Passive components for RF and microwave circuits: Distributed elements

Microwave Electronics

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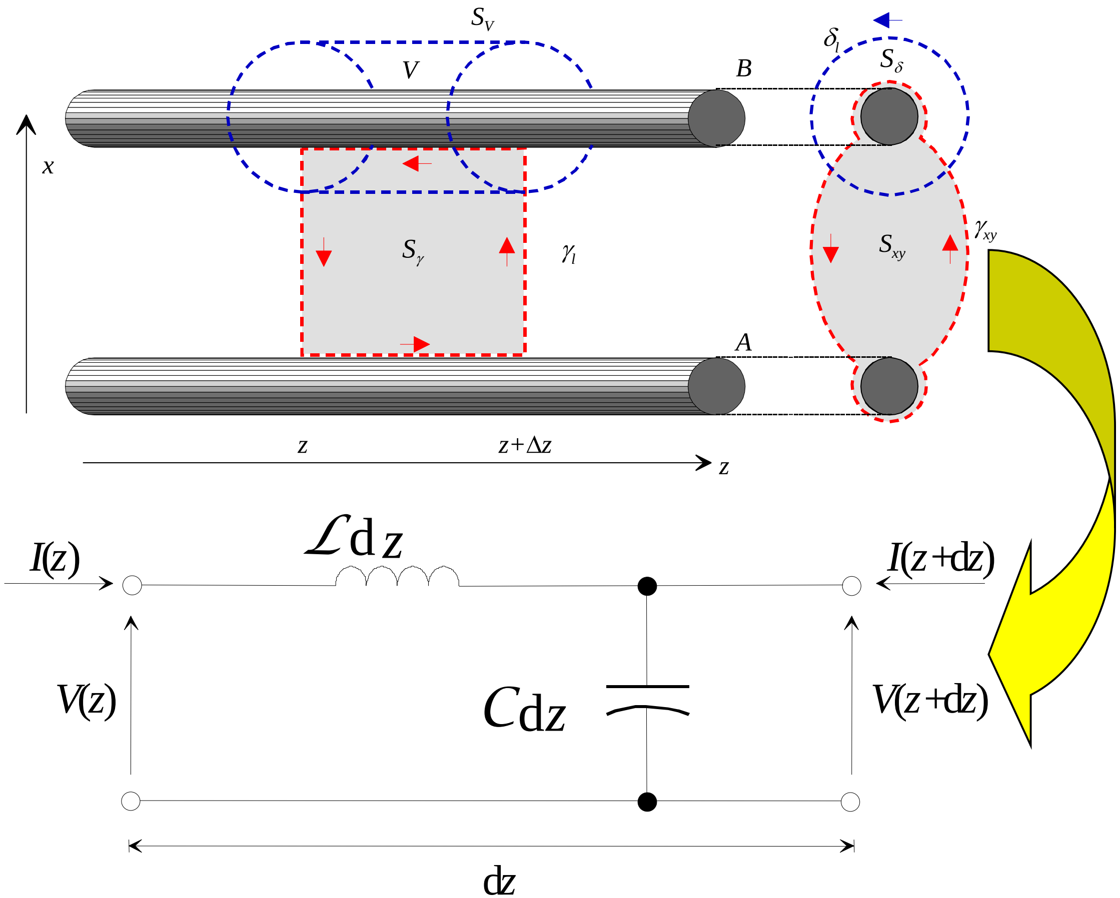
Transmission line review



- *LC* (lossless) line
- Telegraphers' equation:

$$\boxed{\begin{cases} \dfrac{\partial}{\partial z} i(z,t) = -\mathcal{C} \dfrac{\partial}{\partial t} v(z,t) \\ \dfrac{\partial}{\partial z} v(z,t) = -\mathcal{L} \dfrac{\partial}{\partial t} i(z,t) \end{cases}}$$

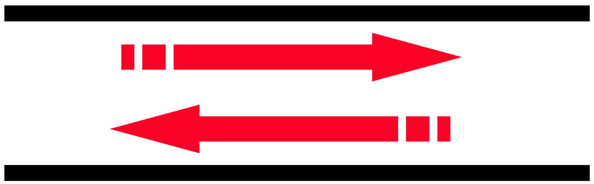
- The solution can be expressed in terms of *forward* and *backward propagating* waves with (phase) velocity:

$$v_f = \sqrt{\frac{1}{\mathcal{L}\mathcal{C}}}$$

Forward and backward waves

$$v^+(z,t) = f^+(z - v_f t)$$

$$i^+(z,t) = f^+(z - v_f t) / Z_0$$



$$v^-(z,t) = f^-(z + v_f t)$$

$$i^-(z,t) = -f^-(z + v_f t) / Z_0$$

- For **time-harmonic** waves:

$$v^\pm(z,t) = V_0^\pm \cos\left[\omega\left(t \mp z/v_f\right) + \varphi^\pm\right] = \mathrm{Re}\left[\underbrace{\overbrace{\left(V_0^\pm e^{j\varphi^\pm}\, e^{\mp j\beta z}\right)}^{V^\pm(z)}}_{\text{Voltage phasor}} e^{j\omega t}\right]$$

$$i^\pm(z,t) = I_0^\pm \cos\left[\omega\left(t \mp z/v_f\right) + \varphi^\pm\right] = \mathrm{Re}\left[\underbrace{\overbrace{\left(I_0^\pm e^{j\varphi^\pm}\, e^{\mp j\beta z}\right)}^{I^\pm(z)}}_{\text{Current phasor}} e^{j\omega t}\right]$$

$$\pm\frac{V_0^\pm}{I_0^\pm} = \pm\frac{V^\pm(z)}{I^\pm(z)} = Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} \quad \text{characteristic impedance}$$

- Therefore a harmonic wave in a lossless line has time and space periodicity

$$\text{time} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\text{space} \Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi v_f}{\omega} = \frac{v_f}{f}$$

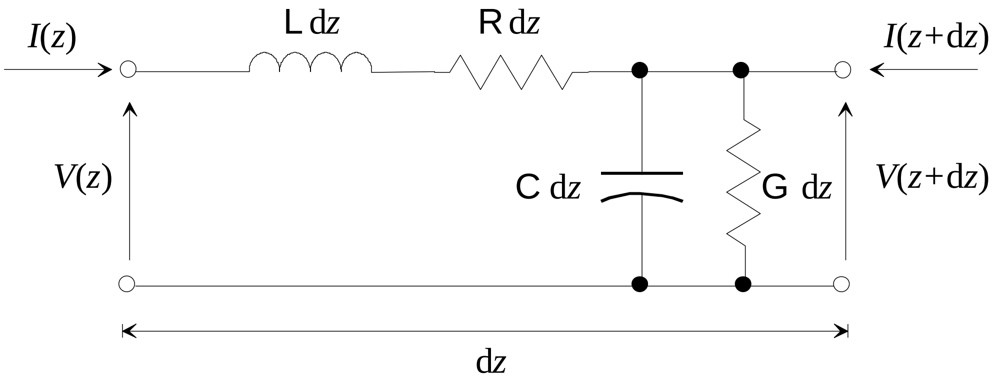
More on the characteristic impedance

- It is defined as the ratio between the progressive voltage phasor and the progressive current phasor (regressive with a minus sign)

$$Z_\infty \equiv Z_0 = \pm \frac{V_0^\pm}{I_0^\pm} = \sqrt{\frac{\mathcal{L}}{C}}$$

- If the line only support a progressive wave (i.e. it is matched at the end) this coincides with the **driving point impedance**
- The charateristic impedance is also the driving point impedance of a infinitely long line **<u>provided there are losses</u>**
- In the general case of a lossy line the characteristic impedance may become complex

Line with series and shunt losses



- Frequency-domain (phasor) solution easier
- The line equations become in *frequency domain*:

$$\begin{cases} \dfrac{\partial}{\partial z} i(z,t) = -\left(\mathcal{C}\dfrac{\partial}{\partial t} v(z,t) + \mathcal{G} v(z,t)\right) \\ \dfrac{\partial}{\partial z} v(z,t) = -\left(\mathcal{L}\dfrac{\partial}{\partial t} i(z,t) + \mathcal{R} i(z,t)\right) \end{cases}$$

$$\begin{cases} \dfrac{\partial}{\partial z} V(z,\omega) = -(\mathrm{j}\omega\mathcal{L} + \mathcal{R}) I(z,\omega) \\ \dfrac{\partial}{\partial z} I(z,\omega) = -(\mathrm{j}\omega\mathcal{C} + \mathcal{G}) V(z,\omega) \end{cases}$$

Frequency-domain (phasor) solution for lossy lines

- Forward and backward line voltages

$$\begin{cases} V(z) = V^{+}(z) + V^{-}(z) = V_0^{+}\,\mathrm{e}^{-\mathrm{j}\beta z - \alpha z} + V_0^{-}\,\mathrm{e}^{\mathrm{j}\beta z + \alpha z} \\ I(z) = I^{+}(z) + I^{-}(z) = I_0^{+}\,\mathrm{e}^{-\mathrm{j}\beta z - \alpha z} + I_0^{-}\,\mathrm{e}^{\mathrm{j}\beta z + \alpha z} = \\ \qquad = \dfrac{V_0^{+}}{Z_\infty}\mathrm{e}^{-\mathrm{j}\beta z - \alpha z} - \dfrac{V_0^{-}}{Z_\infty}\mathrm{e}^{\mathrm{j}\beta z + \alpha z} \end{cases}$$

- Complex propagation constant

$$\alpha + \mathrm{j}\beta = \gamma = \sqrt{(\mathrm{j}\omega\mathcal{L} + \mathcal{R})(\mathrm{j}\omega\mathcal{C} + \mathcal{G})}.$$

