Noise modeling and low-noise amplifier design

Microwave Electronics

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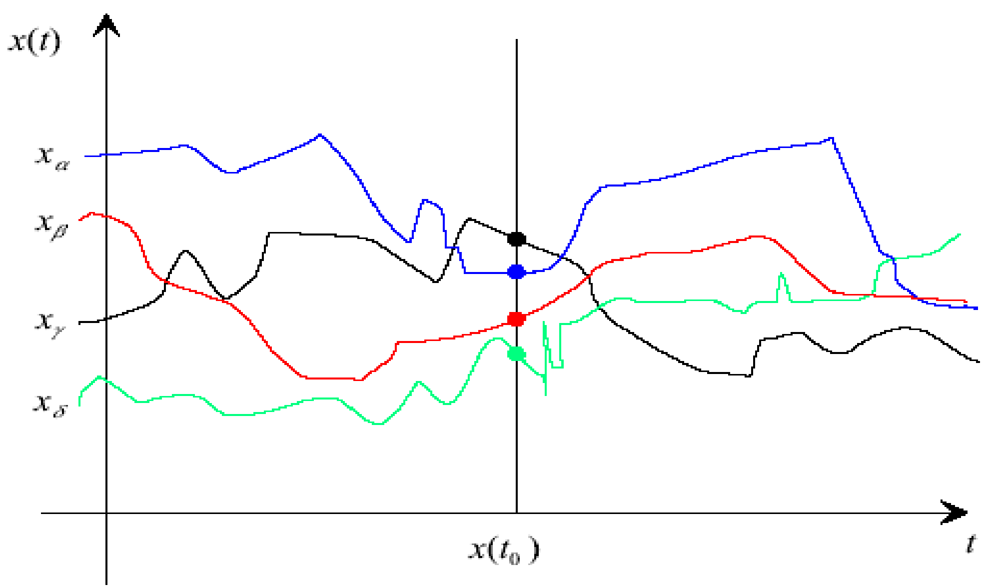
Noise is a stochastic (random) process

- In real world voltages and currents are affected by small-amplitude random fluctuations → the actual signal can be modeled as the ideal signal plus a stochastic process called *noise*
- Stochastic process $x(t)$ → a bundle of random functions of time (*realizations* of the process)



Characterizing noise

- In circuits we decompose "deterministic" voltages and currents from their "fluctuations"; fluctuations have by definition *zero average value*.
- V and I fluctuations → noise has zero mean but **nonzero mean square value** and **nonzero power spectrum** → ***power transfer*** within a circuit.
- Since noise is a (zero average) stochastic process it must be characterized statistically through concepts like:
  - **quadratic mean and mean square value**
  - **correlation and self-correlation**
  - **power and correlation spectra**

A reminder on random processes

- A (complex valued) random process $x(t)$ is a set of *process realizations* $x_k(t)$; we can define
  - ***time averages*** $\langle x_k(t)\rangle$ (random numbers obtained averaging each realization in time)
  - ***ensemble averages*** $E[x(t)]$ (a function of time obtained taking the statistical average of all realizations at a given time)
  - for an ***ergodic & stationary process*** time and ensemble averages coincide → this will be our case
- Mutual ($m \neq n$) and self ($m = n$) correlation:

$$R_{x_m x_n}(t,\tau) = \left\langle x_m(t)\, x_n^*(t+\tau)\right\rangle \underset{\substack{\text{stationary}\\ \text{process}}}{=} R_{x_m x_n}(\tau)$$

- Quadratic mean and root mean square (rms) value:

$$R_{xx}(0) = \left\langle x(t) x^*(t)\right\rangle = \left\langle |x(t)|^2 \right\rangle \equiv \overline{x^2}, \quad \boxed{x_{rms} = \sqrt{\overline{x^2}}}$$

Power and correlation spectra

- Stationary processes are better characterized in the spectral domain → correlation spectrum derived as the Fourier transform of the correlation function

$$S_{x_m x_n}(\omega)=2\int_{-\infty}^{+\infty} R_{x_m x_n}(\tau)e^{j\omega\tau}d\tau,\quad \omega\geq 0 \text{ (single-sided!)}$$

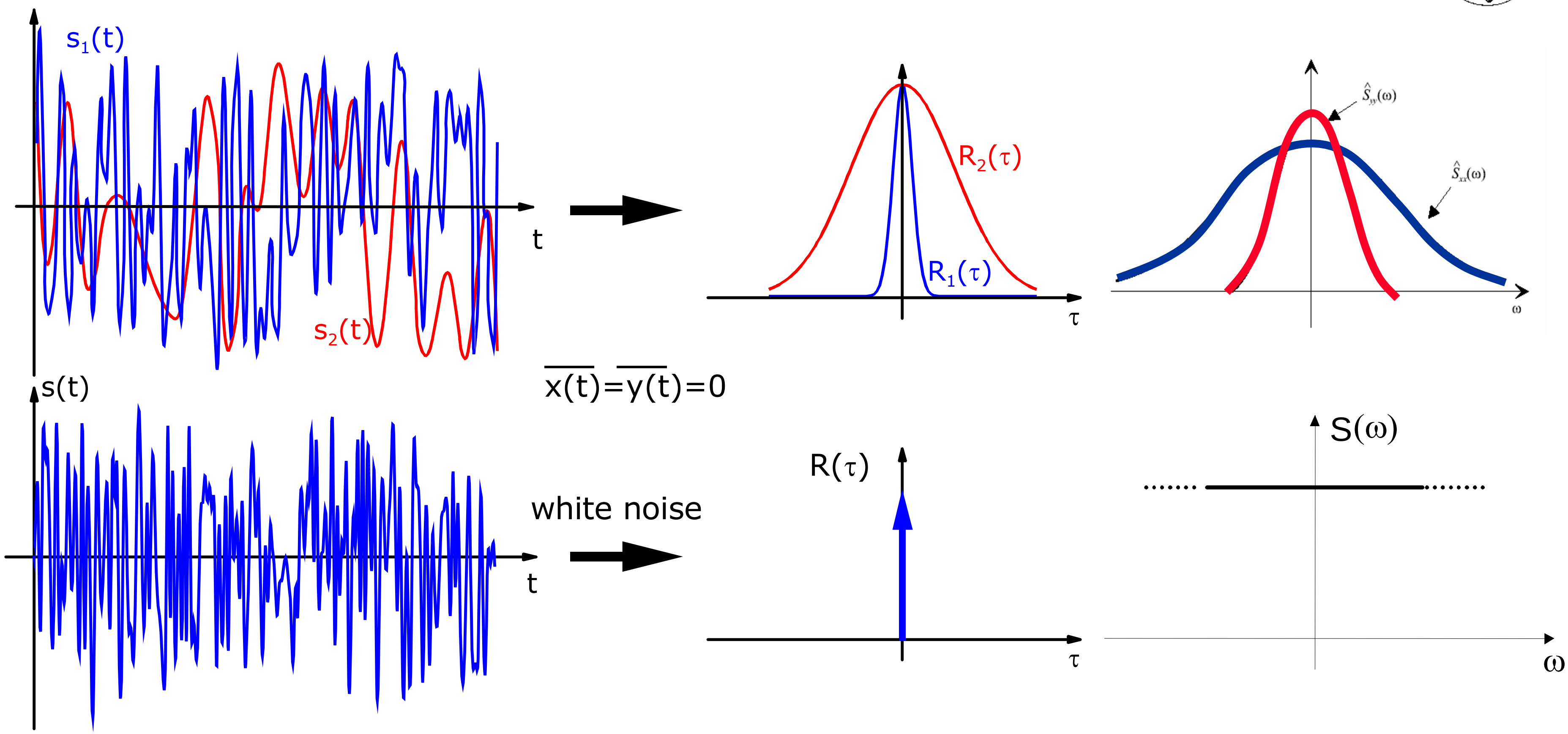
- Power spectrum (Fourier transform of self-correlation):

$$S_{xx}(\omega)\equiv S_x(\omega)=2\int_{-\infty}^{+\infty} R_{xx}(\tau)e^{j\omega\tau}d\tau,\quad \omega\geq 0;\qquad R_{xx}(\tau)=\frac{1}{2\pi}\int_{0}^{+\infty} S_{xx}(\omega)e^{-j\omega\tau}d\omega$$

- Spectral formulation of signal power:

$$P_{x,av}=\overline{x^2}=R_{xx}(0)=\frac{1}{2\pi}\int_0^{+\infty} S_{xx}(\omega)d\omega=\int_0^{+\infty} S_{xx}(f)df=\int_0^{+\infty} p_x(f)df$$

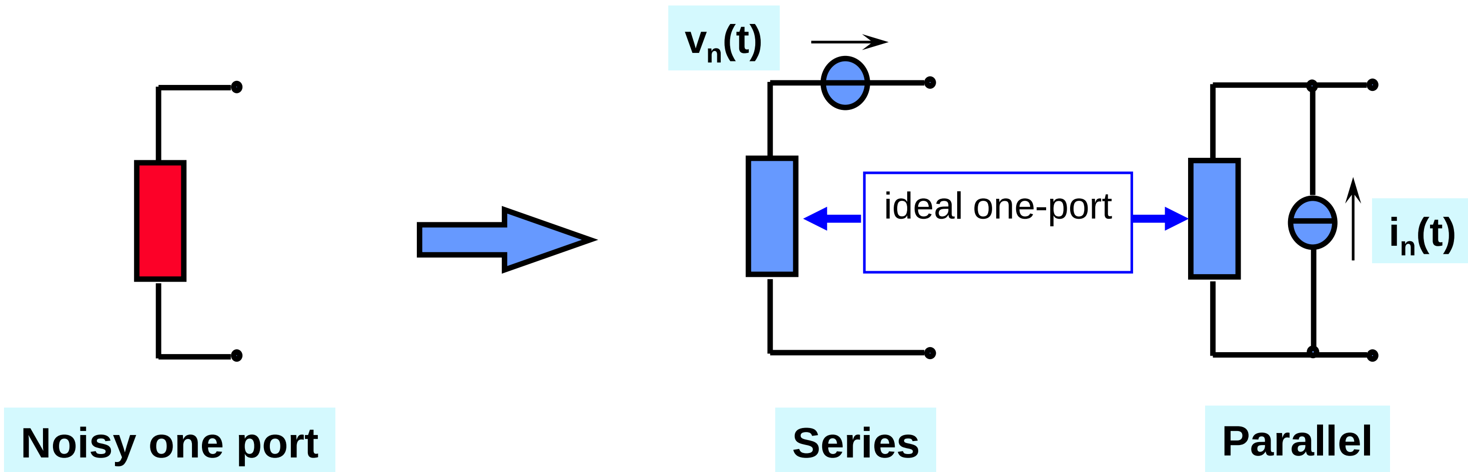
Noise spectral density and white noise



- White noise has (theoretically) a constant spectral density
- Coloured noise a low-pass spectral density

Equivalent circuit of a noisy one-port

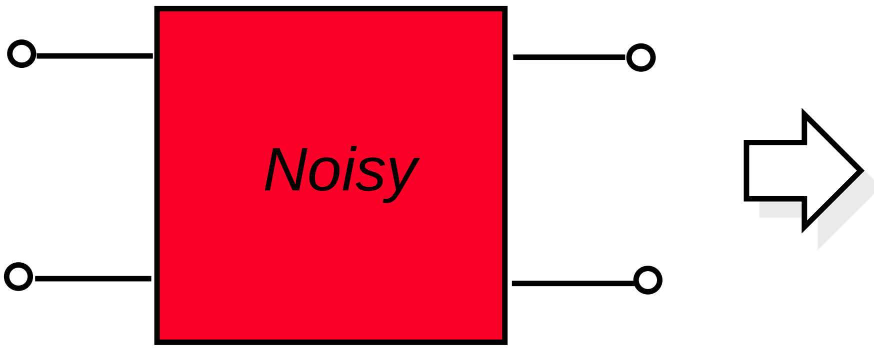
- A noisy one-port is a noiseless equivalent plus a random noise generator (series → Thevenin, parallel → Norton)

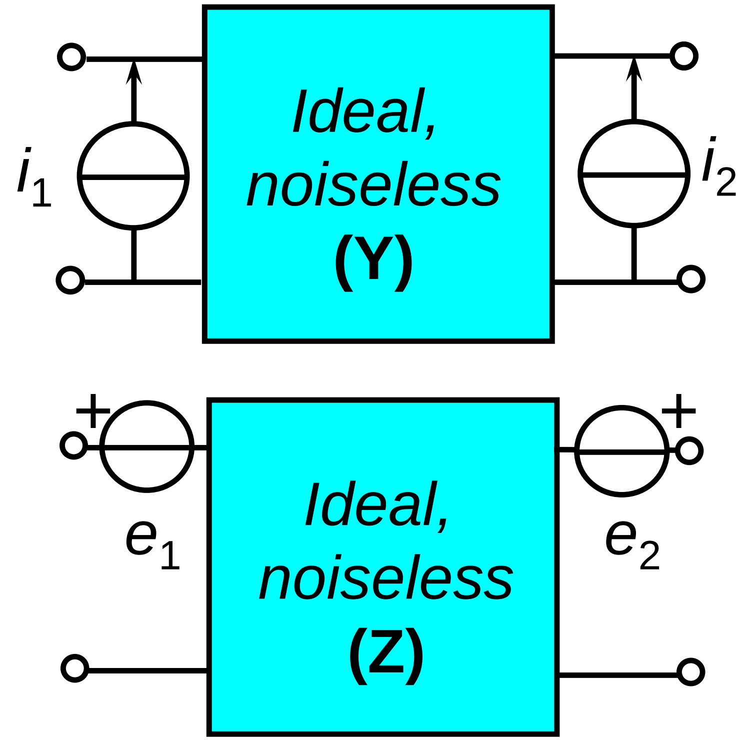


- Available noise power and noise generator spectra (V^2/Hz, A^2/Hz):

$$P_{n,av} = \frac{\overline{v_n^2}}{4R} = \int_0^{+\infty} \frac{S_{v_n}(f)}{4R} df = \frac{\overline{i_n^2}}{4G} = \int_0^{+\infty} \frac{S_{i_n}(f)}{4G} df = \int_0^{+\infty} p_n(f) df$$

