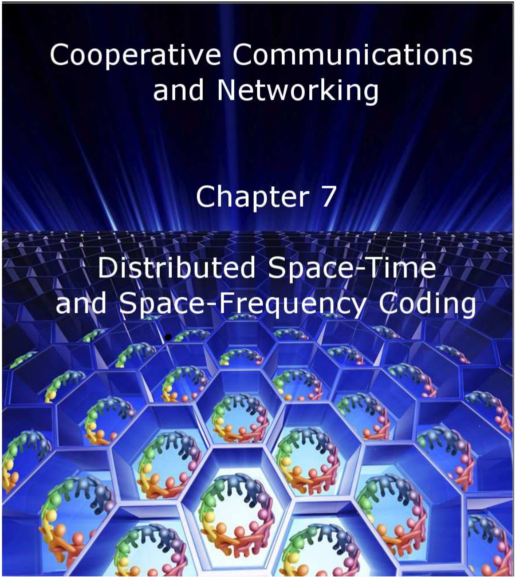
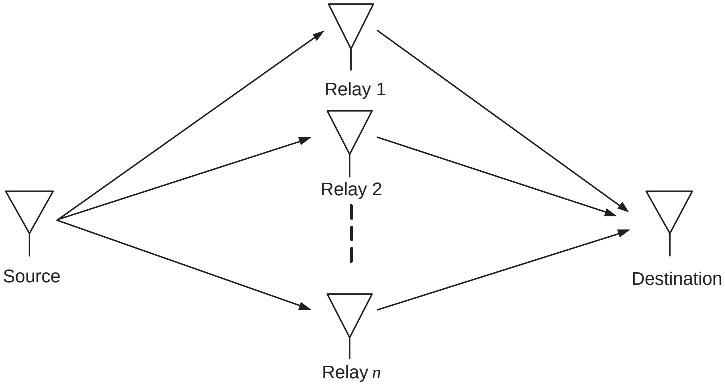

Cooperative Communications
and Networking
Chapter 7
Distributed Space-Time
and Space-Frequency Coding

Introduction

- Designing protocols that allow several single-antenna terminals to cooperate via forwarding each others data can increase the system reliability through achieving spatial diversity.
- The problem with the classical protocols, DAF and AAF, is the loss in the data rate as the number of relays increases.
- Leads to the use of what is known as distributed space-time coding.
- The term distributed comes from the fact that the virtual multi-antenna transmitter is distributed between randomly placed relay nodes.

System model



(we assume that there is no direct link between the source and the destination node)

Decode-and-Forward DSTC System Model

- In phase 1, the source broadcasts its information the n relay nodes

$$\mathbf{y}_{s,r_i} = \sqrt{P_1} h_{s,r_i} \mathbf{s} + \mathbf{v}_{s,r_i}, \quad i = 1, 2, \cdots, n,$$

- In phase 2, the relays that **decodes correctly** re-encodes the data vector **s** with a pre-assigned code structure. Each relay will emulate a single antenna in a multiple antenna transmitter. The signal received at the destination from all relays can be modeled as

$$\mathbf{y_d} = \sqrt{P_2}\left[I_1 \mathbf{x}_{r_1}, I_2 \mathbf{x}_{r_2}, \cdots, I_n \mathbf{x}_{r_n}\right] \mathbf{h}_d + \mathbf{v}_d,$$

System Model (Cont.)

- The state of the k-th relay, i.e., whether it decoded correctly or not, is denoted by the random variable I_k $(1 \leq k \leq n)$ which takes values 1 or 0 if the relay decodes correctly or erroneously, respectively.
- The random variables I_k's $(1 \leq k \leq n)$ are **statistically independent** as the state of each relay depends only on its channel conditions to the source which are independent from other relays.

System Model (Cont.)

- The received signal model at the destination can be rewritten as follows

$$\mathbf{y_d} = \sqrt{P_2}\mathbf{X_r}\mathbf{h}_{d,\mathbf{I}} + \mathbf{v}_d.$$

- The new channel $\mathbf{h}_{d,\mathbf{I}}$ is defined as follows

$$\mathbf{h}_{d,\mathbf{I}} = \left[I_1 h_{r_1,d}, I_2 h_{r_2,d}, \cdots, I_n h_{r_n,d}\right]^T.$$

- The random variable I_k is a Bernoulli random variable with a distribution given by

$$I_k = \begin{cases} 0 & \text{with probability} \simeq L_n\mathbf{SER} \\ 1 & \text{with probability} \simeq 1 - L_n\mathbf{SER}. \end{cases}$$

