Two-port Stability

Microwave Electronics

Giovanni Ghione, Marco Pirola

Politecnico di Torino, DET

Summary

- **Stability**
- Stability criteria
- Examples

Stability issues in a two-port

- A loaded two-port is here a model for a linear amplifier, including an active device → in certain conditions the structure can be unstable, i.e. it can oscillate
- Instability and oscillations correspond to **nonzero solution with zero forcing term**, i.e. zero system determinant:

$$\begin{vmatrix} 1 & -\Gamma_G & 0 & 0 \\ -S_{11} & 1 & -S_{12} & 0 \\ -S_{21} & 0 & -S_{22} & 1 \\ 0 & 0 & 1 & -\Gamma_L \end{vmatrix} = (1 - S_{11}\Gamma_G)(1 - S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_G\Gamma_L = 0$$

- This condition can be expressed equivalently:

$$\boxed{(1 - S_{11}\Gamma_G)(1 - \Gamma_L\Gamma_{out}) \quad = \quad 0} \Longleftrightarrow \boxed{(1 - S_{22}\Gamma_L)(1 - \Gamma_G\Gamma_{in}) \quad = \quad 0}$$

Unconditional & conditional stability - I

- We assume that load and generator are *passive* (gamma with magnitude <1) and that the two-port is *stable when closed on the normalization impedances* (reasonable, why?), i.e.:

$$|S_{11}| < 1 \quad |S_{22}| < 1$$

- it follows that:

$$\left|S_{11}\Gamma_G\right| < 1, \quad \left|S_{22}\Gamma_L\right| < 1 \rightarrow$$

$$\rightarrow \left|1 - S_{11}\Gamma_G\right| > 0, \ \left|1 - S_{22}\Gamma_L\right| > 0$$

- Therefore the instability condition **can only derive from**:

$$\left(1 - \Gamma_G \Gamma_{in}\right) = 0 \Leftrightarrow \left(1 - \Gamma_L \Gamma_{out}\right) = 0$$

Unconditional & conditional stability - II

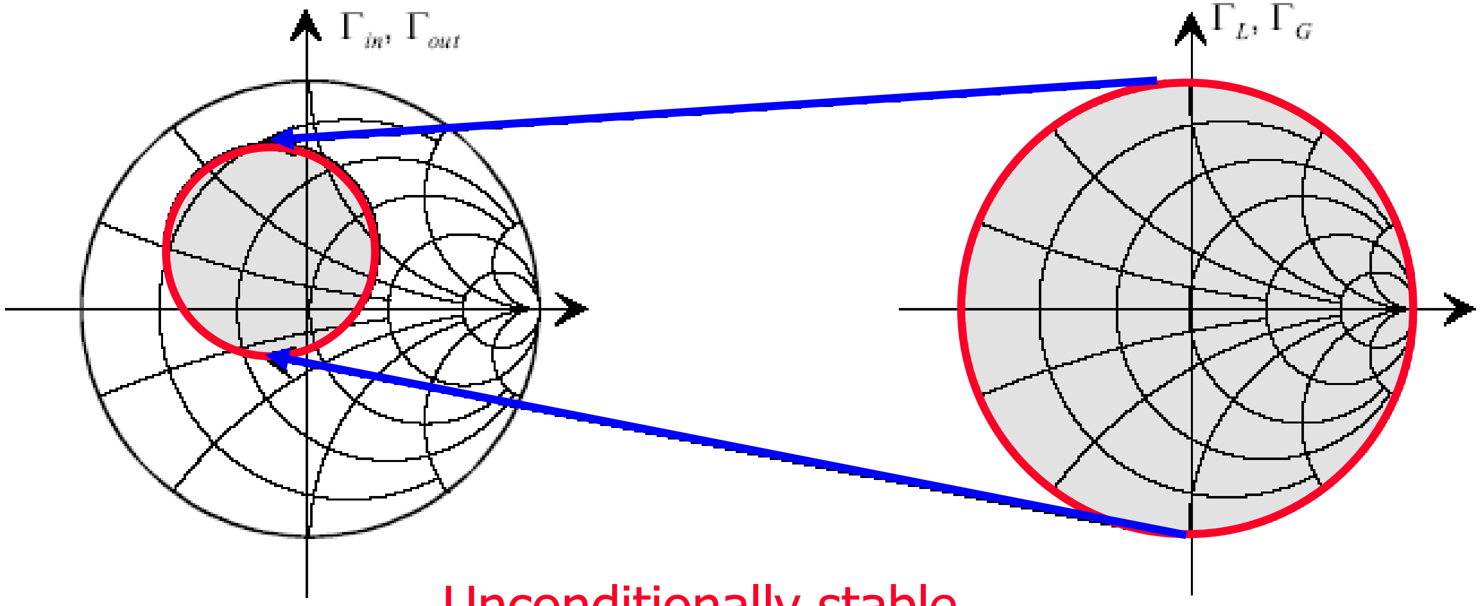
$$\Gamma_G \Gamma_{in} = 1 \quad \Longleftrightarrow \quad \Gamma_L \Gamma_{out} = 1$$

$$\Gamma_{in} = \frac{S_{11} - \Delta_S \Gamma_L}{1 - S_{22}\Gamma_L} \qquad \Gamma_{out} = \frac{S_{22} - \Delta_S \Gamma_G}{1 - S_{11}\Gamma_G}$$

- Two possibilities (generator and load gamma are considered as **passive** → magnitude <1, within the unit circle in Smith chart):
  - For every possible Γ_L we have $|\Gamma_{in}| < 1$ & for every possible Γ_G we have $|\Gamma_{out}| < 1$ → **unconditional stability**
  - For some Γ_L → $|\Gamma_{in}| \geq 1$ or for some Γ_G → $|\Gamma_{out}| \geq 1$ → **conditional stability or potential instability**

