Z_2 = Z_1 + jX$$

$$Z_2 = \frac{-jX \times Z_L}{-jX + Z_L} + jX = \frac{X^2}{-jX + Z_L}; \qquad Y_2 = -j\frac{1}{X} + \frac{Z_L}{X^2};$$

$$Y_{in} = Y_2 + j\frac{1}{X} = \frac{Z_L}{X^2} \qquad Z_{in} = \frac{X^2}{Z_L}$$

Example: 50→200 Ohm transformer at 5 GHz

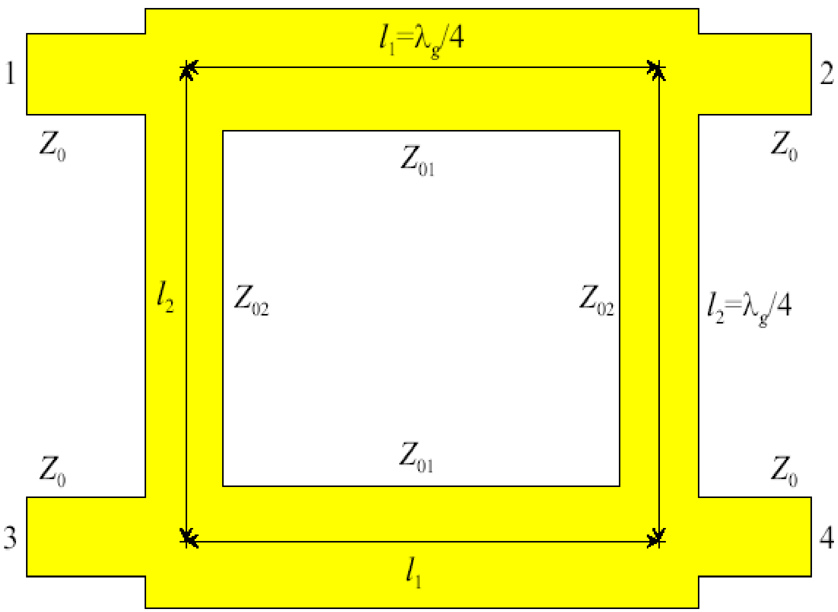
- 50 Ohm load, 100 Ohm line, lambda/4 @5 GHz



Features of interference couplers

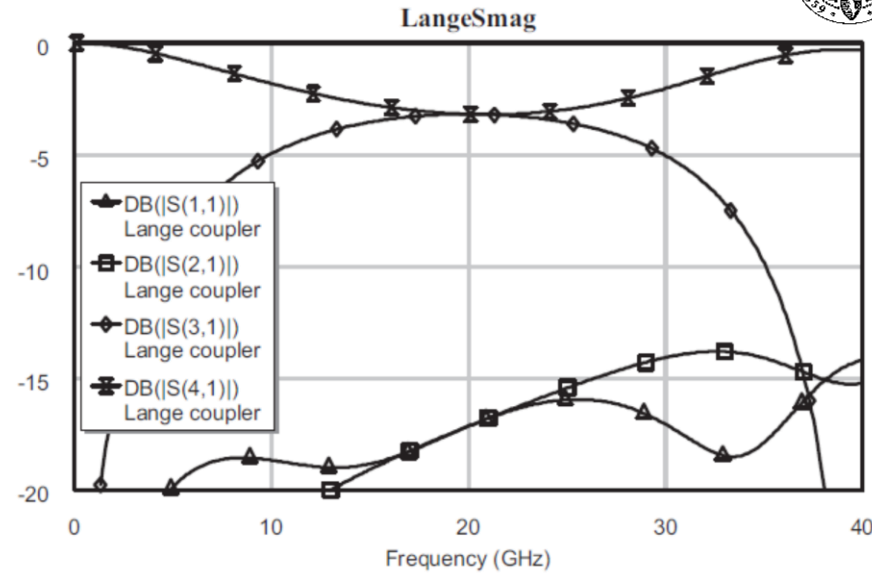
- Large layout size (square or circular layout, side of the order of lambda/4 at centerband)
- Lumped implementations possibile in integrated form till ~25-30 GHz
- Compact implementations based on coupled transmission line sections are also possible
- **180 degrees hybrids possible besides 90 degrees hybrids**
- 3dB couplers easy to implement, low coupling couplers almost impossible to implement
- Narrowband couplers anyway.