Decode-and-Forward DSTC Performance Analysis

- For M-QAM, the exact expression for the SER can be upper bounded by

$$\mathbf{SER} \leq \frac{N_o g(2)}{b P_1 \delta_{s,r}^2}.$$

- The destination applies a **maximum likelihood (ML)** receiver which will be a **minimum distance** rule as follows

$$\mathbf{X} = \arg \min_{\mathbf{X_r} \in \mathcal{X}} \| \mathbf{y_d} - \sqrt{P_2} \mathbf{X_r} \mathbf{h}_{d,\mathbf{l}} \|^2,$$

- The conditional PEP can be upper bounded as

$$P(\mathbf{X} \rightarrow \hat{\mathbf{X}} \,|\, \mathbf{h}_{d,I}) \leq \exp\left(-\frac{P_2 \| \mathbf{\Phi}(\mathbf{X}, \hat{\mathbf{X}}) \mathbf{h}_{d,I} \|^2}{4 N_o} \right).$$

Performance Analysis (Cont.)

- Averaging the conditional PEP over all channel realizations, we get

$$P(\mathbf{X} \to \hat{\mathbf{X}} \,|\, \mathbf{I}) \leq \prod_{i=1}^{r_{\mathbf{I}}} \frac{1}{1 + \frac{P_2 \delta_{r,d}^2}{4N_o} \lambda_{\mathbf{I}_i}},$$

  where $\{\lambda_{\mathbf{I}_i}\}_{i=1}^{r_{\mathbf{I}}}$ is a subset of the eigenvalues of the matrix $\mathbf{D}_{\mathbf{I}} \mathbf{\Phi}(\mathbf{X}, \hat{\mathbf{X}})^{\mathcal{H}} \mathbf{\Phi}(\mathbf{X}, \hat{\mathbf{X}}) \mathbf{D}_{\mathbf{I}}$ that depends on the realization of $\mathbf{I}$.
- We can upper bound the PEP as

$$P(\mathbf{X} \to \hat{\mathbf{X}} \,|\, \mathbf{I}, c_{\mathbf{I}} = k) \leq \max_{\mathcal{I}_k : c_{\mathbf{I}} = k} \prod_{i=1}^{r_{\mathbf{I}}} \frac{1}{1 + \frac{P_2 \delta_{r,d}^2}{4N_o} \lambda_{\mathbf{I}_i}},$$

  where $\mathcal{I}_k$ denotes the set of realizations $\mathbf{I}$ which have the same number of relays that decoded correctly k.

Performance Analysis (Cont.)

- The I_k's are i.i.d. Bernoulli r.v.'s, the number of relays that decoded correctly $c_{\mathbf{I}}$ has a binomial distribution given by

$$P_{c_{\mathbf{I}}}(k) = \binom{n}{k} (1 - L_n\mathbf{SER})^k (L_n\mathbf{SER})^{n-k}, \; k = 0, 1, \cdots, n.$$

- Define $SNR = P/N_o$, where $P = P_1 + P_2$. Let $P_1 = a_1 P$ and $P_2 = a_2 P$
  We get at high SNR

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq \sum_{k=0}^{n} \binom{n}{k} SNR^{-n+(k-r_k)} \left(\frac{L_n g(2)}{b a_1 \delta_{s,r}^2} \right)^{n-k} \times \prod_{i=1}^{r_k} \left(\frac{a_2 \delta_{r,d}^2}{4} \lambda_{i,k} \right)^{-1}.$$

Decode-and-Forward System Code Design Criteria

- To achieve full diversity of order n the code matrix $\mathbf{\Phi}(\mathbf{X},\hat{\mathbf{X}})^{\mathcal{H}}\mathbf{\Phi}(\mathbf{X},\hat{\mathbf{X}})$ must be of full column rank over all pairs of distinct codewords $\mathbf{X}$ and $\hat{\mathbf{X}}$.
- This is the same code design criterion as for the space-time codes designed for the MIMO channels to achieve full diversity.
- To maximize the coding gain of the distributed space-time code we need for each $k \in \{1,\cdots,n\}$ to maximize $\min_{\aleph:\aleph\subset\{1,\cdots,n\},|\aleph|=k}\left(\prod_{i=1}^{k}\lambda_{i,\aleph}\right)$. The maximization is over all distinct pairs of codewords $\mathbf{X}$ and $\hat{\mathbf{X}}$.
- This is different from the determinant criterion in the case of MIMO channels.