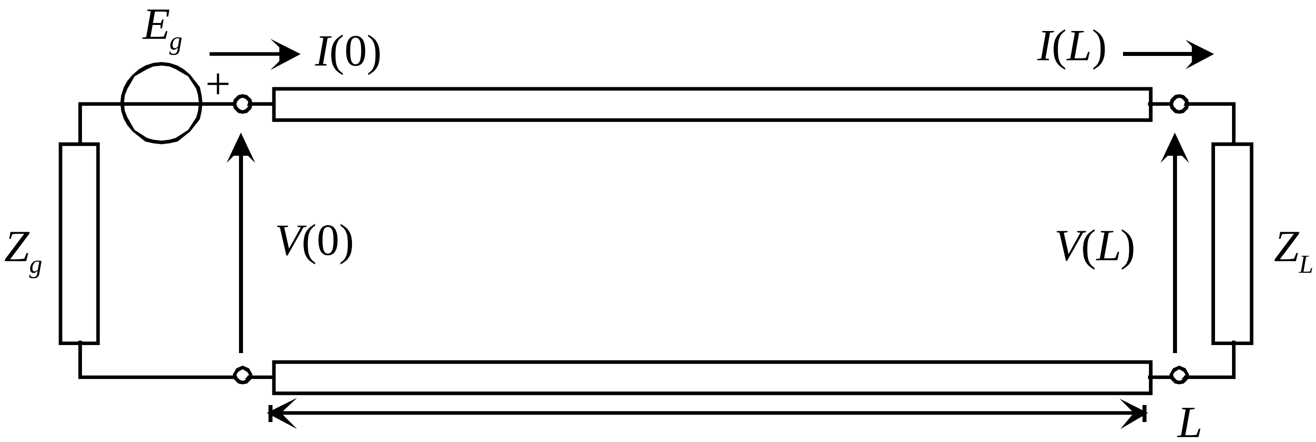
- Characteristic impedance

$$\sqrt{\frac{\mathrm{j}\omega\mathcal{L} + \mathcal{R}}{\mathrm{j}\omega\mathcal{C} + \mathcal{G}}} \equiv Z_\infty \qquad Z_\infty = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} \quad \textit{lossless}$$

Interpretation of solution for lossy line

- The solution is the superposition of two terms:
  - A forward (progressive) wave propagating towards right and attenuating in the propagation direction
  - A backward (regressive) wave propagating towards left and attenuating in the propagation direction
- The **characteristic impedance** is the voltage/current ratio for a forward and backward wave
- The voltage is defined only in the line cross section!!
- Imposing load and generator conditions we can evaluate the amplitude of forward and backward waves for voltage → also for current; mathematically speaking the problem is well posed.

Example of loaded line solution



2 unknowns: $V_0^+ \; V_0^-$,

2 equations

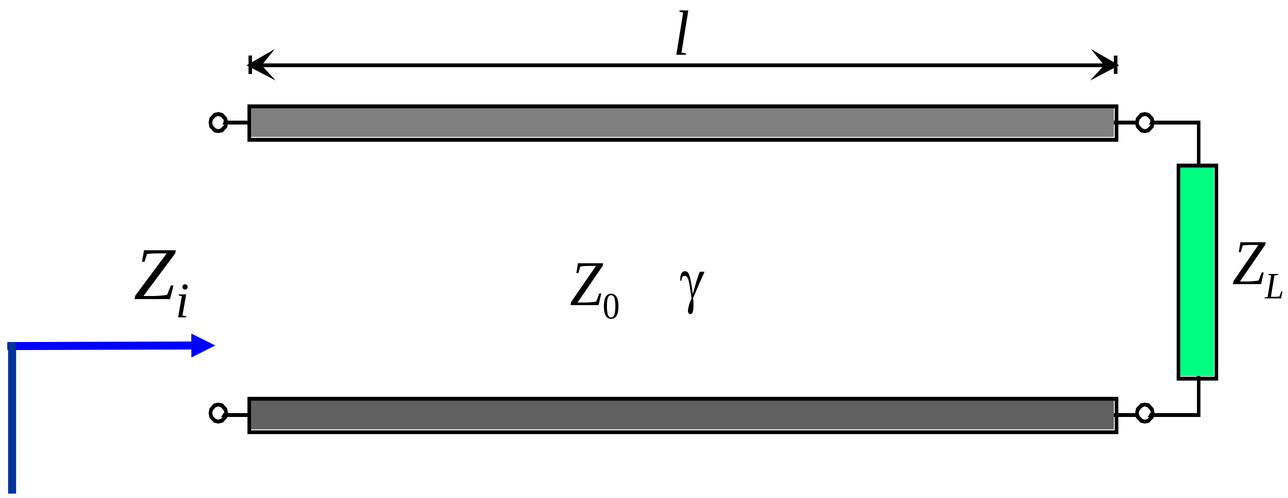
$$\underbrace{V_0^+ + V_0^-}_{V(0)} = E_g - \underbrace{\left(\frac{V_0^+}{Z_\infty} - \frac{V_0^-}{Z_\infty} \right)}_{I(0)} Z_g$$

Generator condition

$$\underbrace{V_0^+ \, \mathrm{e}^{-\mathrm{j}\beta L - \alpha L} + V_0^- \, \mathrm{e}^{\mathrm{j}\beta L + \alpha L}}_{V(L)} = \underbrace{\left(\frac{V_0^+}{Z_\infty} \mathrm{e}^{-\mathrm{j}\beta L - \alpha L} - \frac{V_0^-}{Z_\infty} \mathrm{e}^{\mathrm{j}\beta L + \alpha L} \right)}_{I(L)} Z_L$$

Load condition

Input impedance of loaded line



- Generic loaded line: $$Z_i = Z_0 \left[\frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_L \sinh(\gamma l)} \right]$$

Particular cases

- Z_L=0 $\quad \boxed{Z_{\text{in}}(z) = Z_\infty \tanh(\gamma z)}$ $\quad \boxed{\text{lossless } Z_{\text{in}}(z) = jZ_\infty \tan(\beta z)}$
- Z_L=∞ $\quad \boxed{Z_{\text{in}}(z) = Z_\infty \coth(\gamma z)}$ $\quad \boxed{\text{lossless } Z_{\text{in}}(z) = -jZ_\infty \cot(\beta z)}$
- Z_L=Z_∞ $\quad \boxed{Z_{\text{in}}(z) = Z_\infty}$
- (Lossy) infinite-length line, load whatsoever:

$$\boxed{Z_{\text{in}}(z \to \infty) = Z_\infty}$$

Short lines in short and open

Short line closed by a short circuit $\rightarrow$ an inductor:

$$Z_{in} = jZ_0 \tan \beta l \approx jZ_0 \beta l = j\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} \times \omega\sqrt{\mathcal{L}\mathcal{C}}\, l = j\omega\mathcal{L}l$$

Short line closed by an open circuit $\rightarrow$ a capacitor:

$$Z_{in} = -jZ_0 \cot \beta l \approx -jZ_0 \frac{1}{\beta l} = -j\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} \times \frac{1}{\omega\sqrt{\mathcal{L}\mathcal{C}}\, l} = \frac{1}{j\omega\mathcal{C}l}$$

More on line parameters

- From the propagation constant β the **guided wavelength** is:

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda_0}{n_{eff}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}, \quad \begin{cases} n_{eff} & \text{effective refractive index, also "microwave" or "RF" index} \\ \varepsilon_{eff} & \text{effective permittivity} \end{cases}$$

- This describes the spatial periodicity of voltages and currents on the transmission line
- The line attenuation can be expressed in natural (Np/m) or log (dB/m, dB/cm) units; given α in Np/m one has:

$$\alpha|_{\mathrm{dB/m}} = 8.6859\alpha$$
$$\alpha|_{\mathrm{dB/cm}} = 0.086859\alpha$$

Parameter dispersion due to losses

- Line losses introduce frequency dependence of the line parameters
- Moreover, the p.u.l. resistance depends on frequency on its own due to skin effect (?) and the same does the p.u.l. conductance related to the loss angle (??)
- We can separate:
  - the low-frequency (RG) regime → negligible reactive effects, the line is a distributed lossy system
  - the intermediate frequency (RC) → reactive capacitive effects prevail over inductive (typically, may be the opposite)
  - and the *high-frequency* (LC) regime → reactive elements prevail and the behavior is similar to a lossless line

The high-frequency (*LC*) regime

- The characteristic impedance is (almost) real and constant in frequency (non-dispersive line)

$$Z_0 \approx \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$$

- The phase velocity is contant in frequency (again no dispersion)

$$v_f = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$$

- Losses can be separated into **series losses** (R) and **parallel losses** (G) also called metal and dielectric losses, usually series losses prevail
- Losses are frequency dependent because (see later) R and G **increase with frequency**

$$\alpha + \mathrm{j}\beta \approx \underbrace{\boxed{\frac{\mathcal{R}}{2Z_0}}}_{\text{conductor attenuation } \alpha_c} + \underbrace{\boxed{\frac{\mathcal{G}Z_0}{2}}}_{\text{dielectric attenuation } \alpha_d} + \mathrm{j}\omega\sqrt{\mathcal{L}\mathcal{C}}$$

The high-frequency approximation

$$\alpha + j\beta = \sqrt{(\mathcal{G} + j\omega C)(\mathcal{R} + j\omega \mathcal{L})} =$$

$$= \sqrt{(j\omega C)(j\omega \mathcal{L})}\sqrt{\left(1+\frac{\mathcal{G}}{j\omega C}\right)\left(1+\frac{\mathcal{R}}{j\omega \mathcal{L}}\right)} =$$

$$= \sqrt{(j\omega C)(j\omega \mathcal{L})}\sqrt{\left(1+\frac{\mathcal{G}}{j\omega C}+\frac{\mathcal{R}}{j\omega \mathcal{L}}+\frac{\mathcal{G}\mathcal{R}}{(j\omega)^2 \mathcal{L}C}\right)} \underset{\omega\to\infty}{\approx}$$

$$\approx \sqrt{(j\omega C)(j\omega \mathcal{L})}\left[1+\frac{\mathcal{G}}{2j\omega C}+\frac{\mathcal{R}}{2j\omega \mathcal{L}}\right] = j\omega\sqrt{\mathcal{L}C}+\frac{\mathcal{G}}{2}\sqrt{\frac{\mathcal{L}}{C}}+\frac{\mathcal{R}}{2}\sqrt{\frac{C}{\mathcal{L}}} =$$

$$= j\omega\sqrt{\mathcal{L}C}+\frac{\mathcal{G}Z_0}{2}+\frac{\mathcal{R}}{2Z_0} = j\beta+\alpha_d+\alpha_c$$