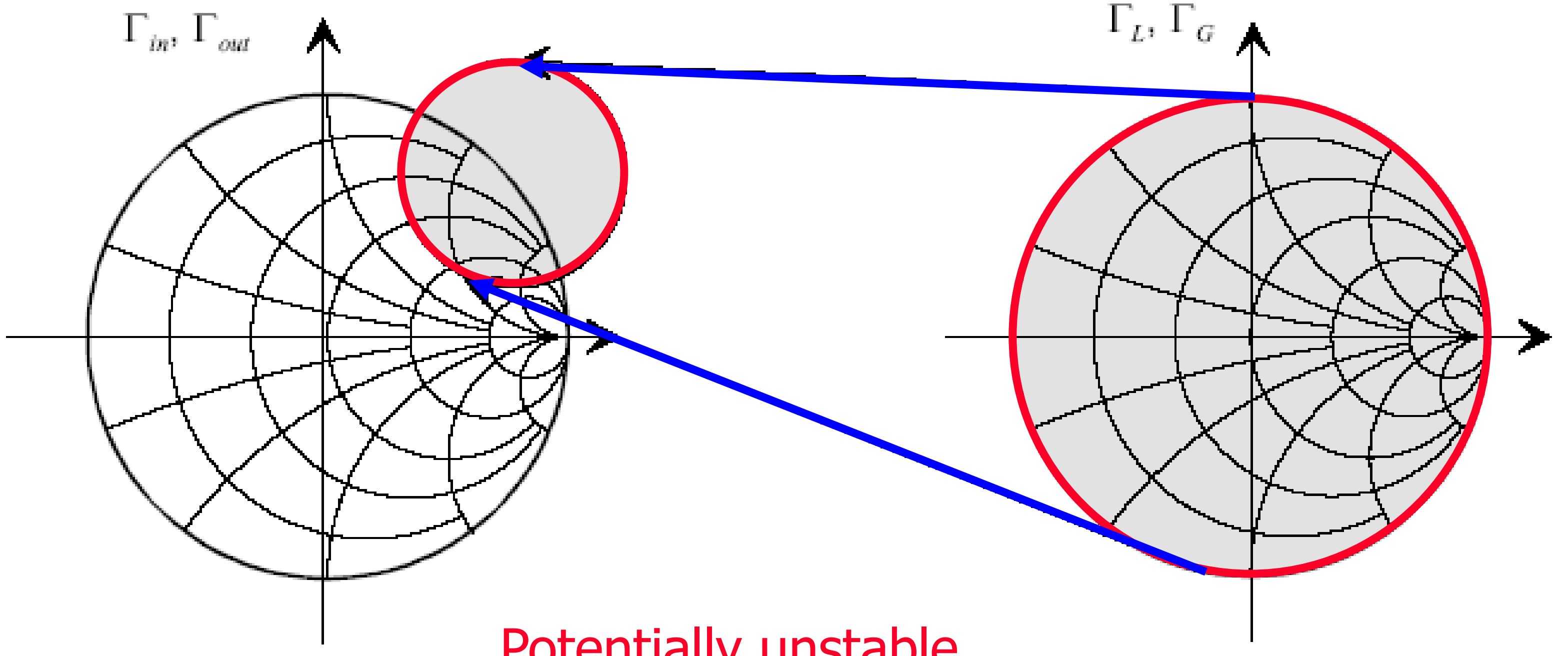
Graphical representation: unconditional stability

- Taking into account that $\Gamma_{in}(\Gamma_L)$ and $\Gamma_{out}(\Gamma_G)$ are *bilinear transformations of complex variables* that turn circles into circles in the respective complex planes we have the graphical interpretation:



Unconditionally stable

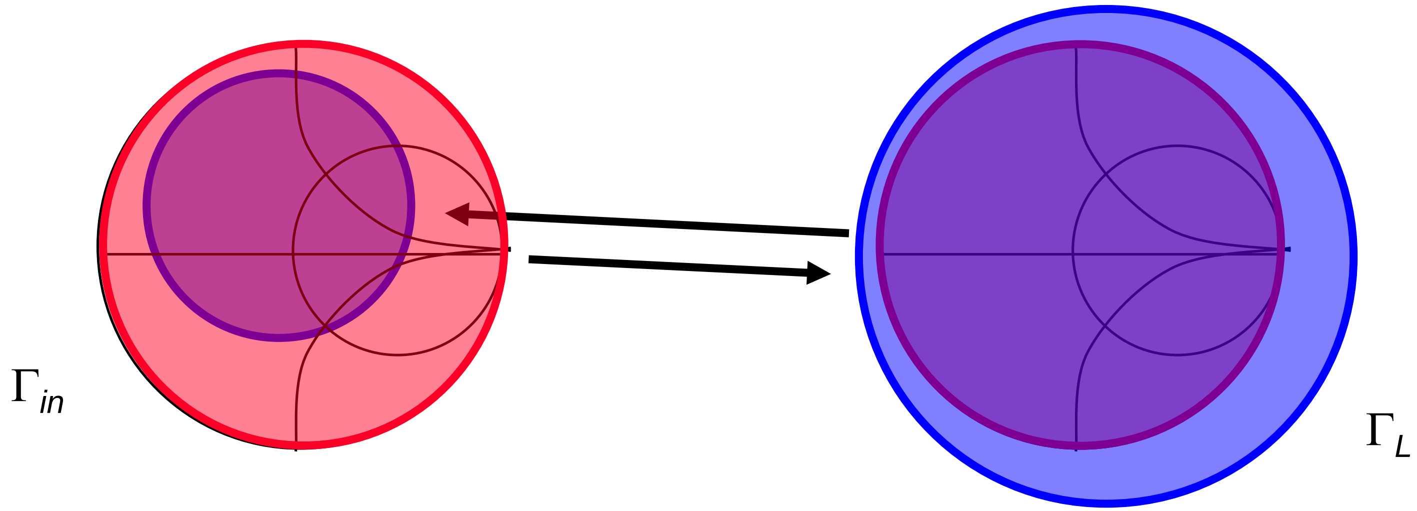
Graphical representation: conditional stability



Potentially unstable

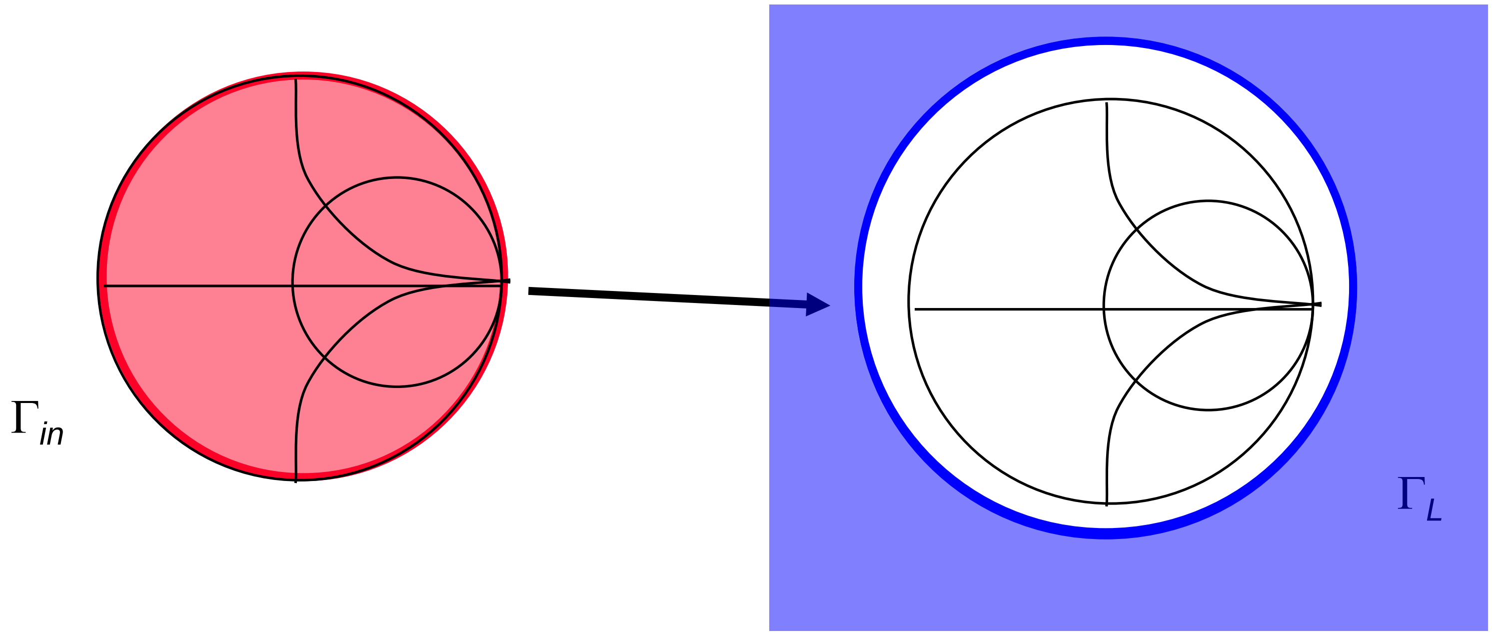
Stability circles - I

- In practice we prefer to consider the *counterimage* of the interior of the unit circle in the Γ_{in} (Γ_{out}) in the plane Γ_L (Γ_G) → **output (input) stability circles,** e.g.:



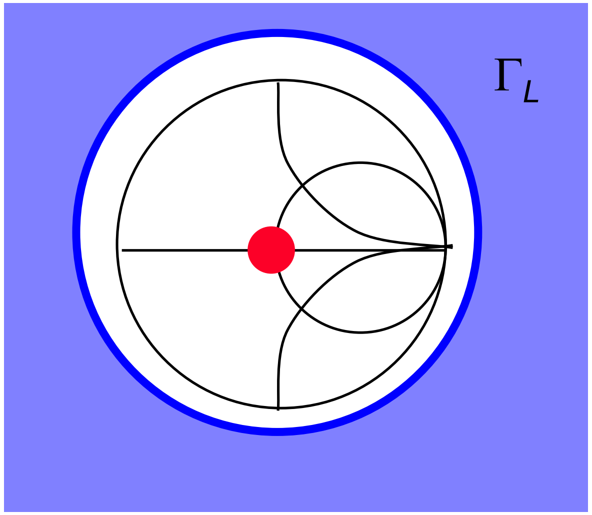
Stability circles - II

- **!!! WARNING !!!** The counterimage of the interior of the unit circle of the plane Γ_{in} (Γ_{out}) in plane Γ_L (Γ_G) can be **the interior** but also **the exterior** of the output (input) stability circle:

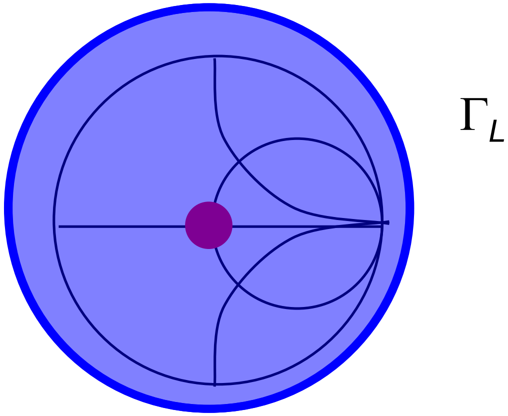


Stability circles - III

- To solve the ambiguity, take into account that:
  - to Γ_L=0 (Γ_G =0) a **|**Γ_{in}**|** < 1, |Γ_{out} | < 1 should correspond.
  - Why? because in this case Γ_{in}**=** S_{11}, Γ_{out}**=** S_{22} and we suppose that those have **magnitude <1**
- The center of the Smith chart is in the stable zone (in light blue)



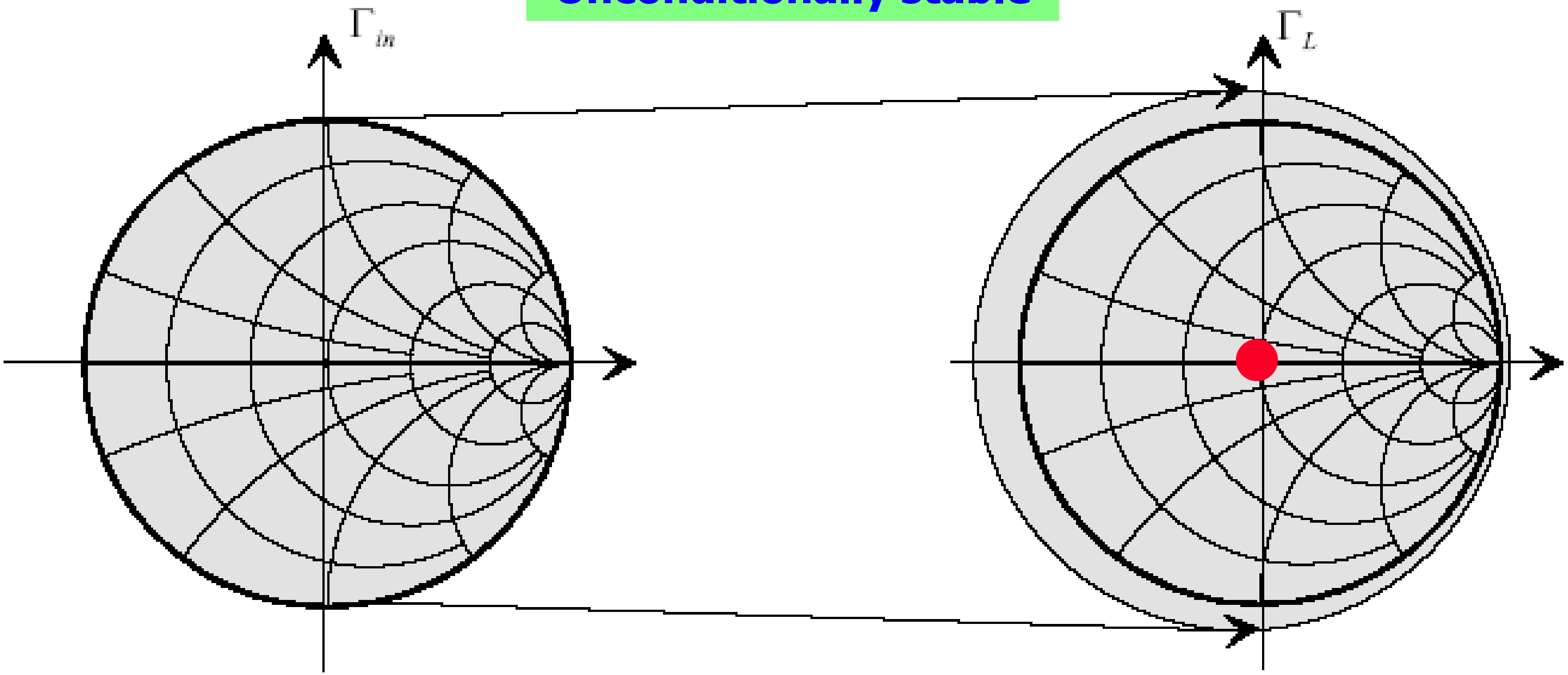
NO!



OK!