DSTC with Amplify-and-Forward Cooperation Protocol

- The system has two phases, in phase 1, the signal received at the i-th relay can be modeled as

$$\mathbf{y}_{s,r_i} = \sqrt{P_1} h_{s,r_i} \mathbf{s} + \mathbf{v}_{s,r_i}, \quad i = 1, 2, \cdots, n,$$

- In the amplify-and-forward protocol, the relays can only amplify the received signal and perform simple operations such as permutations of the received symbols or other forms of linear transformations.
- Each relay will multiply the received signal by the factor $\sqrt{\frac{P_2/K_n}{P_1 \delta_{s,r}^2 + N_0}}$. It can be easily shown that this normalization will give a transmitted power per symbol $P = P_1 + P_2$.

Amplify-and-Forward System Model

- the signal received at the destination from all relays can be modeled as

$$\mathbf{y_d} = \sqrt{\frac{P_2/K_n}{P_1\delta_{s,r}^2 + N_0}}\tilde{\mathbf{X}}\mathbf{h}_d + \mathbf{v}_d.$$

- Each element of $\mathbf{v}_d$ is $\mathcal{CN}(0, N_0\left(1 + \frac{P_2/K_n}{P_1\delta_{s,r}^2+N_o}\sum_{i=1}^{n}|h_{r_i,d}|^2\right))$, and $\mathbf{v}_d$ accounts for both the noise propagated from the relay nodes as well as the noise generated at the destination.
- The received vector can be written as

$$\mathbf{y_d} = \sqrt{\frac{P_2 P_1/K_n}{P_1\delta_{s,r}^2 + N_0}}\mathbf{X}\mathbf{H} + \mathbf{v}_d,$$

where $\mathbf{H} = [\ h_{s,r_1}h_{r_1,d}, h_{s,r_2}h_{r_2,d}, \cdots, h_{s,r_n}h_{r_n,d}]^T$.

DSTC with Amplify-and-Forward Protocol Performance Analysis

- With the ML decoder, the PEP can be upper bounded by the following Chernoff bound

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq E_{\mathbf{H}} \exp\left(- \frac{P_1 P_2 / K_n}{4N_0 \left(P_1 \delta_{s,r}^2 + N_o + \frac{P_2}{K_n} \sum_{i=1}^{n} |h_{r_i,d}|^2 \right)} \mathbf{H}^{\mathcal{H}} (\mathbf{X} - \hat{\mathbf{X}})^{\mathcal{H}} \right.$$
$$\left. \times (\mathbf{X} - \hat{\mathbf{X}}) \mathbf{H} \right).$$

- By averaging over the source to relay channels we get

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq E_{h_{r_1,d}, \cdots, h_{r_n,d}} \det^{-1} \left[\mathbf{I}_n + \frac{\delta_{s,r}^2 P_1 P_2 / K_n}{4N_0 \left(P_1 \delta_{s,r}^2 + N_o + \frac{P_2}{K_n} \sum_{i=1}^{n} |h_{r_i,d}|^2 \right)} \right.$$
$$\left. (\mathbf{X} - \hat{\mathbf{X}})^{\mathcal{H}} (\mathbf{X} - \hat{\mathbf{X}}) \mathtt{diag}(|h_{r_1,d}|^2, |h_{r_2,d}|^2, \cdots, |h_{r_n,d}|^2) \right].$$

Amplify-and-Forward Performance Analysis (Cont.)

- The PEP bound can be written in terms of the eigenvalues of $\mathbf{M}$ as

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq E_{h_{r_1,d},\cdots,h_{r_n,d}} \frac{1}{\prod_{i=1}^{n}(1+\lambda_{M_i})}.$$

- At high SNR (high P) we can get the bound as

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq \prod_{i=1}^{n} \left(\frac{(\delta_{s,r}^2 P_1 P_2 / K_n)\lambda_i}{4N_0 \left(P_1 \delta_{s,r}^2 + \frac{P_2}{K_n}\delta_{r,d}^2 n \right)} \right)^{-1} \prod_{i=1}^{n} \ln \left(\frac{(\delta_{s,r}^2 P_1 P_2 / K_n)\lambda_i}{4N_0 \left(P_1 \delta_{s,r}^2 + \frac{P_2}{K_n}\delta_{r,d}^2 n \right)} \right)$$

- Let $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where P is the power per symbol, for some $\alpha \in (0,1)$. With the definition of the SNR as $SNR = P/N_0$, the bound can be given as

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq a_{AF} \frac{1}{\prod_{i=1}^{n} \lambda_i} SNR^{-n} \left(\ln(SNR) \right)^n .$$

Amplify-and-Forward Performance Analysis (Cont.)