Low frequency regime and intermediate frequencies

- For very low frequency the line is a distributed attenuator → no propagation, just attenuation

$$Z_0 \approx \sqrt{\frac{\mathcal{R}}{\mathcal{G}}}$$

$$\gamma \approx \sqrt{\mathcal{R}\mathcal{G}}$$

- For intermediate frequencies capacitive effects tipically prevail over inductive ones and the line is strongly dispersive → RC regime typical of Si-based integrated circuits

$$Z_0 \approx \sqrt{\frac{\mathcal{R} + \mathrm{j}\cancel{\omega\mathcal{L}}}{\mathrm{j}\omega\mathcal{C}}} \approx \frac{1-\mathrm{j}}{\sqrt{2}}\sqrt{\frac{\mathcal{R}}{\omega\mathcal{C}}}$$

$$\gamma \approx \sqrt{(\mathcal{R} + \mathrm{j}\cancel{\omega\mathcal{L}})\,(\mathrm{j}\omega\mathcal{C})} \approx \frac{1+\mathrm{j}}{\sqrt{2}}\sqrt{\omega\mathcal{R}\mathcal{C}}$$

The skin effect: frequency behaviour of *R* and skin depth

- The per unit length (p.u.l.) resistance increases at high frequency owing to the *skin effect* as:

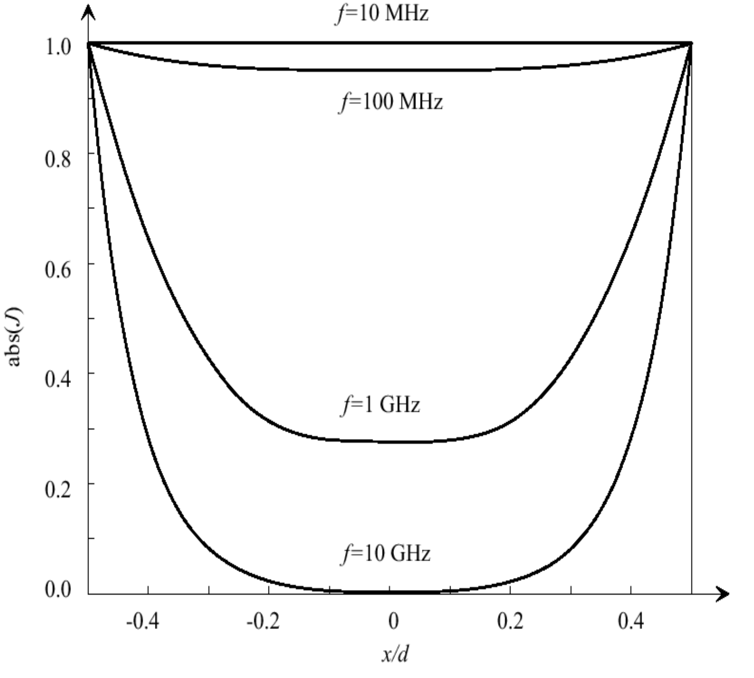
$$\mathcal{R}(f) \approx \mathcal{R}(f_0)\sqrt{f / f_0}$$

- At high frequency the surface impedance (see next slide) of the conductor becomes complex (resistive and inductive):

$$Z_s(\omega) = R_s + \mathrm{j}X_s = (1+\mathrm{j})\frac{1}{\sigma\delta}$$

- Skin depth:

$$\boxed{\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = \sqrt{\frac{1}{\pi\mu\sigma f}}}$$



- Skin depth values for good conductors → a few microns at a few GHz

Surface impedance and p.u.l. impedance - I

- Surface or sheet resistance → impedance of a metal or semiconductor layer **square patch** of thickness *t*:

$$R_s = \frac{L}{Wt\sigma} \quad \Omega/\square$$

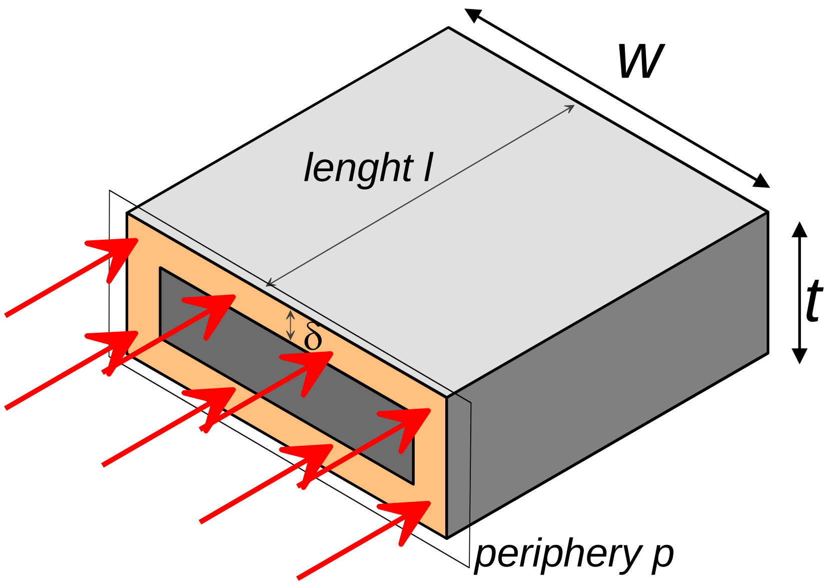
- Measured in Ohm/square. In a high frequency conductor:
  - The equivalent thickness of the layer is the skin penetration depth
  - Reactive effects add an inductive reactance
- Thus: the skin-effect surface impedance Z_s is the impedance of a **metal patch** of width *W* and length *L*=*W* (measured in Ohm/square):

$$Z_s(\omega) = R_s + \mathrm{j}X_s = (1+\mathrm{j})\frac{1}{\sigma\delta}$$

Surface impedance and p.u.l. impedance - II

- For a conductor of periphery *p* and length *l=p* (the conductive surface is a square) the total impedance input impedance is derived from the p.u.l. impedance $\mathcal{Z}$ as:

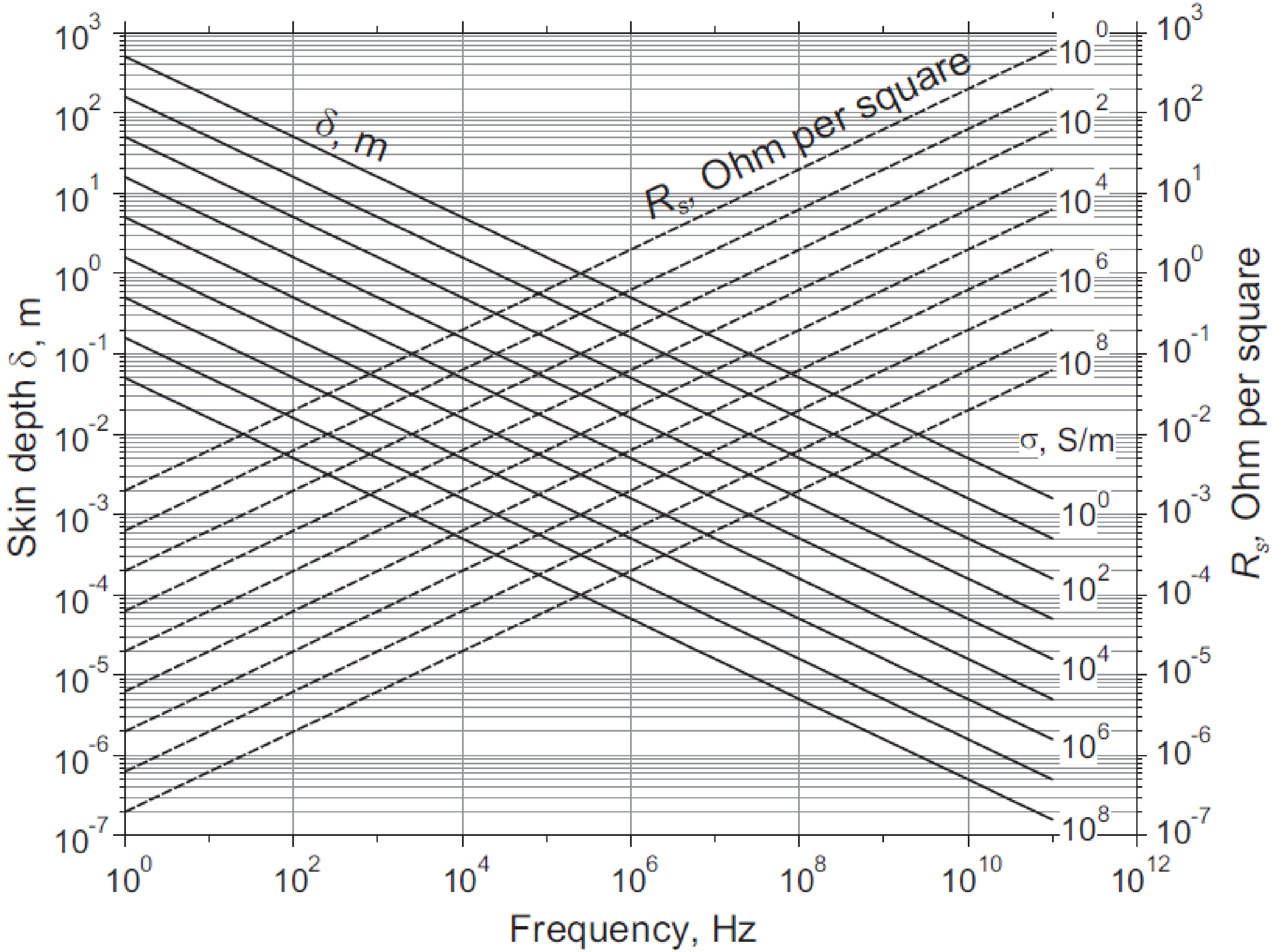
$$Z = \mathcal{Z}l \equiv \mathcal{Z}p = Z_s \rightarrow \boxed{\mathcal{Z} = \frac{Z_s}{p}}$$