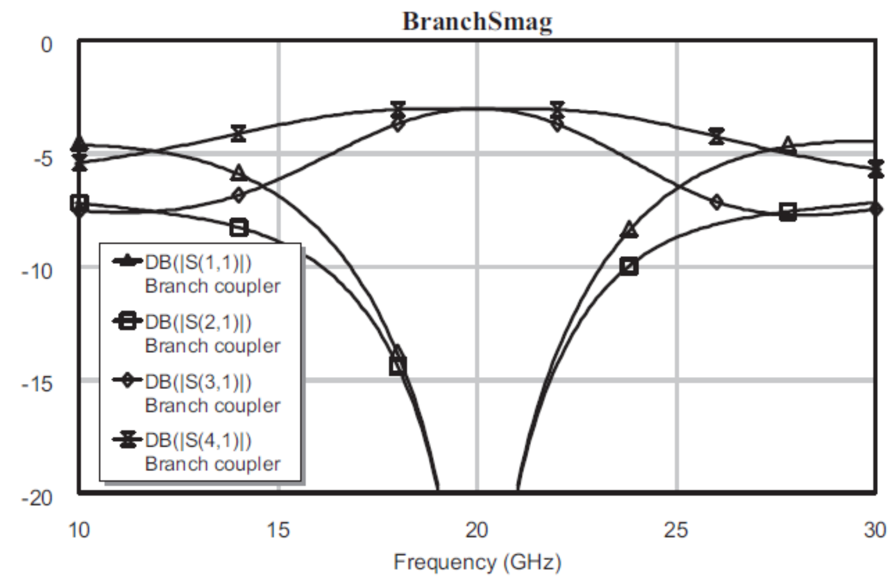
Branch-line couplers, frequency behavior vs. Lange couper



$$Z_{01} = CZ_0$$

$$Z_{02} = \frac{CZ_0}{\sqrt{1-C^2}}.$$





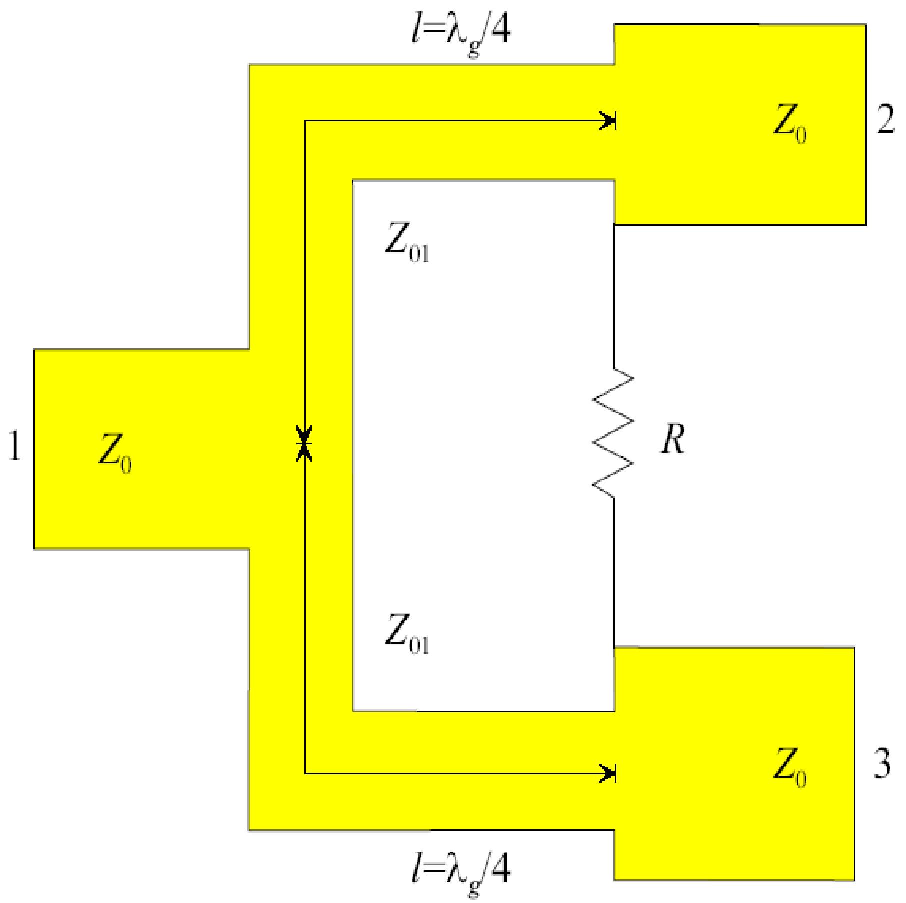
Outline

- Coupled lines
- Directional couplers
- **Power dividers and combiners**

Power dividers and combiners

- The aim is to **divide the input power** (dividers) or **combine several output powers** (combiners) uniformly, granting impedance matching at all ports.
- Usually narrowband, can be made wideband by multisection structures
- Directional couplers can be exploited but ad hoc combiners are preferred → several solutions available, a classical example is **the Wilkinson combiner / divider**.

