Microwave Measurements

Microwave Electronics

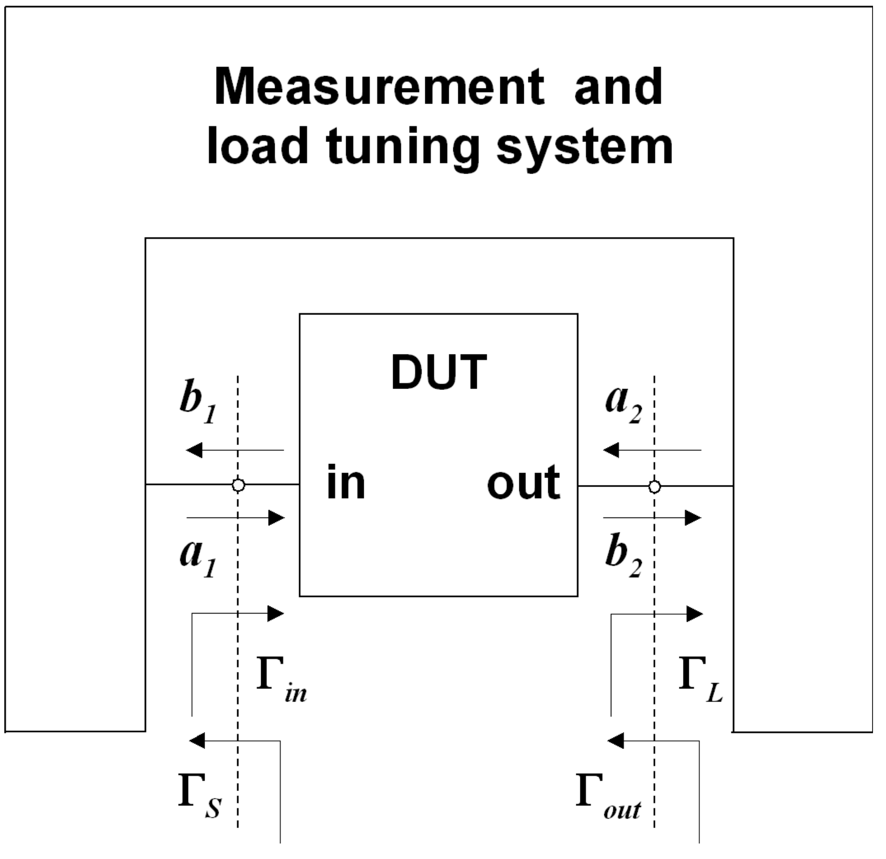
Giovanni Ghione, Marco Pirola

Politecnico di Torino, DET

Summary

- Introduction
- Linear and Non linear Measurement concepts
- Load/source-pull fundamentals
- Scalar and real time load-pull systems
- Passive and Active load-pull systems
- Harmonic load-pull systems
- Advanced NL Characterization techniques
- Conclusions

Some Definitions



- Reflection coefficients
  - » $\Gamma_{in} \equiv b_1/a_1$
  - » $\Gamma_L \equiv a_2/b_2$
- Input and output power
  - » $P_{av} \equiv |a_1|^2/(1-|\Gamma_S|^2)$
  - » $P_{in} \equiv |a_1|^2(1-|\Gamma_{in}|^2)$
  - » $P_{out} \equiv |b_2|^2(1-|\Gamma_L|^2)$
- Gains and PAE
  - » $G_{av} \equiv P_{out}/P_{av}$
  - » $G_{op} \equiv P_{out}/P_{in}$
  - » $PAE \equiv (P_{out}-P_{in})/P_{DC}$

Our Devices Under Test

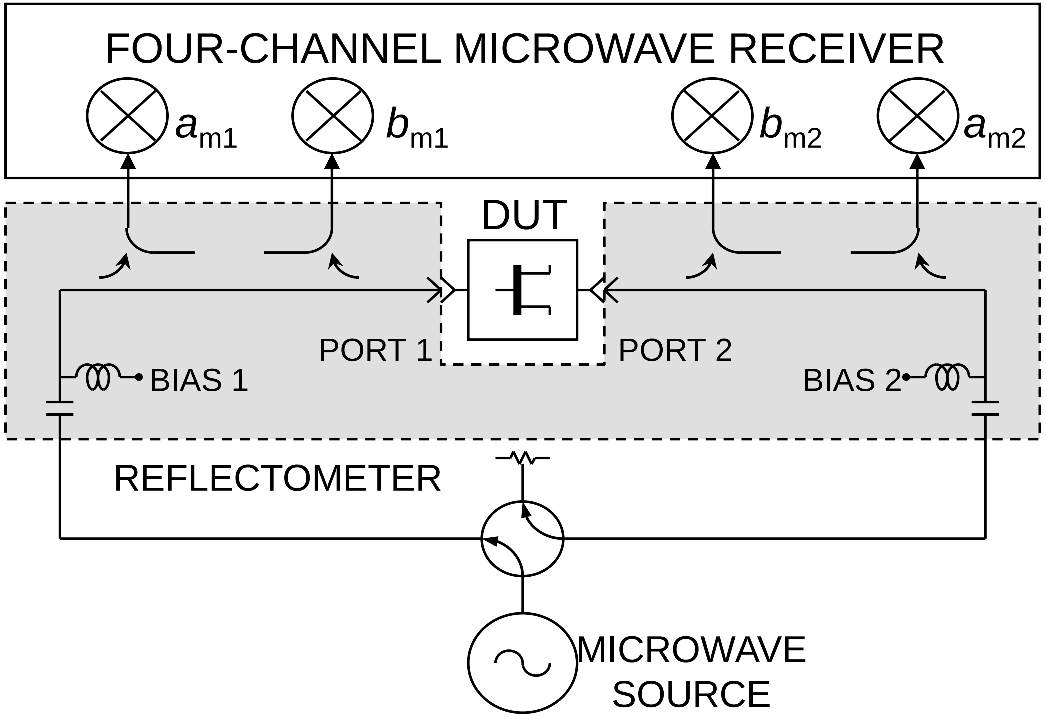
- Passive elements (Linear Characterization)
  - can be considered linear → S-parameters vs. frequency enough for complete DUT characterization.
- Active elements (NonLinear - Power Characterization)
  - mildly or deeply driven in nonlinear regime → spectral components arise (harmonics, intermodulation products) not present in the stimulus;
  - input and output terminations deeply affect the nonlinear behaviour (e.g. P_{OUT} and PAE);
  - harmonic termination control needed e.g. in class F amplifier design based on measurements.

Our Measurements

- Linear
  - VNA based → accurate, simple, fast, repeatable;
  - relative measurements → only ratios between causes and effects;
  - consolidated since many years;
  - measurements (Scattering Parameters) on 50 Ω completely determine the behaviour for whatsoever source/load terminations;
- Non-Linear
  - Rather new characterizations;
  - absolute measurements;
  - spectrum, P_{IN}, P_{OUT}, PAE, source and load termination at the DUT ports both at fundamental and at the harmonics;
  - sharing the same NVA to preserve its intrinsic accuracy, dynamic range, by taking a small part of the signal, etc.;
  - Load/Source Terminations (fundamental and harmonics) strongly affect the amplifier performances.

Vector Network Analyzer (VNA)

- Key element for Linear and Non-linear high frequency (RF, microwave, mm-wave,…) measurements
- Amp & phase (e.g. vector) frequency voltage measurements



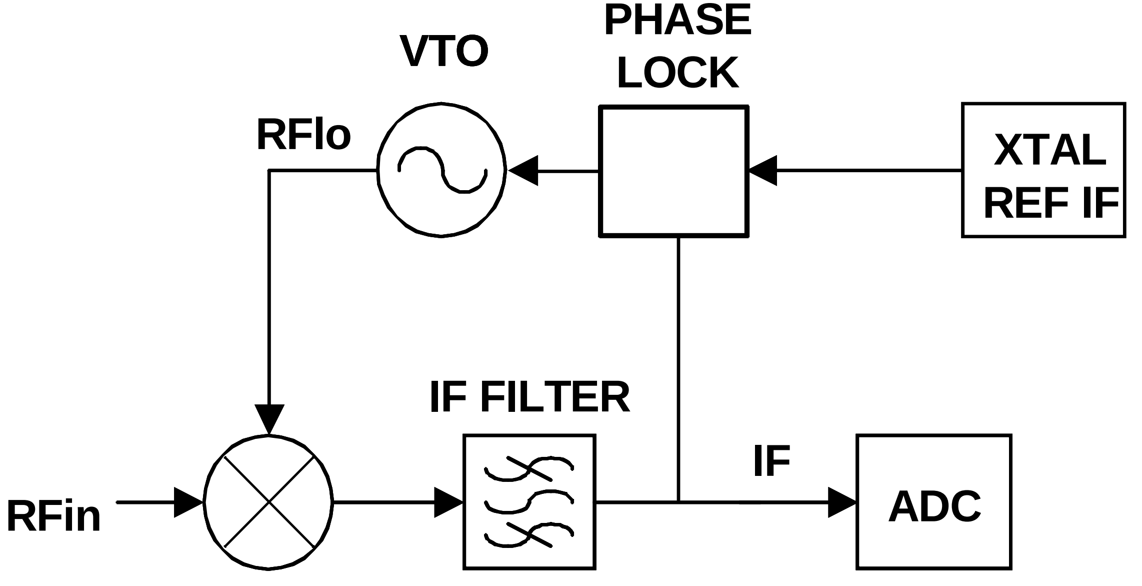
Everything looks easy, but...

- Direct phase measurements unfeasible at RF (GHz)
  - ❑ Signals *synchronously* downconverted→phases and moduli measured at IF (ratios only matter)

- Directional couplers with finite directivity and frequency dependent behavior
  - ❑ Systematic errors→ Error Box Model → calibration algorithms → calibration standards

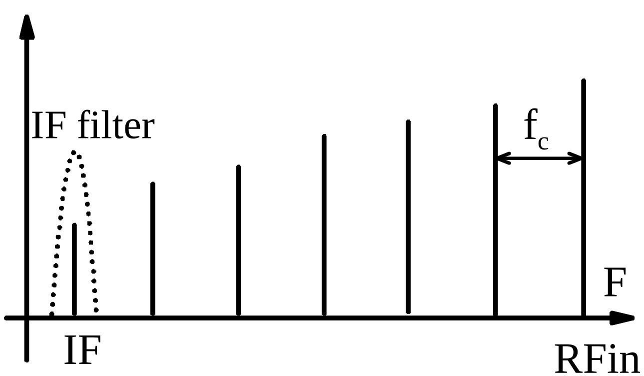
Mixer based downconversion

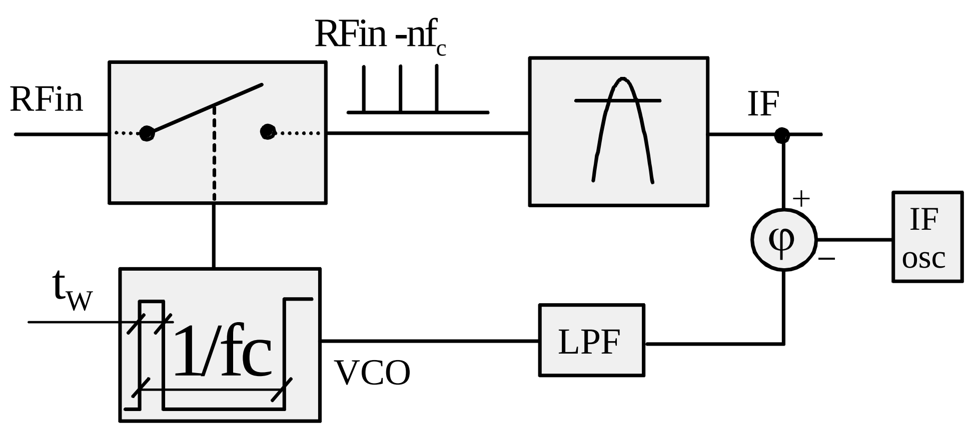
- Two RF sources needed: RFin and RFlo
- RFlo *tracked* to RFin
- In first VNAs dynamic range and noise were issues
- MIXER technology today greatly improved →issues solved→ preferred solution by all VNAs producers