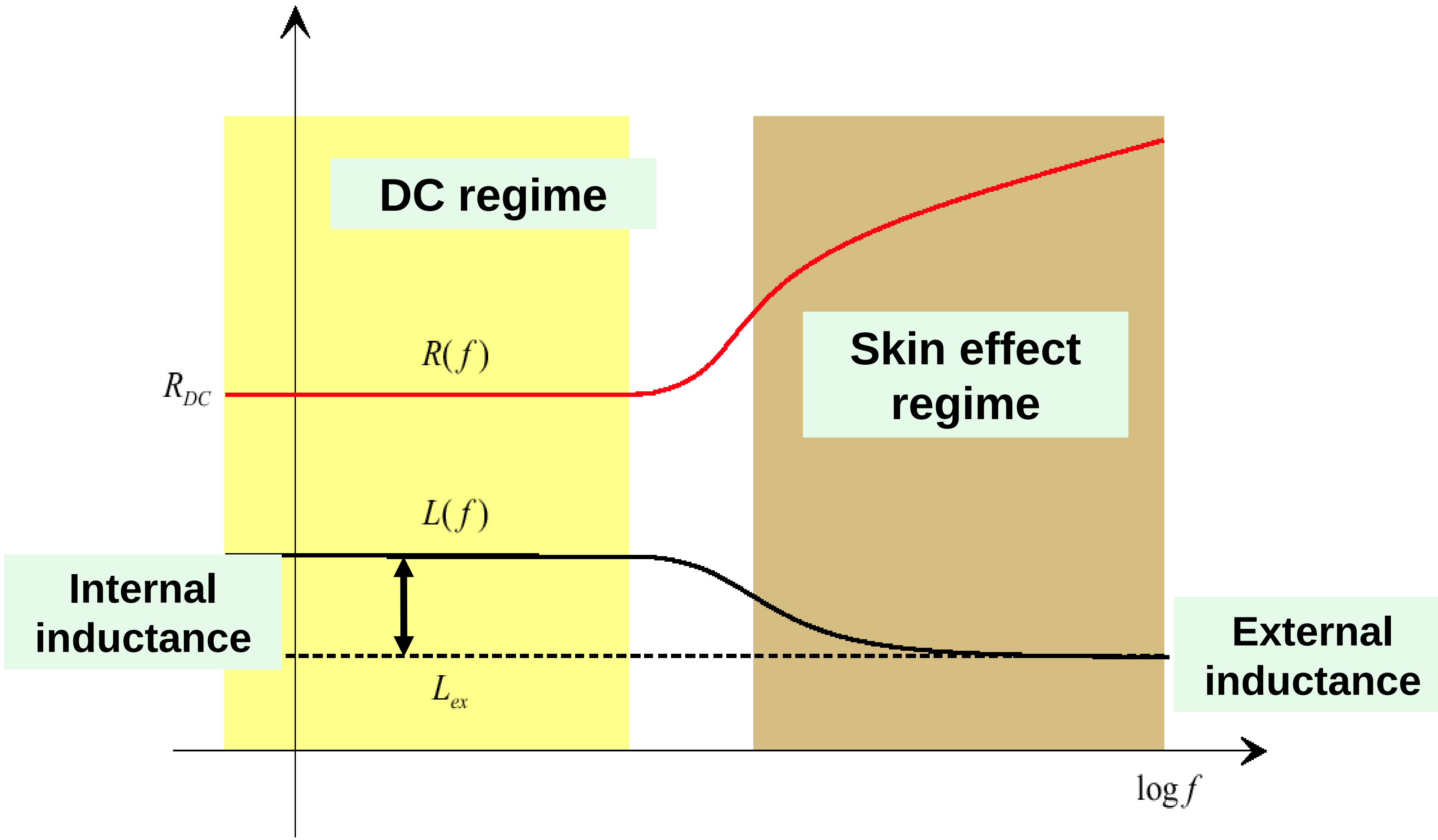
- For example, for a circular wire of radius *r* and for a strip of width *w* and thickness *t* we have:

$$\mathcal{Z}_{\text{wire}} = \frac{Z_s}{2\pi r}, \mathcal{Z}_{\text{strip}} = \frac{Z_s}{2(w+t)}$$

Skin depth and *R* vs. frequency for different conductivities



Skin effect →p.u.l resistance and inductance



Frequency behaviour of *G*

- A lossy dielectric has complex permittivity:

$$\epsilon_{rc} = \epsilon_r - \mathrm{j}\epsilon_2 = \epsilon_r(1 - \mathrm{j}\tan\delta)$$

- Parallel admittance (C_a is the p.u.l. capacitance in air):

$$\mathcal{Y} = \mathrm{j}\omega\mathcal{C} = \mathrm{j}\omega\epsilon_{rc}\mathcal{C}_a = \mathrm{j}\omega\epsilon_r\mathcal{C}_a + \omega\epsilon_r\tan(\delta)\mathcal{C}_a$$

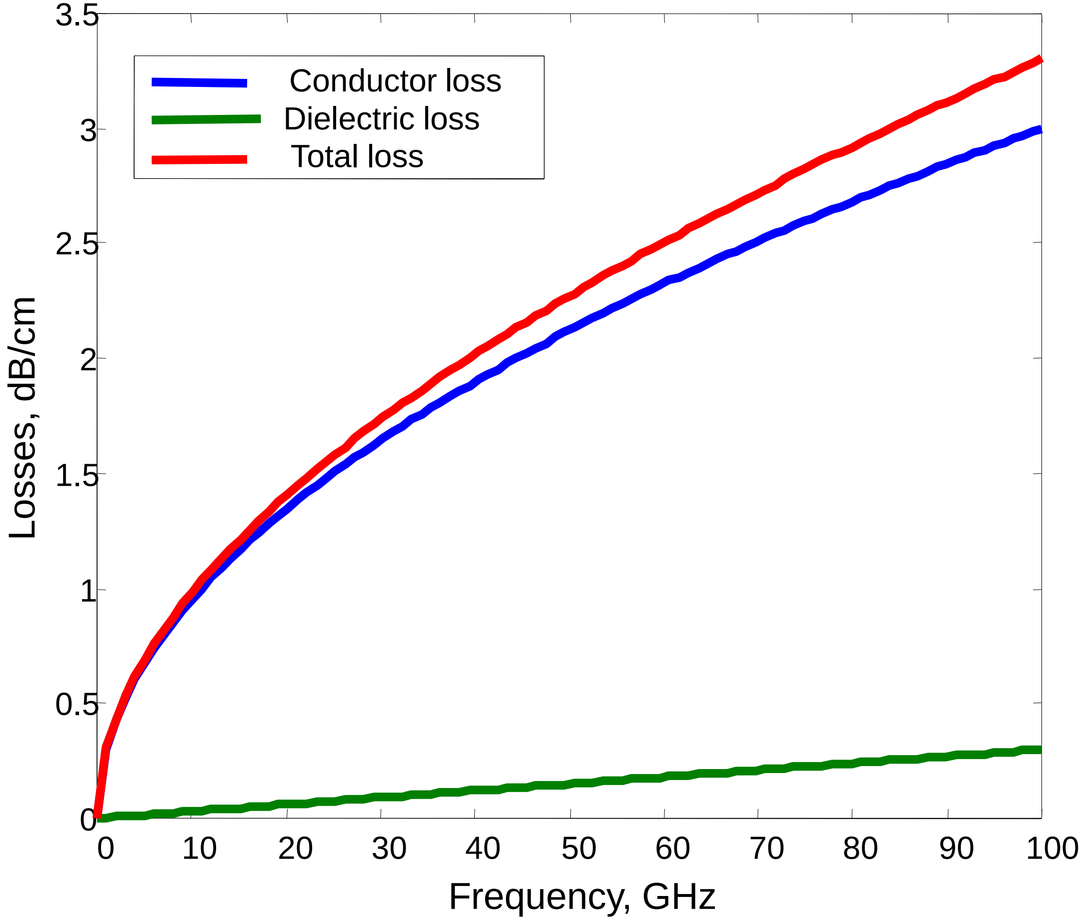
- Conductance:

$$\boxed{\mathcal{G} = \omega\epsilon_r\tan(\delta)\mathcal{C}_a = \sigma\mathcal{C}_a/\epsilon_0}$$

- Substrate examples:

Material	Alumina	Quartz	Teflon	Beryl oxide	GaAs	InP	Si
ϵ_r	9.8	3.78	2	6	12.9	12.4	11.9
$\tan\delta$	10^{-3}	10^{-4}	10^{-4}	10^{-3}	10^{-3}	10^{-3}	10^{-2}