Wilkinson divider – Layout and *Y* matrix



$$\mathbf{Y} = \begin{pmatrix} 2Y_{11}^l & Y_{12}^l & Y_{12}^l \\ Y_{12}^l & Y_{11}^l + Y_{11}^R & Y_{12}^R \\ Y_{12}^l & Y_{12}^R & Y_{11}^l + Y_{11}^R \end{pmatrix}$$

$$Y_{11}^l = Y_{22}^l = -\mathrm{j}Y_{01}\cot\theta$$

$$Y_{12}^l = Y_{21}^l = \mathrm{j}Y_{01}/\sin\theta$$

$$Y_{11}^R = Y_{22}^R = G$$

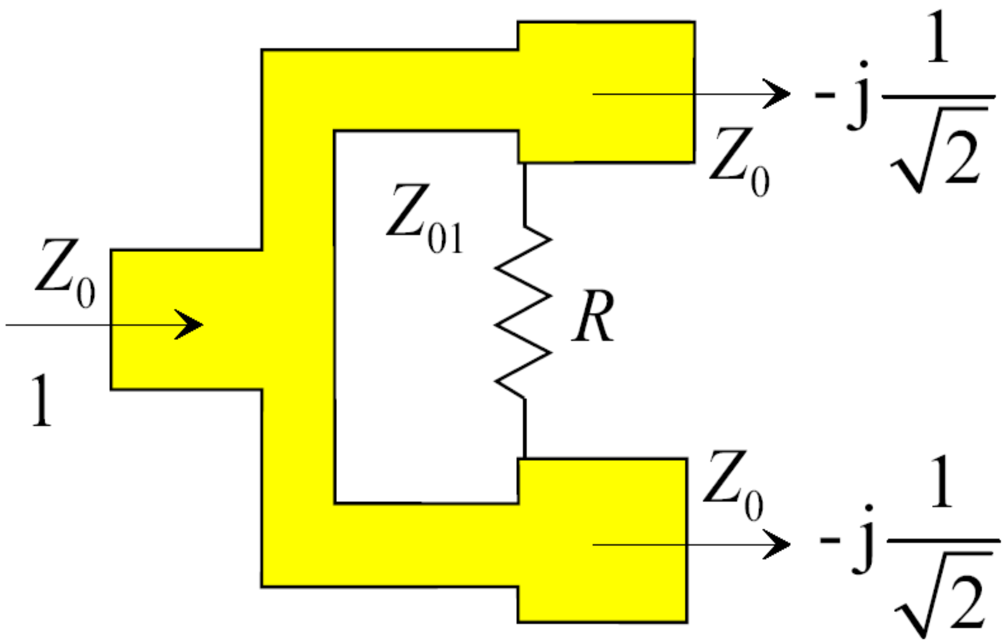
$$Y_{12}^R = Y_{21}^R = -G.$$

Wilkinson power divider (combiner)

- The two lines are a quarter wavelength at centerband
- The divider is narrowband and splits in two the input power
- If the structure is perfecty balanced at the output the resistor is redundant