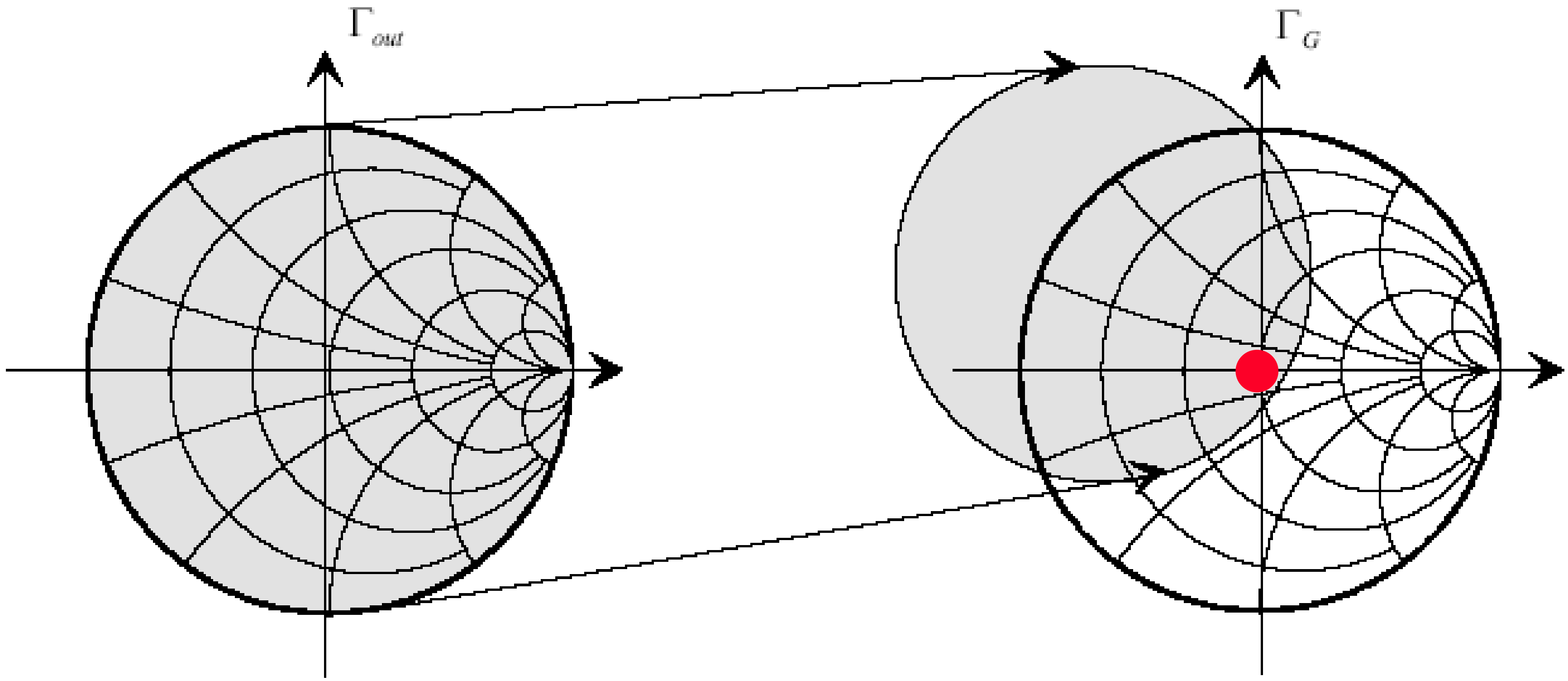
Example – output stability circle

Unconditionally stable

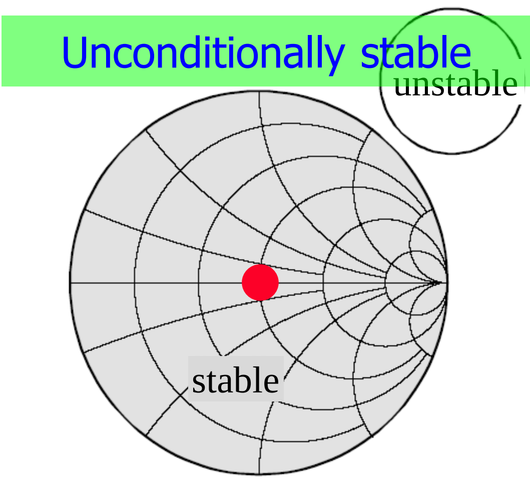


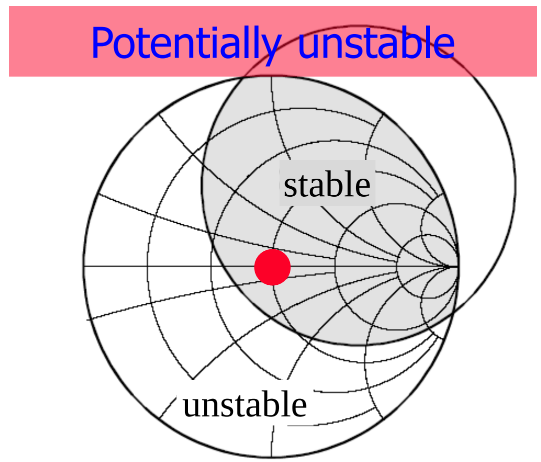
Example – input stability circle

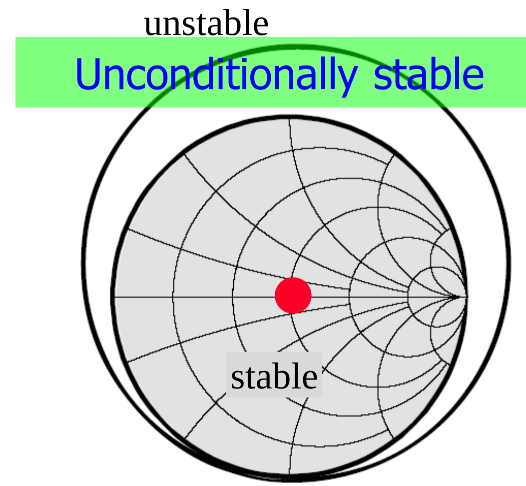
Potentially unstable

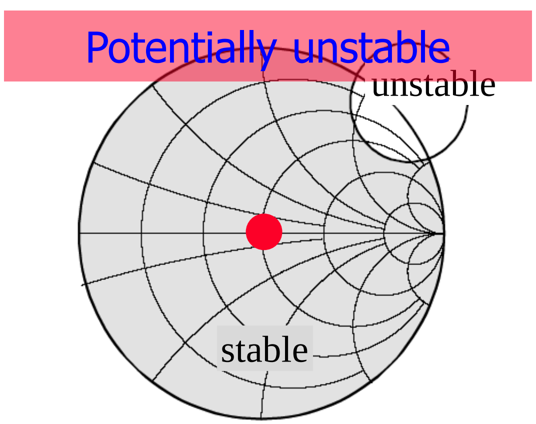


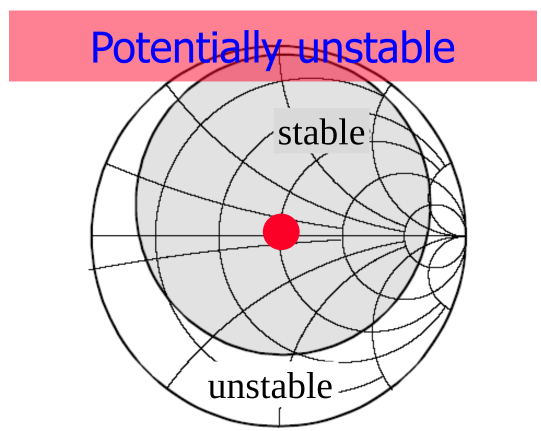
Other examples

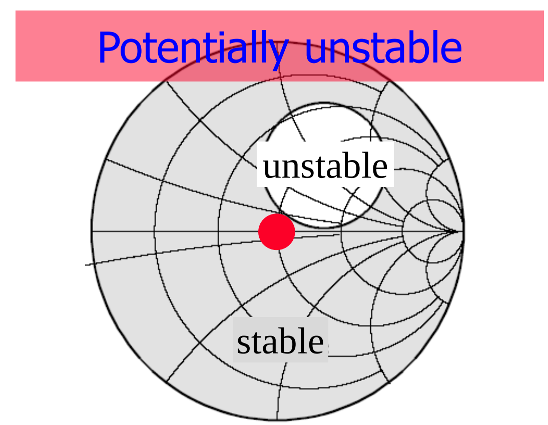












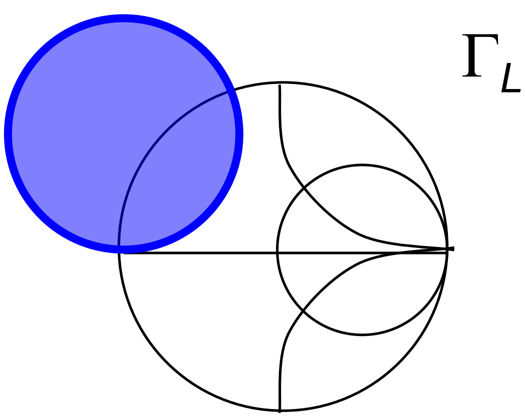
Stability and gain - I

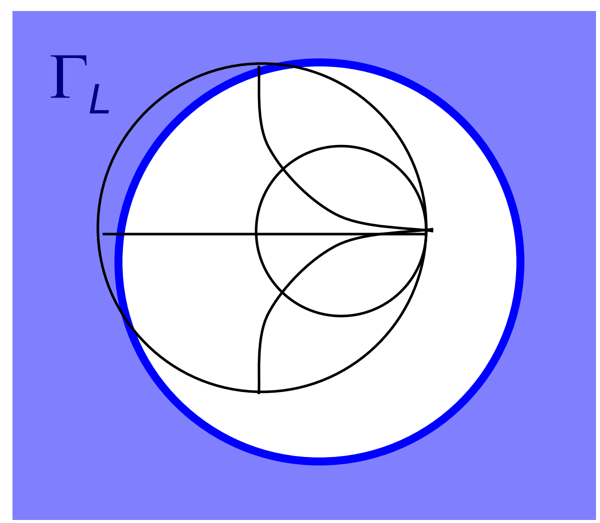
- The two sets corresponding to the potential instability condition:

$$\left|\Gamma_{in}\left(\Gamma_L\right)\right| = \left|\frac{S_{11} - \Delta_S \Gamma_L}{1 - S_{22}\Gamma_L}\right| \geq 1 \qquad \left|\Gamma_{out}\left(\Gamma_G\right)\right| = \left|\frac{S_{22} - \Delta_S \Gamma_G}{1 - S_{11}\Gamma_G}\right| \geq 1$$

are (in the Γ_L or Γ_G plane) **circles ($|\Gamma|$=1) + their internal or external region**

- Examples:





Stability and gain - II

- On the circles corresponding to the limit of potential instability condition the related gains → infinity (check, see next slide):

$$\left|\Gamma_{in}\left(\Gamma_L\right)\right| = 1 \rightarrow G_{op} = \infty \qquad \left|\Gamma_{out}\left(\Gamma_G\right)\right| = 1 \rightarrow G_{av} = \infty$$

- In an unconditionally stable two-port the "unstable" region fall <u>outside the Smith chart</u> and the gains have a **well defined maximum (MAG)**
- In a potentially unstable two-port the "unstable" region falls in part <u>within the Smith chart</u> and therefore a maximum gain does not exist any more, **gain becomes infinity on a set of points within the Smith chart**

Stability and gain - III

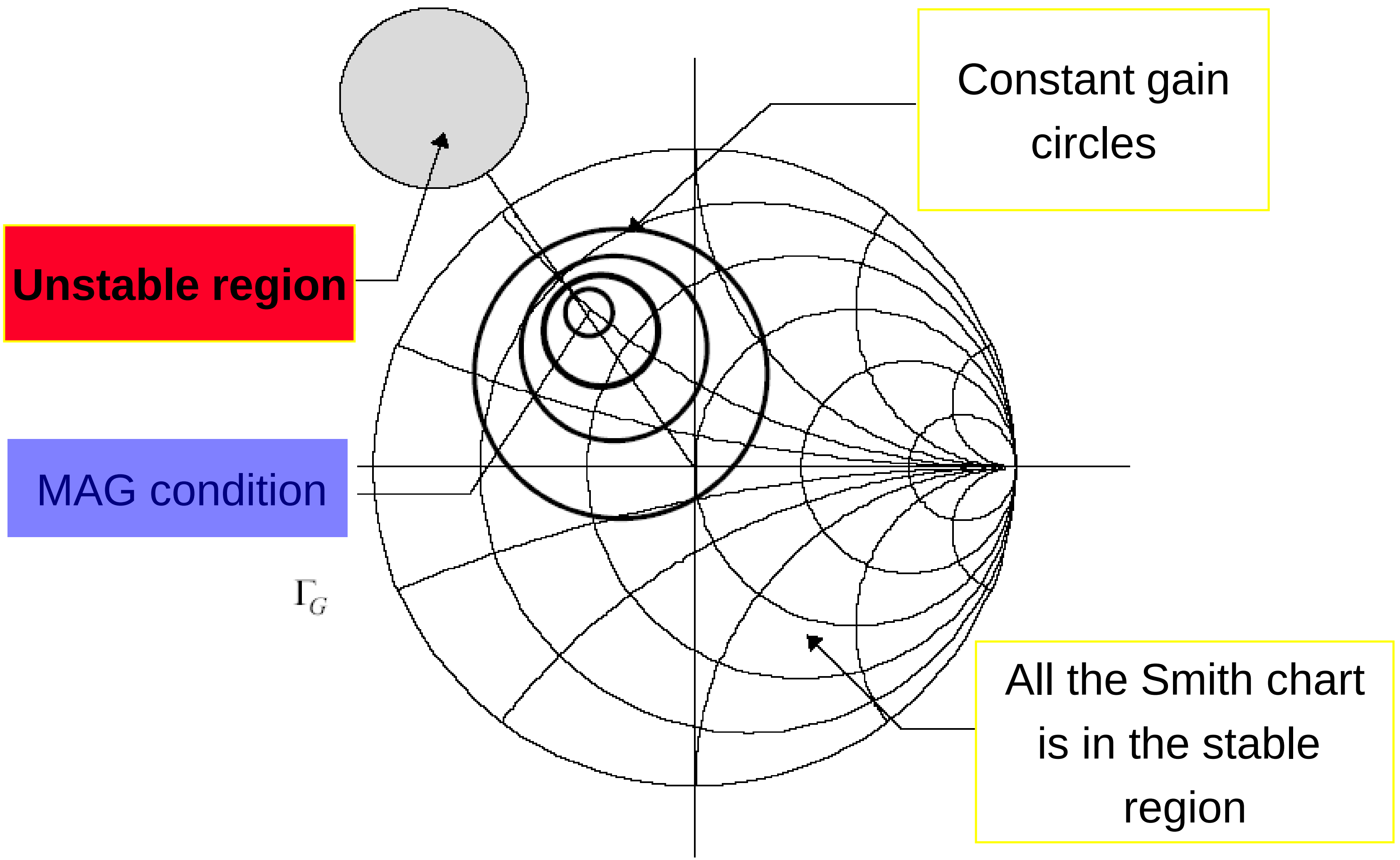
$$G_{op} = \frac{P_L}{P_{in}}$$

$$P_{in} = \frac{|b_0|^2 \left(1 - |\Gamma_{in}|^2\right)}{|1 - \Gamma_{in}\Gamma_G|^2} \rightarrow P_{in} \underset{|\Gamma_{in}|=1}{=} 0, \quad P_L \neq 0, \rightarrow G_{op} = \infty$$