- The diversity order of the system can now be calculated as

$$d_{AF} = \lim_{SNR\to\infty} -\frac{\log(PEP)}{\log(SNR)} = n.$$

  The system will achieve a full diversity of order n, if the matrix $\mathbf{M}$ is full rank, that is the code matrix $\Psi(\mathbf{X}, \hat{\mathbf{X}})$ must have a full rank of order n over all distinct pairs of codewords $\mathbf{X}$ and $\hat{\mathbf{X}}$.
- So any code that is designed to achieve full diversity in MIMO channels will achieve full diversity in the case of amplify-and-forward distributed space-time coding.

Amplify-and-Forward Performance Analysis (Cont.)

- If the full diversity is achieved, the coding gain is $C_{AF} = \left(a_{AF} \frac{1}{\prod_{i=1}^{n} \lambda_i} \right)^{-\frac{1}{n}}$.
- To maximize the coding gain of the amplify-and-forward distributed space-time codes we need to maximize the term $\prod_{i=1}^{n} \lambda_i$ which is the same as the determinant criterion used for MIMO channels.
- If the space-time code is designed to maximize the coding gain in the MIMO channels it will also maximize the coding gain if it is used in a distributed fashion with the amplify-and-forward protocol.

Simulation Results

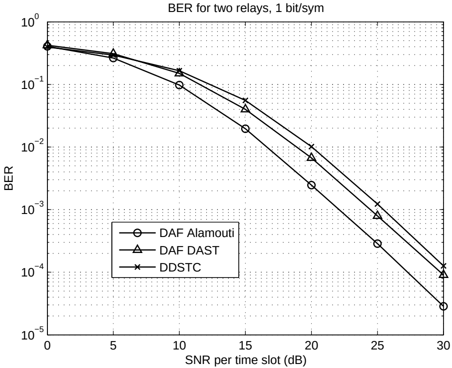


Figure shows the simulations for two decode-and-forward systems using the Alamouti scheme (DAF Alamouti) and the diagonal STC (DAF DAST), and the DDSTC.

Summary

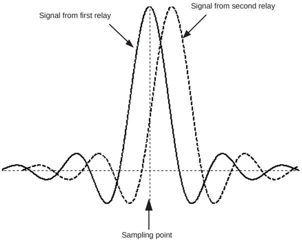
- For the decode-and-forward distributed space-time codes we find that any code that is designed to achieve full diversity in the MIMO channels will achieve full diversity.
- A code that maximizes the coding gain over the MIMO channels is not guaranteed to maximize the coding gain in the decode-and-forward distributed space-time coding.
- For the amplify-and-forward distributed space-time codes the code designed to achieve full diversity in the MIMO channels will also achieve full diversity.
- Furthermore, the code that maximizes the coding gain over the MIMO channels will also maximize the coding gain in the amplify-and-forward distributed space-time system.

Motivation of DDSTC

- Most of the previous works on cooperative transmission, including DSTC, assume perfect synchronization between the nodes.
- Perfect synchronization means that the users' timings, carrier frequencies, and propagation delays are identical.
- Perfect synchronization is almost impossible to be achieved in wireless relay networks!
- To simplify the synchronization in the network a diagonal structure is imposed on the space-time code used.
- The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay to transmit at any time slot.

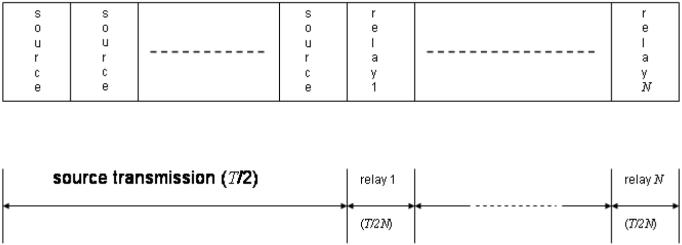
Motivation of DDSTC (Cont.)

- Nodes can maintain slot synchronization, which means that coarse slot synchronization is available.
- However, fine synchronization (synchronization within the time slot) is more difficult to be achieved.



Baseband signals (each is raised cosine pulse-shaped) from two relays at the receiver.

DDSTC Time Frame Structure



Each relay will transmit in one time slot and there is no need to synchronize simultaneous transmissions.