Noisy two-ports





i_1, i_2 e_1,e_2 *random correlated sources*

$$S_{i_1}(\omega), \quad S_{i_1 i_2}(\omega), \quad S_{i_2}(\omega)$$

$$S_{e_1}(\omega), \quad S_{e_1 e_2}(\omega), \quad S_{e_2}(\omega)$$

Power spectra are **real**, correlation spectra **complex** → 4 real numbers each frequency

The cause of noise

- In a semiconductors noise is caused by *velocity fluctuations* and *population fluctuations* of carriers, e.g. in *n*-type conductor:

$$\underline{J}_n + \delta \underline{J}_n = q(n+\delta n)\left(\underline{v}_n + \delta \underline{v}_n\right) \approx qn\underline{v}_n + \underbrace{q\delta n \underline{v}_n}_{\text{population fluctuations}} + \underbrace{qn\delta \underline{v}_n}_{\text{velocity fluctuations}}$$

- Velocity fluctuations exist in any conductor → thermal noise (diffusion noise out of equilibrium) → white
- Population fluctuations (also connected with surfaces) are the main cause of 1/f and flicker noise → coloured
- Noise due to population fluctuations vanishes if no current (excess noise)
- In linear RF applications **white noise** dominates

Noise models: resistor (Nyquist law)

- In a resistor the power spectrum of (thermal) noise generators is given by Nyquist law (*T* temperature, k_B Boltzmann constant):

$$S_{v_n} = 4k_BTR \ \ \mathrm{V}^2/\mathrm{Hz}, \qquad S_{i_n} = 4k_BTG \ \ \mathrm{A}^2/\mathrm{Hz}$$

- Noise power spectral density and rms values:

$$p_n(f) = \frac{S_{v_n}(f)}{4R} = \frac{S_{i_n}(f)}{4G} = k_BT \ \ \mathrm{W/Hz}$$

$$v_{n,rms} = \sqrt{4k_BTRB}, \quad i_{n,rms} = \sqrt{4k_BTGB} \quad B \text{ system bandwidth}$$

- In a passive one-port one has the generalyzed Nyquist law:

$$S_{v_n} = 4k_BT\Re\left[Z(\omega)\right] \ \ \mathrm{V}^2/\mathrm{Hz}, \qquad S_{i_n} = 4k_BT\Re\left[Y(\omega)\right] \ \ \mathrm{A}^2/\mathrm{Hz}$$

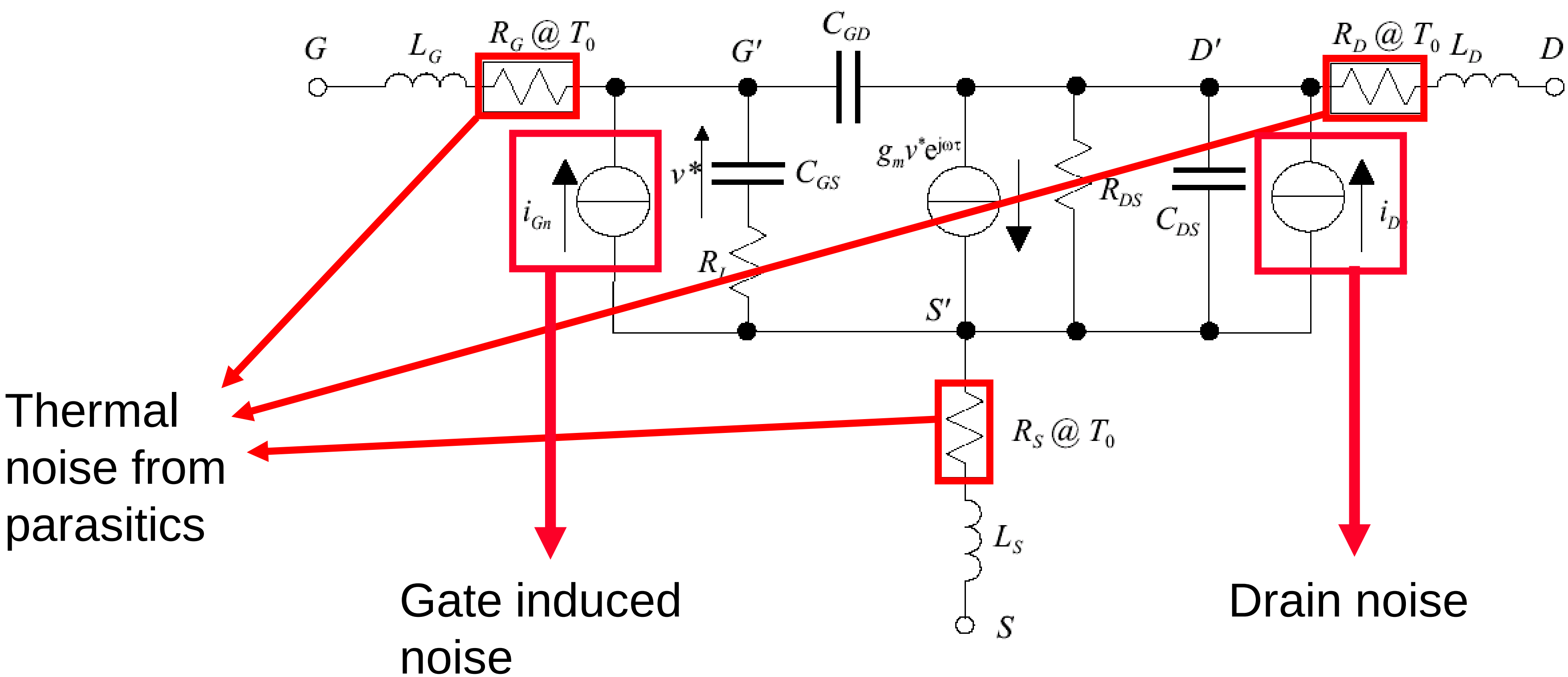
Semiconductor device noise models

- For passive networks the Nyquist law holds
- Junction devices (diodes, bipolar transistors) exhibit *shot noise,* e.g. for a diode the short-circuit noise current is:

$$S_i = 2q\left(I_D + 2I_0\right) \approx 2qI_D$$

- In FET devices noise is thermal or diffusion, but no exact theoretical model exists relating noise generators to small-signal parameters; however, compact noise FET models synthesize the noise generators with a low number of parameters (e.g. the PRC model)

FET noise model (PRC)



$$S_{i_{Dn}i_{Dn}} \approx 4k_B T_0 g_m \boxed{P}$$

γ (~2/3)

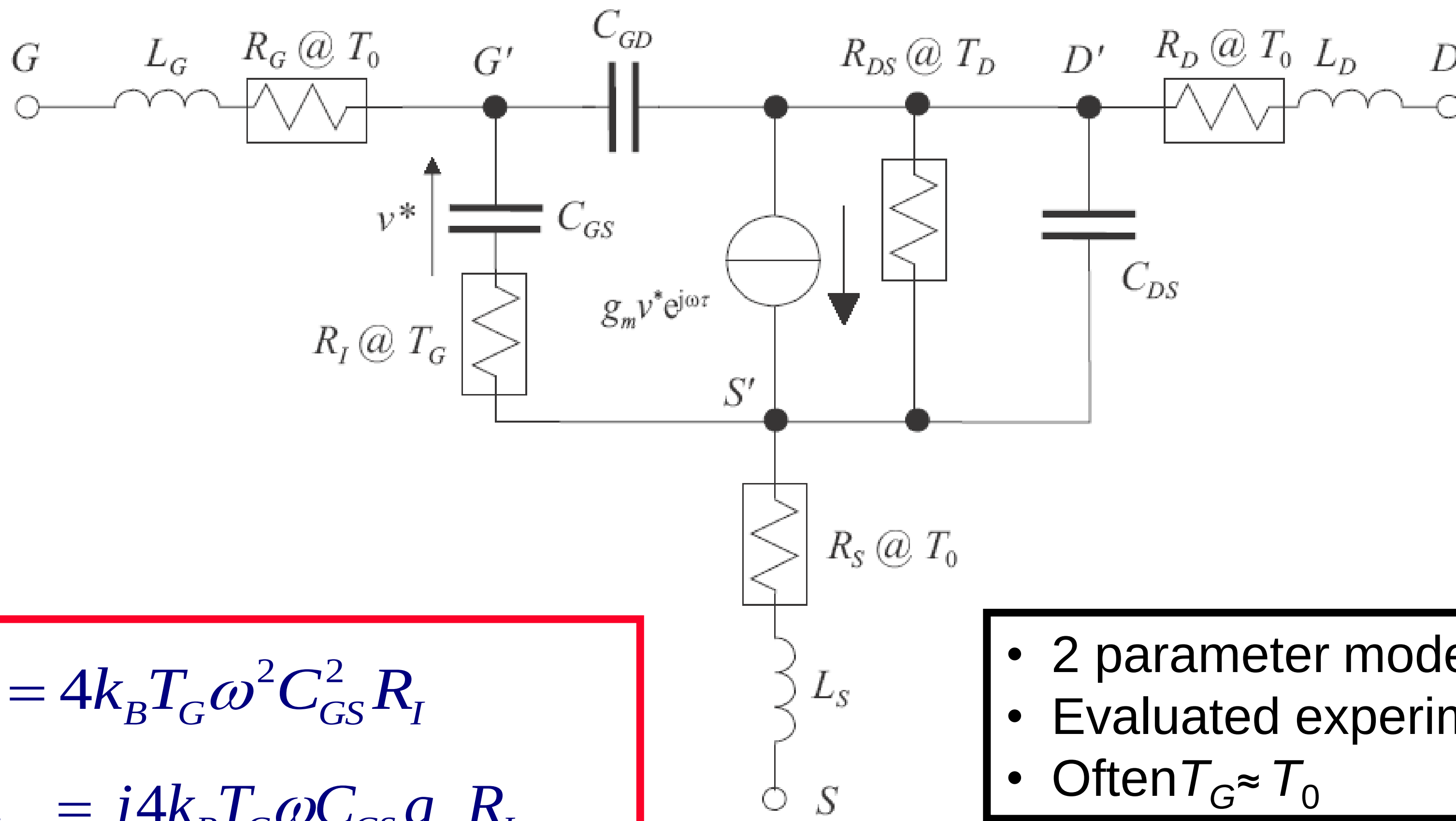
$$S_{i_{Gn}i_{Gn}} \approx 4k_B T_0 \frac{\omega^2 C_{GS}^2}{g_m} \boxed{R}$$

~β,δ ~ 0.12

$$S_{i_{Gn}i_{Dn}} \approx \boxed{jC} \sqrt{S_{i_{Dn}i_{Dn}} S_{i_{Gn}i_{Gn}}}$$

Correlation

Two-temperature noise model



$$S_{i_{nG}} = 4k_B T_G \omega^2 C_{GS}^2 R_I$$

$$S_{i_{nG},i_{nD}} = j4k_B T_G \omega C_{GS} g_m R_I$$

$$S_{i_{nD}} \approx 4k_B \left(T_G\, g_m^2 R_I + T_D G_{DS} \right)$$

- 2 parameter model
- Evaluated experimentally
- Often $T_G \approx T_0$

1 parameter model

Solving circuits with noise generators - I

- Suppose a linear circuit includes a set of (correlated and/or uncorrelated) noise generators, how can we compute the total noise power on a load or the power spectrum of a given voltage or current?
- We have better work in the frequency domain associating to each process $x(t)$ its "Fourier transform" or "associated phasor" $X(\omega)$ and exploiting the *symbolic definitions*:

$$S_{xx}(\omega) = \overline{XX^*}, \quad S_{xy}(\omega) = \overline{XY^*} = S^*_{yx}(\omega)$$

- This definition is consistent with the *transformation rules* of the power spectrum of a random process through a linear system:

$$Y(\omega) = H(\omega)X(\omega); \quad S_{yy} = \overline{YY^*} = \overline{HXH^*X^*} = HH^*\overline{XX^*} = |H|^2 S_{xx}$$

Solving circuits with noise generators - II

- Procedure:
  - Associate to each random voltage or current generator a voltage or current phasor
  - The circuit is solved in frequency domain with the phasor technique
  - Power and correlation spectra are evaluated through the simbolic definitions as a function of the (supposedly known) power and correlation spectra of the noise sources
  - The average power exchanged are recovered by integrating the power densities on the system bandwidth (often to be specified separately → remember the presence of bandpass filters!)