Sampler based downconversion

- RFin sampled at f_c<<2RFin (*sub-Nyquist* sampling)
- Comb sampled signal with lines equispaced by f_c
- $f_c \approx$ MHz, $t_W < 5$ ps, non critical f_c but ultrafast t_W
- VCO range to ensure line at IF (e.g. 20 – 100 MHz)
- With respect to mixer solution: cheap, no local oscillator a RFlo (only RFin)
- preferred in the first VNAs, today almost abandoned





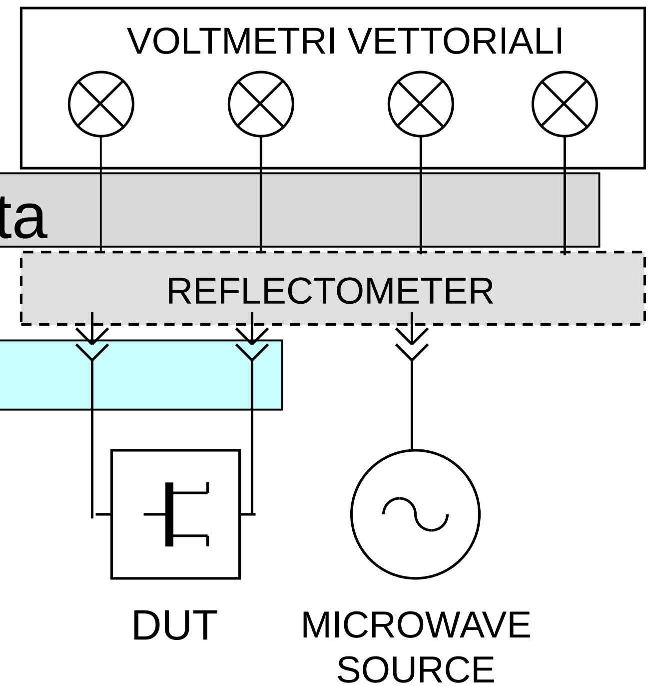
Systematic Error Correction: *Calibration*

Fundamental problem at high frequency

starting from the measured ***raw*** data

FIND

those at the ***DUT*** ports



How to proceed:

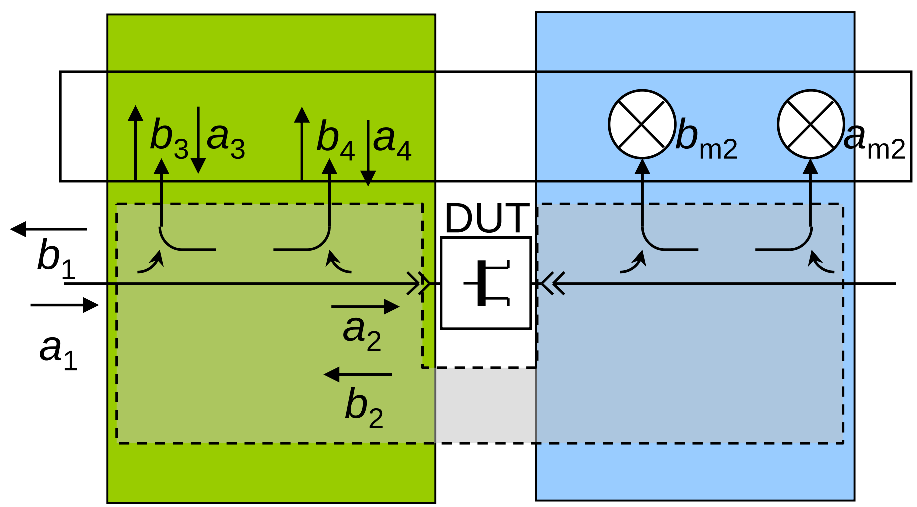
1. Define the error model
2. Find the *calibration* procedure
3. De-embedding of the raw data

Calibration Error Model Definition I

- Assumptions
  1. all reflectometer components **linear and invariant**
  2. Parts ▇ and ▇ of the reflectometer only *talk* through the DUT (leakless VNAs) and can hence be handled separately
- Lets consider half refectometer part ▇
  1. 8 unknowns: a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4
  2. Vector voltmeter readings $\propto b_3, b_4$

$$V_{m1}=k_1 b_3\,,\; V_{m2}=k_2 b_4$$

$$a_{m1}=V_{m1},\; b_{m1}=V_{m2}$$



Calibration Error Model Definition II

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Half reflectometer ■ constitutive equations

Voltmeter

$$a_3 = \Gamma_3 b_3 \qquad a_4 = \Gamma_4 b_4$$

reflections

readings

$$a_{m1} = k_3 b_3 \qquad b_{m1} = k_4 b_4$$

8 equations with 10 unknowns, 8 can be parameterized as functions of the measured waves a_{m1} and b_{m1}

Deriving $a_{DUT}=a_2(a_{m1},b_{m1})$ and $b_{DUT}=b_2(a_{m1},b_{m1})$ the model of half ■ reflectometer is found

Calibration Error Model Definition III

$$\begin{bmatrix} b_{1DUT} \\ a_{1DUT} \end{bmatrix} = \begin{pmatrix} T_{A11} & T_{A12} \\ T_{A21} & T_{A22} \end{pmatrix}^{-1} \begin{bmatrix} b_{1m} \\ a_{1m} \end{bmatrix} = T_A^{-1} \begin{bmatrix} b_{1m} \\ a_{1m} \end{bmatrix}$$

$$\begin{bmatrix} a_{2DUT} \\ b_{2DUT} \end{bmatrix} = \begin{pmatrix} T_{B11} & T_{B12} \\ T_{B21} & T_{B22} \end{pmatrix}^{-1} \begin{bmatrix} a_{2m} \\ b_{2m} \end{bmatrix} = T_B^{-1} \begin{bmatrix} a_{2m} \\ b_{2m} \end{bmatrix}$$

T_A, T_B equivalent of a 2port transmission matrix

T_{DUT} is the DUT representation in terms of transmission matrix

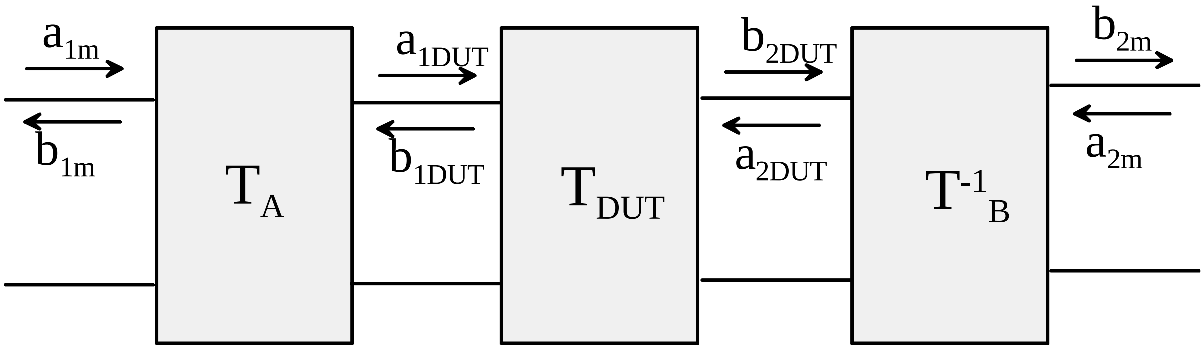
$$\begin{bmatrix} b_{1DUT} \\ a_{1DUT} \end{bmatrix} = T_{DUT} \begin{bmatrix} a_{2m} \\ b_{2m} \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} b_{1m} \\ a_{1m} \end{bmatrix} = T_A T_{DUT} T_B^{-1} \begin{bmatrix} a_{2m} \\ b_{2m} \end{bmatrix}}$$

Calibration Error Model Definition IV

- Analitical Model:

$$\begin{bmatrix} b_{1m} \\ a_{1m} \end{bmatrix} = T_A T_{DUT} T_B^{-1} \begin{bmatrix} a_{2m} \\ b_{2m} \end{bmatrix}$$

- Electrical Model:

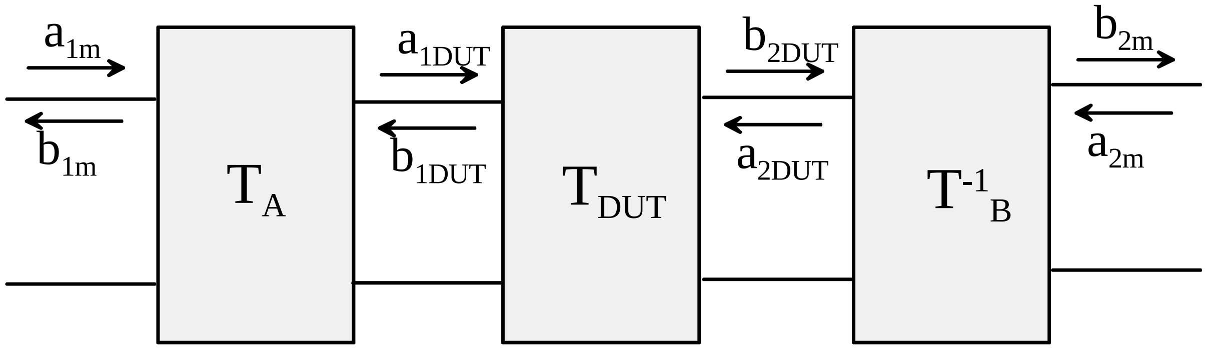


- T_A, T_B **Error Box**

T_A, T_B virtual 2ports linking the wanted waves at the DUT ports and those read by the instrumentation

a_{1m}, b_{1m}, a_{2m}, b_{2m}

VNA Calibration I



- Measure defined and well known DUT (standards) to identify error box model matrices T_A and T_B
- Once T_A and T_B have been found the T_{DUT} of an arbitrary DUT can be measured (deeembedded fro the measured *raw* data)

$$T_{DUT} = T_A^{-1} T_m T_B$$

- Calibration procedure must be carried out to find T_A e T_B for any frequency of the characterization