DDSTC System Model with the AF Protocol

- In phase 1, the source transmits the data $\mathbf{s} = [s_1, s_2, ..., s_n]^T$ to the n relay nodes. The received signal at the kth relay is

$$\mathbf{y}_{s,r_k} = \sqrt{P_1} h_{s,r_k} \mathbf{s} + \mathbf{v}_{s,r_k}, \quad \forall k \in \{1, 2, ..., n\}.$$

- In phase 2, the k-th relay applies a linear transformation $\mathbf{t}_k$ to the received data vector, as

$$\begin{aligned} y_{r_k} &= \mathbf{t}_k \mathbf{y}_{s,r_k} \\ &= \sqrt{P_1} h_{s,r_k} \mathbf{t}_k \mathbf{s} + \mathbf{t}_k \mathbf{v}_{s,r_k} \\ &= \sqrt{P_1} h_{s,r_k} x_k + \mathbf{v}_{r_k}, \end{aligned}$$

where $x_k = \mathbf{t}_k \mathbf{s}$ and $\mathbf{v}_{r_k} = \mathbf{t}_k \mathbf{v}_{s,r_k}$.

DDSTC System Model with the AF Protocol (Cont.)

- Then, the relay multiplies y_{r_k} by the factor β_k. And the received signal at the destination due to the k-th relay transmission is given by

$$\begin{aligned} y_k &= h_{r_k,d}\beta_k\sqrt{P_1}h_{s,r_k}x_k + h_{r_k,d}\beta_k v_{r_k} + \tilde{v}_k \\ &= h_{r_k,d}\beta_k\sqrt{P_1}h_{s,r_k}x_k + z_k, \quad k \in [1,n]. \end{aligned}$$

- The maximum likelihood (ML) decoder can be expressed as

$$\arg\max_{\mathbf{s}_i} p(\mathbf{y}/\mathbf{s}_i) =$$
$$\arg\min_{\mathbf{s}_i} \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left|y_i - \sqrt{\frac{P_1 P_2}{P_1|h_{s,r_i}|^2 + N_0}} h_{s,r_i} h_{r_i,d} x_i\right|^2.$$

DDSTC with the AF Protocol

- The PEP of mistaking $\mathbf{X}_1$ by $\mathbf{X}_2$ can be upper-bounded as

$$Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq N_0^n \prod_{i=1, x_{1i} \neq x_{2i}}^{n} \left(\frac{1}{P_1 \delta_{s,r}^2} + \frac{1}{P_2 \delta_{r,d}^2} \right) \times \left(\prod_{i=1, x_{1i} \neq x_{2i}}^{n} \frac{1}{4} |x_{1i} - x_{2i}|^2 \right)^{-1}.$$

- The diversity order d_{DDSTC} is

$$d_{DDSTC} = \lim_{SNR \to \infty} -\frac{\log(PEP)}{\log(SNR)} = \min_{m \neq j} rank(\mathbf{X}_m, \mathbf{X}_j),$$

where $\mathbf{X}_m$ and $\mathbf{X}_j$ are two possible code matrices.

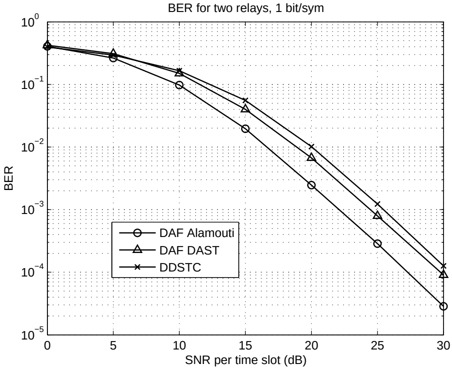
DDSTC with the AF Protocol (Cont.)

- To achieve a **diversity order of** n, the matrix $\mathbf{X}_m - \mathbf{X}_j$ should be of **full rank** for any $m \neq j$ (that is $x_{mi} \neq x_{ji}$, $\forall m \neq j, \forall i \in [1, n]$).
- To minimize the PEP bound we need to maximize

$$\min_{m \neq j} \left(\prod_{i=1}^{n} |x_{mi} - x_{ji}|^2 \right)^{1/n},$$

which is called **the minimum product distance** of the set of symbols $\mathbf{s} = [s_1, s_2, ..., s_n]^T$. This is the same criteria used to design full-rate full-diversity space-frequency codes in Chapter 3. We can use the design presented in Chapter 3 to design the DDSTC.