Example

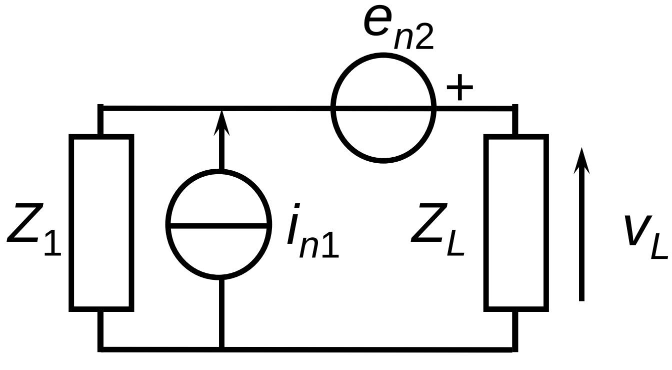
- Association random process & phasor $e_{n2} \leftrightarrow E_{n2}$ e $i_{n1} \leftrightarrow I_{n1}, v_L \leftrightarrow V_L$;
- The power spectra $S_{e_{n2}}, S_{i_{n1}}, S_{e_{n2} i_{n1}}$ are known
- From the circuit:

$$V_L = \frac{\left(I_{n1} Z_1 + E_{n2}\right) Z_L}{Z_1 + Z_L} \rightarrow S_{v_L} = \overline{V_L V_L^*} = \frac{\overline{\left(I_{n1} Z_1 + E_{n2}\right)\left(I_{n1} Z_1 + E_{n2}\right)^*} \left|Z_L\right|^2}{\left|Z_1 + Z_L\right|^2}$$

- i.e.:

$$S_{v_L} = \overline{V_L V_L^*} = \frac{\left(\overline{I_{n1} I_{n1}^*} \left|Z_1\right|^2 + \overline{E_{n2} E_{n2}^*} + Z_1 \overline{I_{n1} E_{n2}^*} + Z_1^* \overline{E_{n2} I_{n1}^*}\right) \left|Z_L\right|^2}{\left|Z_1 + Z_L\right|^2} =$$

$$= \frac{\left|Z_L\right|^2}{\left|Z_1 + Z_L\right|^2} \left(\left|Z_1\right|^2 S_{i_{n1}} + S_{e_{n2}} + Z_1 S_{e_{n2} i_{n1}}^* + Z_1^* S_{e_{n2} i_{n1}}\right)$$



- To evaluate the total noise power one must integrate on the system bandwidth

The two-port noise figure

- From a system standpoint a commonly used amplifier noise figure of merit is the *noise figure* NF:

$$\mathrm{NF}=\frac{p_{nav,L}}{p'_{nav,L}}=\frac{\textbf{Noise available power spectral density on load}}{\textbf{Noise available power spectral density on load with noiseless two-port}}$$

- In the standard NF definition the input noise is thermal noise at T_0 (conventionally ~290 K, at that temperature kT=25 meV)
- This definition as the ratio of power spectral densities is called the **spot noise figure**; we can also define NF considering power and noise on a certain system bandwidth:

$$\mathrm{NF}=\frac{\int_{f_0-B/2}^{f_0+B/2} p_{nav,L}(f)\,df}{\int_{f_0-B/2}^{f_0+B/2} p'_{nav,L}(f)\,df}=\frac{P_{nav,L}}{P'_{nav,L}}$$

System definition of noise figure

- The noise figure can be also shown to admit the alternative system definition involving the input and output ***signal over noise ratios***:

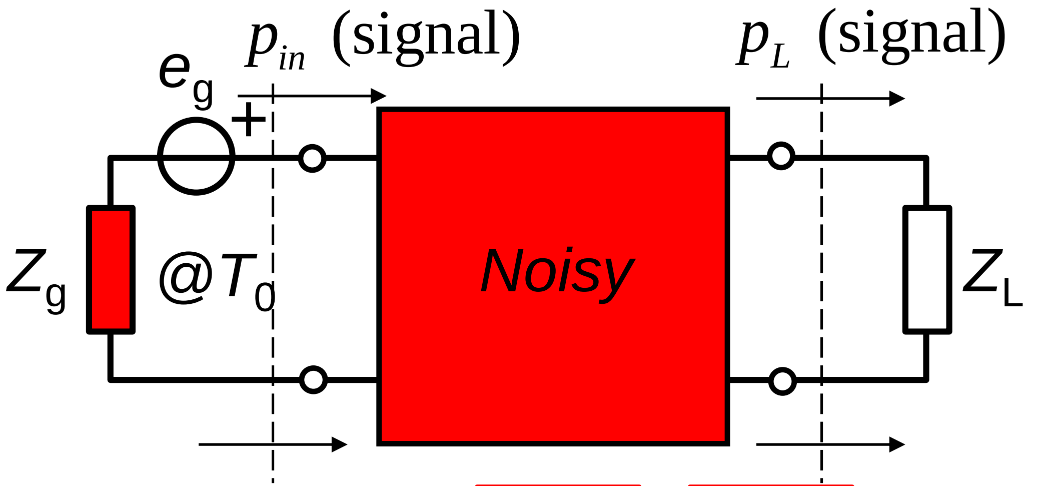
$$\boxed{\mathrm{NF} = \frac{(S/N)_{in}}{(S/N)_L}}$$

- In fact:

$$\mathrm{NF} = \frac{P_{nav,L}}{P'_{nav,L}} \equiv \frac{N_L}{N_{in} G_{av}} = \frac{N_L S_{in}}{N_{in} G_{av} S_{in}} =$$

$$= \frac{N_L S_{in}}{N_{in} S_L} = \frac{(S/N)_{in}}{(S/N)_L}$$

Spot noise figure in terms of (S/N)



$p'_{nav,in}$

$$\boxed{p_{nav,L}} = \boxed{p'_{nav,L}} + p''_{nav,L}$$

Total output noise available power

Output noise available power generated from generator noise available power

- All powers (signal and noise) are available powers
- Definition of the output available noise power due to generator:

$$p'_{nav,L} = G_{av} p'_{nav,in}$$

$$\mathrm{NF} = \frac{p_{nav,L}}{p'_{nav,L}} = \frac{p_{nav,L}}{p'_{nav,in} G_{av}} = \frac{p_{nav,L} \, p_{in}}{p'_{nav,in} G_{av} \, p_{in}} =$$

$$= \frac{p_{nav,L} \, p_{in}}{p'_{nav,in} \, p_L} = \frac{p_{in}}{p'_{nav,in}} \times \frac{p_{nav,L}}{p_L} = \frac{(S/N)_{in}}{(S/N)_L}$$

Noise temperature

- In a noiseless two-port *NF*=1 (e.g.: reactive two-port)
- If $p''_{nav,L}$ is the noise available power density on the load *because of the effect of the noisy two port only:*

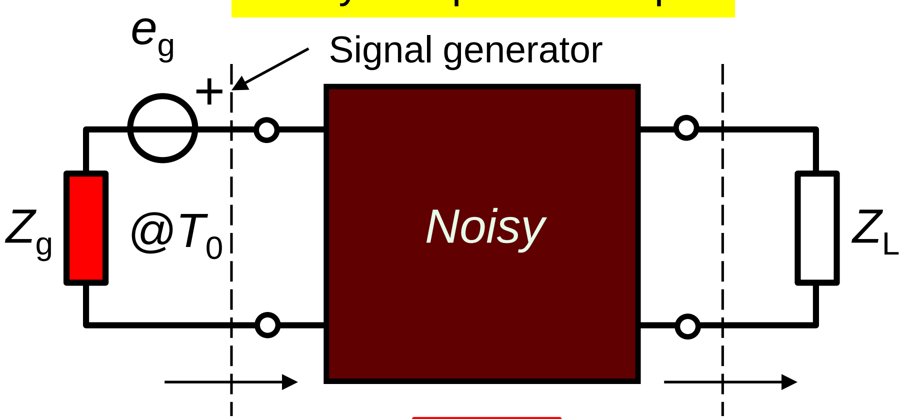
$$\boxed{\mathrm{NF} = 1 + \frac{p''_{nav,L}}{p'_{nav,L}}} \Rightarrow \boxed{\mathrm{NF} = 1 + \frac{G_{av}\, p''_{nav,in}}{G_{av}\, p'_{nav,in}} = 1 + \frac{p''_{nav,in}}{k_{\mathrm{B}} T_0}}$$

- Supposing that the input referred noise available power density $p''_{nav,in}$ be expressed as: $p''_{nav,in} = k_{\mathrm{B}} T_n$

$$\boxed{T_n = T_0 \left(\mathrm{NF} - 1 \right)} \leftarrow \text{Noise temperature of two-port}$$

Noise temperature

Noisy two-port and input



$p'_{nav,in}$

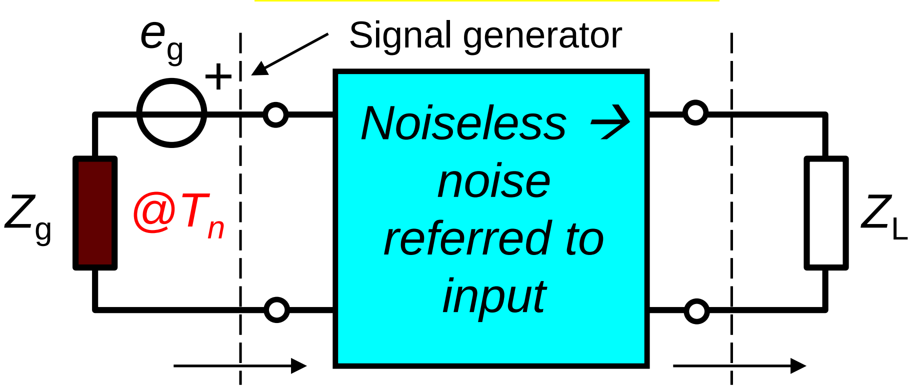
$\boxed{p_{nav,L}} = p'_{nav,L} + p''_{nav,L}$

$$\mathrm{NF} = \frac{p_{nav,L}}{p'_{nav,L}} = \frac{p'_{nav,L} + p''_{nav,L}}{p'_{nav,L}} =$$

$$= 1 + \frac{p''_{nav,L}}{p'_{nav,L}} = 1 + \frac{p''_{nav,L}/G_{av}}{p'_{nav,L}/G_{av}} =$$

$$= 1 + \frac{p''_{nav,in}}{p'_{nav,in}} = 1 + \frac{k_B T_n}{k_B T_0} = 1 + \frac{T_n}{T_0}$$

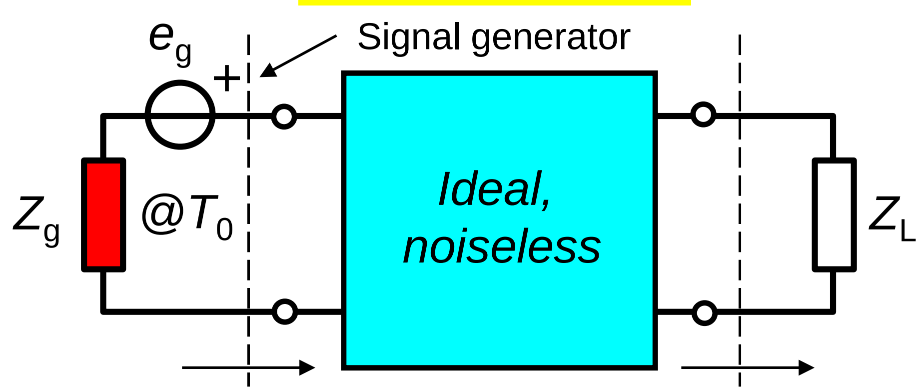
Noise referred to input



$p''_{nav,in}$

$$p''_{nav,L} = G_{av} p''_{nav,in} = = G_{av} k_B T_n$$

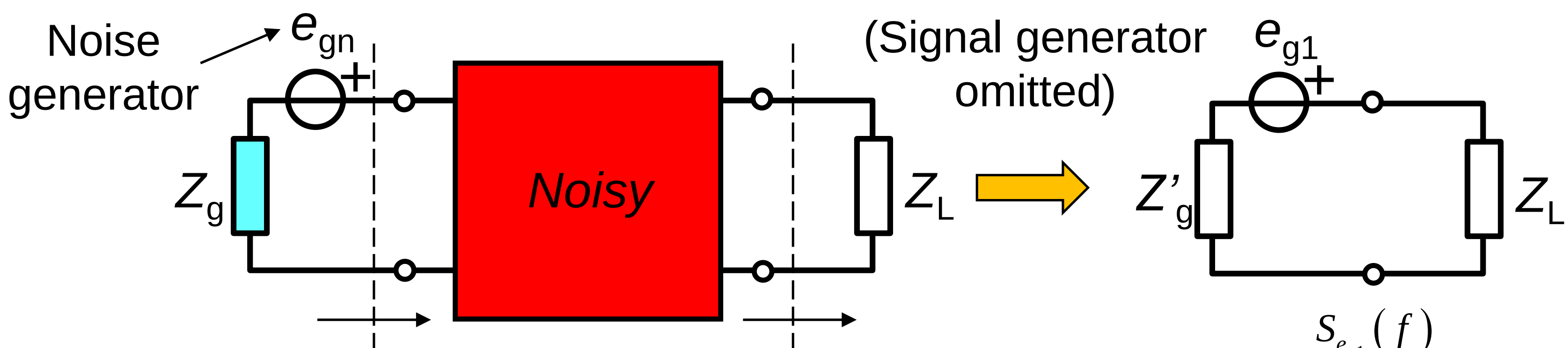
Noiseless two-port



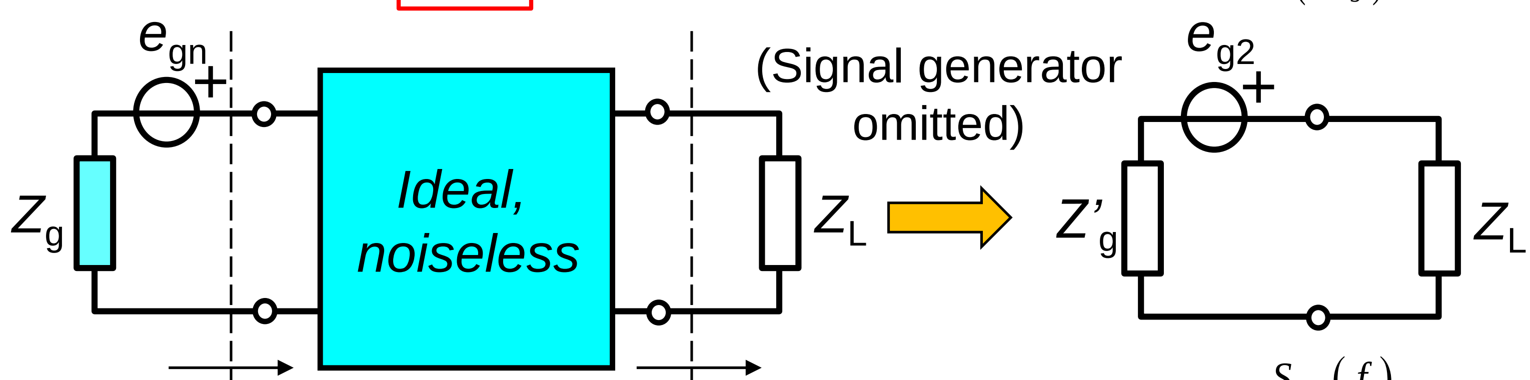
$p'_{nav,in}$

$$p'_{nav,L} = G_{av} p'_{nav,in} = = G_{av} k_B T_0$$

Noise figure in terms of I/V spectra - I



$p'_{nav,in}$ $\qquad$ $\boxed{p_{nav,L}} = p'_{nav,L} + p''_{nav,L}$ $\qquad$ $p_{nav,L} = \dfrac{S_{e_{g1}}(f)}{4\mathrm{Re}(Z'_g)}$