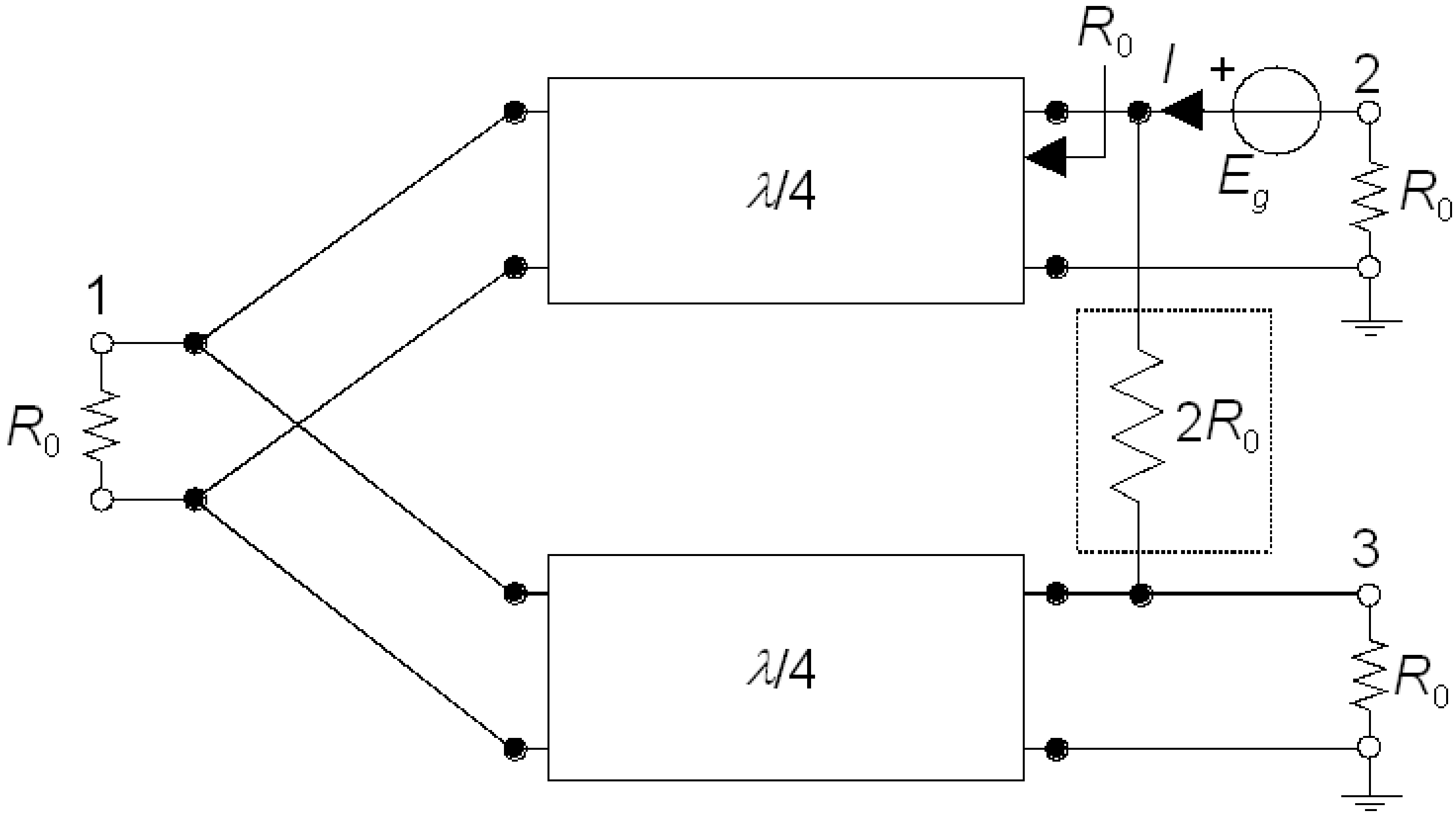
$$Z_{01} = Z_0\sqrt{2}$$

$$R = 2Z_0$$



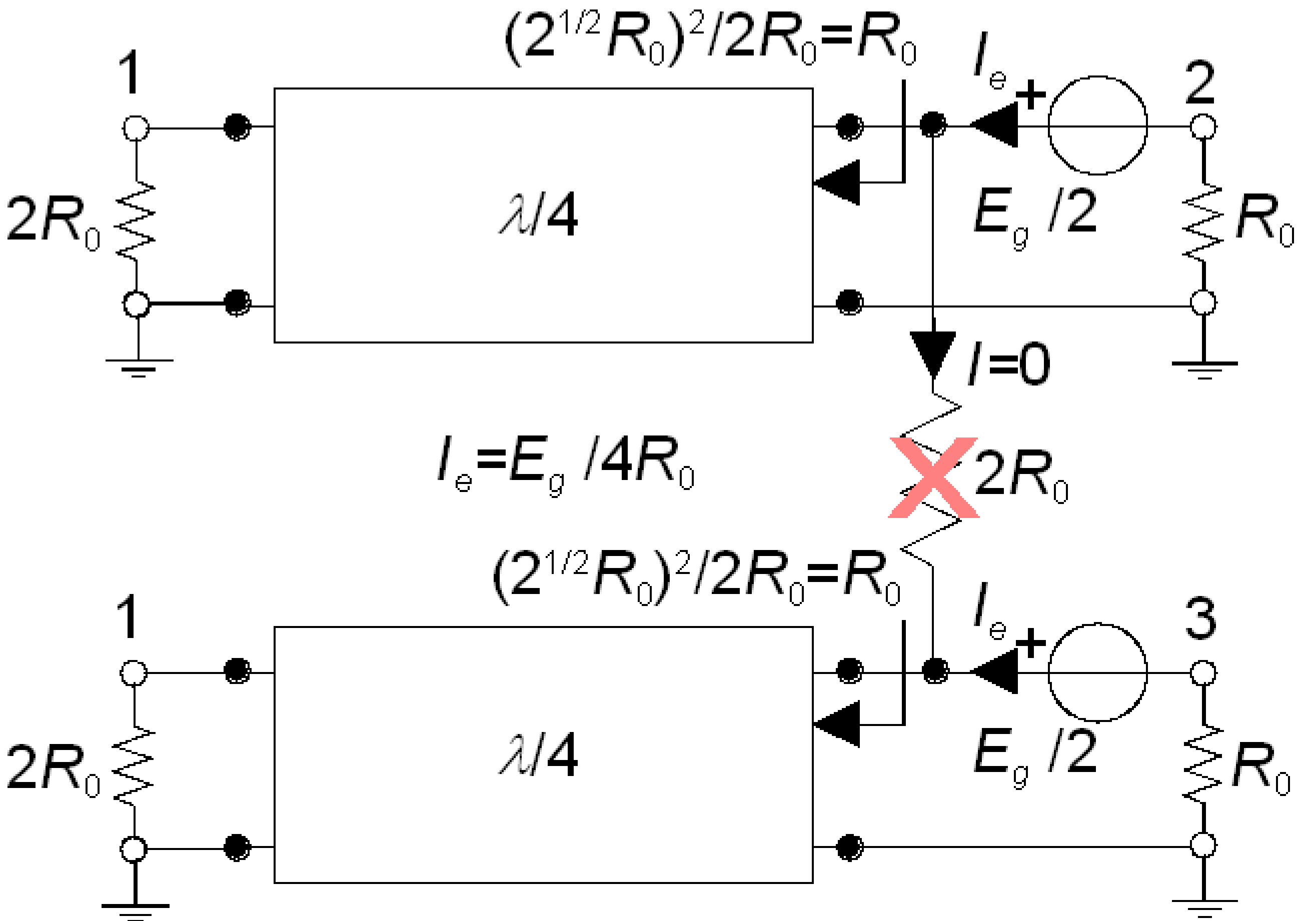
Even-odd proof of $R = 2Z_0$ - I

- We want to check that the