VNA Calibration II

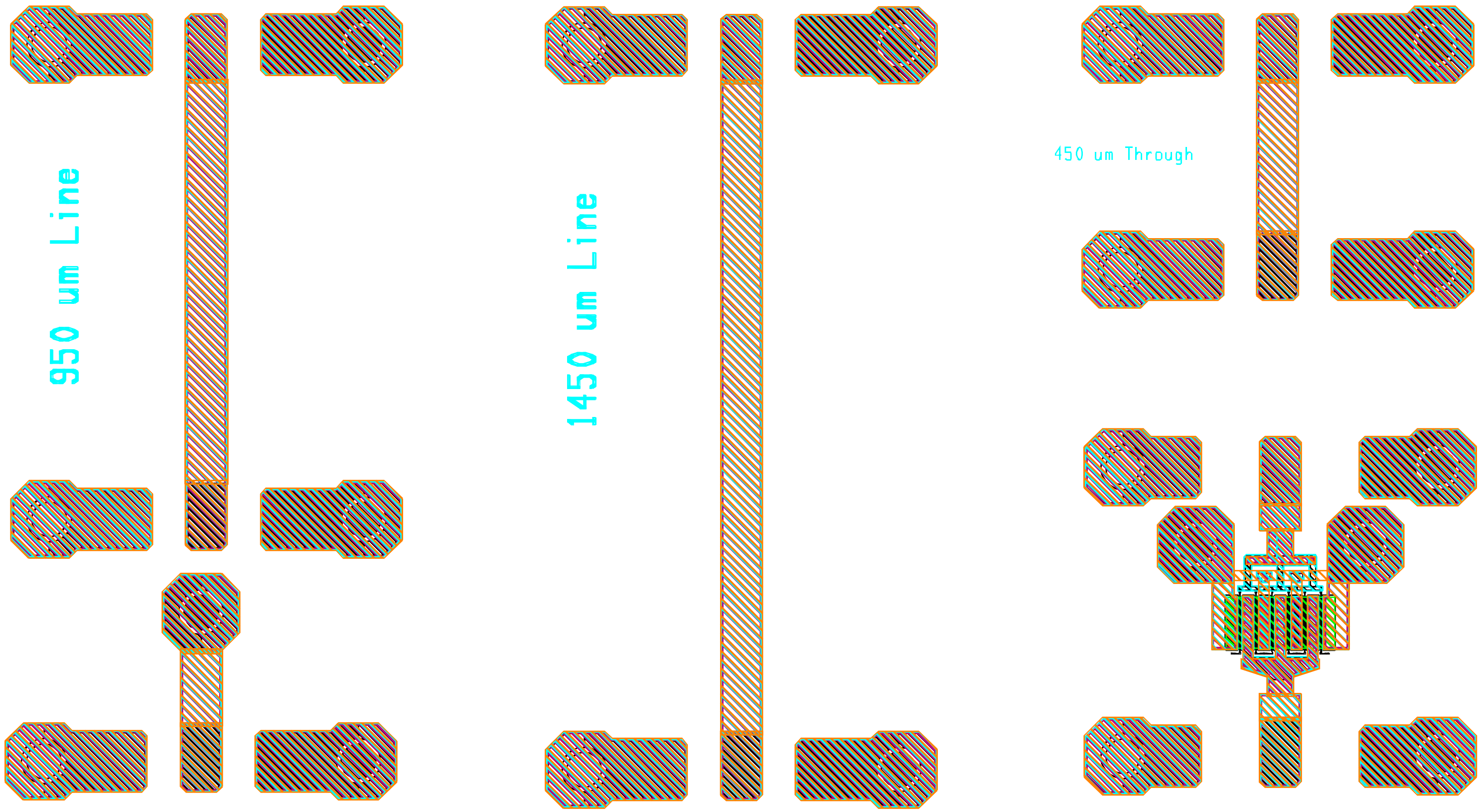
- The two error boxes have 8 parameters to be identified measuring the calibration standards
- The standards (e.g. short, open, line) force a well defined wave **ratios (ratiometric standars)** at their ports
- Calibration sets the **ratio** among the power waves at the DUT port and one chosen as the reference
- The error box parameters to be identified reduce to **7**
- At least 7 independent measurements as a function of the frequency are needed for the calibration
- The several calibration algorithms differ according to the number and kind of the adopted standards
- VNA calibration algorithms use one-port standards and at least a two-port standards (transmission measures)

Most Used Calibration techniques

- **SOLT** (Short, Open, Load, Thru): mandatory for old/cheap 3 sampler VNAs
- **TSD-TRL** (Thru, Short, Delay o Thru, Reflect, Line): used for on wafer and on guide calibrations
- **LRM** (Line, Reflect, Match): the today preferred for on wafer calibrations
- **SOLR** (Short, Open, Load, Reciprocal) allows the calibration of DUT with non compatible ports, e.g different connectors (sex and kind)

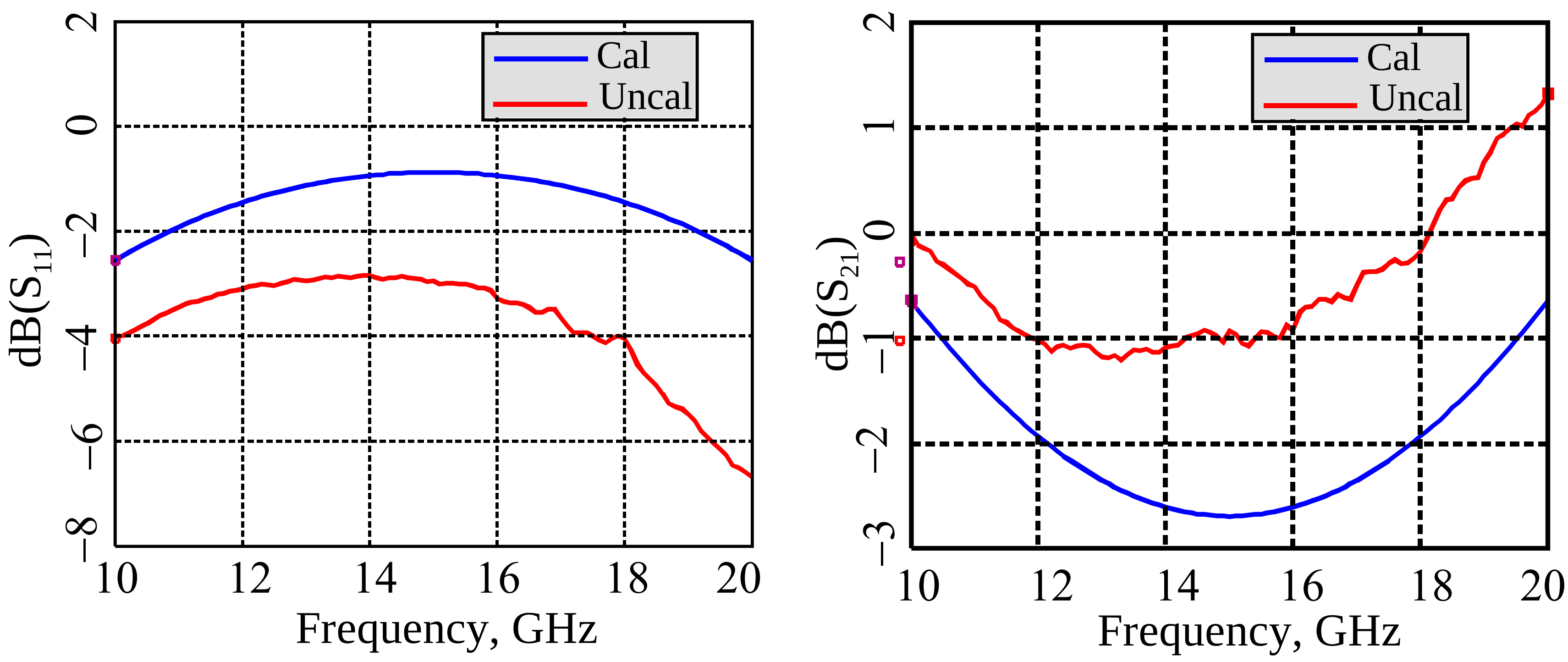
Example of on-wafer standards and DUT

Through Long Line Short Line



Short On wafer HEMT

Comparison of Calibrated and Raw data



NonLinear characterization VS Design

- Indirect design info
  - Use NL data for the model set-up and verification

→

- Model based design
  - ☺ Bias point refinement
  - ☺ Use scalable device
  - ☹ NonLinear cumbersome Optimization (ADS, AWR,…)

- Direct design info
  - Experimentally evaluate the optimum loads

→

- Meas. based design
  - ☺ High Accuracy
  - ☺ Simple CAD for load synthesize
  - ☹ Fixed Bias point
  - ☹ Difficult scalability

NL Amplifier Measurements System

- The load/source, fund/harm terminations determine the amplifier behaviour and performances
- 50 Ω fixed impedance systems
  - Of some help for performance analysis but low/no added value to the designer
  - Low cost, straightforward usage
  - Power Meter and Spectrum Analyzer Based
- Load-pull systems
  - Of great help to the designer
  - High Cost, difficult usage, skilled/trained staff
  - VNA based

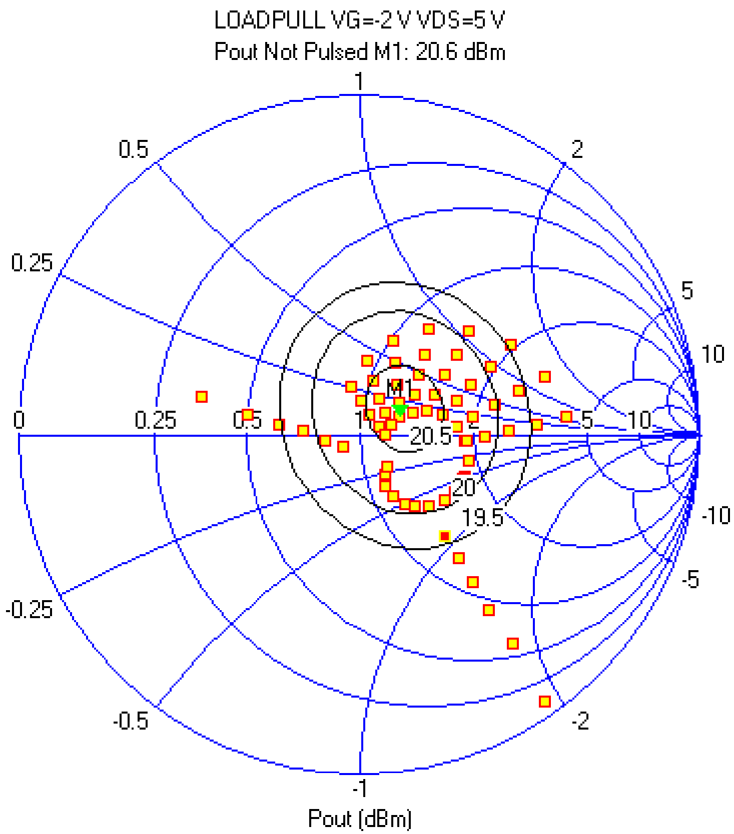
Some definitions

- Load-pull : The output termination of an active device is controlled at the output port
- Source-pull : The input termination of an active device is controlled at the input port
- Fundamental load/source-pull : Load/source are controlled only at the fundamental frequency
- Harmonic load/source-pull : Load/source controlled at fundamental and one or more harmonic frequencies

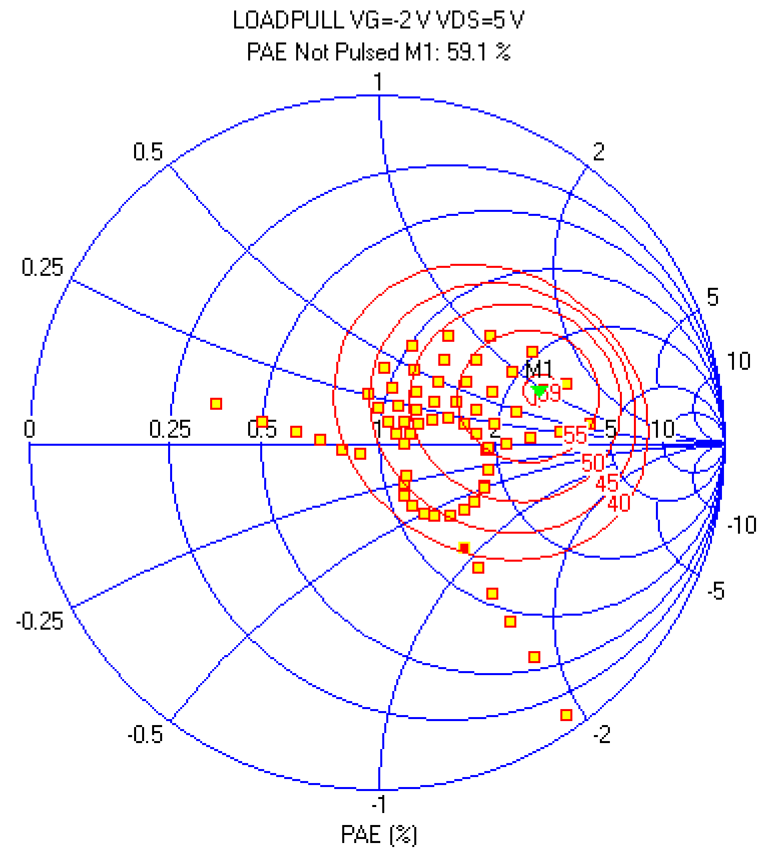
Load/Source-Pull Systems

- Capabilities to explore the complete Smith Chart
- Stability, resolution, high power handling capabilities
- Scalar (power meter based) systems
  - Information on scalar quantity only, low accuracy, cheap
- Vectorial (VNA based) systems
  - Modulus and phase DUT information VS termination conditions
  - High accuracy thanks to the use of calibrated VNAs
  - exensive
- Non Real Time (power measurements based)
  - Non real time characterization, accuracy based on tuner preliminary calibration
  - Lower-cost
- Real Time (Vector VNA measurements based)
  - Vector and more complete information on DUT performances
  - High accuracy, thanks to vector calibration
  - Higher Cost