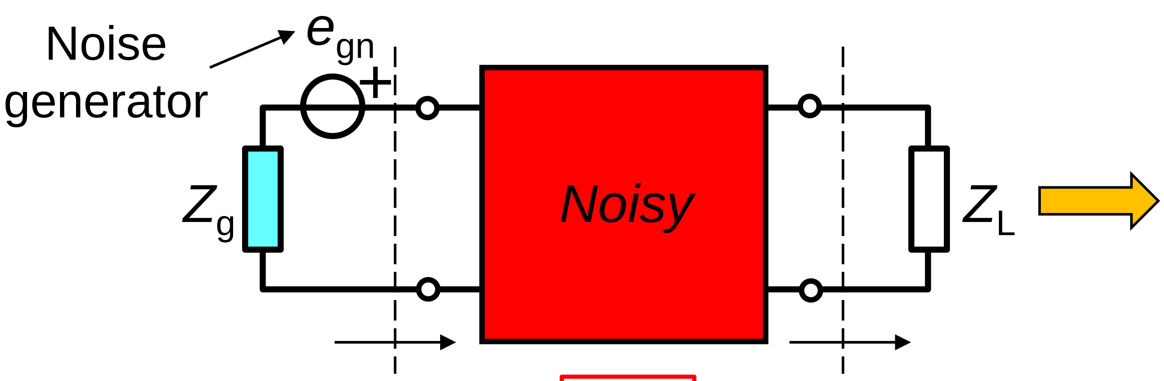


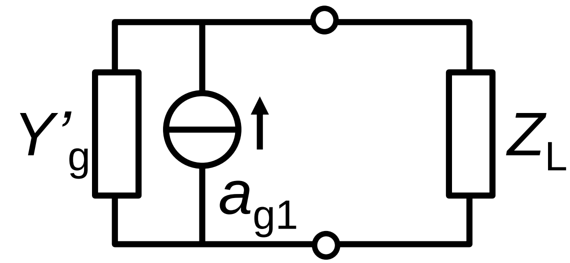
$p'_{nav,in}$ $\qquad$ $p_{nav,L} = \boxed{p'_{nav,L}} = G_{av} p'_{nav,in}$ $\qquad$ $p'_{nav,L} = \dfrac{S_{e_{g2}}(f)}{4\mathrm{Re}(Z'_g)}$

$$\mathrm{NF} = \frac{p_{nav,L}}{p'_{nav,L}} = \frac{S_{e_{g1}}(f)/4\mathrm{Re}(Z'_g)}{S_{e_{g2}}(f)/4\mathrm{Re}(Z'_g)} = \frac{S_{e_{g1}}(f)}{S_{e_{g2}}(f)}$$

Ratio of open-circuit output voltage spectra

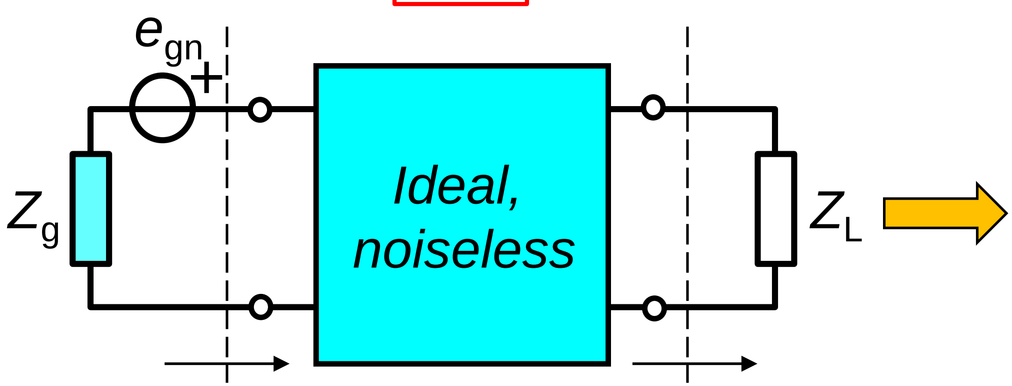
Noise figure in terms of I/V spectra - I

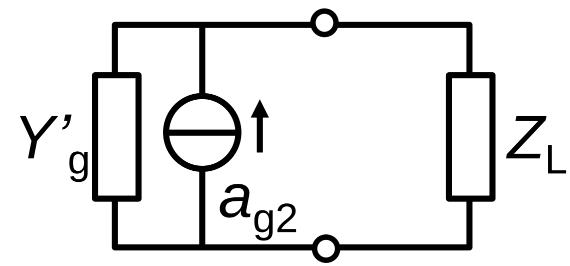




$$p'_{nav,in} \qquad \boxed{p_{nav,L}} = p'_{nav,L} + p''_{nav,L}$$

$$p_{nav,L} = \frac{S_{a_{g1}}(f)}{4\,\mathrm{Re}(Y'_g)}$$





$$p'_{nav,in} \qquad p_{nav,L} = \boxed{p'_{nav,L}} = G_{av}\,p'_{nav,in}$$

$$p_{nav,L} = \frac{S_{a_{g2}}(f)}{4\,\mathrm{Re}(Y'_g)}$$

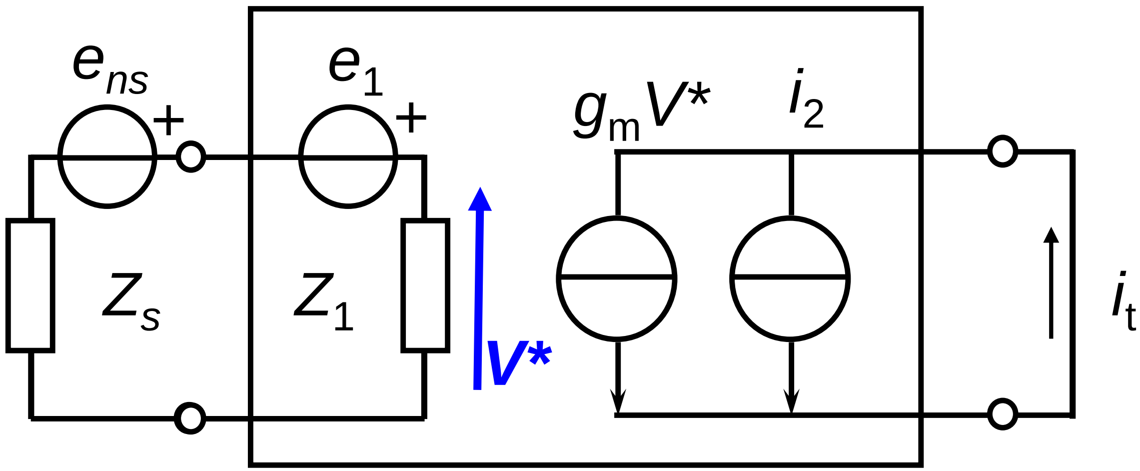
$$\mathrm{NF} = \frac{p_{nav,L}}{p'_{nav,L}} = \frac{S_{a_{g1}}(f)/4\,\mathrm{Re}(Y'_g)}{S_{a_{g2}}(f)/4\,\mathrm{Re}(Y'_g)} = \frac{S_{a_{g1}}(f)}{S_{a_{g2}}(f)}$$

Ratio of short-circuit output current spectra

Looking for the minimum noise figure

- The noise figure of a two-port is a ratio of available powers at port 2 → it only depends on the two port SS and noise parameters and on the source impedance
- It can be shown that an optimum source impedance exists which **minimizes the noise figure** (but of course does not maximize the available power gain!)
- The sensitivity of the NF with respect to the minimum is a further noise parameter (see later)
- A simplified example of the minimization process is presented in the next slides

Example of minimum NF - I



All noise sources are <u>uncorrelated</u>

$$I_t = g_m v^* + I_2 = (E_{ns} + E_1)\frac{g_m Z_1}{Z_s + Z_1} + I_2$$

$$S_{i_t} = \overline{I_t I_t^*} = (\overline{E_{ns}E_{ns}^*} + \overline{E_1 E_1^*})\frac{g_m^2 |Z_1|^2}{|Z_s + Z_1|^2} + \overline{I_2 I_2^*}$$

$$\overline{E_{ns}E_{ns}^*} = 4k_B T_0 R_s$$

$$\overline{E_1 E_1^*} = 4k_B T_0 R_n$$

$$\overline{I_2 I_2^*} = 4k_B T_0 G_n$$

Nyquist for resistor, pseudo-Nyquist for 2-port sources

Example of minimum NF - II

NF as the ratio of short-circuit current power spectra

$$\text{NF} = \frac{(4k_BT_0R_s + 4k_BT_0R_n)\frac{g_m^2|Z_1|^2}{|Z_s+Z_1|^2} + 4k_BT_0G_n}{4k_BT_0R_s\frac{g_m^2|Z_1|^2}{|Z_s+Z_1|^2}} =$$

$$= 1 + \frac{R_n}{R_s} + \frac{|Z_s+Z_1|^2}{g_m^2|Z_1|^2}\frac{G_n}{R_s} =$$

$$= 1 + \frac{R_n}{R_s} + \frac{(R_s+R_1)^2 + (X_s+X_1)^2}{g_m^2|Z_1|^2}\frac{G_n}{R_s}$$

optimum source reactance

$$\boxed{X_{so} = -X_1}$$

Resulting NF:

$$\text{NF} = 1 + \frac{R_n}{R_s} + \frac{(R_s+R_1)^2}{g_m^2|Z_1|^2}\frac{G_n}{R_s} =$$

$$= 1 + \frac{R_n}{R_s} + \frac{R_1^2G_n}{R_sg_m^2|Z_1|^2} + \frac{2R_1G_n}{g_m^2|Z_1|^2} + \frac{G_nR_s}{g_m^2|Z_1|^2}$$

Example of minimum NF - III

Function:

$$\text{NF} = 1 + \frac{a}{R_s} + b + cR_s$$

$$\frac{d\text{NF}}{dR_s} = -\frac{a}{R_s^2} + c = 0 \rightarrow R_{so} = \sqrt{\frac{a}{c}}$$

$$\text{NF}_{\min} = 1 + 2\sqrt{ac} + b$$

Introducing values:

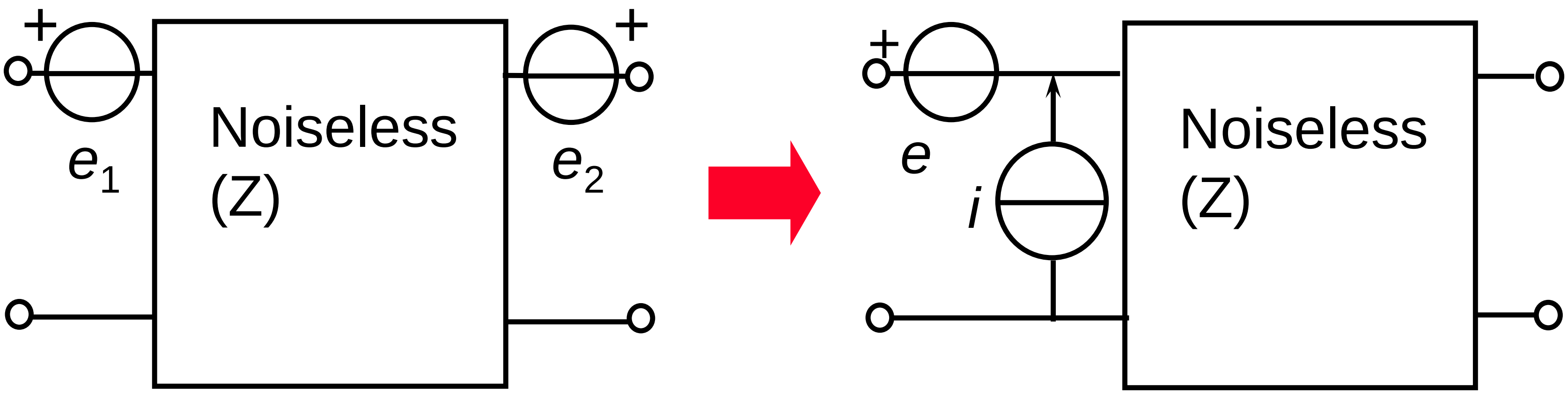
$$\text{NF}_{\min} = 1 + 2\sqrt{\left(R_n + \frac{R_1^2 G_n}{g_m^2|Z_1|^2}\right)\frac{G_n}{g_m^2|Z_1|^2}} + \frac{2R_1 G_n}{g_m^2|Z_1|^2}$$

$$R_{so} = \sqrt{\frac{R_n + \frac{R_1^2 G_n}{g_m^2|Z_1|^2}}{\frac{G_n}{g_m^2|Z_1|^2}}} = \sqrt{R_1^2 + \frac{g_m^2|Z_1|^2 R_n}{G_n}}$$

- if the device input is capacitive (FETs) the minimum noise figure increases with *f*, if it has a resistive component (BJT) with *f2*