parallel resistance grants matching in port 2 and 3



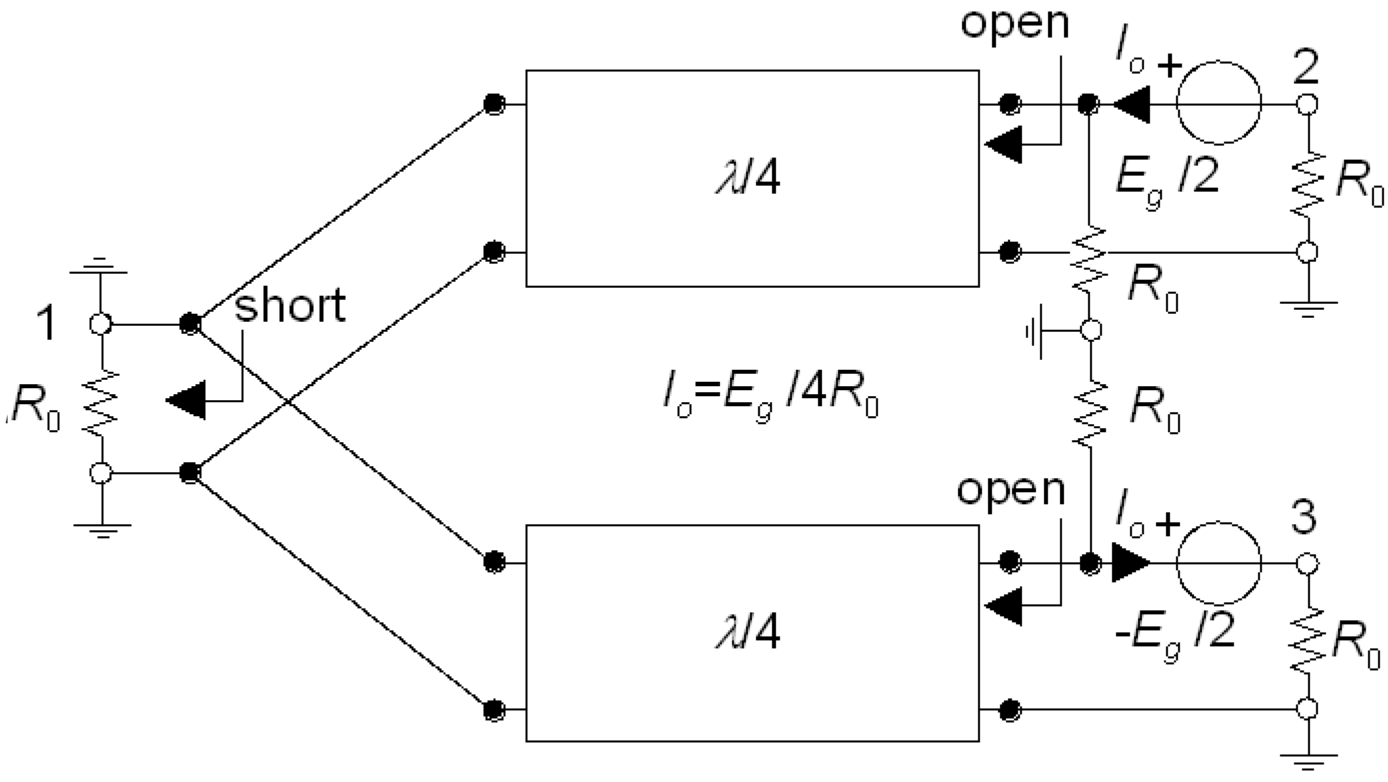
Even-odd proof of $R = 2Z_0$ - II

- Decomposition into even and odd mode excitation: **even**