Example of LP data meas. on a power MESFET



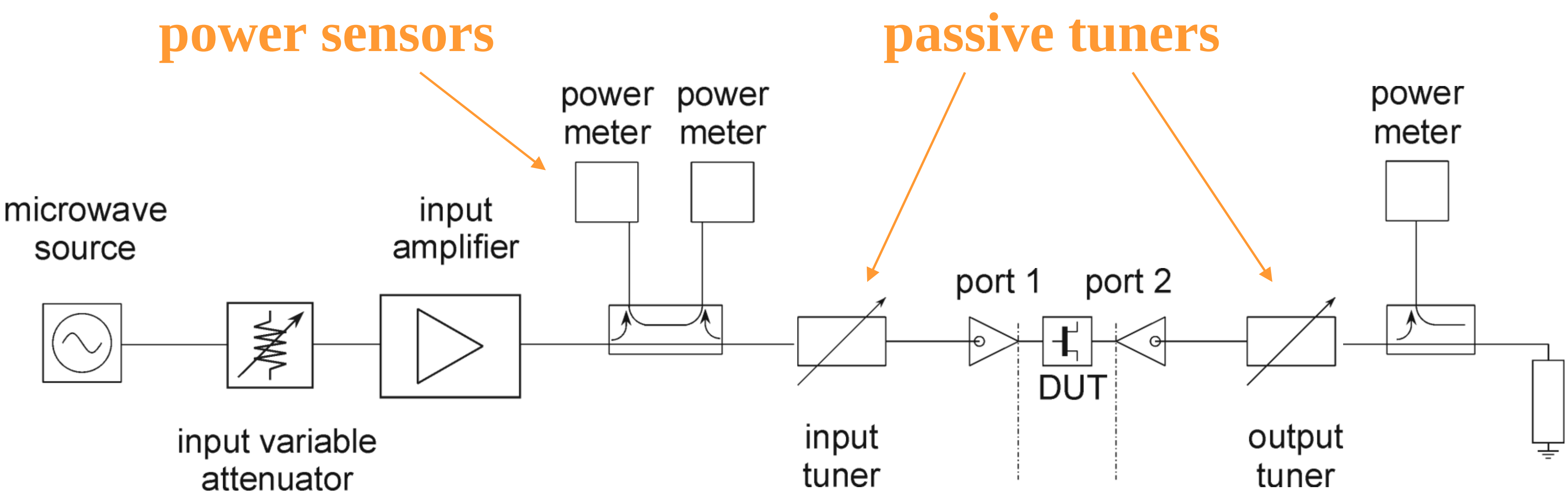
Output Power, dBm
@ 1dB gain compression



Power Added Efficiency PAE
@ 2dB gain compression

Non real-time load-pull systems I

- Passive loads
  - Mechanical tuners
  - Electronic tuners (PIN diode-based)



Non real-time load-pull systems II

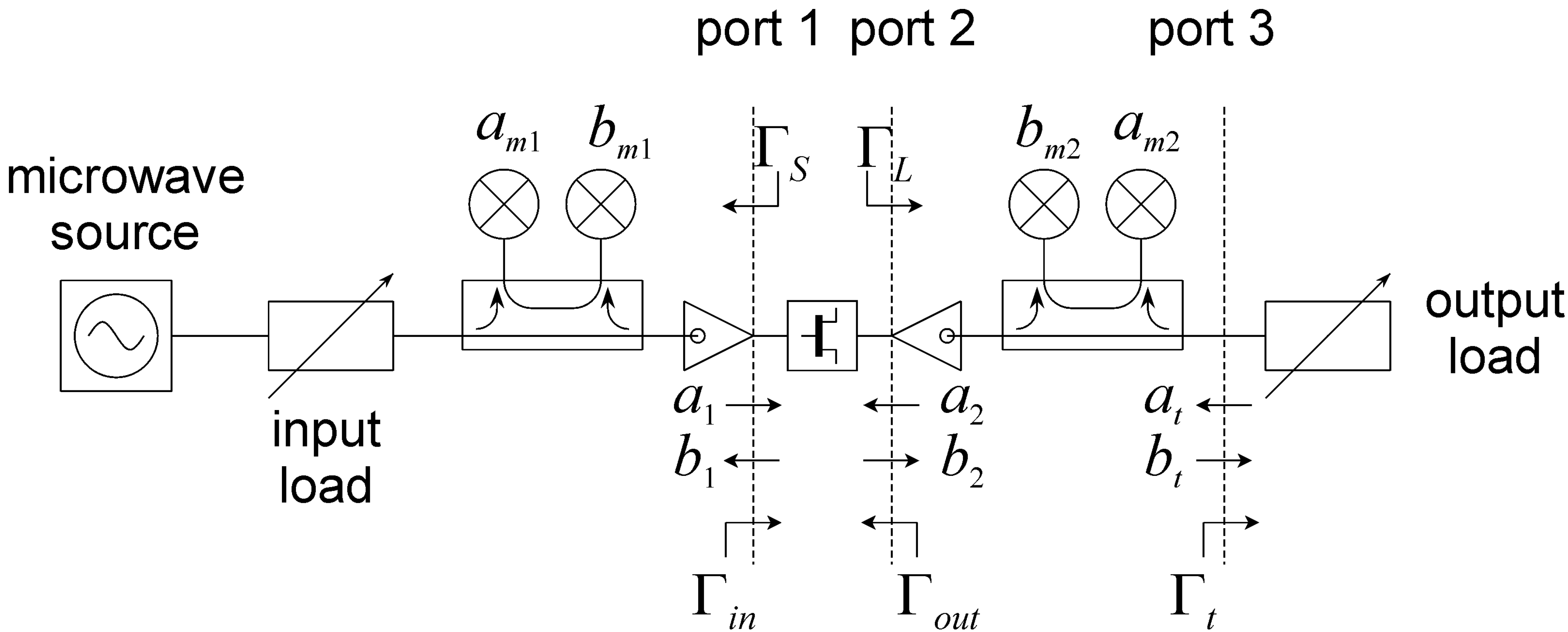
☺PRO

- High power handling
- Relatively inexpensive

☹CON

- Tuner pre-characterization with a network analyzer required
- High repeatability tuner behaviour as a function of position

Real-Time VNA-based LP system I



Real-Time VNA-based LP system II

☺PROS

- All complex wave simultaneously available
- $\Gamma_{LOAD/SOURCE}$ measured in real time: no precal required
- Active load available

☹CONS

- Rather expensive set-up
- Power calibration required
- Effective Γ attainable at the device port significantly less than the external one due to losses (active load to recover losses)

Non real-time load-pull systems II

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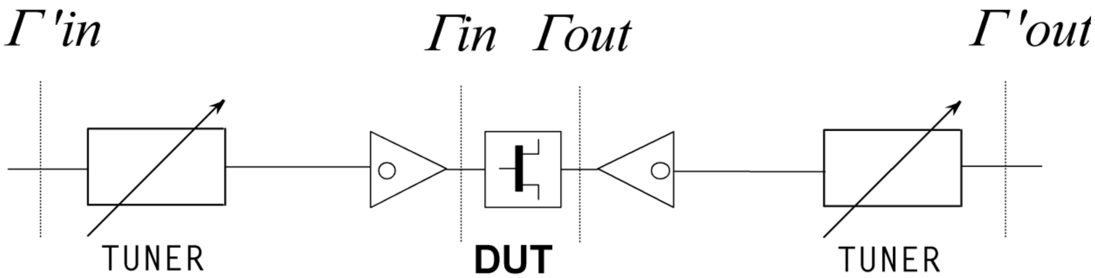
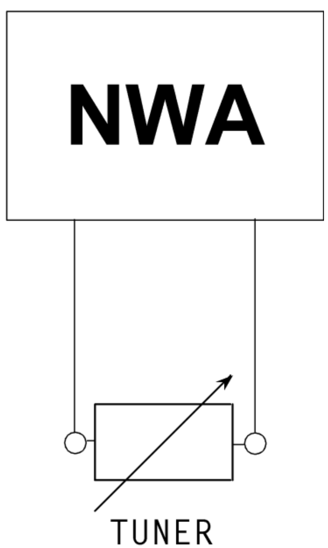
Non Real-Time LP Calibration

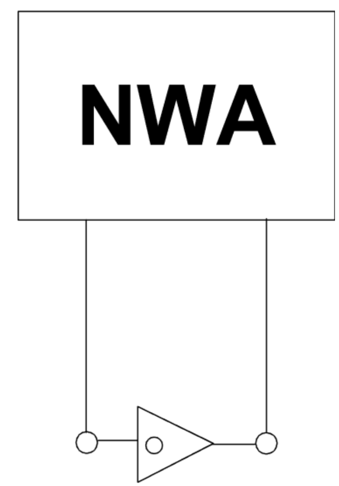


Passive system

Tuner Pre-calibration



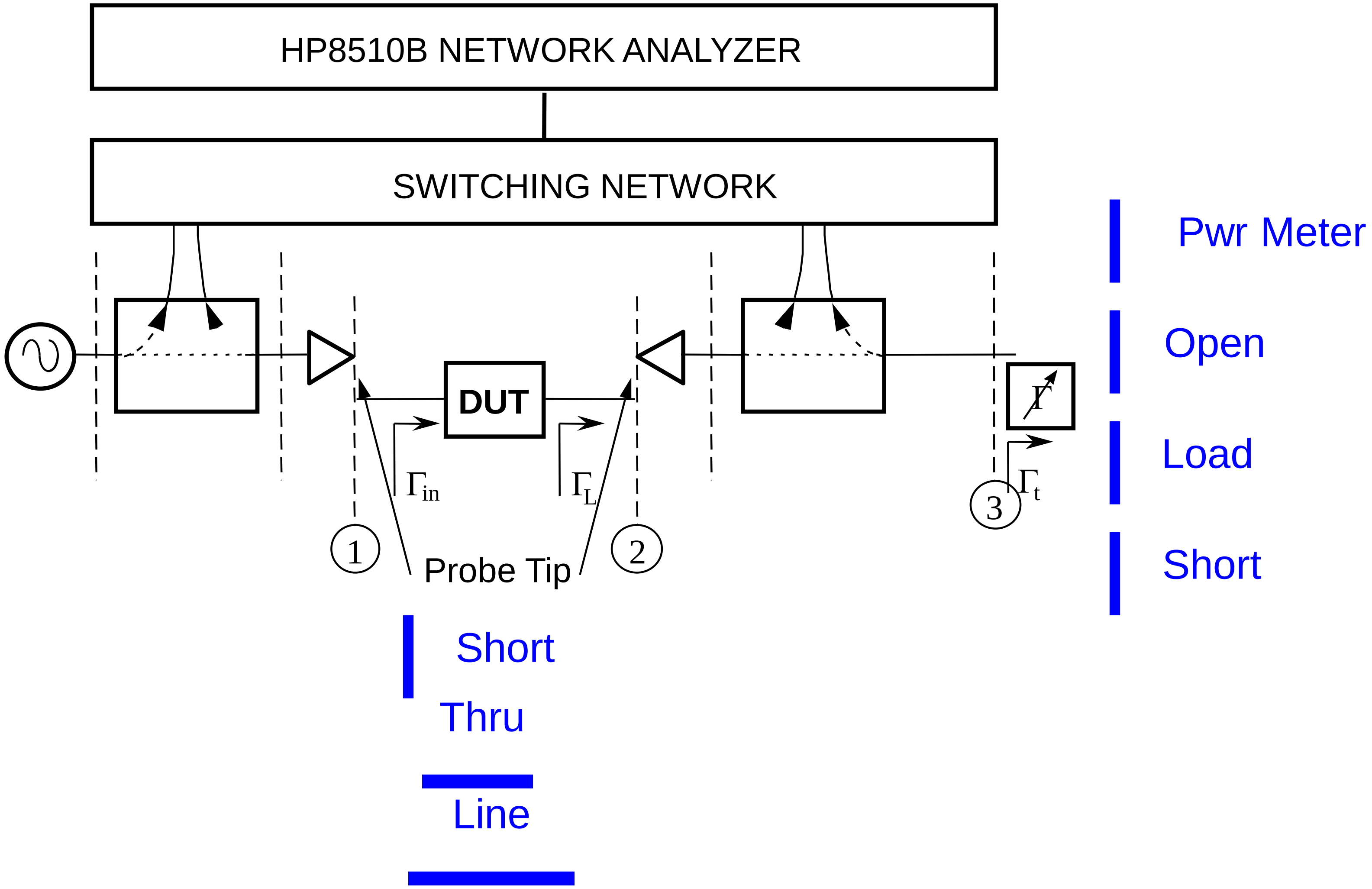
Probe de-embedding



Real-Time LP Calibration

- Load-Pull Calibrations are derived from more traditional *S*-parameter calibration like
  - Through Reflect Line(to obtain the best accuracy)
  - QSOLT (to accomplish traditional solution)
  - R Short Open Load (to avoid the thru)
  - Line Reflect Match (to obtain the best on wafer)
- An absolute power calibration is also needed