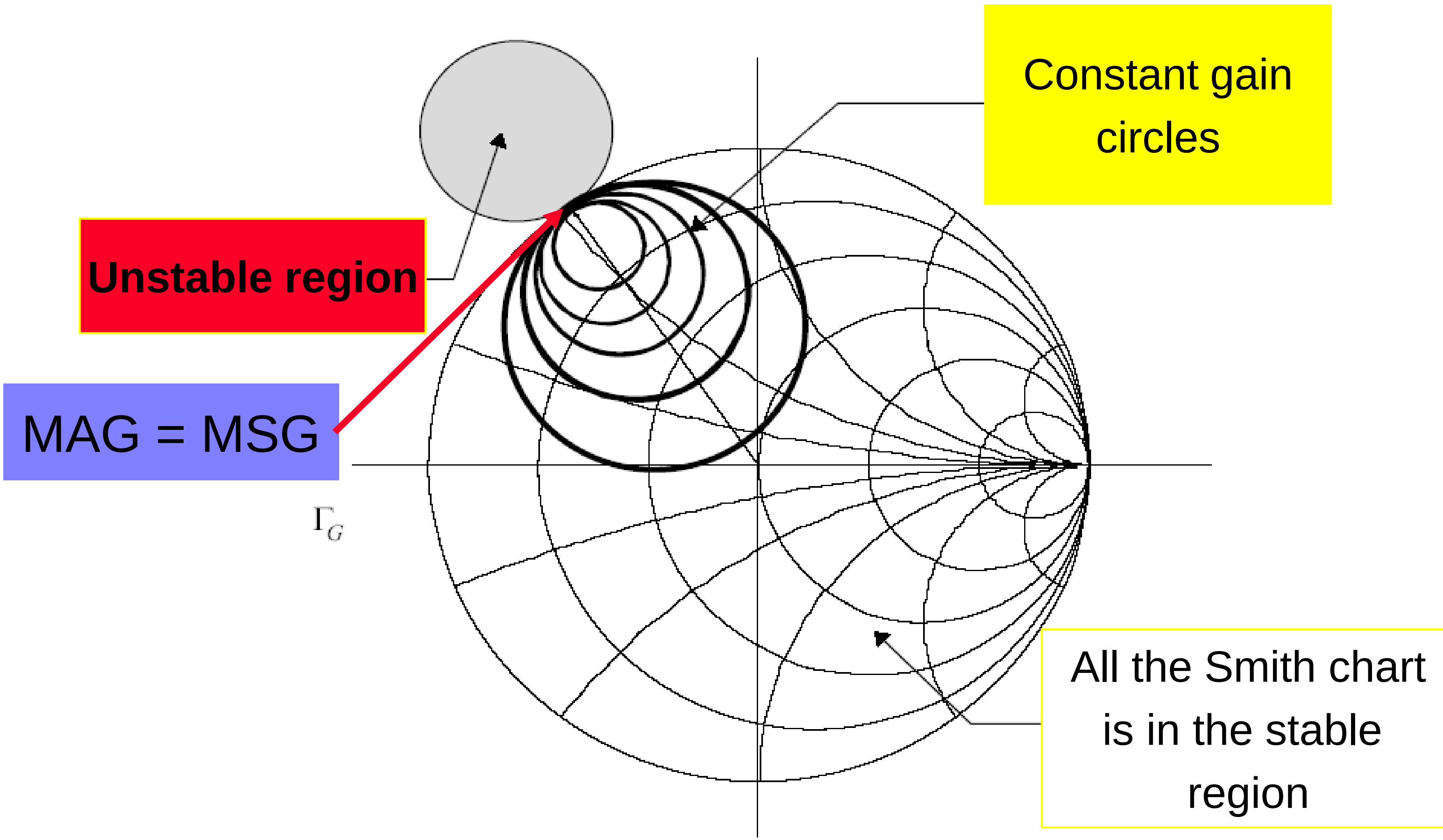
$$G_{av} = \frac{P_{av,\mathrm{out}}}{P_{av,in}}$$

$$P_{av,\mathrm{out}} = \frac{|b_0|^2 |S_{21}|^2}{\left(1 - |\Gamma_{out}|^2\right)|1 - S_{11}\Gamma_G|^2} \rightarrow P_{av,\mathrm{out}} \underset{|\Gamma_{out}|=1}{=} \infty, \rightarrow G_{av} = \infty$$

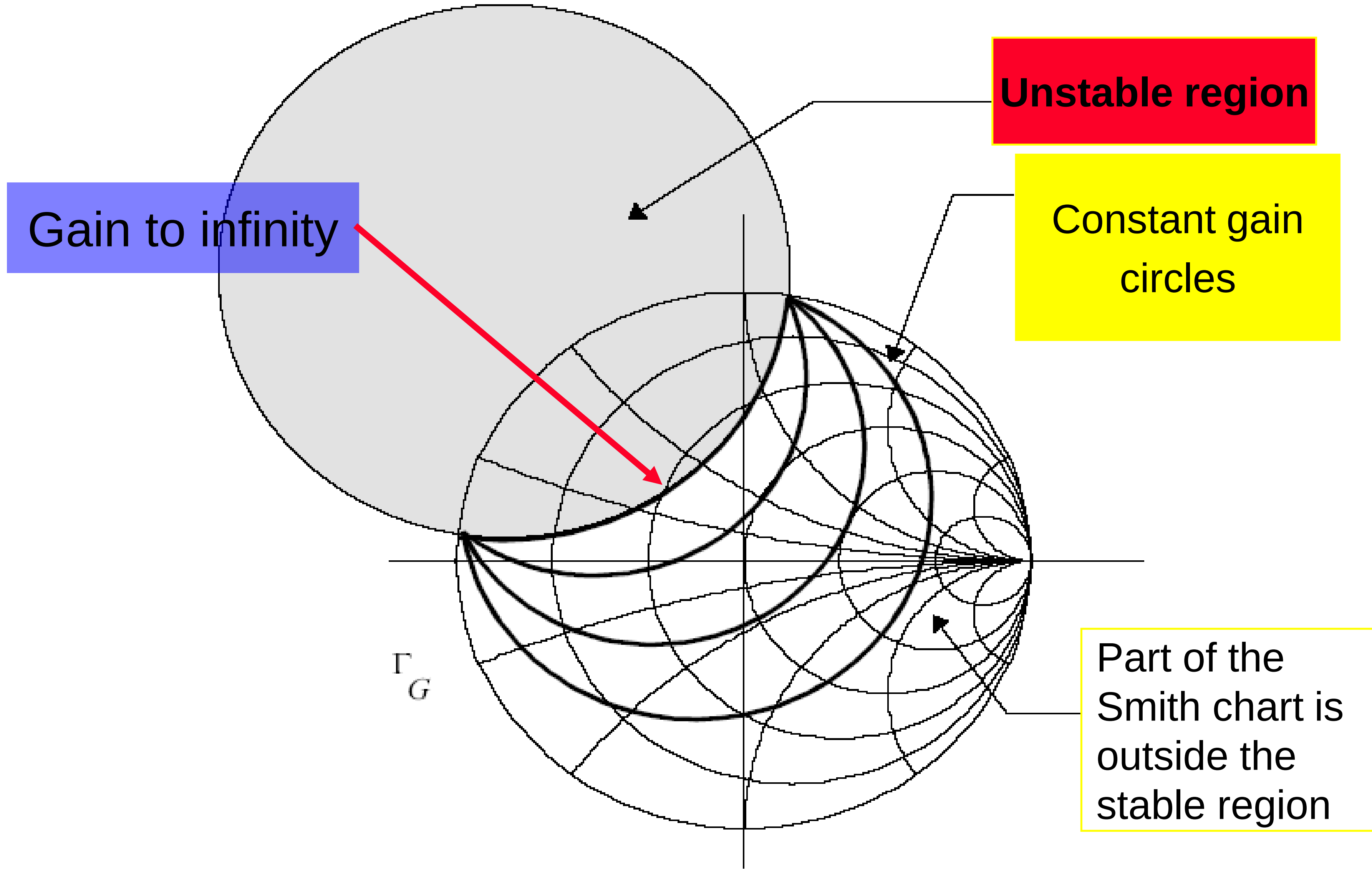
Example: available gain, unconditionally stable