Evaluating and minimizing NF: how to do it

- Referring noise generators to the input:



$$S_{e_1 e_1}(\omega), S_{e_1 e_2}(\omega), S_{e_2 e_2}(\omega)$$

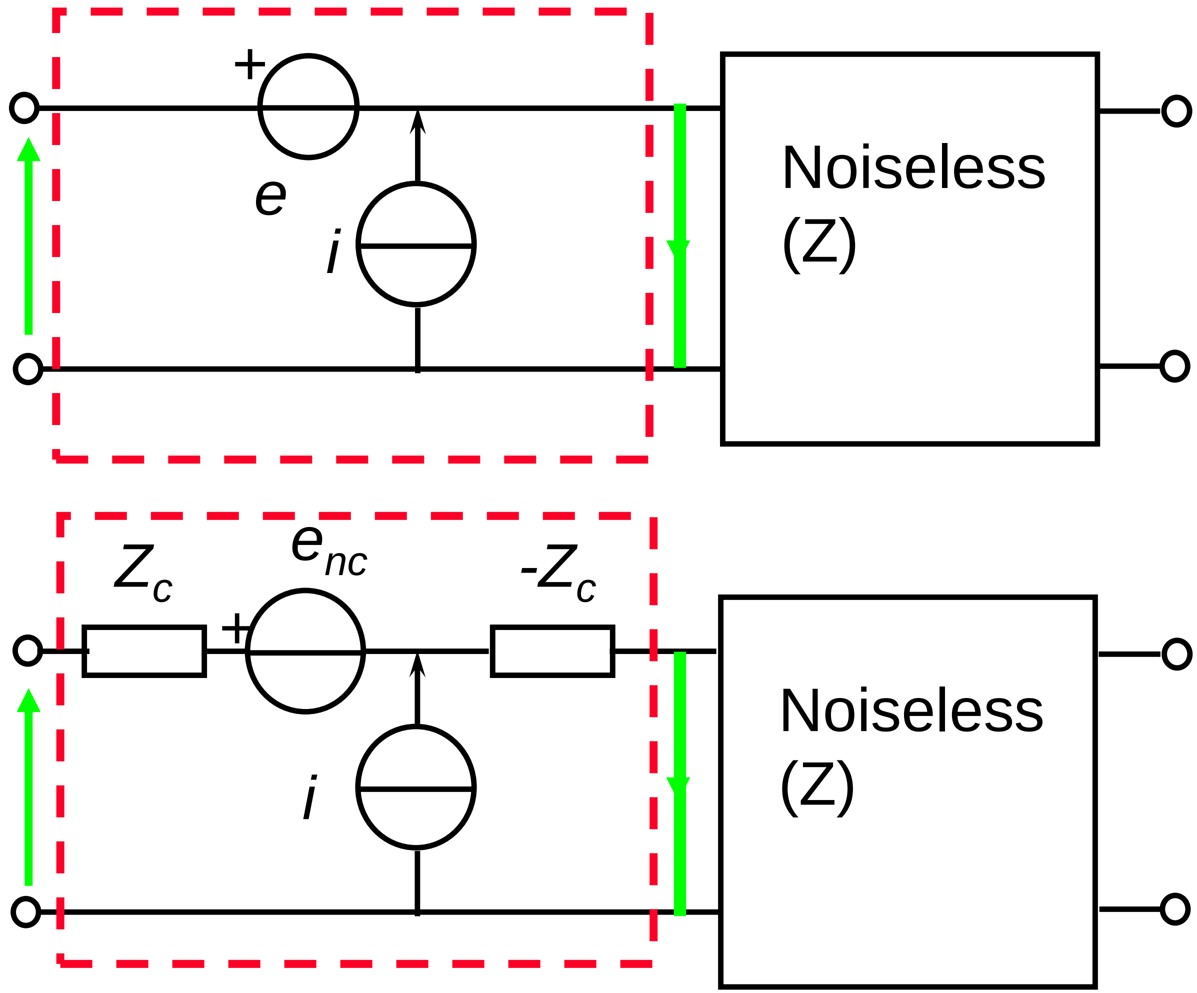
$$S_{ee}(\omega), S_{ei}(\omega), S_{ii}(\omega)$$

- By imposing the equivalence of the two representations we obtain the correlation matrix of the two generators *i* and *e.*

Representation with input uncorrelated generators

- It is expedient to introduce instead of *e* and *i* (correlated) a voltage generator uncorrelated from *I*
- The equivalence is obtained by imposing that the short-circuit current and open circuit-voltage of the two sets is the same
- Imposing that e_{nc} and *i* be uncorrelated we obtain:

Z_c correlation impedance



Power spectra of uncorrelated generators

- The power spectra of the two uncorrelated generators can be for brevity expressed introducing the **series noise resistance and conductance**; the info on correlation is in Z_c

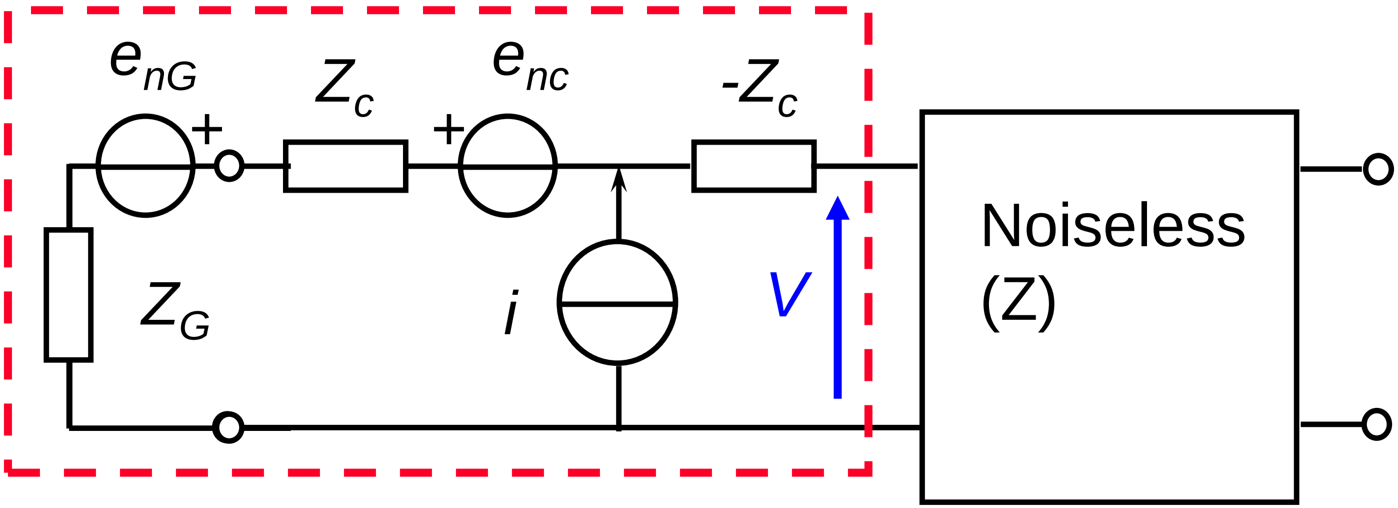
$$\boxed{S_{e_{nc},e_{nc}} = 4k_{\mathrm{B}}T_0 r_n} \qquad \boxed{S_{i,i} = 4k_{\mathrm{B}}T_0 g_n} \qquad \boxed{Z_c}$$

- Starting from a parallel (Norton) two-port representation we obtain a dual circuit with uncorrelated generators having as parameters the **parallel noise resistance and conductance and the correlation admittance:**

$$\boxed{R_n, \quad G_n, \quad Y_c \neq 1/Z_c}$$

Noise figure

- Starting from the input referred generators we find the noise figure as the ratio of the input open-circuit voltage power spectra: full/due to generator noise only:



$$\mathrm{NF} = \frac{\dfrac{S_{v,v}}{4R_G}}{\dfrac{S_{e_{nG},e_{nG}}}{4R_G}}$$

$$\boxed{\mathrm{NF} = \frac{S_{e_{nG},e_{nG}} + S_{e_{nc},e_{nc}} + S_i \left|Z_c + Z_G\right|^2}{S_{e_{nG},e_{nG}}}} \Rightarrow \boxed{\mathrm{NF} = 1 + \frac{r_n}{R_G} + \frac{g_n}{R_G}\left|Z_c + Z_G\right|^2}$$

Minimum noise figure & Z_{Go}

- NF does not depend on load
- Minimizing NF vs. R_G and X_G we generally find the *optimum source impedance Z_{Go}*
- The minimum noise figure results:

$$\boxed{\begin{aligned} R_{Go} &= \sqrt{\frac{r_n}{g_n} + R_c^2} \\ X_{Go} &= -X_c \end{aligned}}$$

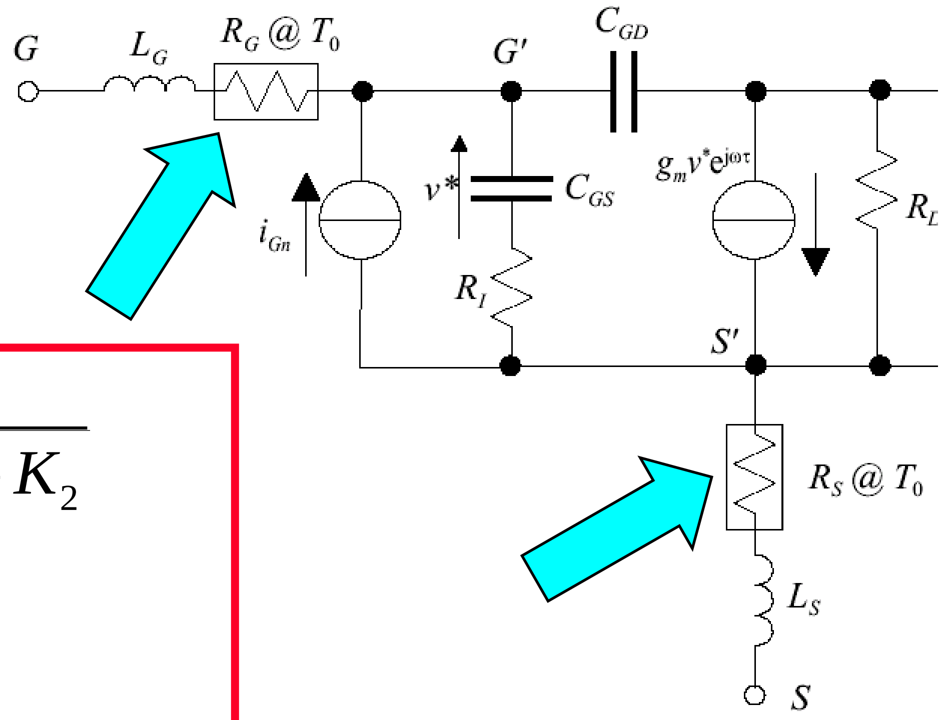
$$\boxed{\mathrm{NF}_{min} = 1 + 2g_n R_c + 2g_n\sqrt{\frac{r_n}{g_n} + R_c^2}}$$

System characterization of two-port

- The two-port noise is therefore characterized by four real parameters, which can be also evaluated directly through measurements:
  - ***Minimum noise figure*** $NF_{\min}$
  - ***Series noise conductance*** g_n
  - ***Optimum source impedance*** $Z_{sopt}=R_{sopt}+jX_{sopt}$
- Meaning of series noise conductance: sensitivity of NF with respect to minimum:

$$\boxed{NF = NF_{\min} + \frac{4g_n}{R_G}\frac{\left|Z_G - Z_{Go}\right|^2}{\left|Z_{Go}\right|^2}}$$

FET noise parameters with PRC model



$$NF_{\min} \approx 1 + 2K_1 \frac{f}{f_T}\sqrt{g_m\left(R_G + R_S\right) + K_2}$$

$$g_n \approx g_m K_1 \left(\frac{f}{f_T}\right)^2$$

$$Z_{So} = \frac{\sqrt{g_m\left(R_G + R_S\right) + K_2}}{\omega C_{GS} K_1} + \mathbf{j}\frac{P - C\sqrt{RP}}{\omega C_{GS} K_1^2}$$

$$K_1 = \sqrt{P + R - 2C\sqrt{PR}}$$

$$K_2 = PR\left(1 - C^2\right) / K_1^2$$

$$f_T = g_m / 2\pi C_{GS}$$

Fukui formula

- If the contribution of the R_G and R_S parasitic resistances is dominant, the noise figure can be approximated as (**Fukui formula**)

$$\boxed{NF_{\min} \approx 1 + 2K_1 \frac{f}{f_T}\sqrt{g_m\left(R_G + R_S\right)} = 1 + 2KL_G f\sqrt{g_m\left(R_G + R_S\right)}}$$

- For a HEMT K ~0.1, for a MESFET ~0.3 same gate length
- NF deteriorates with frequency and with resistive parasitics, improves by increasing the cutoff frequency

Frequency behaviour of noise figure and associate gain

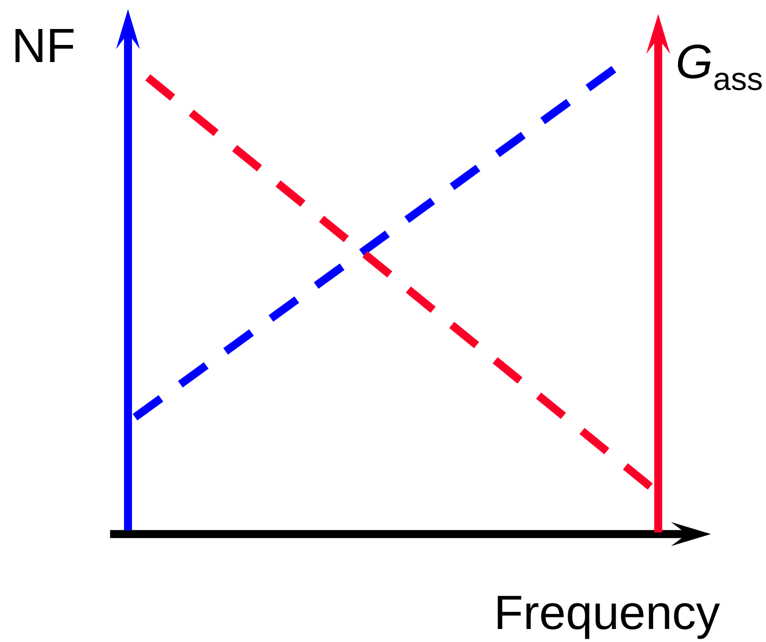
One-temperature model, neglecting gate noise:

$$p_{n,L} = k_B T_D + G_{ass} k_B T_0$$

$$p'_{n,L} = G_{ass} k_B T_0$$

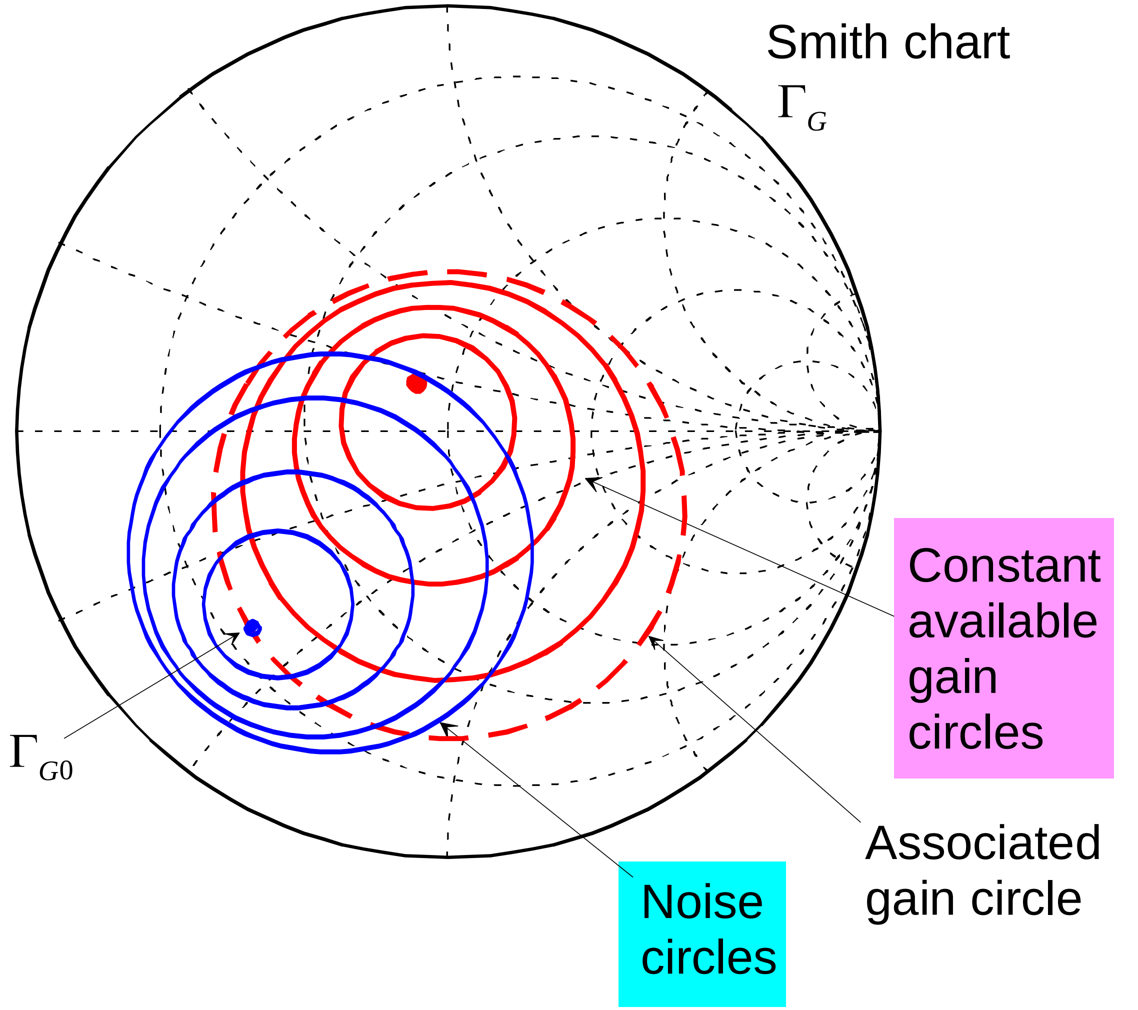
$$\mathrm{NF}_{\min} = 1 + \frac{T_D}{G_{ass} T_0}$$

$$\boxed{G_{ass} \cdot \left(NF_{\min} - 1 \right) \approx \frac{T_D}{T_0}}$$

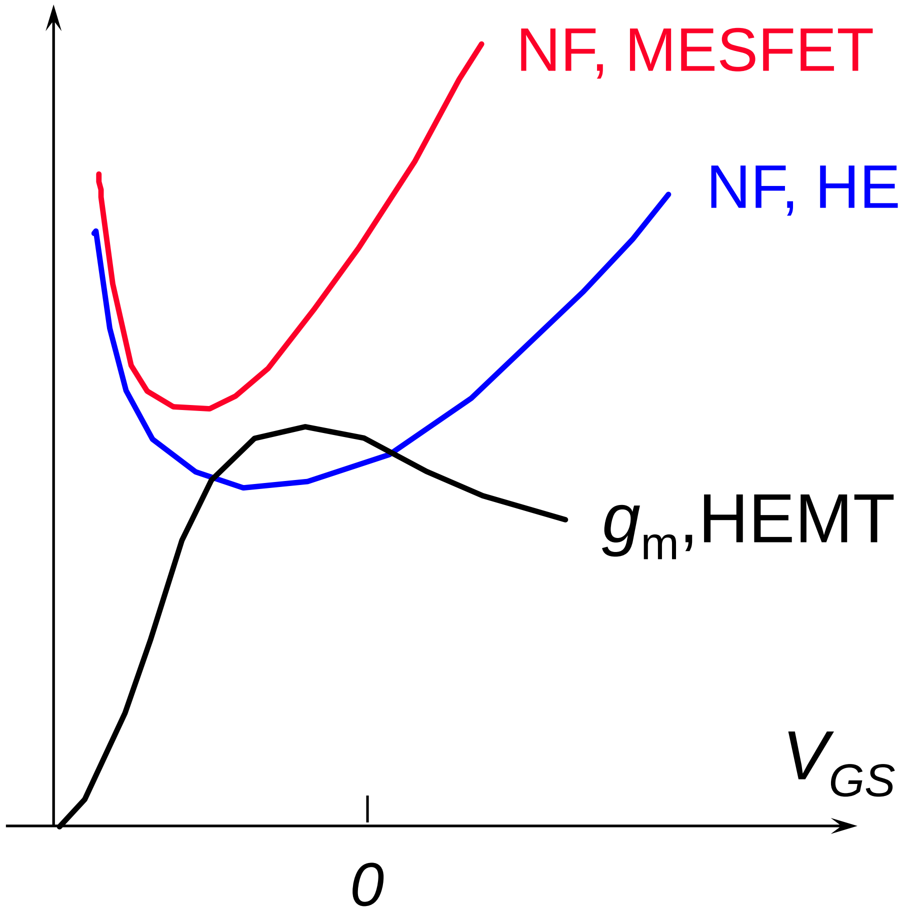


Constant noise circles and noise-gain tradeoff

- On the Γ_G Smith chart constant NF curves are again circles
- Constant noise circles and constant available gain circles allow for a tradeoff between gain and noise

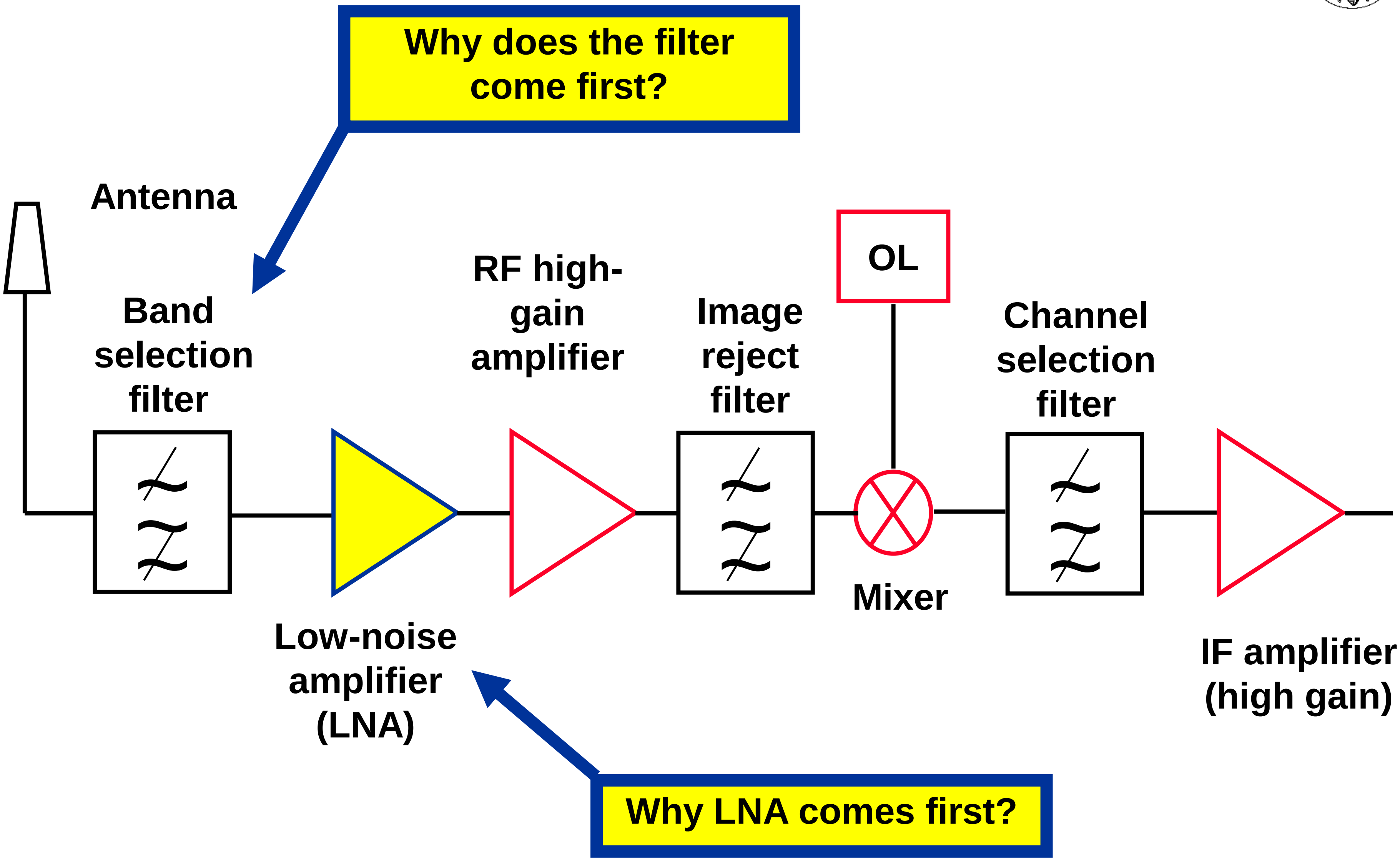


Low-noise active device bias



- Typically the optimum low-noise bias corresponds to low to average currents and low drain-to-source voltage
- Better compromise in some devices (e.g. HEMTs) with respect to gain

Noise in a receiver chain



Noise in cascaded two-ports - The Friis formula

$$\mathrm{NF}_{\mathrm{tot}} = 1 + (\mathrm{NF}_1 - 1) + \frac{(\mathrm{NF}_2 - 1)}{G_1} + \frac{(\mathrm{NF}_3 - 1)}{G_1 G_2} + \frac{(\mathrm{NF}_4 - 1)}{G_1 G_2 G_3} + \ldots.$$

- From the Friis formula, the first stage should be low noise, high gain
- Lossy stages in the first position are usually unavoidable but should be minimized (for a lossy two-port NF=L where L is the loss > 1 – why? See next slide)
- A passband filter before the LNA is indispensable because it reduces the noise bandwidth of the amplifier (and also rejects spurious interferers from outside the system bandwidth)

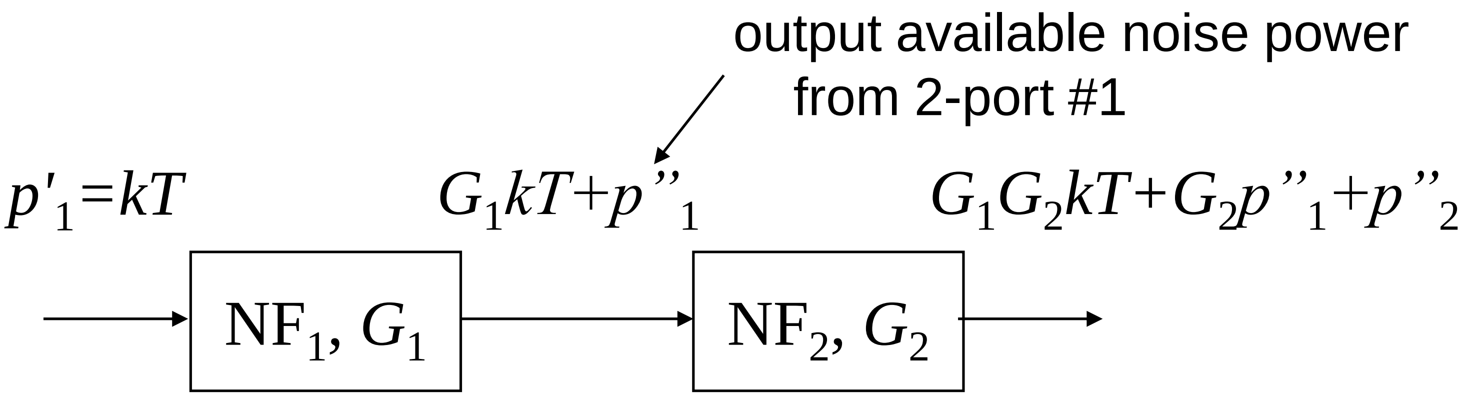
The noise figure of a lossy two-port

$$\mathrm{NF} = \frac{p_{nav,L}}{p'_{nav,L}} = \frac{k_B T_0}{G_{av}\, p'_{nav,in}} = \frac{k_B T_0}{G_{av} k_B T_0} = L_{av}$$

$L_{av} = \dfrac{1}{G_{av}}$ available power loss of the two port

$p_{nav,L} = k_B T_0$ since the output behaves as a resistor

Proof of Friis formula



$$\mathrm{NF}_1 = 1 + \frac{p''_1}{G_1 kT} \rightarrow p''_1 = G_1 kT(\mathrm{NF}_1 - 1)$$

$$\mathrm{NF}_2 = 1 + \frac{p''_2}{G_2 kT} \rightarrow p''_2 = G_2 kT(\mathrm{NF}_2 - 1)$$

$$\mathrm{NF}_{\mathrm{tot}} = \frac{G_1 G_2 kT + G_2 p''_1 + p''_2}{G_1 G_2 kT} = 1 + \frac{p''_1}{G_1 kT} + \frac{p''_2}{G_1 G_2 kT} =$$