Simulation Results



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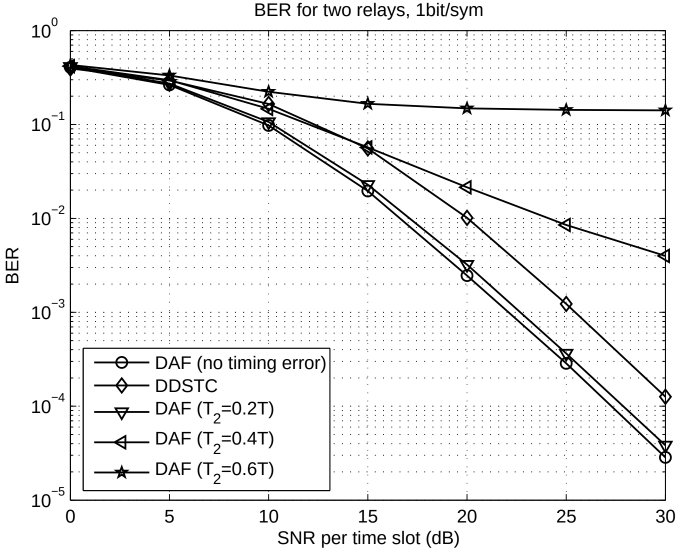
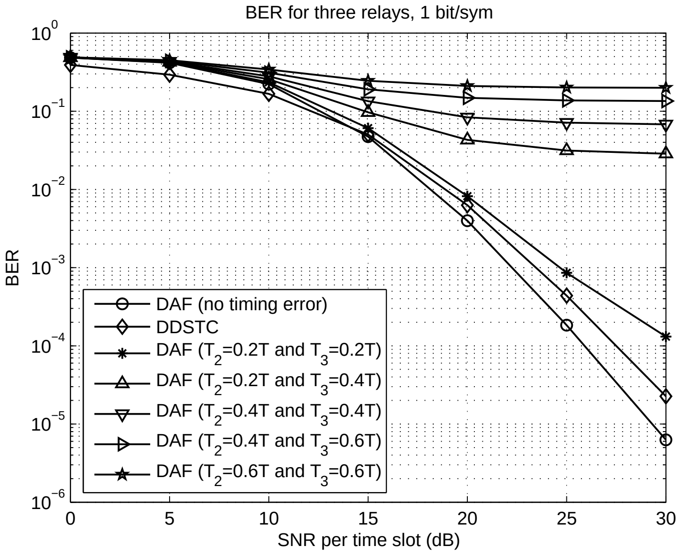


Simulation Results



Summary

- For the decode-and-forward distributed space-time codes we find that any code that is designed to achieve full diversity in the MIMO channels will achieve full diversity.
- A code that maximizes the coding gain over the MIMO channels is not guaranteed to maximize the coding gain in the decode-and-forward distributed space-time coding.
- For the amplify-and-forward distributed space-time codes the code designed to achieve full diversity in the MIMO channels will also achieve full diversity.
- Furthermore, the code that maximizes the coding gain over the MIMO channels will also maximize the coding gain in the amplify-and-forward distributed space-time system.
- With DDSTC, the stringent synchronization between randomly located relay nodes is simplified.

Introduction to Distributed Space-Frequency Coding

- Most of previous works have considered Distributed Space-Time Coding (DSTC)
- For **multi-path channels**, design of Distributed Space-Frequency Codes (DSFCs) is needed to exploit the multi-path (frequency) diversity of the channel
- Exploiting the frequency axis diversity can highly improve the system performance
- We consider the design of DSFCs with the decode-and-forward (DAF) cooperation protocol

System Model

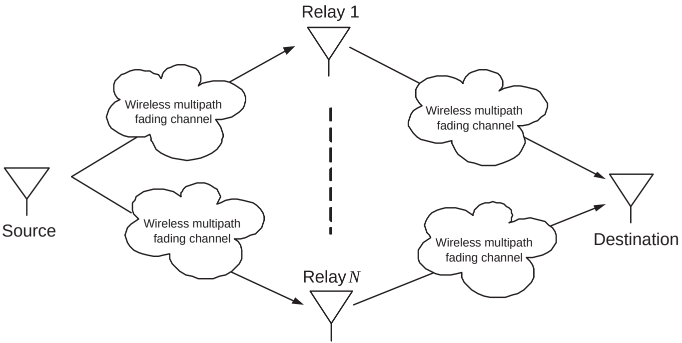


Figure: Simplified **two-hop** system model for the distributed space-frequency codes.