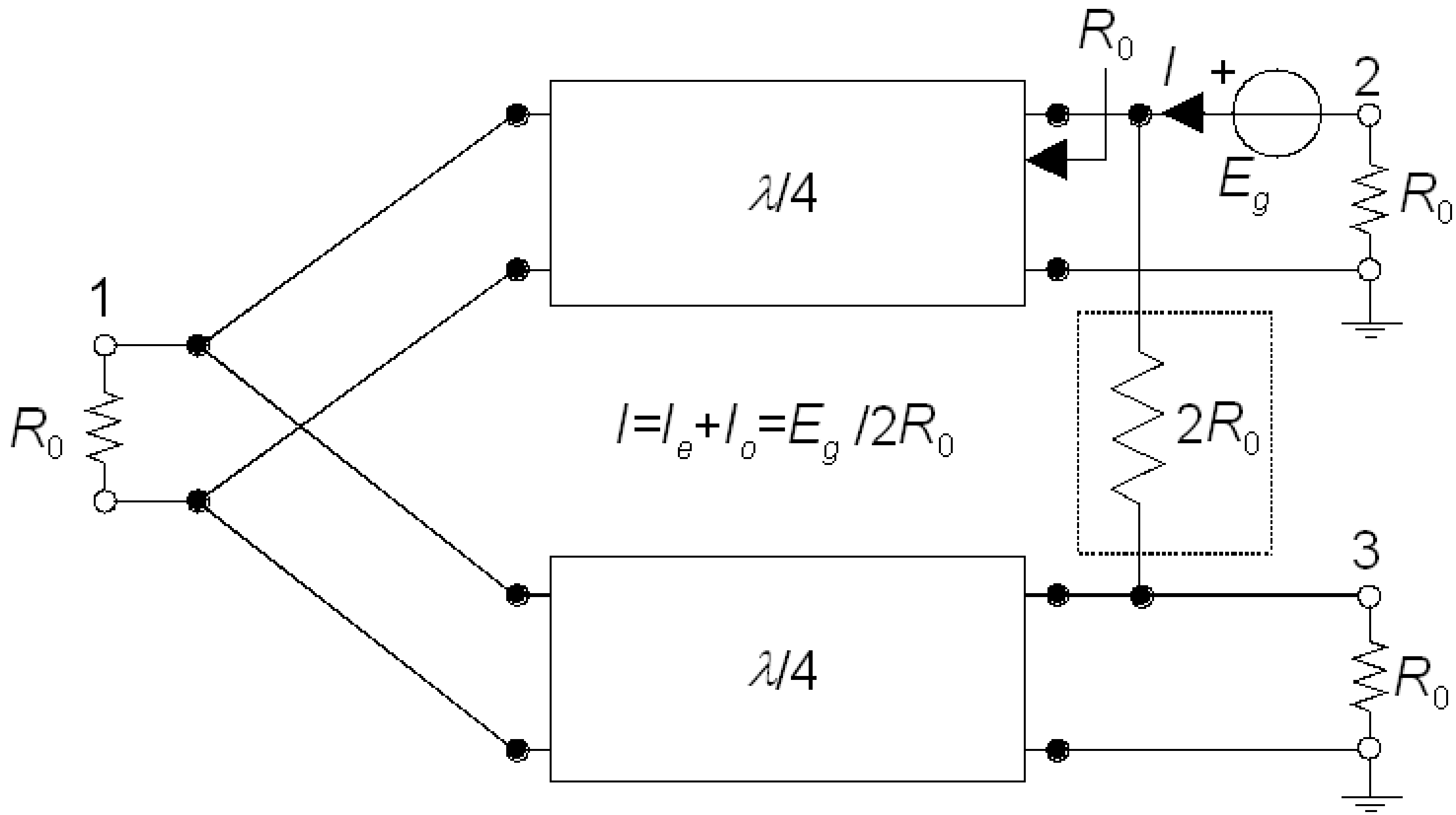
Even-odd proof of $R = 2Z_0$ - III

- Decomposition into even and odd mode excitation: **odd**

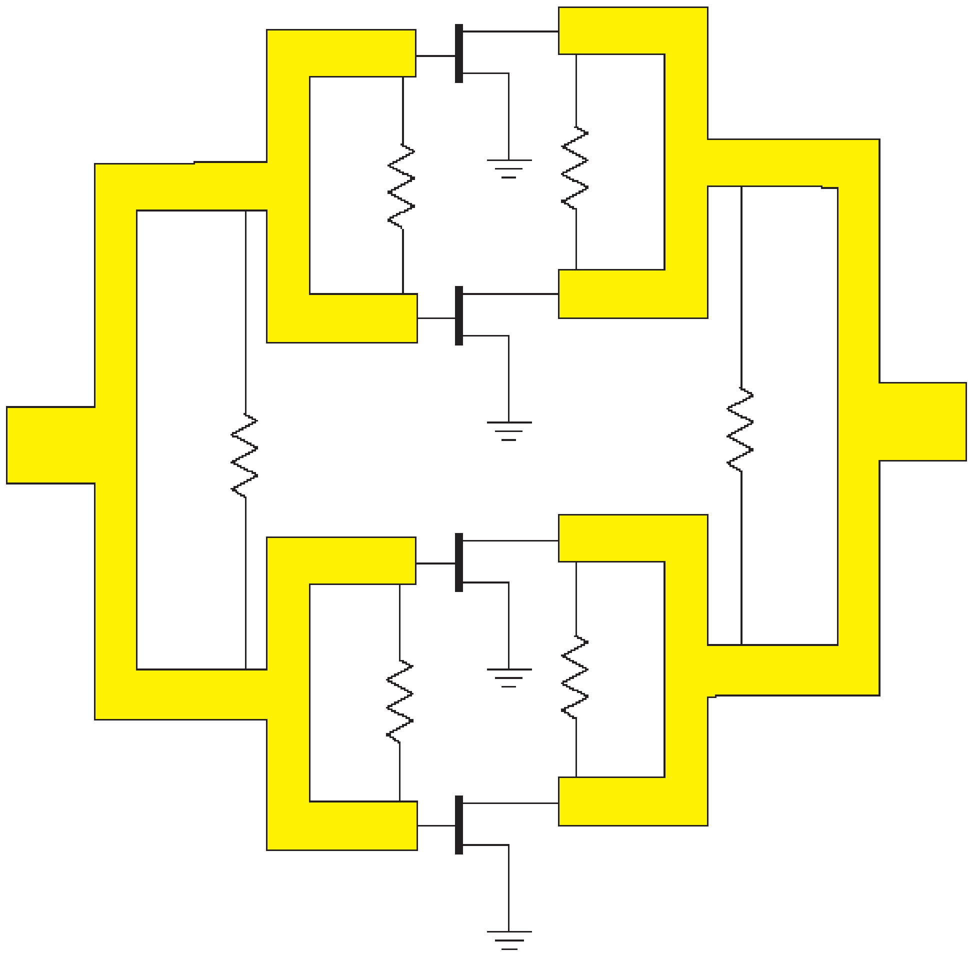