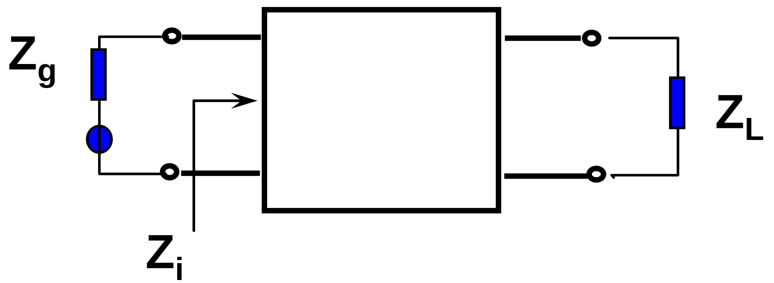
$$= 1 + \frac{G_1 kT(\mathrm{NF}_1 - 1)}{G_1 kT} + \frac{G_2 kT(\mathrm{NF}_2 - 1)}{G_1 G_2 kT} \rightarrow \boxed{\mathrm{NF}_{\mathrm{tot}} = 1 + (\mathrm{NF}_1 - 1) + \frac{\mathrm{NF}_2 - 1}{G_1}}$$

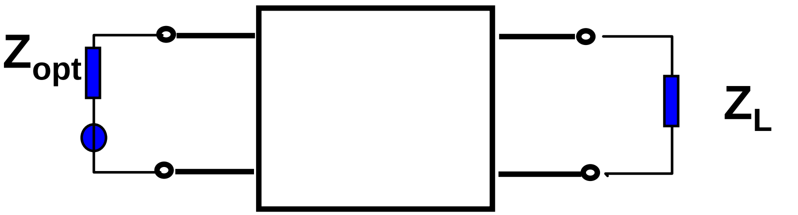
Low noise design and Associated gain

- In LNA design the *source impedance* is the optimum impedance, the *load is power matched*
- The optimum source impedance for noise does not generally yield input power match and therefore the corresponding available power gain ("associate gain") is lower than the MAG

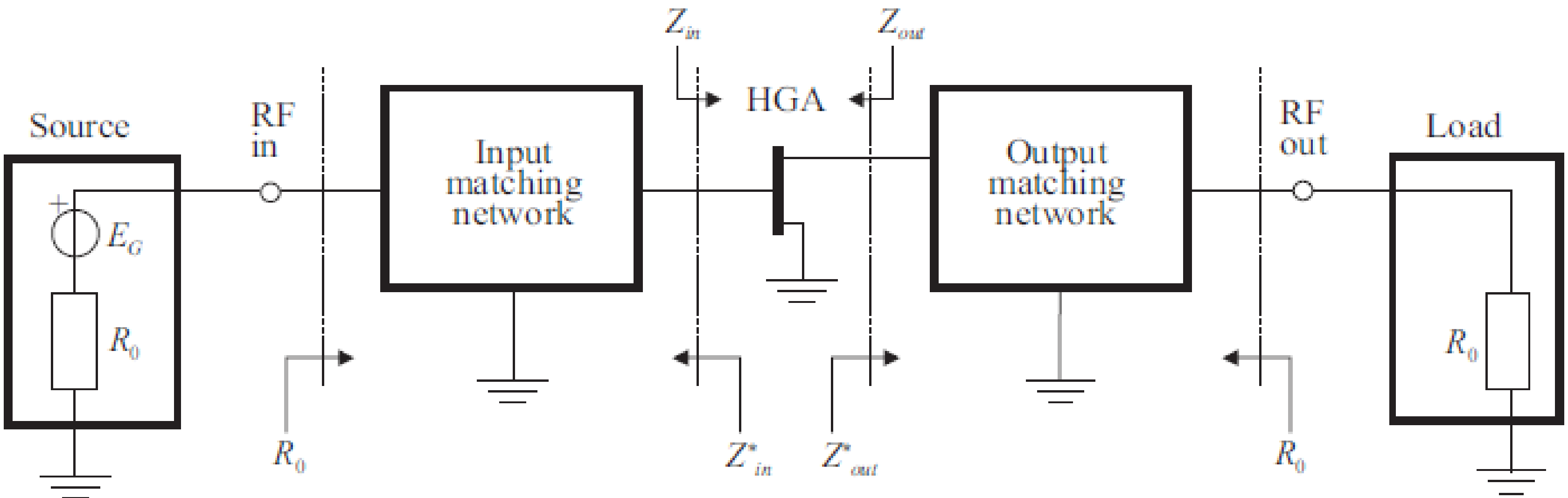
- **Power match** at input: $Z_g = Z_i^*$
- **Power match** at output
- Gain = MAG

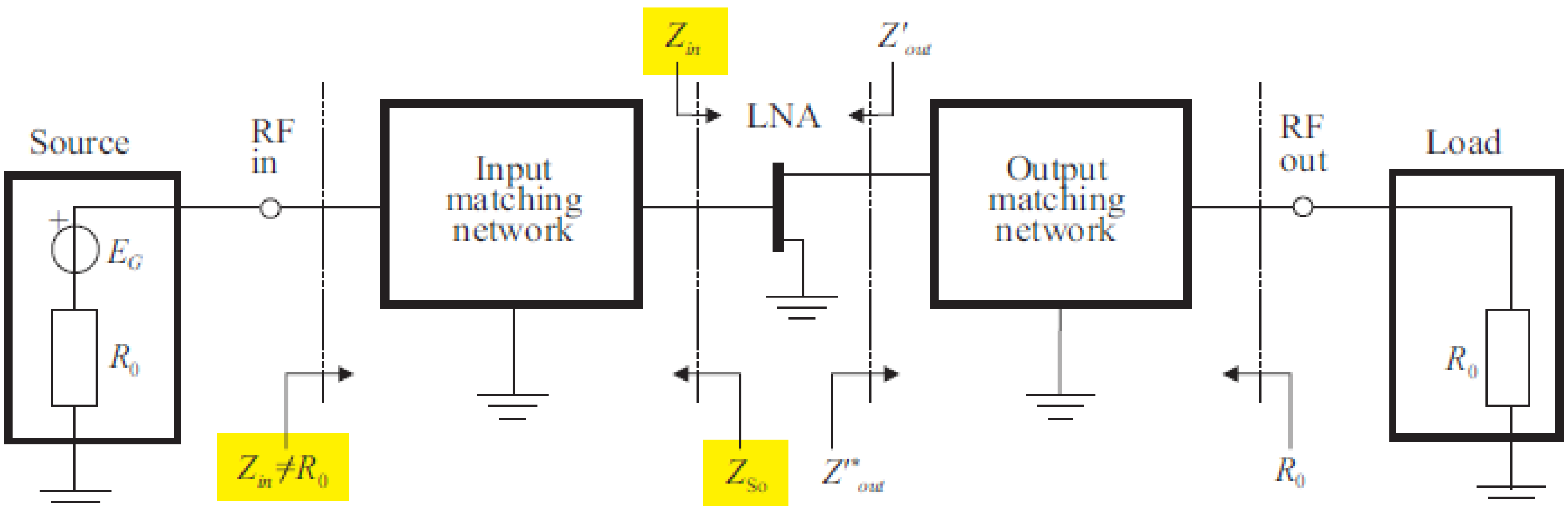
- **Noise match** at input: $Z_g = Z_{opt}$
- **Power match** at output
- Associate gain < MAG





Low-noise design does not grant input matching!!





The traditional low-noise design strategy

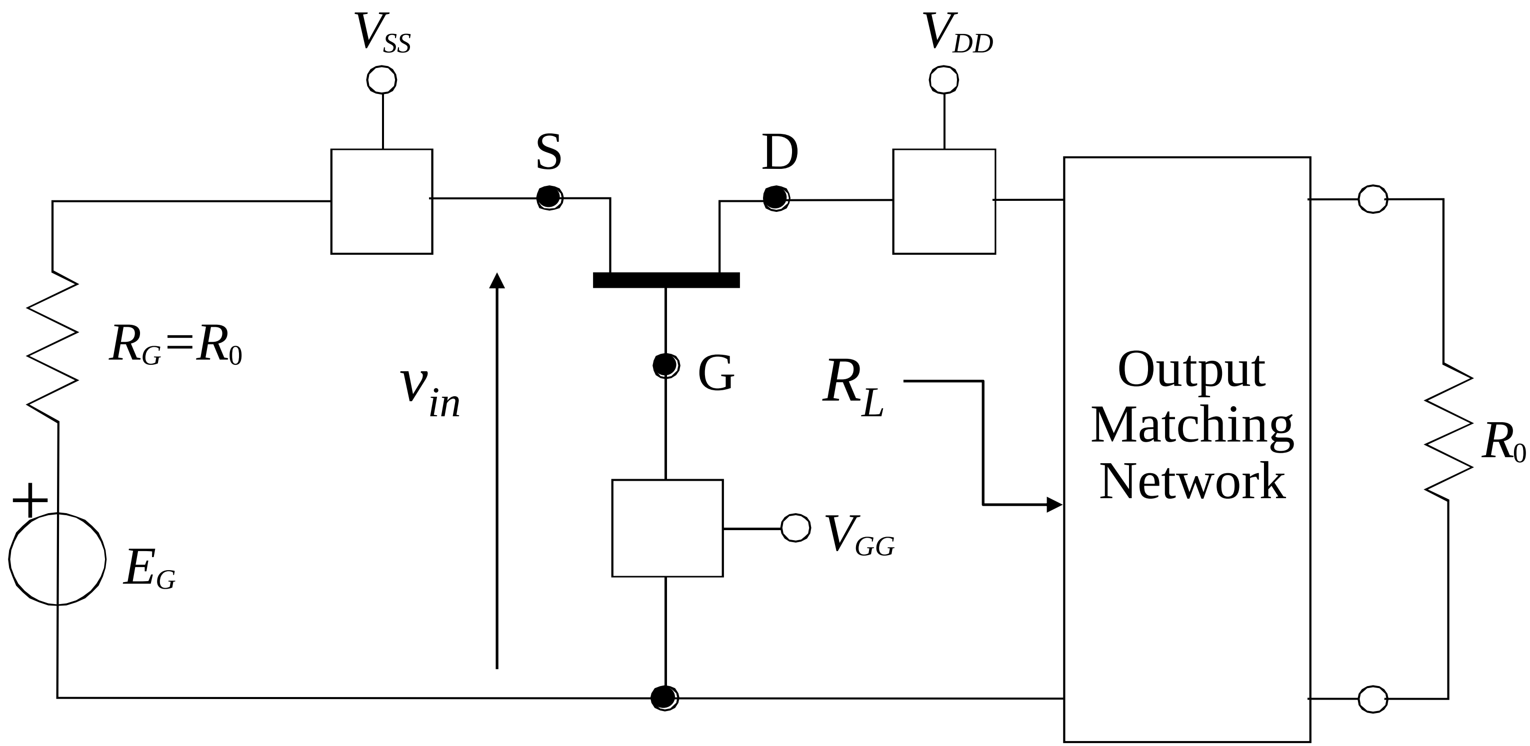
- In traditional low-noise design we have $NF=NF_{\mathrm{min}}$
- 1st step: choice of device and device optimum working point
- 2nd step: input matching network so as to obtain from the generator impedance (e.g. 50 Ω) the optimum noise impedance Z_{Go}
- 3rd step: output matching network implementing output power match
- The amplifier has minimum NF, associated gain
- In many cases the classical strategy is unsatisfactory because input mismatching is unacceptable → alternative input matching strategies allow for improvements in the input match without overly deteriorating NF → particularly popular in RF

Popular FET LNA topologies

- Common gate (base) stage → input matching through bias or periphery design, high noise figure
- Inductive source (emitter) feedback → noise figure marginally worse than the minimum, reasonable gain, narrowband input matching; can also use a cascade stage
- The cascode configuration has advantages in LNAs thanks to the better isolation → improved stability → however in some cases LNAs use potentially unstable devices (in-band!)
- The classical design can be input matched by using a balanced configuration or through an input circulator

Common gate FET stage

- Noninverting amplifier, can be input matched by proper transconductance design (→ bias)
- For ideal parameters the noise figure is 2.2 dB ☹



Common gate FET stage

- The formulae hold if the input capacitance is negligible
- For ideal parameters the noise figure is 2.2 dB ☹

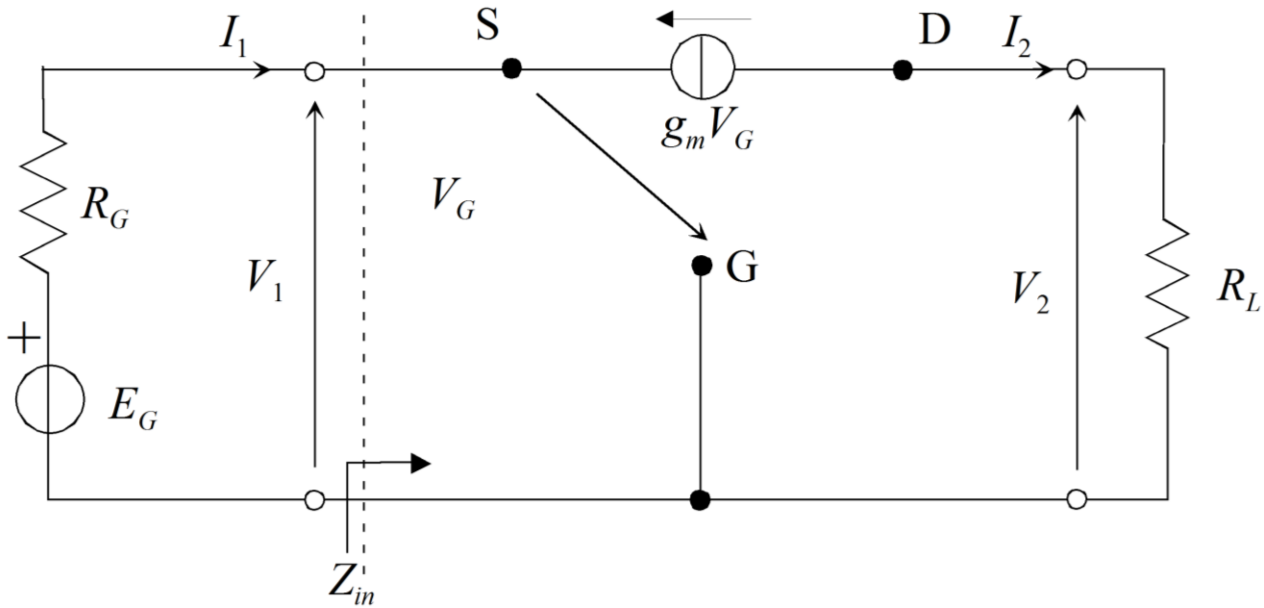
$$Z_{in} = \frac{1}{g_m}$$

$$A_V = \frac{V_D}{V_{in}} = g_m R_L$$

$$NF = \frac{4k_B T R_G g_m^{\;2} + 4k_B T P g_m}{4k_B T R_G g_m^{\;2}} = 1 + \frac{P}{R_G g_m}$$

$$\xrightarrow{1/g_m = R_G} 1 + P \approx 1 + \frac{2}{3} = \frac{5}{3} \rightarrow \mathrm{NF_{dB}} = 2.22\ \mathrm{dB}$$