System Model (cont'd)

- OFDM with K subcarriers is used
- The channel is modeled as

$$h_{s,r_n}(\tau) = \sum_{l=1}^{L} \alpha_{s,r_n}(l)\delta(\tau - \tau_l)$$

- The received signal in the frequency domain on the k-th subcarrier at the n-th relay node is given as

$$y_{s,r_n}(k) = \sqrt{P_s} H_{s,r_n}(k) s(k) + \eta_{s,r_n}(k)$$

- $H_{s,r_n}(k)$ is given by

$$H_{s,r_n}(k) = \sum_{l=1}^{L} \alpha_{s,r_n}(l) e^{-j2\pi(k-1)\Delta f \tau_l}$$

DSFC with the DAF protocol

- Each relay tries to decode the source symbols before retransmission
- Two stages of coding are needed
  - **Stage 1:** coding at the source node to ensure a diversity of order L at relay nodes
  - **Stage 2:** coding at the relay nodes to guarantee full diversity of order NL
- We assume that each relay will be able to decide whether it has decoded the "block" correctly or not

DSFC with the DAF protocol: Source Node Encoding

- For two distinct transmitted source symbols the PEP at any relay node can tightly upper bounded as

$$PEP(\mathbf{s} \rightarrow \tilde{\mathbf{s}}) \leq \begin{pmatrix} 2\nu - 1 \\ \nu \end{pmatrix} \left(\prod_{i=1}^{\nu} \lambda_i \right) \left(\frac{P_s}{N_0} \right)^{\nu}$$

and ν is the rank of the matrix $\mathbf{C} \circ \mathbf{R}$ where

$$\mathbf{C} = (\mathbf{s} - \tilde{\mathbf{s}})(\mathbf{s} - \tilde{\mathbf{s}})^{\mathcal{H}},$$

$$\mathbf{R} = E \left\{ \mathbf{H}_{s,r_n} \mathbf{H}_{s,r_n}^{\mathcal{H}} \right\},$$

and $\mathbf{H}_{s,r_n} = [H_{s,r_n}(1), \cdots, H_{s,r_n}(K)]^T$
λ_i's are the non-zero eigenvalues of the matrix $\mathbf{C} \circ \mathbf{R}$

DSFC with the DAF protocol: Source Node Encoding (cont'd)

- We propose to partition the transmitted $K \times 1$ source codeword as

$$\mathbf{s} = [s(1), s(2), \cdots, s(K)]^T = [\mathbf{F}_1^T, \mathbf{F}_2^T, \cdots, \mathbf{F}_M^T, \mathbf{0}_{K-ML}^T]^T,$$

  where $\mathbf{F}_i = [F_i(1), \cdots, F_i(L)]^T$
- For any two distinct source codewords, $\mathbf{s}$ and $\tilde{\mathbf{s}}$, there exists at least one index p_0 for which $\mathbf{F}_{p_0} \neq \tilde{\mathbf{F}}_{p_0}$
- We assume for $\mathbf{s}$ and $\tilde{\mathbf{s}}$ that $\mathbf{F}_p = \tilde{\mathbf{F}}_p$ for all $p \neq p_0$ (worst case PEP)

DSFC with the DAF protocol: Source Node Encoding (cont'd)

- We can prove that if the product $\prod_{l=1}^{L} \left| F_{p_0}(l) - \tilde{F}_{p_0}(l) \right|^2$ is **non-zero over all the possible pairs of transmitted codewords**, $\mathbf{s}$ and $\tilde{\mathbf{s}}$, then, a diversity of order L will be achieved at any relay node

DSFC with the DAF protocol: Relay Nodes Encoding

- The transmitted $K \times N$ SF codeword from the relay nodes is given by

$$\mathbf{C}_r = \begin{pmatrix} C_r(1,1) & C_r(1,2) & \cdots & C_r(1,N) \\ C_r(2,1) & C_r(2,2) & \cdots & C_r(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ C_r(K,1) & C_r(K,2) & \cdots & C_r(K,N) \end{pmatrix}$$

- The received signal at the destination on the k-th subcarrier is given by

$$y_d(k) = \sqrt{P_r} \sum_{n=1}^{N} H_{r_n,d}(k) C_r(k,n) I_n + \eta_{r_n,d}(k)$$

- I_n is the state of the n-th relay. $I_n = 1$ if the n-th relay has decoded correctly in phase 1 and $I_n = 0$, otherwise

DSFC with the DAF protocol: Relay Nodes Encoding (cont'd)

- The state of the n-th relay node, I_n, is a Bernoulli random variable with pmf

$$I_n = \begin{cases} 0 & \text{with probability} = SER \\ 1 & \text{with probability} = 1 - SER \end{cases}$$

- The SER at relay nodes can be upper bounded as

$$\begin{aligned} SER &= \sum_{\mathbf{s}\in\mathcal{S}} \Pr(\mathbf{s}) \Pr\{\text{error given that } \mathbf{s} \text{ was transmitted}\} \\ &\leq \sum_{\mathbf{s}\in\mathcal{S}} \Pr(\mathbf{s}) \sum_{\tilde{\mathbf{s}}\in\mathcal{S},\tilde{\mathbf{s}}\neq\mathbf{s}} PEP(\mathbf{s}\to\tilde{\mathbf{s}}) \\ &\leq \mathit{constant} \times SNR^{-L} \end{aligned}$$

DSFC with the DAF protocol: Relay Nodes Encoding (Cont.)