Real Time LP Calibration Example



Active and Passive Load-Pull Systems

- Passive loads:
  - Mechanical tuners with single or double slug
  - Electronic tuners (PIN diode-based)
  - Γ_{LOAD}<<1
  - Configuration intrinsically stable
- Active loads (Electronically Synthesized load):
  - Part of output power is amplified and injected back to the input/output device port
  - The phase load is controlled through mechanical/electronic phase shifter
  - Whatsoever load amplitude available, (Γ_{LOAD}>1)→ full coverage of theSmith Chart thank to the loop amplifier loss recovery
  - The loop must be carefully controlled to avoid oscillation

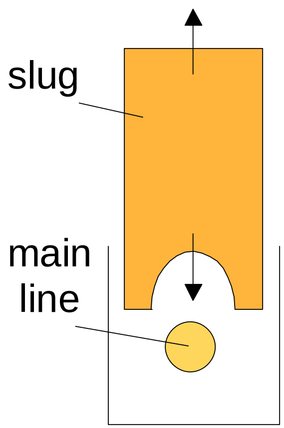
Traditional Passive Tuners I

- Features
  - Single or double slug tuners,
  - If harmonic, one tuner for each harmonic or harmonic resonator
  - Pre-characterization with a network analyzer
- Pros
  - High repeatability of tuner positions
  - High power handling
- Constraints
  - Load reflection coefficient limited in magnitude by tuner and test-set losses
  - Harmonic tuning difficult (higher frequency and optimum load on the edge of the Smith chart)
  - Cumbersome and long calibration procedure

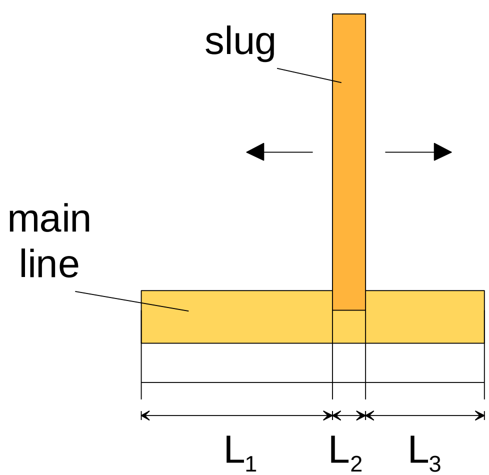
Traditional Passive Tuners II

- Sketch and electrical equivalent

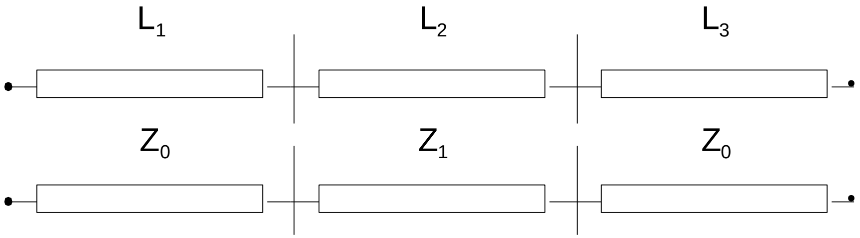
vertical movement (magnitude)



longitudinal movement (phase)

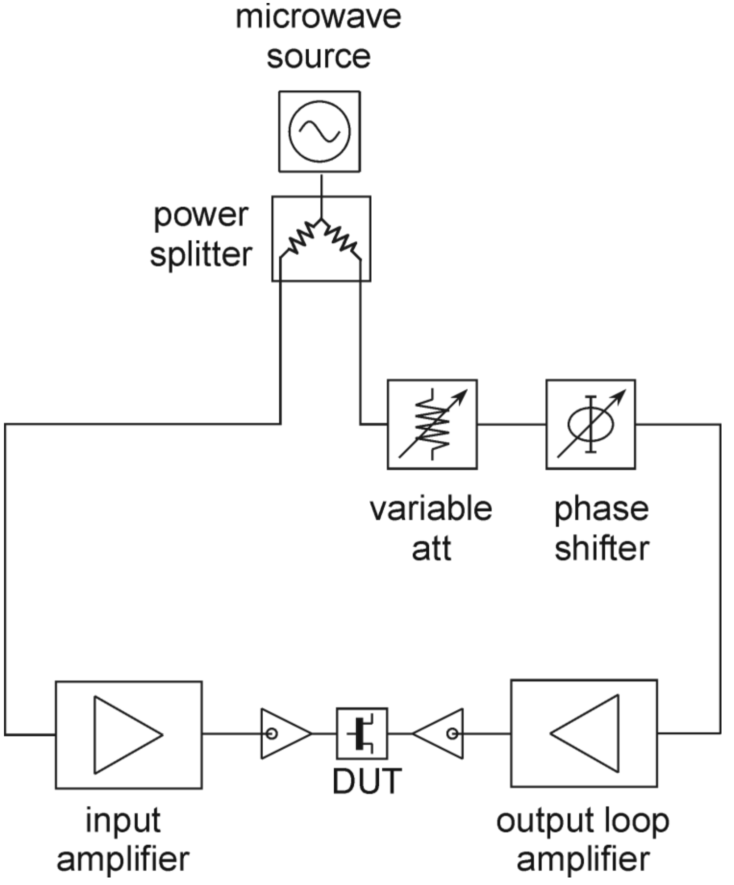


electrical equivalent



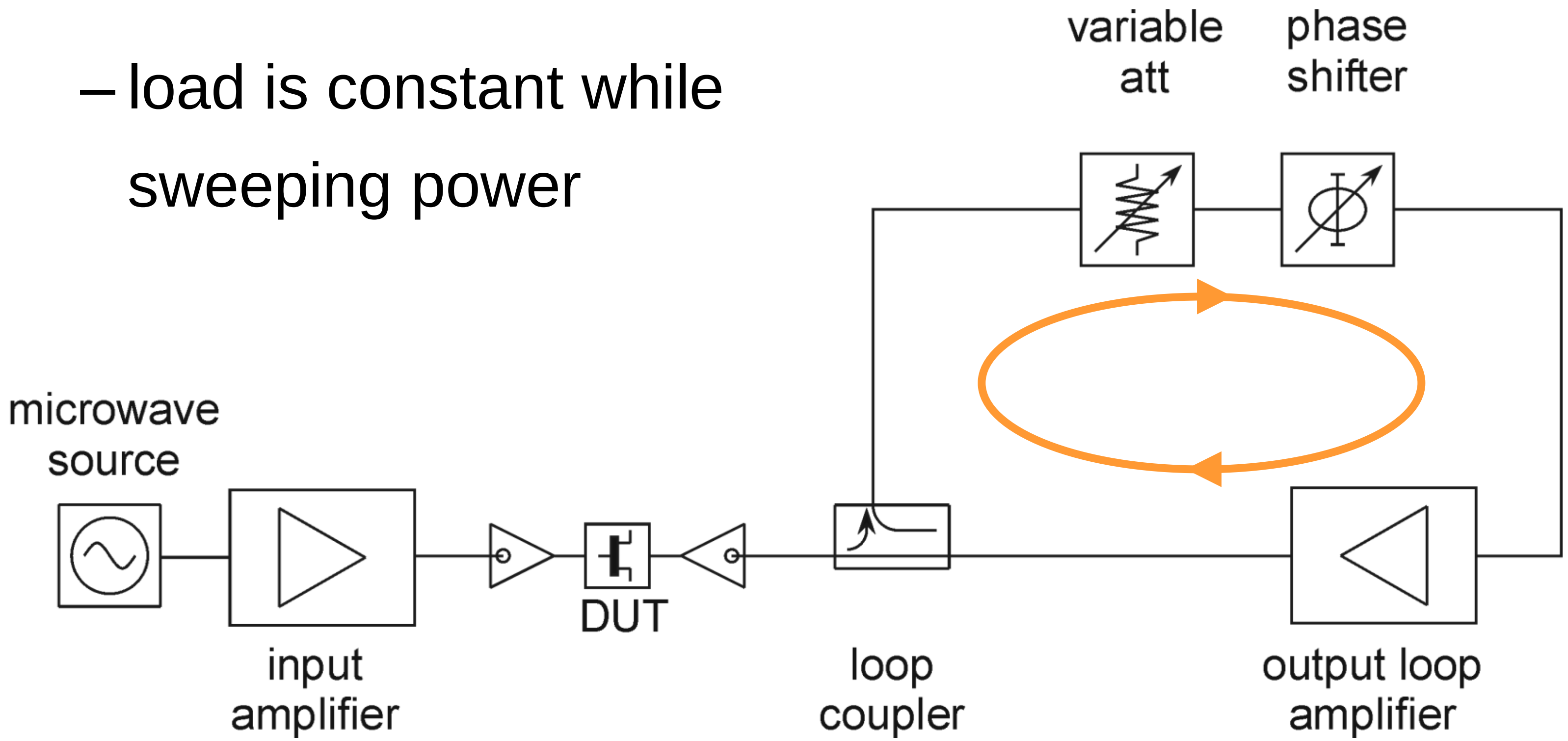
Active loads: two Signal Path

- Two signal path (Takayama)
  - Intrinsically stable, no oscillations
  - difficult to keep load constant while sweeping power



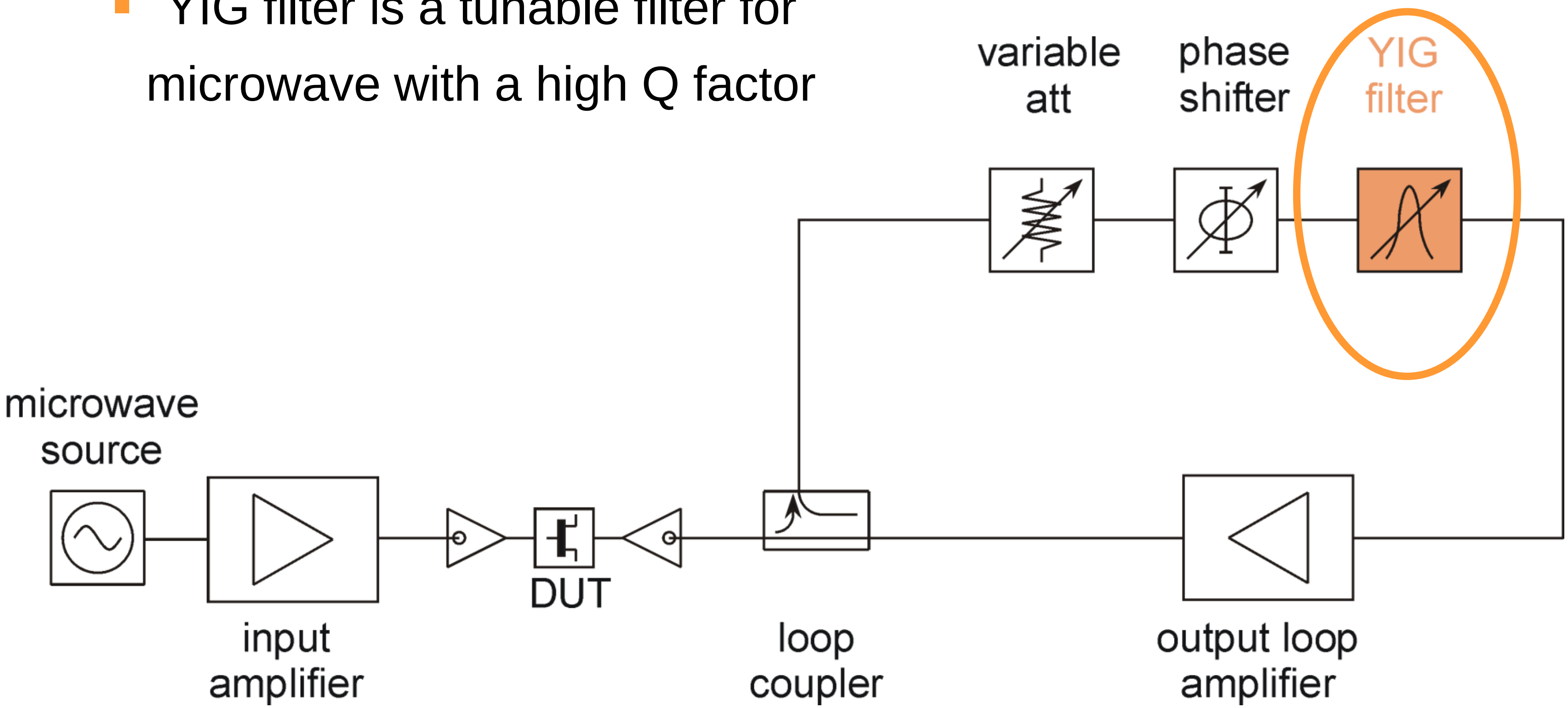
Active loads: active loop

- Active loop
  - risk of oscillations
  - load is constant while sweeping power



Active loads: active loop

- **Yttrium iron garnet filter (YIG) to avoid oscillations**
  - YIG filter is a tunable filter for microwave with a high Q factor