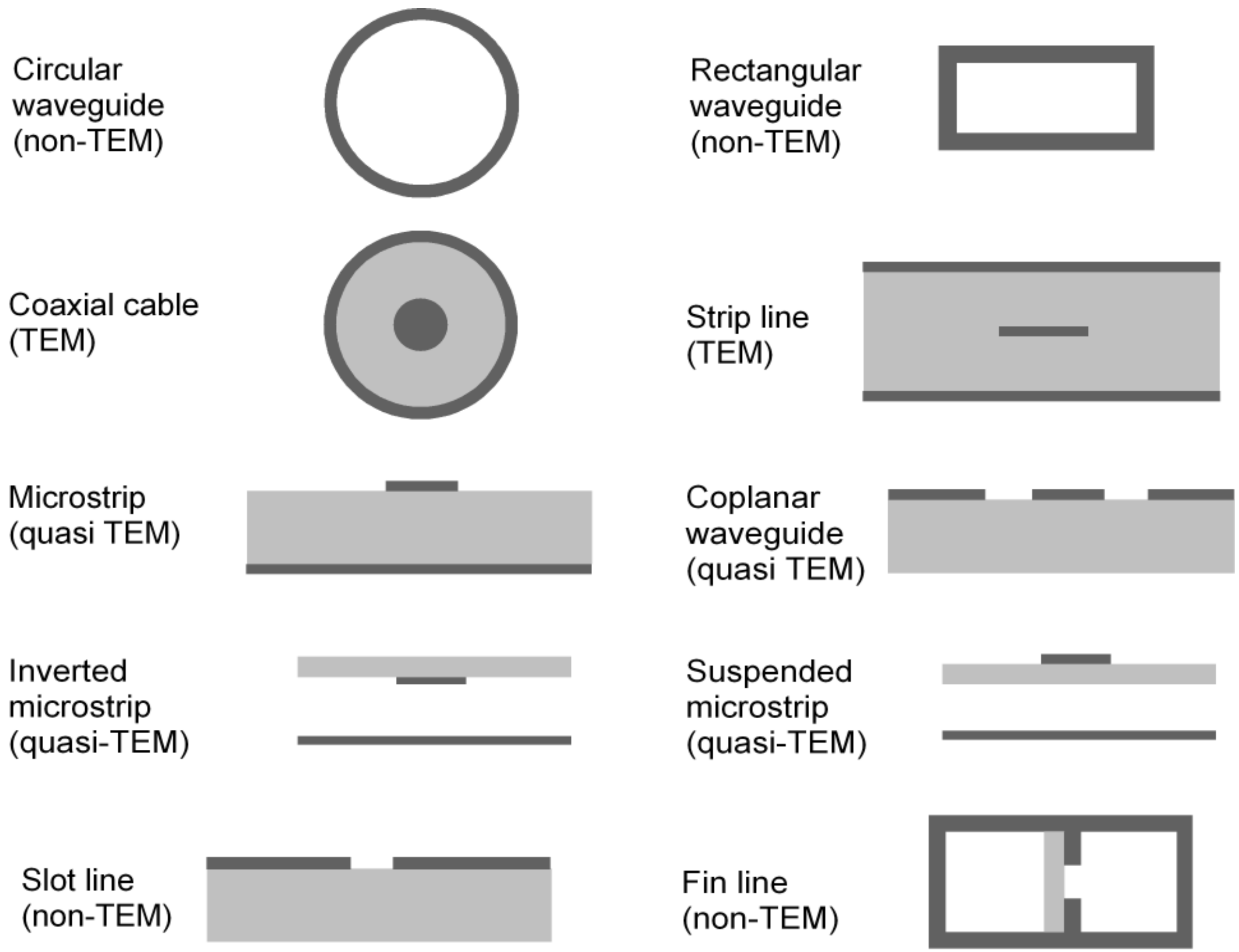
Frequency behaviour of line attenuation due to *R*(*f*), *G*(*f*)



$$\alpha(f) \approx \frac{R(f)}{2Z_0} + \frac{G(f)}{2Y_0}$$

$$= K_1\sqrt{f} + K_2 f$$

Non-TEM (Transverse Electro Magnetic), TEM, quasi-TEM transmission lines



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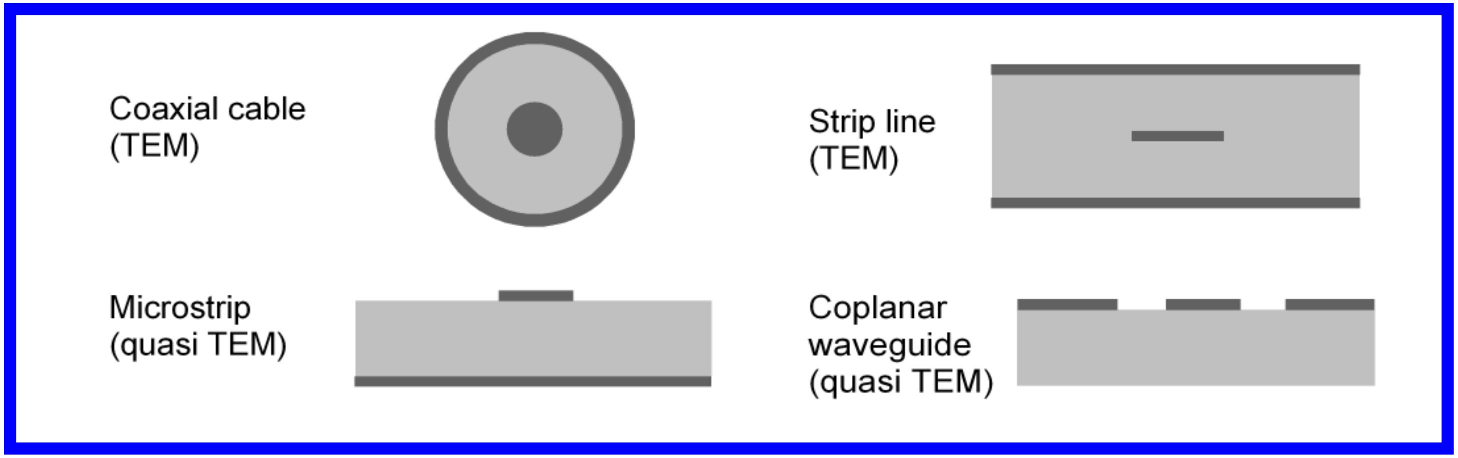

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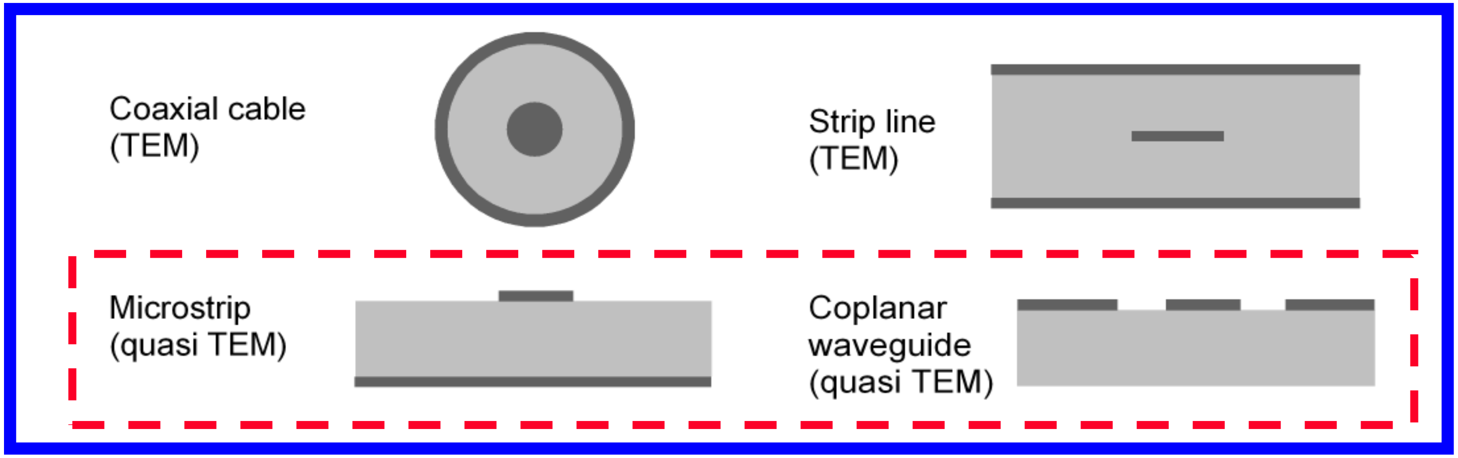


TEM vs. quasi-TEM lines

- TEM: homogeneous cross section, minimum two conductors, E and H fields in the cross section
- No cutoff for fundamental mode, higher-order modes possible
- Frequency-independent propagation parameters
- Low-frequency (RC) dispersion in lossy lines

$$v_f \approx v_g = \frac{c_0}{\sqrt{\varepsilon_r}}$$

- Quasi-TEM: non-homogeneous cross section, E and H field transversal only at low frequency
- No cutoff for fundamental mode, higher-order modes possible
- Slightly dispersive propagation parameters
- Low-frequency (RC) dispersion in lossy lines

$$v_f \approx v_g = \frac{c_0}{\sqrt{\varepsilon_{\text{eff}}}}$$

$\varepsilon_{\text{eff}}(f)$ effective permittivity

Further on Quasi-TEM lines parameters

- In a quasi-TEM line the inductance p.u.l. does not depend on dielectrics → same as in vacuo inductance
- In vacuo the propagation velocity is the same as the light velocity, thus:

$$\mathcal{L} = \mathcal{L}_a = \frac{1}{c_0^2 \mathcal{C}_a}$$

- C_a is the in vauo (in air) capacitance. The effective ("average") permettivity is such as:

$$\mathcal{C} = \epsilon_{\text{eff}} \mathcal{C}_a$$

- The impedance and phase velocity can be expressed as:

$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{c_0 \sqrt{\mathcal{C}\mathcal{C}_a}}$$

$$v_f = \sqrt{\frac{1}{\mathcal{L}\mathcal{C}}} = c_0 \sqrt{\frac{\mathcal{C}_a}{\mathcal{C}}}$$

- Or, through the effective permittivity:

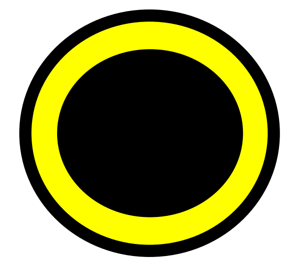
$$Z_0 = \frac{1}{c_0 \mathcal{C}_a \sqrt{\epsilon_{\text{eff}}}} = \frac{Z_{0a}}{\sqrt{\epsilon_{\text{eff}}}}$$

$$v_f = \frac{c_0}{\sqrt{\epsilon_{\text{eff}}}}$$

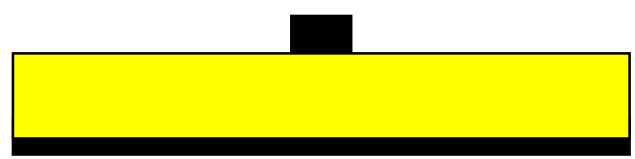
High and low impedance quasi-TEM lines

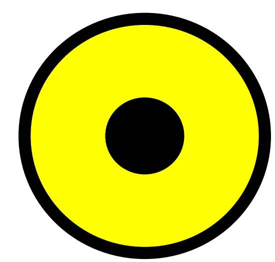
$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{c_0\sqrt{\mathcal{C}\mathcal{C}_a}}$$

- High capacitance, low inductance, low impedance line



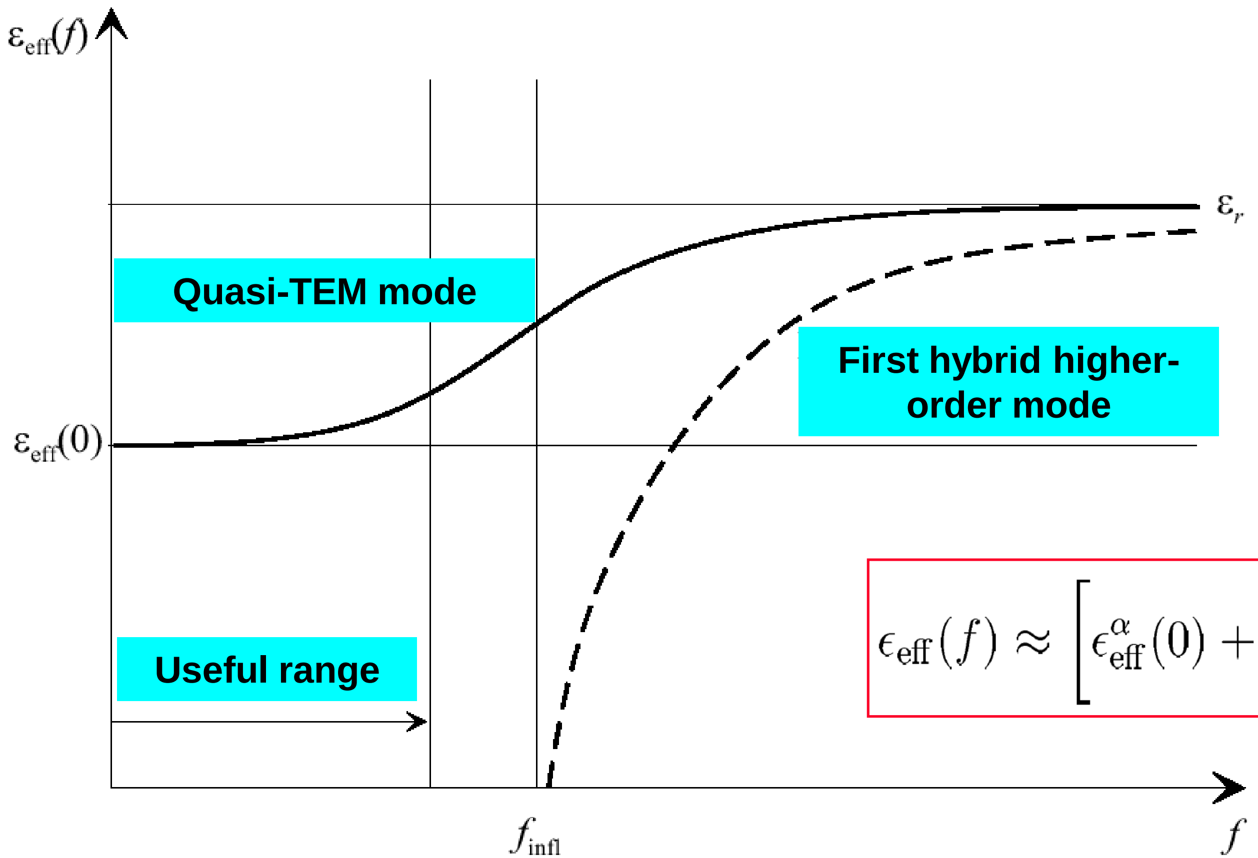
- Low capacitance, high inductance, high impedance line





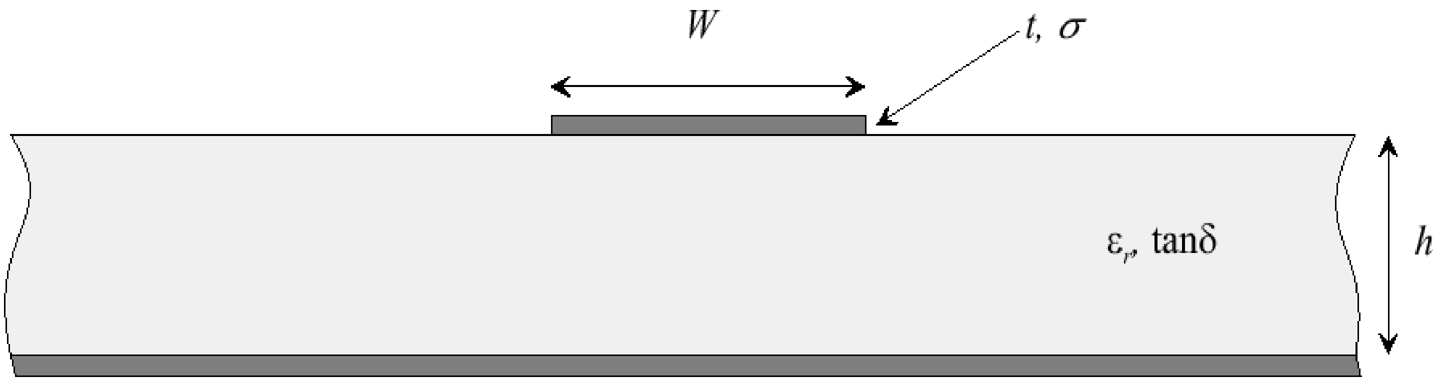
Quasi-TEM frequency-dependent effective permittivity

- The quasi-TEM mode exhibits a certain amount of *modal dispersion*→ frequency dependent effective permittivity



$$\epsilon_{\rm eff}(f) \approx \left[\epsilon_{\rm eff}^{\alpha}(0) + \frac{\epsilon_r^{\alpha} - \epsilon_{\rm eff}^{\alpha}(0)}{1 + (f_{\rm infl}/f)^{\beta}}\right]^{1/\alpha}$$

Planar transmission lines: the microstrip



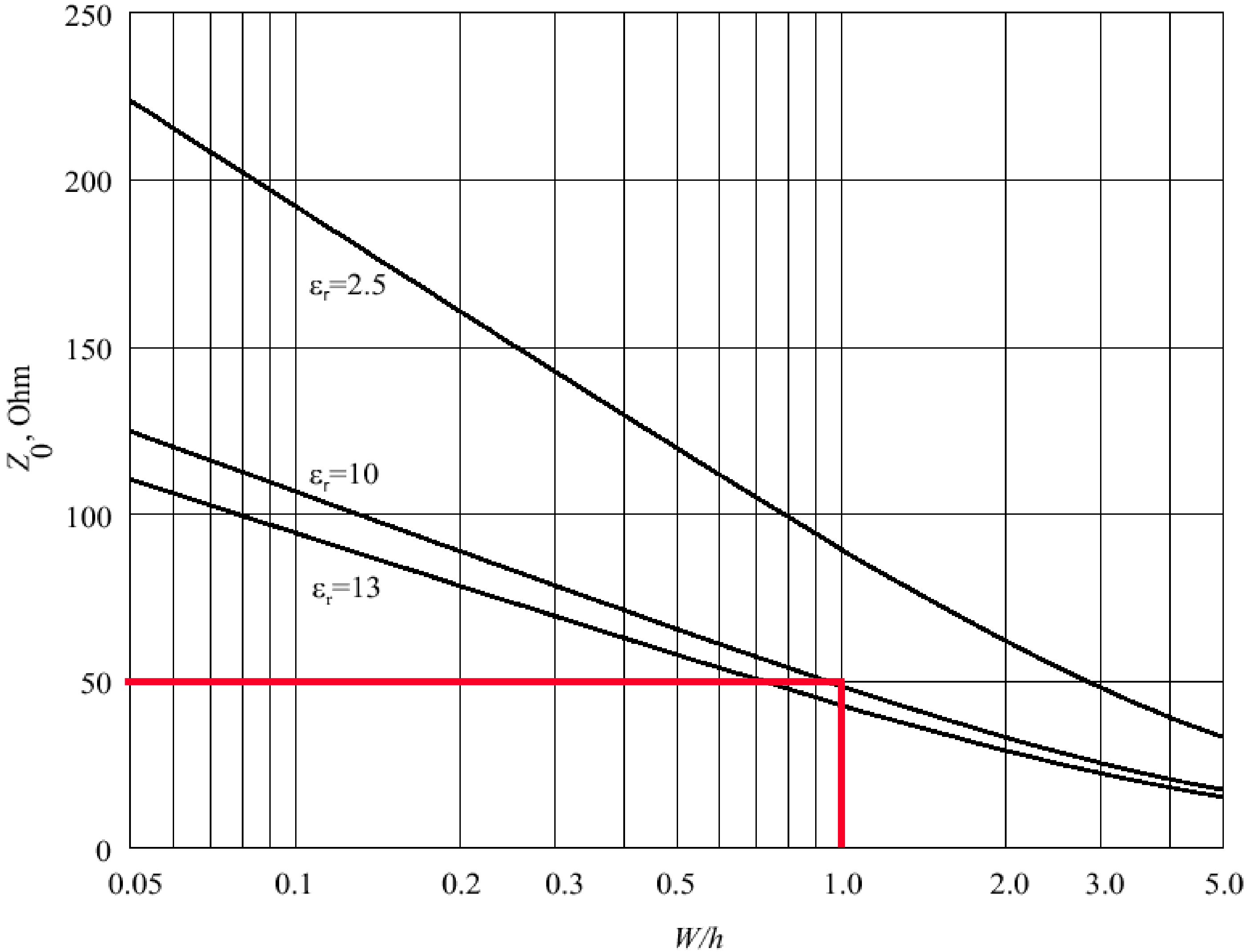
$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_{\mathrm{eff}}}} \log\left[\dfrac{8h}{W'} + \dfrac{W'}{4h}\right], & \dfrac{W'}{h} \le 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_{\mathrm{eff}}}} \left[\dfrac{W'}{h} + 1.393 + 0.667 \log\left(\dfrac{W'}{h} + 1.444\right)\right]^{-1}, & \dfrac{W'}{h} > 1 \end{cases}$$