Input impedance and amplification

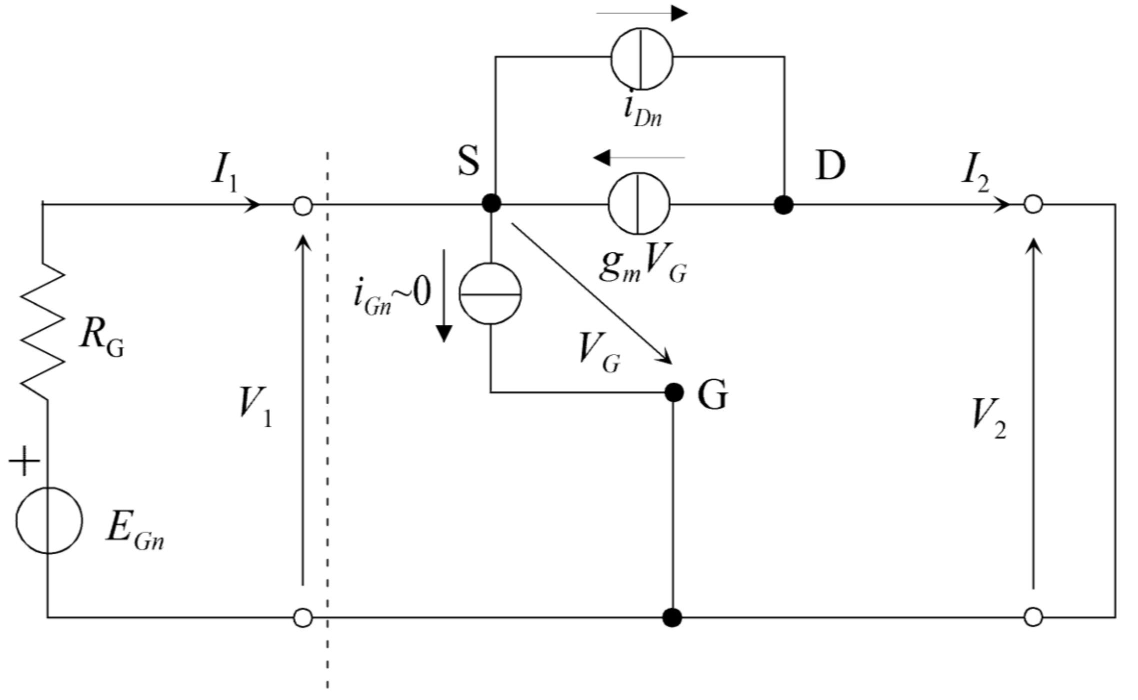


$$I_1 = -g_m V_G \quad V_2 = -g_m V_G R_L \quad V_1 = -V_G$$

$$Z'_{in} = I_2 R_L = \frac{V_1}{I_1} = \frac{-V_G}{-g_m V_G} = \frac{1}{g_m}$$

$$A_V = \frac{V_2}{V_1} = \frac{-g_m V_G R_L}{-V_G} = g_m R_L$$

Noise figure - I

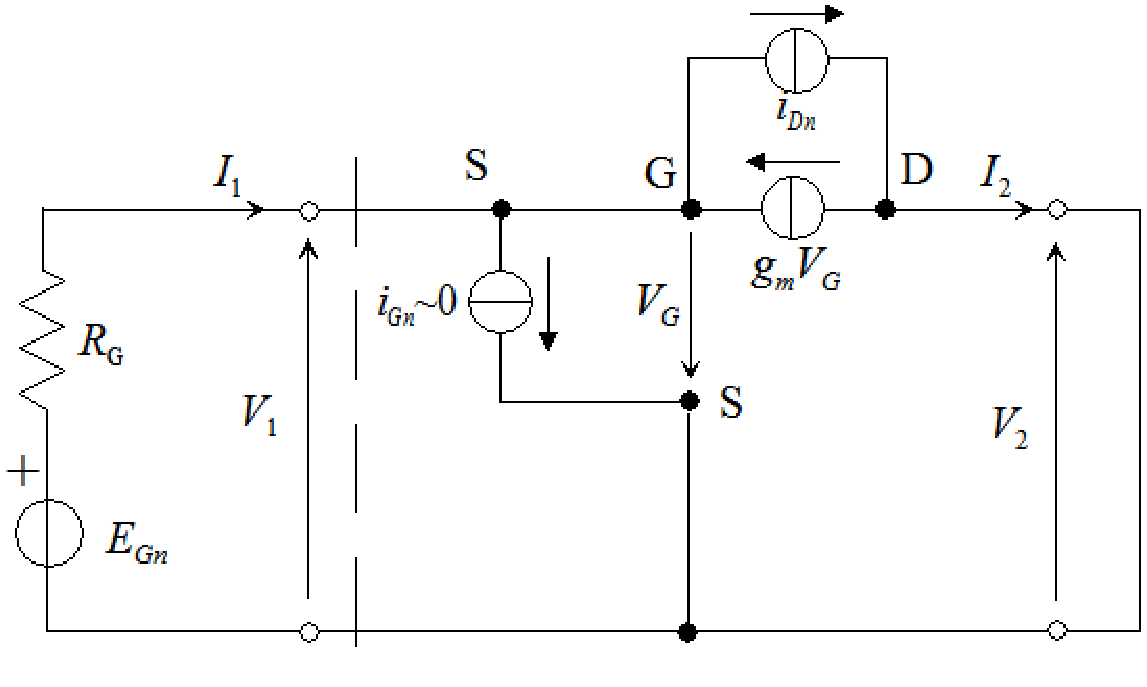


$$I_2 = I_1 = -g_m V_G + I_{Dn}$$

$$V_G = R_G I_1 - E_{Gn} \rightarrow I_1 = -g_m R_G I_1 + g_m E_{Gn} + I_{Dn}$$

$$I_1 = I_2 = \frac{g_m E_{Gn} + I_{Dn}}{1 + g_m R_G}$$

Noise figure - II

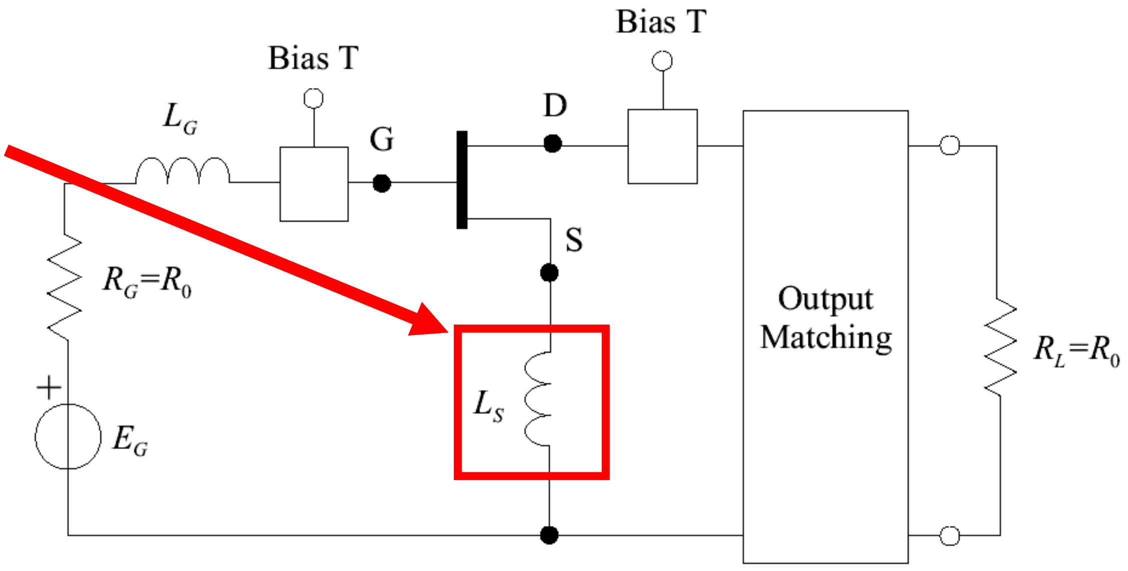


$$\overline{I_2 I_2^*} = \frac{g_m^2 \overline{E_{Gn} E_{Gn}^*} + \overline{I_{Dn} I_{Dn}^*}}{\left(1 + R_G g_m\right)^2}$$

$$\mathrm{NF} = \frac{g_m^2 \overline{E_{Gn} E_{Gn}^*} + \overline{I_{Dn} I_{Dn}^*}}{g_m^2 \overline{E_{Gn} E_{Gn}^*}} = 1 + \frac{4 k_B T_0 g_m P}{g_m^2 4 k_B T_0 R_G} = 1 + \frac{P}{g_m R_G} \xrightarrow{R_G = 1/g_m} \mathrm{NF} = 1 + P$$

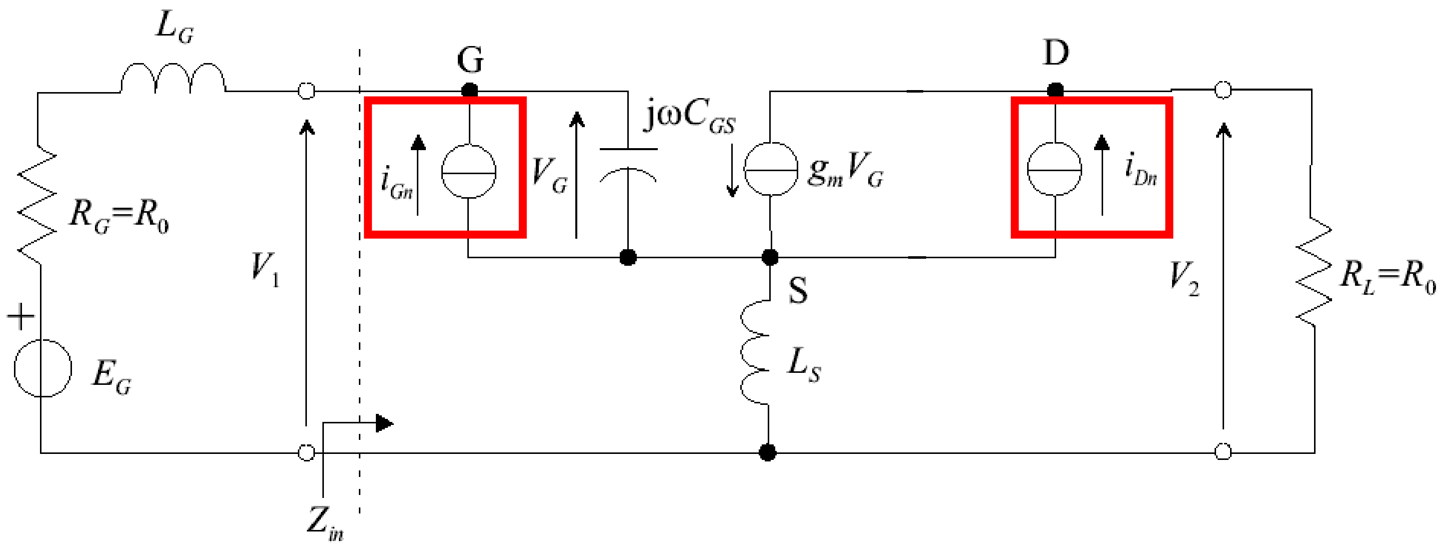
Series inductive feedback LNA

Series inductor on FET source



- An example of popular RF LNA design approach, where the input matching is obtained through a lossless network and the noise figure can be only marginally worse than the minimum one, but with reasonable and low input reflection coefficient
- Remember that cascading a stage with reactive networks does not change the minimum NF, but a source inductor does (sometimes marginally)!

Series Inductive Feedback LNA



- Input impedance:

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+R_0} = -\frac{\mathrm{j}}{\omega C_{GS}} + \mathrm{j}\omega L_S - \frac{\mathrm{j}\omega L_S \left(\mathrm{j}\frac{g_m}{\omega C_{GS} G_{DS}} + \mathrm{j}\omega L_S\right) G_{DS}}{1+\left(\mathrm{j}\omega L_S + R_0\right) G_{DS}} \approx$$

$$\approx \boxed{-\frac{\mathrm{j}}{\omega C_{GS}} + \mathrm{j}\omega L_S} + \boxed{\frac{g_m L_S}{C_{GS}}}.$$

can be designed to 50 Ohm

can be cancelled through resonance → L_G

LNA design

- Matching corresponds to conditions:

$$\boxed{\begin{aligned} L_S &= \frac{C_{GS}R_0}{g_m} = \frac{R_0}{\omega_T} \\ \omega\left(L_G + L_S\right) &= \frac{1}{\omega C_{GS}} \end{aligned}}$$

- In input matching conditions the NF can be shown to be:

$$\boxed{\mathrm{NF} = 1 + \frac{\omega}{\omega_T}\left[Q_L R + \frac{1}{Q_L}\left(R - 2C\sqrt{PR} + P\right)\right]}$$

- where:

$$\boxed{Q_L = \frac{\omega\left(L_G + L_S\right)}{R_0} = \frac{1}{\omega C_{GS} R_0}}$$

“Power optimization”

- The NF can be now be optimized with respect to the Q factor → since the gate noise contribution *R* is small usually this corresponds to large values of Q
- This can be obtained by changing the device periphery in order to set the input capacitance → reducing this also the power dissipation is reduced
- Optimum (not minimum!) Q & NF:

$$\boxed{\begin{aligned} Q_{Lo} &= \sqrt{\frac{R - 2C\sqrt{PR} + P}{R}} \\ \mathrm{NF}_o &= 1 + 2\frac{\omega}{\omega_T}\sqrt{R\left(R - 2C\sqrt{PR} + P\right)} \end{aligned}}$$