Example: available gain, almost potentially unstable



Example: available gain, potentially unstable



Stability and amplifier design

- The maximum gain design is possible through **simultaneous conjugate matching** only if the two-port is unconditionally stable on the operating bandwidth
- If the two-port is potentially unstable we have to choices:
  – Choose a load and generator condition far enough away from the unstable region (critical!)
  – Stabilize the two-port through resistive networks (safer, our choice always)
- Low-frequency (out-of-band) stability is always to be checked and made sure of (why? See next slides, the problem lies with amplifier saturation due to “self jamming”)
- How do you know that a two-port is unconditionally stable? See “stability criteria”

An amplifier (approximate) system model

- The model is nonlinear and is often called “descriptive function model”; it goes beyond the small-signal linear approximation

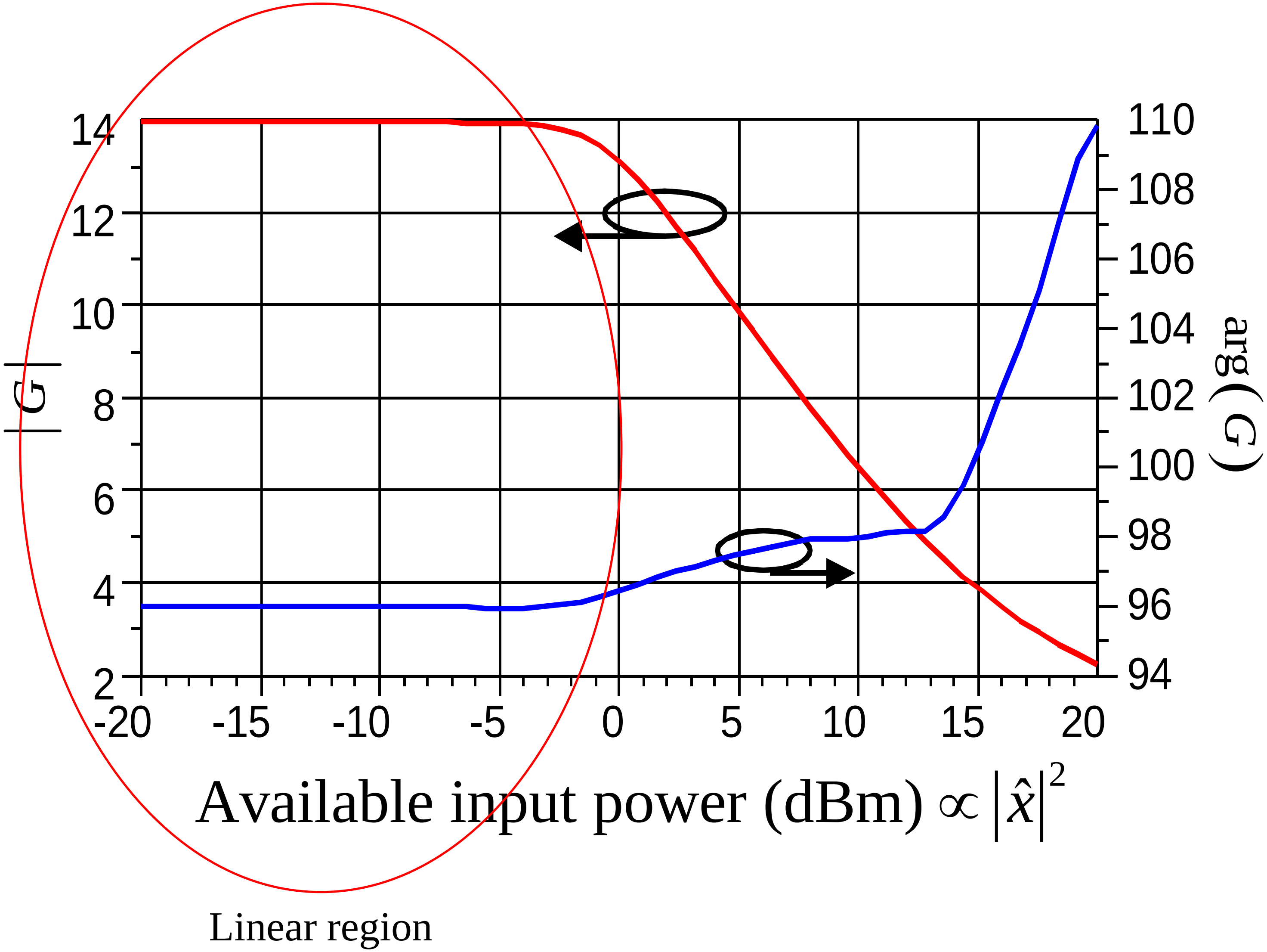
$$\text{input} \rightarrow x(t) = \mathrm{Re}\left\{\hat{x}(t)\exp(j\omega_c t)\right\}$$

$$\text{output} \rightarrow y(t) = \mathrm{Re}\left\{\hat{y}(t)\exp(j\omega_c t)\right\}$$

$$\hat{y}(t) = G\left(\left|\hat{x}(t)\right|\right)\hat{x}(t)$$

- ***G*** is the complex “descriptive function” (of a real variable) relating the input and output signal **envelopes**.
- Note that the **input envelope** can be a **constant + a sinusoidal signal** so that the total input is made of **two frequencies**.
- The model accounts for **gain compression** and **intermodulation distortion**

Descriptive function - example



Effect of out-of-band oscillations

- Suppose the signal input *x* of the amplifier is affected by a large interferer *X* generated by self-oscillations:

$$\hat{x}_{tot} = \hat{x}(t) + \hat{X}(t)$$

- While the gain for *x* is large, the gain for *X* is small:

$$G\left(\left|\hat{x}(t)\right|\right) = G_{ss}, \qquad G\left(\left|\hat{X}(t)\right|\right) = G_1, \qquad \left|G_1\right| \ll \left|G_{ss}\right|$$

- The gain for the “good” part of the total signal will be also small → the amplifier is desensitized, i.e. dominated by out-of-band oscillations:

$$G\left(\left|\hat{x} + \hat{X}\right|\right) \approx G_1 \rightarrow \hat{y} \approx \underbrace{G_1 \hat{x}(t)}_{\text{negligible}} + \underbrace{G_1 \hat{X}(t)}_{\text{dominant}}$$