$$\epsilon_{\mathrm{eff}} = \frac{1+\epsilon_r}{2} + \frac{\epsilon_r - 1}{2} F\left(\frac{W}{h}\right) - \frac{\epsilon_r - 1}{4.6} \frac{t}{h} \sqrt{\frac{h}{W}}$$

$$F\left(\frac{W}{h}\right) = \begin{cases} \left[1 + \dfrac{12h}{W}\right]^{-1/2} + 0.04\left[1 - \dfrac{W}{h}\right]^2, & \dfrac{W}{h} \le 1 \\ \left[1 + \dfrac{12h}{W}\right]^{-1/2} & \dfrac{W}{h} > 1 \end{cases}$$

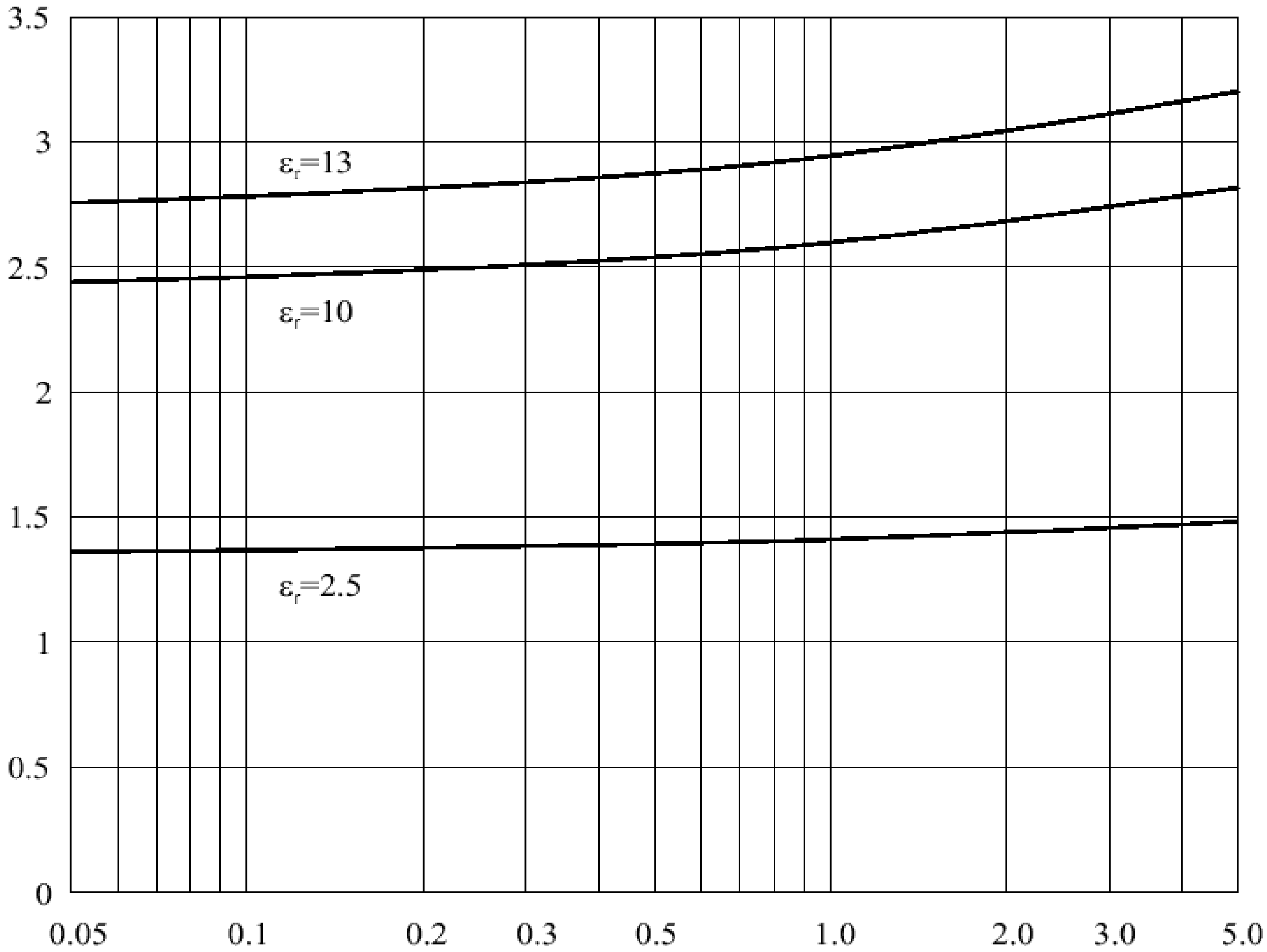
Off-the-shelf models in any CAD tool!

Microstrip impedance vs. *w/h*



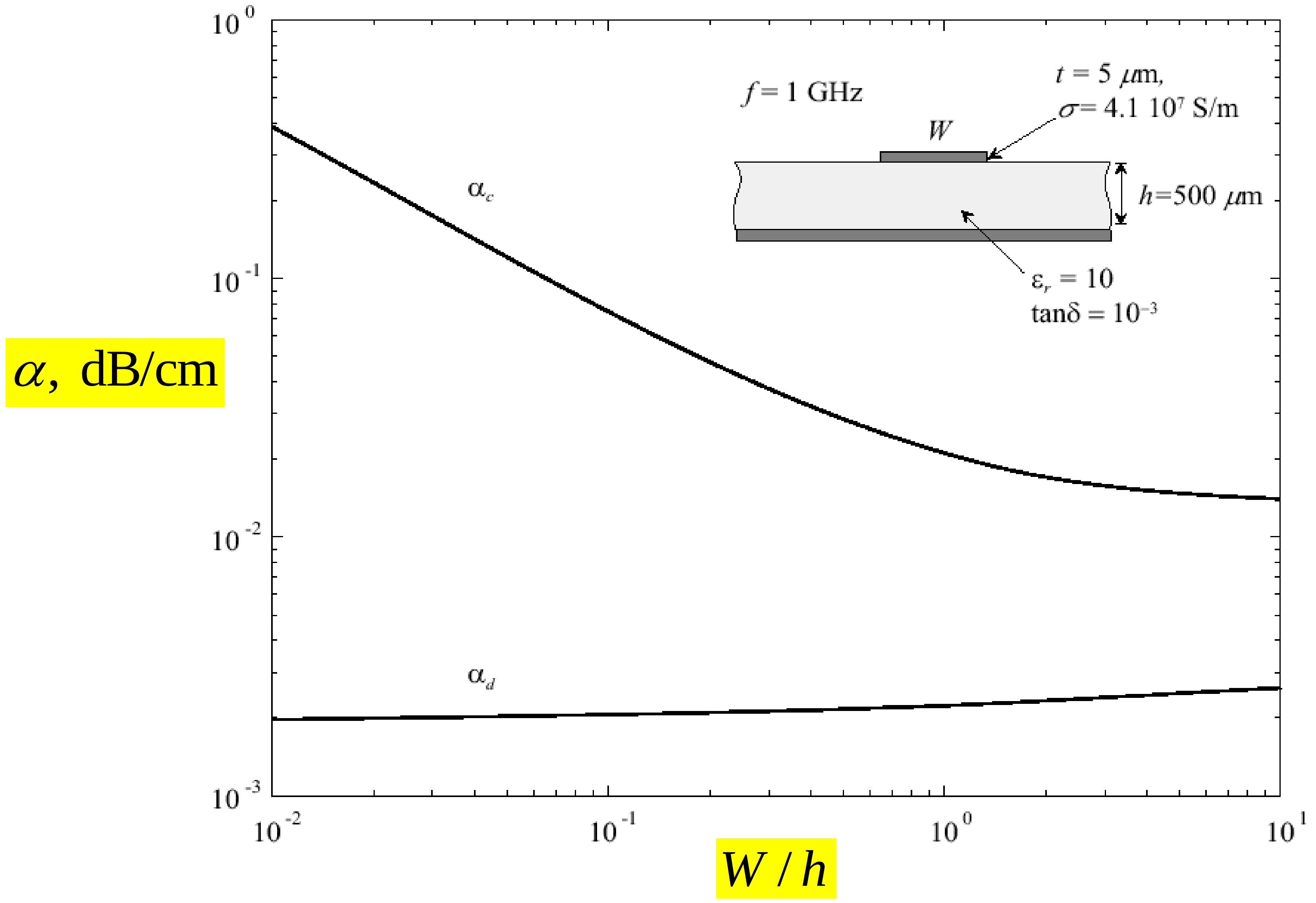
Microstrip effective refractive index vs. *w/h*

$$n_{eff} = \sqrt{\varepsilon_{eff}}$$



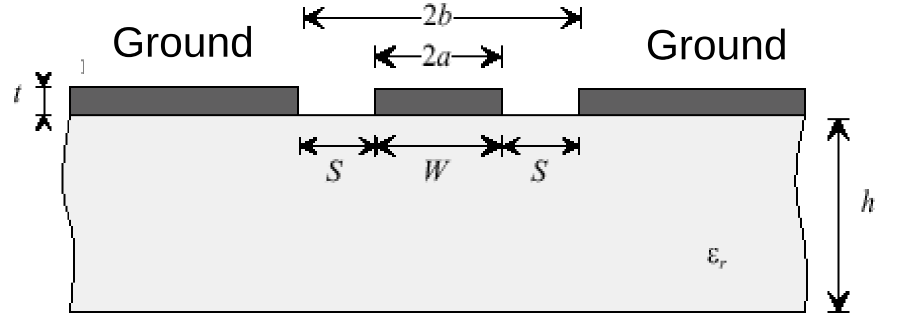
W / h

Microstrip attenuation vs. *w/h*



Planar transmission lines: the coplanar waveguide (CPW)