Load-pull Information

- Classical PA design information available:
  - Power Sweep
  - Optimum Loads
- Map based info
- Additional info with Active Real Time System
  - Gamma In
  - AM/PM conversion
- Optimum (P_{out}, Efficiency., …) harmonic load terminations

Load-pull and PA Design

Data Set Example

Microsoft Excel - temp.xls

File Edit View Insert Format Tools Data Window Help

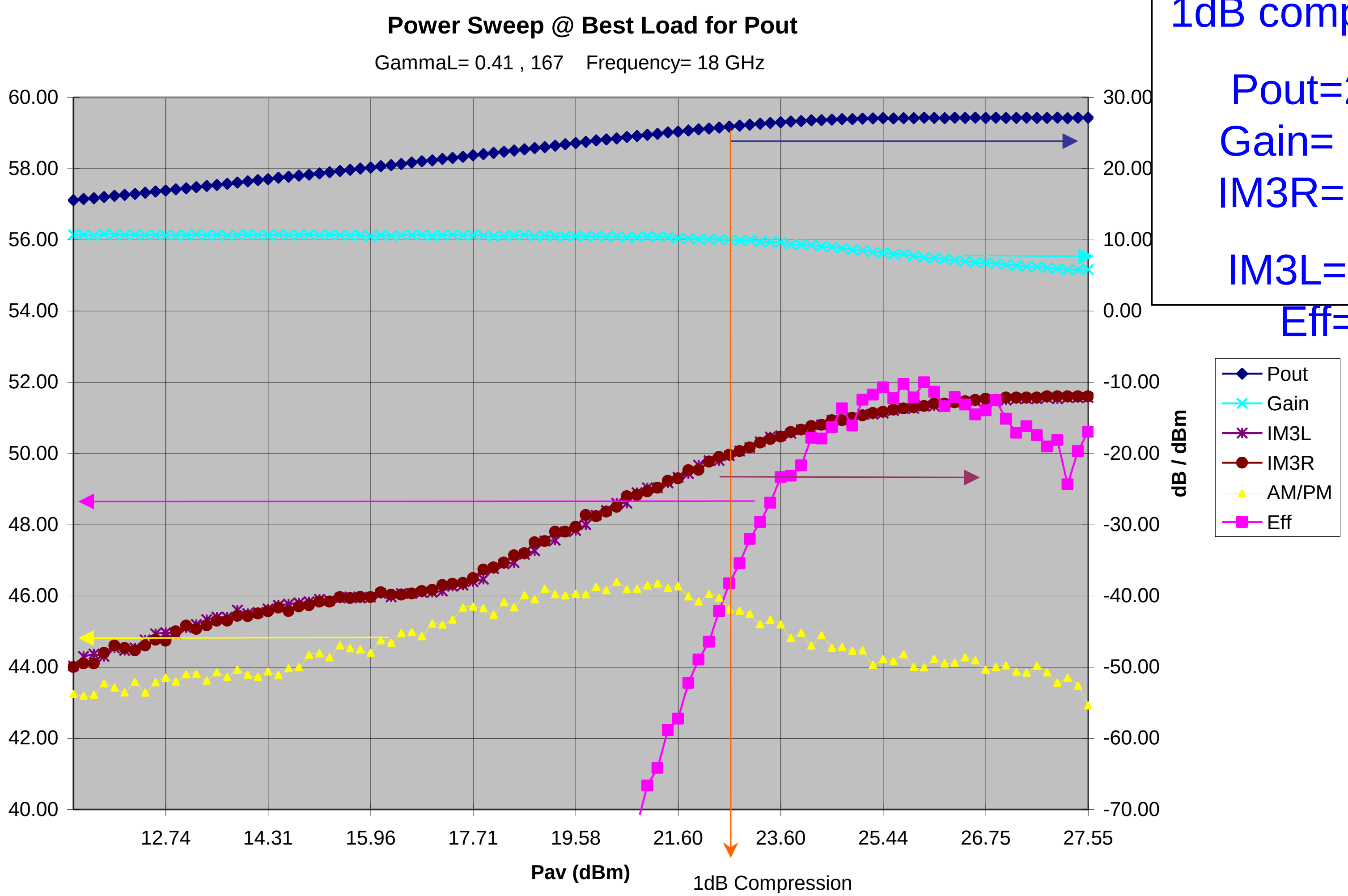
A1 = POLIPULL2000 SYSTEM

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	W
1	POLIPULL2000 SYSTEM																				
2	Date:																				
3		Operator:																			
4		Device:																			
5		Comment:			MAP with IM on PWR MAX, 1dB COMPR and 18dBm OUTPUT PWR																
6																					
7	FR:1	GL	GL	GIN	GIN	GS	GS	PAV	Pin	Pout	Gain	Vds	Vgs	Ids	Igs	Eff	S21L	S21L	IM3L	IM3R	
8	(GHz)	(MAG)	(Deg)	(MAG)	(Deg)	(MAG)	(Deg)	(dBm)	(dBm)	(dBm)	(dB)	(V)	(V)	(A)	(A)	(%)	(MAG)	(Deg)	()	()	
9	19.00	0.74	151.34	0.91	-176.17	0.00	0.00	20.43	12.63	22.73	10.10	6.47	-0.54	0.11	0.00	24.31	1.93	60.41	-24.67	-32.17	
10	19.00	0.71	148.31	0.92	-172.11	0.00	0.00	25.22	17.07	26.53	9.47	6.44	-0.67	0.13	0.00	46.23	1.65	57.93	-13.00	-18.83	
11	19.00	0.75	152.82	0.91	-178.31	0.00	0.00	15.47	8.01	17.82	9.80	6.50	-0.54	0.08	0.00	9.90	1.98	59.64	-27.67	-48.67	
12	19.00	0.60	153.97	0.89	-179.71	0.00	0.00	13.75	6.78	17.27	10.49	6.50	-0.54	0.08	0.00	9.19	1.87	54.60	-31.66	-51.00	
13	19.00	0.59	151.74	0.90	-173.87	0.00	0.00	23.69	16.41	26.11	9.70	6.45	-0.54	0.13	0.00	44.54	1.63	56.91	-15.34	-19.67	
14	19.00	0.60	153.53	0.90	-179.23	0.00	0.00	14.20	7.16	17.68	10.52	6.50	-0.54	0.08	0.00	10.11	1.86	55.33	-31.17	-49.34	
15	19.00	0.47	157.15	0.89	179.72	0.00	0.00	11.28	4.61	14.98	10.37	6.50	-0.54	0.08	0.00	5.47	1.73	53.70	-34.50	-49.00	
16	19.00	0.46	155.01	0.89	-174.94	0.00	0.00	22.93	16.15	26.04	9.89	6.45	-0.54	0.13	0.00	44.61	1.61	52.41	-16.66	-20.33	
17	19.00	0.47	156.79	0.88	-179.97	0.00	0.00	13.94	7.37	17.71	10.34	6.50	-0.54	0.08	0.00	10.18	1.75	51.94	-32.50	-49.50	
18	19.00	0.36	161.69	0.88	179.35	0.00	0.00	14.13	7.62	18.02	10.40	6.50	-0.54	0.08	0.00	10.82	1.68	50.26	-33.00	-48.17	
19	19.00	0.36	160.61	0.88	-175.73	0.00	0.00	22.61	16.07	25.83	9.76	6.45	-0.53	0.13	0.00	42.19	1.55	51.19	-17.33	-20.50	
20	19.00	0.36	162.55	0.88	179.48	0.00	0.00	14.25	7.77	18.07	10.30	6.50	-0.54	0.08	0.00	10.93	1.67	52.39	-33.00	-48.17	
21	19.00	0.29	170.42	0.87	179.33	0.00	0.00	13.51	7.32	17.15	9.83	6.50	-0.54	0.08	0.00	8.74	1.59	49.41	-33.67	-48.50	

MeasData / Sheet2 / Sheet3

Ready

Power Sweep and More



1dB compression Point

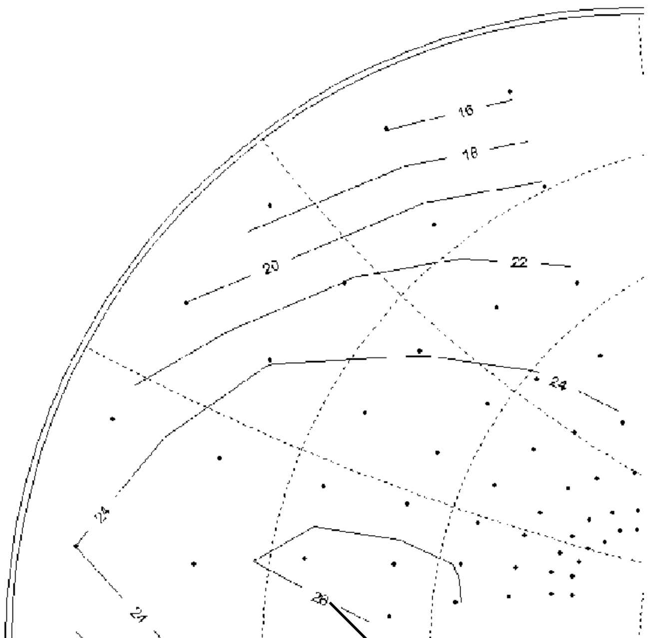
Pout=26.29 dBm
Gain= 9.72 dB
IM3R= -18.34 dBc