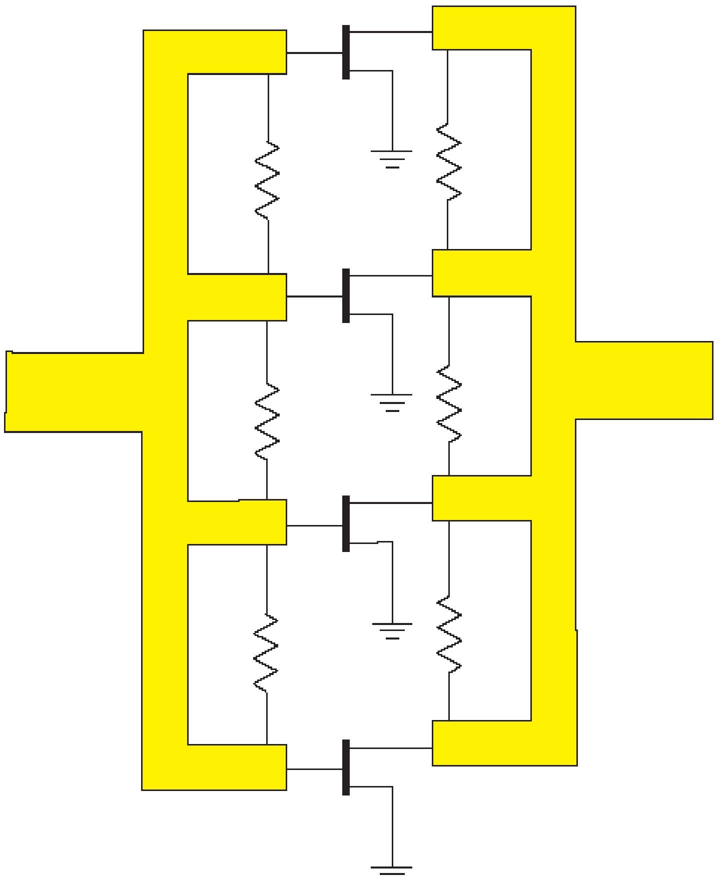
Even-odd proof of $R = 2Z_0$ - IV

- Summing the even and odd mode currents we find $I=E_g/2R_0$ → the input resistance is R_0 at port 2 (same at port 3)