- For thick substrate, the impedance does not depend on the substrate thickness.
- Impedance depends on *w/s* → small and large lines can have the same impedance.
- Same losses as microstrip, layout less compact owing to ground planes; modal dispersion comparable to microstrip.



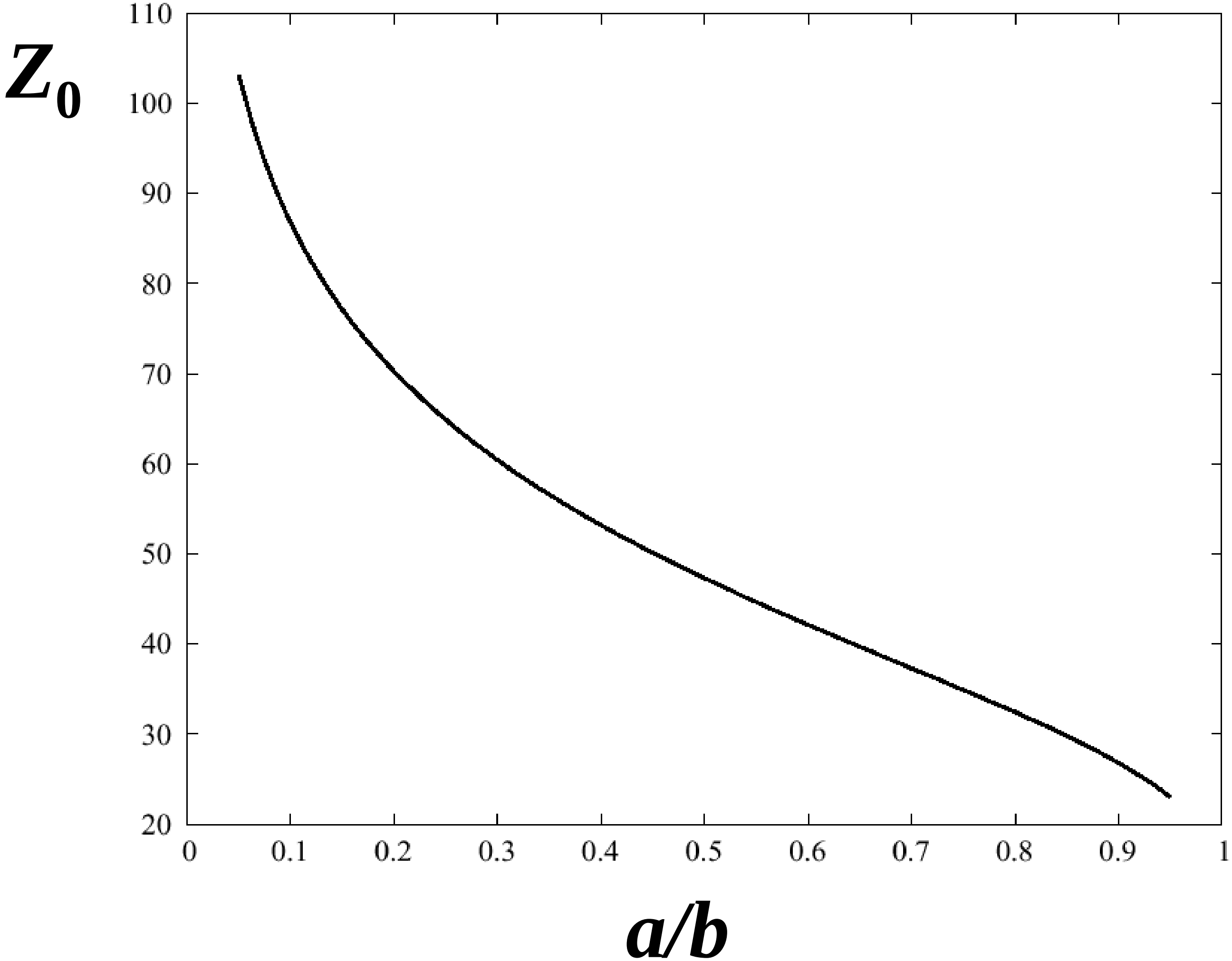
Coplanar waveguide (CPW)

Characteristic parameters for thick substrate

- Impedance $\Rightarrow$ $$Z_0 = \frac{30\pi}{\sqrt{\epsilon_{\text{eff}}}} \frac{K(k')}{K(k)}$$
- Effective permittivity $\Rightarrow$ $$\boxed{\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2}}$$
- Aspect ratio *k* $\Rightarrow$ $$k = a/b$$

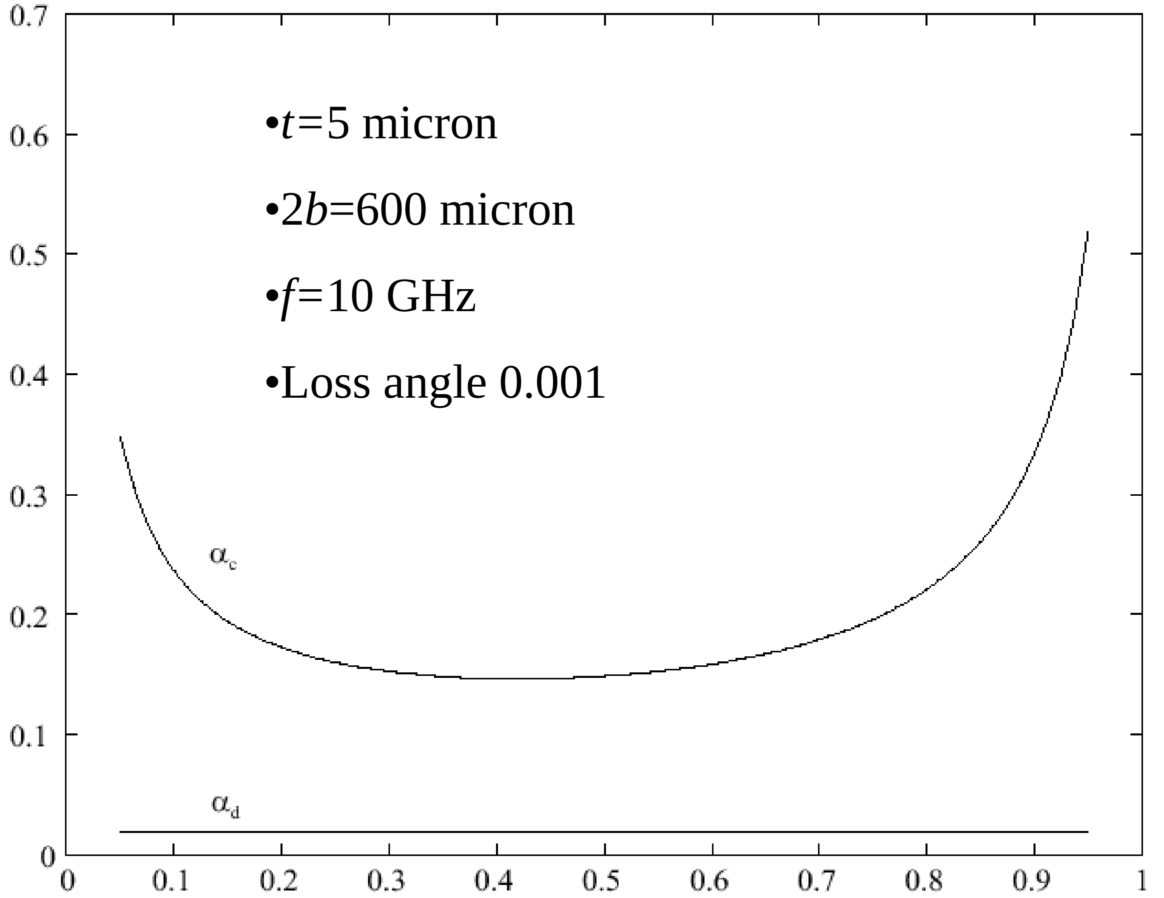
$$\boxed{\begin{aligned} \frac{K(k)}{K(k')} &\approx \frac{1}{\pi} \log\left(2\frac{1+\sqrt{k}}{1-\sqrt{k}}\right), \quad 0.5 \le k^2 < 1 \\ \frac{K(k')}{K(k)} &\approx \frac{1}{\pi} \log\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right), \quad 0 < k^2 \le 0.5 \end{aligned}}$$

Z_0 vs. *a/b* for CPW on allumina



Attenuation *vs. a/b*

α,
dB/cm



a/b