IM3L=-18.50 dBc
Eff=48.07%

Load-pull and PA Design I

COMBINING LOADPULL MAPS

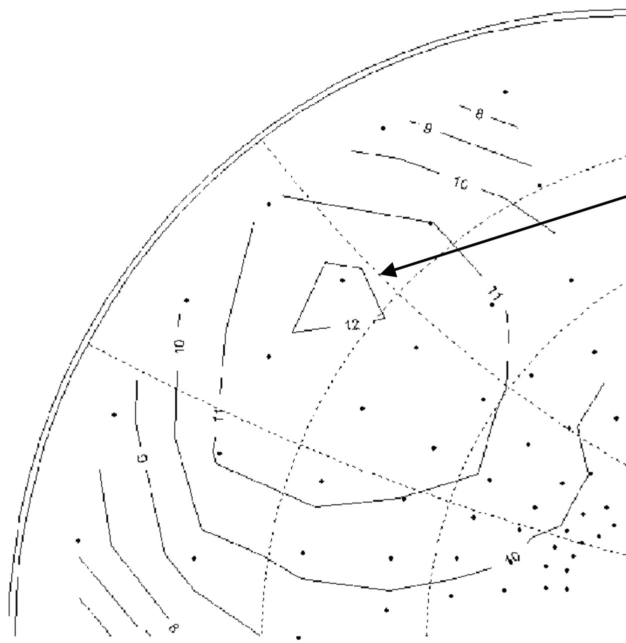
TO OPTIMIZE POWER PERFORMANCES



OUTPUT POWER @ 1 dB GAIN COMPRESSION

26dBm

12 dB

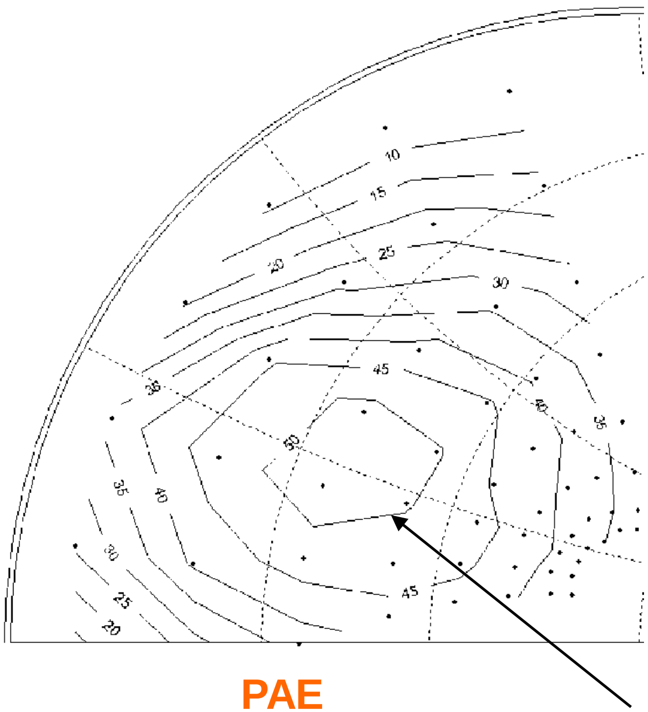


POWER GAIN @ 1 dB GAIN COMPRESSION

Load-pull and PA Design II

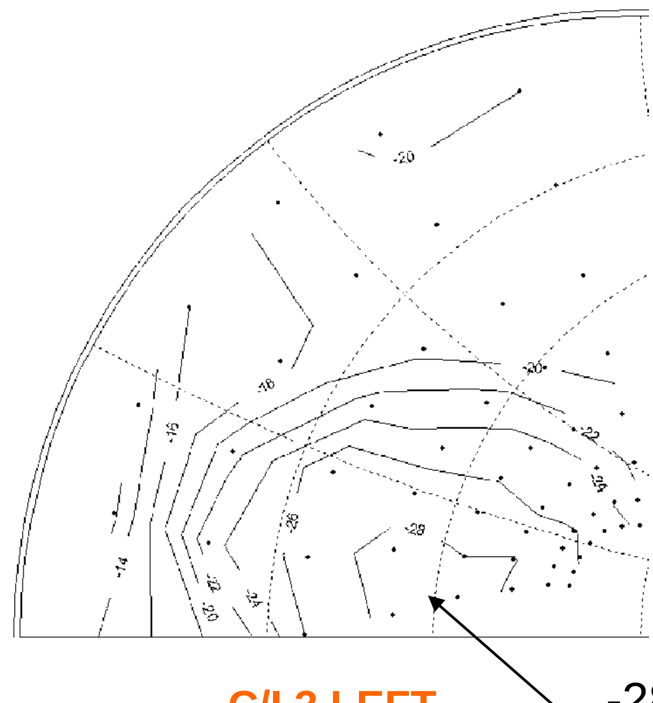
COMBINING LOADPULL MAPS

TO OPTIMIZE LINEARITY PERFORMANCES



50%

PAE
@ 1 dB GAIN COMPRESSION



-28 dBm

C/I 3 LEFT
@ POUT = 24 dBm

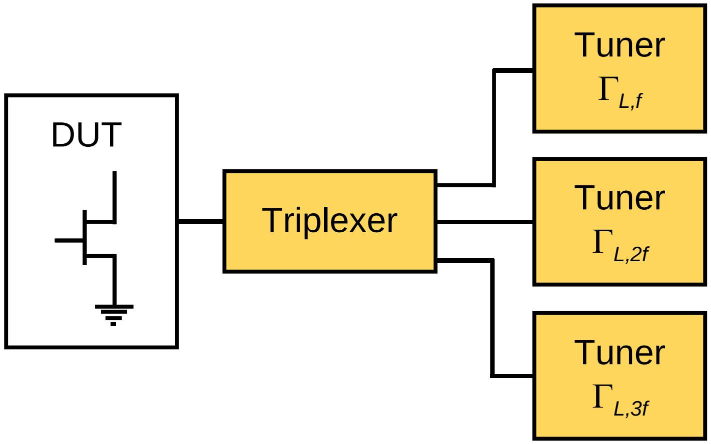
Harmonic load-pull Systems

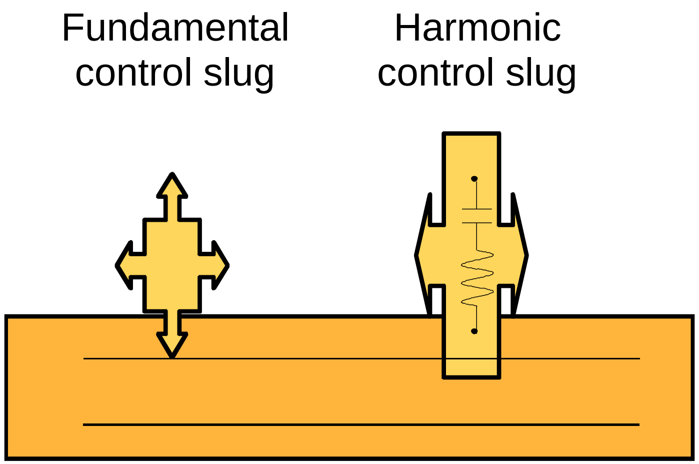
- Controlling the Load/Source condition at harmonic frequencies
- Enable wave-shaping techniques at microwave frequencies
- Great complexity of the system but potential improvement of the performance (e.g. Class–F amplifier)
- Passive and active harmonic termination synthesis

Passive systems - Harmonic Tuners

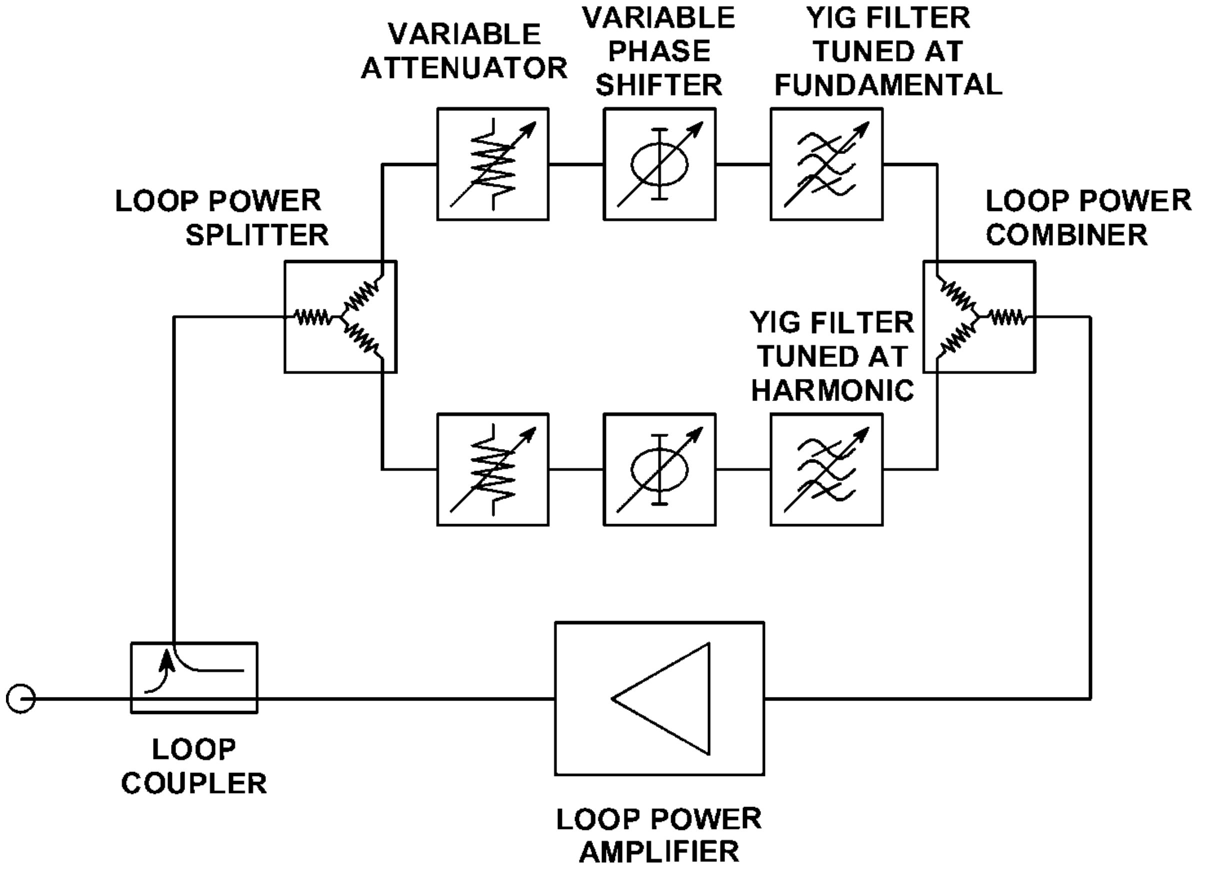
- A Tuner for each harmonic
  - Complex
  - Easy to change frequency
  - More control of the harmonic load

- Harmonic Resonators
  - Difficult to change frequency
  - Only Phase control of the load