Summary

- Stability
- **Stability criteria**
- Examples

Stability criteria (classical)

- It can be shown that a necessary and sufficient condition for stability is the set made by the following two conditions (other equivalent choices exist for the second condition):

$$\boxed{K = \frac{1 - |S_{22}|^2 - |S_{11}|^2 + |\Delta_S|^2}{2\,|S_{21}S_{12}|} > 1}$$

$$\boxed{|\Delta_S| \;<\; 1}$$

Example: reactive two-port

- For a reactive two-port $SS^* = 1$ since the S-matrix is symmetrical
- It follows $|\Delta_S|^2 = 1$ (determinant of product of two matrices)
- Moreover expanding $SS^* = 1$ one has:

$$|S_{11}|^2 + |S_{12}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

- Thus:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta_S|^2}{2|S_{12}|^2} =$$

$$= \frac{1 - (1 - |S_{12}|^2) - (1 - |S_{12}|^2) + 1}{2|S_{12}|^2} = 1$$

Single-parameter stability criterion

- More recently a single-parameter, completely equivalent stability criterion was introduced, based on verifying **one** of the two following conditions (the other one follows):

$$\mu_1 = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta_S|+|S_{12}S_{21}|} > 1$$

$$\mu_2 = \frac{1-|S_{22}|^2}{|S_{11}-S_{22}^*\Delta_S|+|S_{12}S_{21}|} > 1$$

Single-frequency stability: examples

	$\lvert S_{11}\rvert$	ph(S_{11}), degrees	$\lvert S_{12}\rvert$	ph(S_{12}), degrees	$\lvert S_{21}\rvert$	ph(S_{21}), degrees	$\lvert S_{22}\rvert$	ph(S_{22}), degrees
1	0.2	20	0.05	120	3	30	0.5	-50
2	0.75	-60	0.3	70	6	90	0.5	60
3	1.05	20	0.05	120	3	40	0.5	-50
4	0.5	0	0.025	180	2	0	0.1	0
5	0.95	-22	0.04	80	3.5	165	0.61	-13
6	0.69	-123	0.11	48	1.29	78	0.52	-77
7	0.1	0	0	0	0	0	0.3	0
8	1.2	0	0	0	0	0	0.3	0
9	0.1	0	0	0	0	0	1.3	0

Stable or potentially unstable?

	K	$\lvert\Delta_S\rvert$	TYPE	$\lvert\Gamma_{Gopt}\rvert$	ph(Γ_{Gopt}), degrees	$\lvert\Gamma_{Lopt}\rvert$	ph(Γ_{Lopt}), degrees	G_{MAX}, dB
1	2.57	0.249	ST	0.10	-20	0.48	50	10.8
2	1.34	2.156	UNST	undef.	undef.	undef.	undef.	undef.
3	0.34	0.673	UNST	undef.	undef.	undef.	undef.	undef.
4	7.50	0.1	ST	0.50	0	0.07	0	7.3
5	0.19	0.572	UNST	undef.	undef.	undef.	undef.	undef.
6	1.12	0.254	ST	0.88	127	0.82	86	8.6
7	$+\infty$	0.03	ST	0.1	0	0.3	0	0
8	$-\infty$	0.36	UNST	undef.	undef.	undef.	undef.	undef.
9	$-\infty$	0.13	UNST	undef.	undef.	undef.	undef.	undef.

Stability as a function of frequency

- In general stability changes with frequency, at low frequency devices have high gain and are potentially more unstable, gain decreases with frequency finally leading to an unconditionally stable device
- In the transition from the stable to the potentially unstable regions there is a frequency at which K=1 → at that frequency MAG=MSG
- In the potentially unstable frequency range MSG is used as a figure of merit
- High-frequency devices typically need low-frequency stabilization to prevent out-of-band oscillations

Summary

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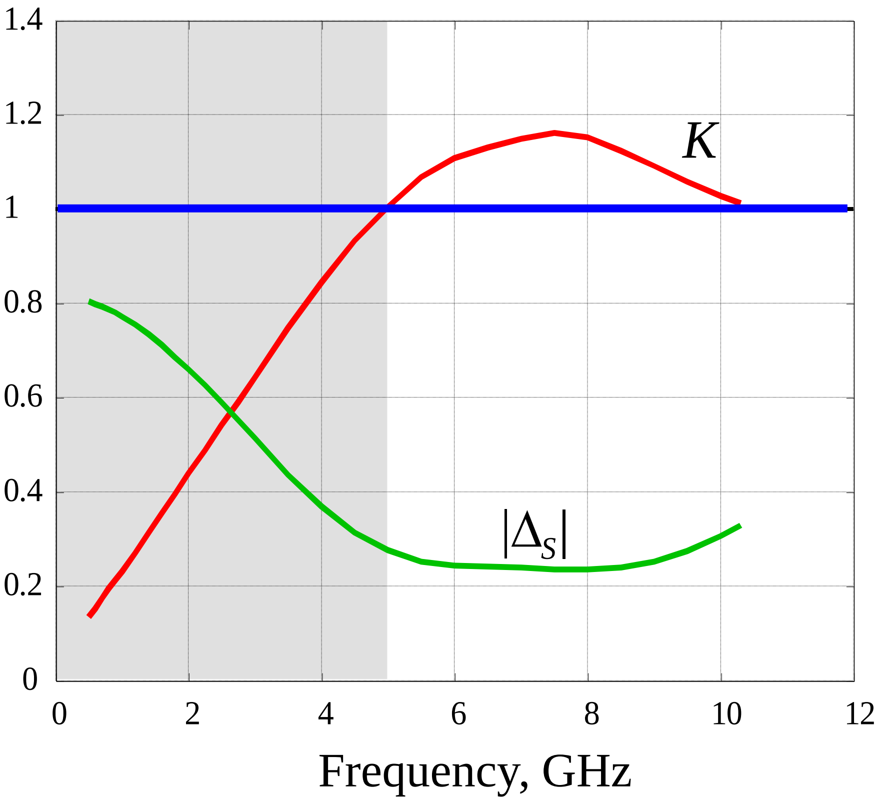
- **Examples**

Stability vs. frequency in a MESFET



f, GHz	$\lvert S_{11}\rvert$	ϕ_{11}	$\lvert S_{21}\rvert$	ϕ_{21}	$\lvert S_{12}\rvert$	ϕ_{12}	$\lvert S_{22}\rvert$	ϕ_{22}
1.000	0.949	-29.8	4.825	151.1	0.038	72.1	0.781	-14.4
2.000	0.821	-59.8	4.531	123.8	0.070	56.0	0.696	-28.9
3.000	0.648	-94.2	4.092	97.6	0.092	41.4	0.600	-42.4
4.000	0.512	-133.0	3.516	73.9	0.102	30.5	0.518	-51.8
5.000	0.472	-165.2	3.025	54.7	0.108	25.3	0.444	-57.8
6.000	0.464	176.0	2.714	38.4	0.118	23.7	0.367	-65.4
7.000	0.441	158.2	2.505	22.1	0.134	20.2	0.302	-80.8
8.000	0.411	127.5	2.321	4.0	0.151	15.0	0.281	-105.9
9.000	0.454	91.4	2.093	-15.1	0.168	7.0	0.300	-134.2
10.000	0.551	66.6	1.836	-34.5	0.181	-2.8	0.328	-169.8

Gain parameters versus frequency