- The transmitted $K \times N$ SF codeword from the relay nodes, if all relays decoded correctly, is given by

$$\mathbf{C}_r = [\mathbf{G}_1^T, \mathbf{G}_2^T, \cdots, \mathbf{G}_P^T, \mathbf{0}_{K-PLN}^T]^T$$

- Each $\mathbf{G}_i$ is a block diagonal matrix that has the structure

$$\mathbf{G}_i = \begin{pmatrix} \mathbf{X}_{1_{L\times 1}} & \mathbf{0}_{L\times 1} & \cdots & \mathbf{0}_{L\times 1} \\ \mathbf{0}_{L\times 1} & \mathbf{X}_{2_{L\times 1}} & \cdots & \mathbf{0}_{L\times 1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{L\times 1} & \mathbf{0}_{L\times 1} & \cdots & \mathbf{X}_{N_{L\times 1}} \end{pmatrix}$$

- Given a realization of the relays states, the PEP is upper bounded as

$$PEP(\mathbf{s} \rightarrow \tilde{\mathbf{s}}/\mathbf{I}) \leq \begin{pmatrix} 2\kappa - 1 \\ \kappa \end{pmatrix} \left(\prod_{i=1}^{\kappa} \eta_i \right) \left(\frac{P_r}{N_0} \right)^{\kappa}$$

DSFC with the DAF protocol: Relay Nodes Encoding (cont'd)

- κ is the rank of the matrix $\mathbf{C_l} \circ \mathbf{R}$ and $\mathbf{C_l}$ is defined as

$$\mathbf{C_l} = (\mathbf{C}(\mathbf{l})_r - \tilde{\mathbf{C}}(\mathbf{l})_r)(\mathbf{C}(\mathbf{l})_r - \tilde{\mathbf{C}}(\mathbf{l})_r)^{\mathcal{H}}$$

- Consider two distinct source codewords for which $\mathbf{G}_{p_0} \neq \tilde{\mathbf{G}}_{p_0}$ for some p_0 and $\mathbf{G}_p = \tilde{\mathbf{G}}_p$ for all $p \neq p_0$ (worst case PEP)
- It can be shown if the product $\prod_{l=1}^{NL} \left| G_{p_0}(l) - \tilde{G}_{p_0}(l) \right|^2$ is non-zero over all the possible pairs of transmitted codewords then, the pairwise error probability decays with exponent $n_l L$
- That rate of decay is guaranteed over all the possible realizations of the relays' states

DSFC with the DAF protocol: Relay Nodes Encoding (cont'd)

- The number of relays that have decoded correctly, c_r, follows a Binomial distribution as

$$\Pr\{c_r = k\} = \left(\begin{array}{c} N \\ k \end{array} \right) (1 - SER)^k SER^{N-k}$$

- The pairwise error probability can now be upper bounded as

$$PEP(\mathbf{s} \rightarrow \tilde{\mathbf{s}}) = \sum_{\mathbf{l}} \Pr(\mathbf{l}) PEP(\mathbf{s} \rightarrow \tilde{\mathbf{s}}/\mathbf{l}) \leq cons. \times SNR^{-NL}$$

where $SNR = P_s/N_0 = P_r/N_0$

Simulation Results

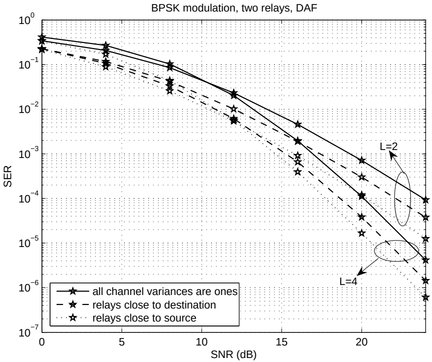


Figure: SER for DSFCs, for BPSK modulation with Vandermonde based linear transformations, versus SNR

Summary

- We proposed a two-stage coding for DSFC in conjunction with the DAF protocol
- We proved that the proposed DSFCs with DAF protocol achieve full diversity over the wireless relay channels
- The proposed DSFCs mitigate relay nodes' synchronization mismatches due to the use of OFDM transmission
- The proposed DSFCs mitigate relay nodes' carriers offsets mismatches since only one relay is transmitting on any subcarrier