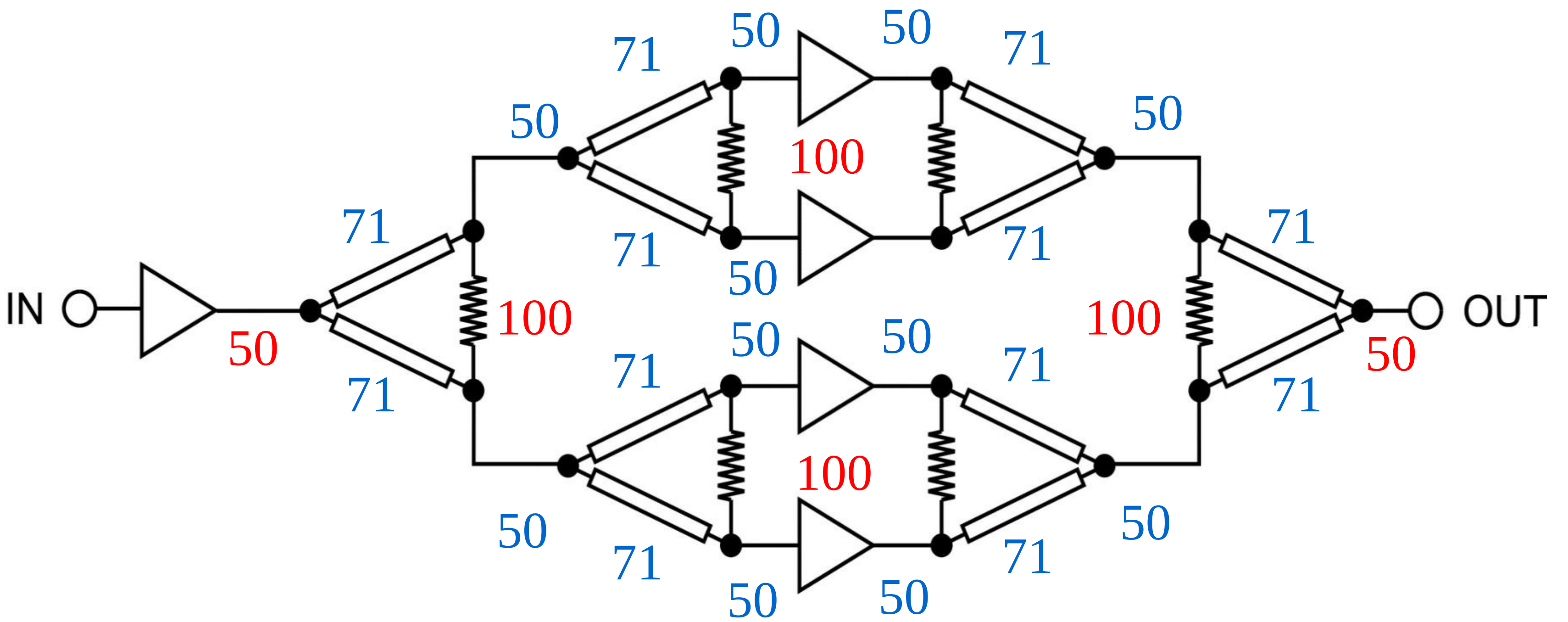


N-branch Wilkinson divider/combiner





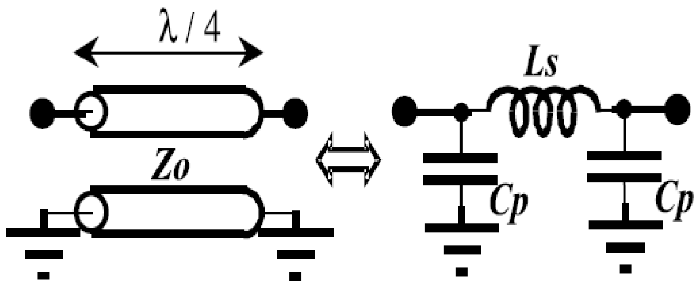
Example of n-branch combiner



- Combining more modules is straightforward but combiner loss will increase

Lumped parameter Wilkinson

lumped
λ/4 line



lumped
Wilkinson