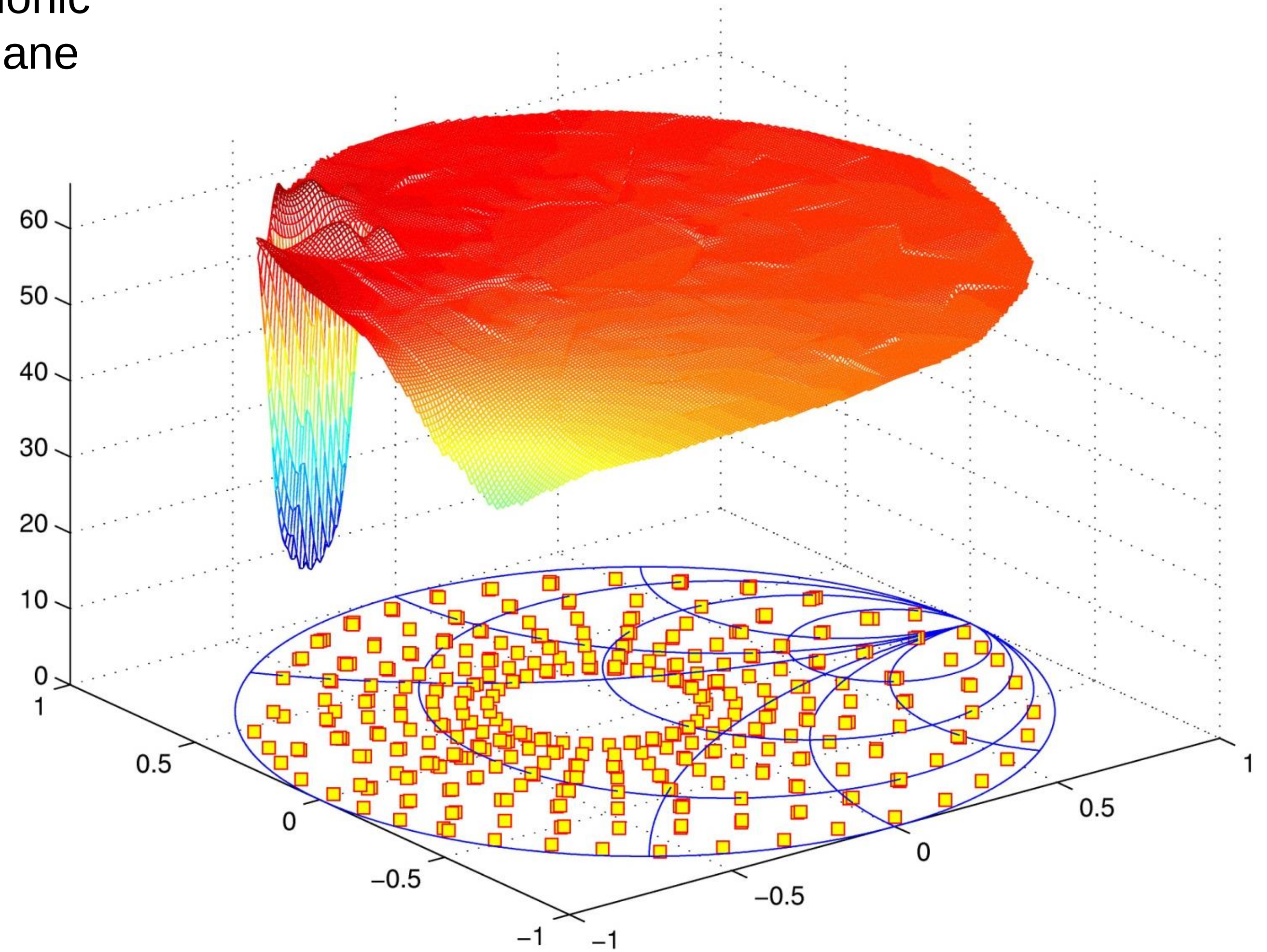
Active Harmonic load-pull



Harmonic LoaPull Example

2nd Harmonic
Load Plane

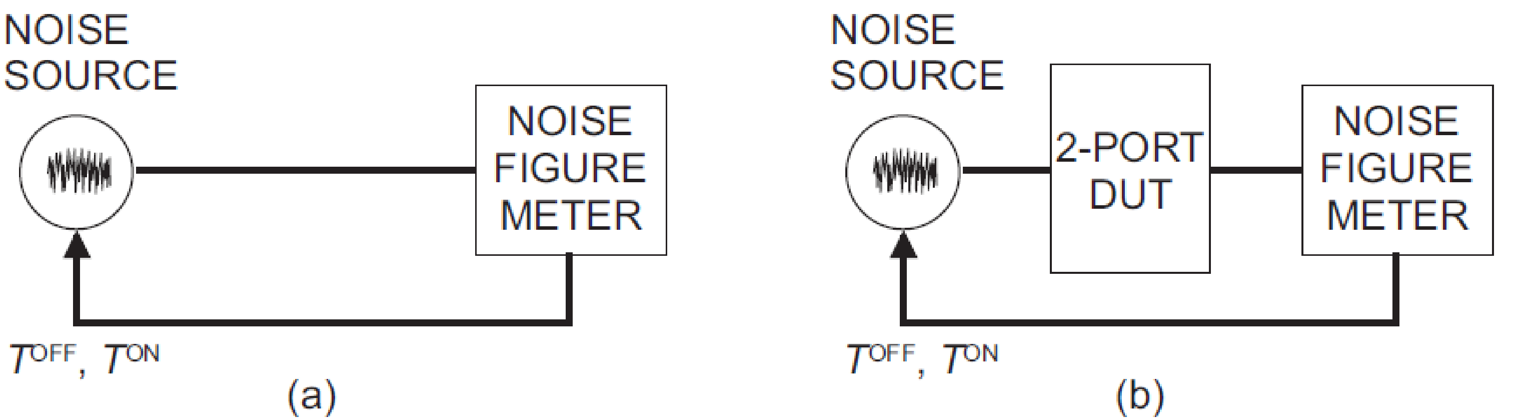
PAE



f: 3.6 GHz

Noise measurements

- Noise figure meter (NFM): includes a power meter or spectrum analyzer providing the noise power on a bandwidth *B*
- The measurement procedure is called *Y-factor technique.*



(a) Y-factor technique, first step, measurement of the noise source only; (b) Y-factor technique, second step, measurement of the DUT with the noise source at the input.

The calibrated noise source

- Allows for two states: high noise temperature (ON state, e.g. of a diode, e.g. 10000 K), low noise temperature (OFF state, e,g. 290 K).
- Excess noise ratio (ENR) and Y-factor:

$$\mathrm{ENR} = \frac{T_s^{\mathrm{ON}} - T_s^{\mathrm{OFF}}}{T_0}$$

$$T_s^{\mathrm{ON}} = T_s^{\mathrm{OFF}} + \mathrm{ENR} \cdot T_0.$$

$$Y = \frac{P_{n,\mathrm{av}\,s}^{\mathrm{ON}}}{P_{n,\mathrm{av}\,s}^{\mathrm{OFF}}} = \frac{P_{n,s}^{\mathrm{ON}}}{P_{n,s}^{\mathrm{OFF}}} = \frac{k_B B T_s^{\mathrm{ON}}}{k_B B T_s^{\mathrm{OFF}}} = \frac{T_s^{\mathrm{ON}}}{T_s^{\mathrm{OFF}}}$$

The noise figure measurement – preliminary remarks

- Also the NFM adds noise and has a noise figure and a noise temperature that should be estimated.
- Since the NFM is cascaded with the DUT, the Friis formula applies (we assume 50 Ohm matching):

$$F = F_{\mathrm{DUT}} + \frac{F_{\mathrm{NFM}} - 1}{G_{\mathrm{DUT}}} = 1 + \frac{T_2}{T_0}$$

- where T_2 is the cascade noise temperature. We also have:

$$\frac{T_2}{T_0} = \frac{T_{\mathrm{DUT}}}{T_0} + \frac{T_{\mathrm{NFM}}}{T_0 G_{\mathrm{DUT}}} \Rightarrow T_2 = T_{\mathrm{DUT}} + \frac{T_{\mathrm{NFM}}}{G_{\mathrm{DUT}}}$$

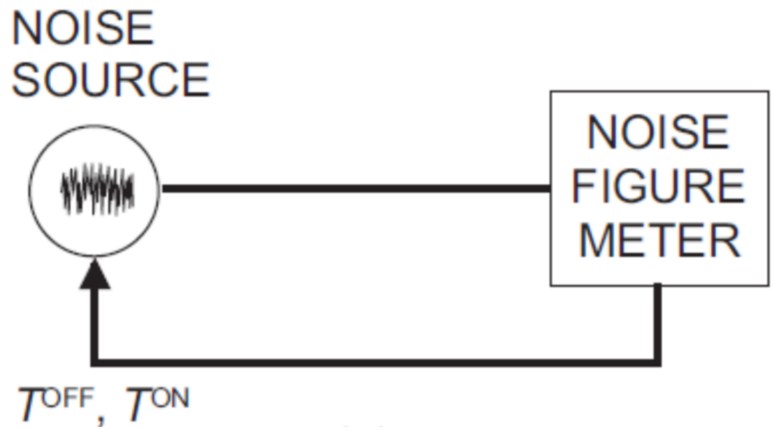
Y-factor: first step

- The noise source is directly connected to the NFM
- Two noise measurements are made in the ON and OFF states; we evaluate the ratio:

$$Y_1 = \frac{P_{n1}^{\mathrm{ON}}}{P_{n1}^{\mathrm{OFF}}} = \frac{T_s^{\mathrm{ON}} + T_{\mathrm{NFM}}}{T_s^{\mathrm{OFF}} + T_{\mathrm{NFM}}}$$

- from which:

$$T_{\mathrm{NFM}} = \frac{T_s^{\mathrm{ON}} - Y_1 T_s^{\mathrm{OFF}}}{Y_1 - 1}$$



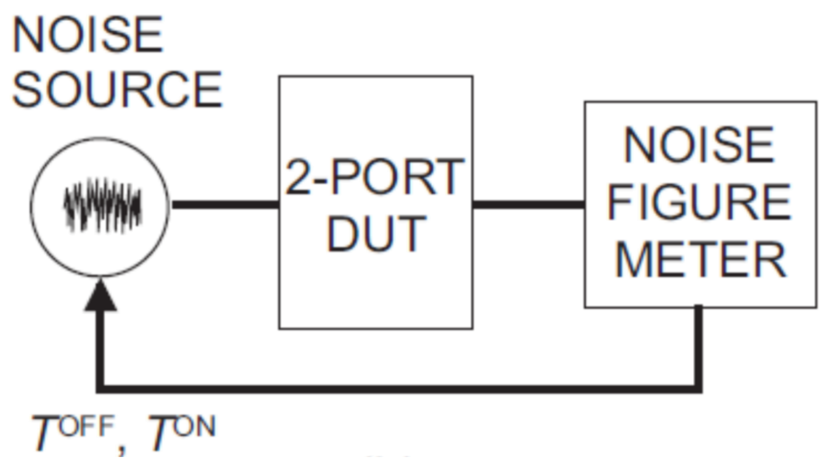
Y-factor: second step

- We now connect the DUT between the noise source and the NFM and we evaluate the ratio:

$$Y_2 = \frac{P_{n2}^{\mathrm{ON}}}{P_{n2}^{\mathrm{OFF}}} = \frac{T_s^{\mathrm{ON}} + T_2}{T_s^{\mathrm{OFF}} + T_2}$$

- T_2 is the total noise temperature of the DUT+NFM cascade; we have:

$$T_2 = \frac{T_s^{\mathrm{ON}} - Y_2 T_s^{\mathrm{OFF}}}{Y_2 - 1}$$



Evaluating the DUT gain and NF

- Since noise powers are uncorrelated we have (1→without DUT; 2→with DUT):

$$
\begin{aligned}
P_{n1}^{\mathrm{ON}} &= P_{n,s}^{\mathrm{ON}} + P_{n,\mathrm{NFM}} \\
P_{n1}^{\mathrm{OFF}} &= P_{n,s}^{\mathrm{OFF}} + P_{n,\mathrm{NFM}} \\
P_{n2}^{\mathrm{ON}} &= G_{\mathrm{DUT}} P_{n,s}^{\mathrm{ON}} + P_{n,\mathrm{DUT}} + P_{n,\mathrm{NFM}} \\
P_{n2}^{\mathrm{OFF}} &= G_{\mathrm{DUT}} P_{n,s}^{\mathrm{OFF}} + P_{n,\mathrm{DUT}} + P_{n,\mathrm{NFM}}
\end{aligned}
$$

- Subtracting:

$$
\begin{aligned}
P_{n1}^{\mathrm{ON}} - P_{n1}^{\mathrm{OFF}} &= P_{n,s}^{\mathrm{ON}} - P_{n,s}^{\mathrm{OFF}} \\
P_{n2}^{\mathrm{ON}} - P_{n2}^{\mathrm{OFF}} &= G_{\mathrm{DUT}} \left(P_{n,s}^{\mathrm{ON}} - P_{n,s}^{\mathrm{OFF}} \right)
\end{aligned}
$$

$$
G_{\mathrm{DUT}} = \frac{P_{n2}^{\mathrm{ON}} - P_{n2}^{\mathrm{OFF}}}{P_{n1}^{\mathrm{ON}} - P_{n1}^{\mathrm{OFF}}} \qquad T_{\mathrm{DUT}} = T_2 - \frac{T_{\mathrm{NFM}}}{G_{\mathrm{DUT}}} \Rightarrow \boxed{F_{\mathrm{DUT}} = 1 + \frac{T_{\mathrm{DUT}}}{T_0}}
$$

Complete noise characterization

- To evaluate the minimum noise figure, optimum noise impedance and the noise resistance we need to perform a source-pull using as a source a noise source; the parameters can be evaluated by numerically approximating the NF surface.

$$F = F_{\min} + 4 g_n R_0 \frac{|\Gamma_G - \Gamma_{Go}|^2}{\left(1 - |\Gamma_G|^2\right) |1 - \Gamma_{Go}|^2} \equiv \frac{4 R_n}{R_0} \frac{|\Gamma_G - \Gamma_{Go}|^2}{\left(1 - |\Gamma_G|^2\right) |1 - \Gamma_{Go}|^2}$$