

Problems for Chapter 7 of 'Ultra Low Power Bioelectronics'

Problem 7.1

- From an online search, find the rainfall current (water volume per second) on the rainiest day in your city in a square meter of area. Assume that all the rain falls uniformly in a six hour period.
- Approximate the size of a raindrop and the amount of water that it contains.
- From a) and b) estimate the Poisson arrival rate of raindrops on a square meter of area.
- Assuming that each raindrop arrival results in an impulsive click of water received by a square-meter collector on the ground, find the power spectral density and power-spectral shape of the rainfall current arriving at the detector.
- Estimate the variance in the number of raindrops collected over a square meter over six hours.

Problem 7.2

By drawing analogies between Equation (7.14) for electronic current and the rainfall current of Problem 7.1, compute the increase/decrease factor in the relative precision of the number of gathered raindrops (variance/(mean²)) if

- The collection area for gathering rain is doubled.
- The volume of each raindrop doubles but the average rainfall current remains the same.
- The collection time is decreased to 3 hours from 6 hours.

Problem 7.3

Two noise sources contribute with identical transfer functions of 1 at all frequencies of interest to a particular output, and have identical power spectral densities at all frequencies. Find the ratio of the power spectral density of noise measured at the output to the power spectral density of either noise source if

- The two noise sources are perfectly correlated.
- The two noise sources are perfectly anti-correlated.
- The two noise sources are uncorrelated.

You may assume that all the noise measured at the output is only caused by these two noise sources.

Problem 7.4

Quanta (discrete energy packets) of sunlight with energy approximated by an energy per quantum of $E_{quant} = hc/\lambda$ with $h = 6.62 \times 10^{-34}$ (Planck's constant), $c = 3 \times 10^8$ ms⁻¹ (speed of light), $\lambda = 555 \times 10^{-9}$ m (average wavelength) fall on a $10 \mu\text{m} \times 10 \mu\text{m}$ photo detector with a power density of 1 W/m^2 . Assuming that 30% of photons that strike the detector result in the generation of an electron in the photodetector, calculate

- The mean number of electrons created per second by light impinging on the photodetector.
- The mean electronic current created by the light.
- The current noise power spectral density of the electronic current.

Problem 7.5

Derive Equation (7.30) from Equation (7.27) and your knowledge of device physics gained in Chapter 5.

Problem 7.6

Noise measurements of a transistor operated at relatively high above-threshold electronic currents reveal a flicker-noise corner frequency of 1 MHz, with $f_T = 10$ GHz. Assuming that all the $1/f$ noise in the transistor is caused by traps in its gate insulator of 10 nm thickness, estimate the trap impurity density (traps per unit area of gate oxide) in the process used to create the transistor. If the f_T of the transistor is reduced to 100 MHz by operating in the subthreshold regime, estimate the new flicker-noise corner frequency.

Problem 7.7

A chemical reaction transforms species A into species B with a forward molecular flux of I_F molecules per second. A backward reaction transforms species B back to A with a backward molecular flux of I_R molecules per second. Compute

- The mean net forward molecular flux of the reaction.
- Assuming that both forward and backward molecular fluxes obey Poisson statistics, compute the noise power spectral density of the molecular flux observed in the reaction.
- Do your answers bear any similarity to that obtained for subthreshold electronic current flow?

Problem 7.8

A mechanical system is composed of a damper with a damping force (F) versus velocity (v) relationship given by

$$F = \eta v$$

with η being a damping constant.

- If F is analogous to current and v is analogous to voltage, what is the electrical parameter that η is analogous to?
- By drawing analogies between force noise and current noise, and by using the fluctuation-dissipation theorem, compute the thermal force noise power spectral density caused by the dissipative damping.

Problem 7.9

A velocity-saturated transistor operating in its saturated regime is measured to have $\gamma = 2$ in Equation (7.32). Explain why its input-referred noise power compared with a subthreshold transistor of large width but with the same bias current as the velocity-saturated transistor is greater than 4x. From the equations that describe velocity-saturated operation in Chapter 6, derive an explicit formula for the input-referred noise in Equation (7.32) as a function of the overdrive voltage ($v_{GS} - V_{TS}$), and the parameters that characterize velocity-saturated operation.

Problem 7.10

A subthreshold transistor with width = W and length = L is modified such that $W \rightarrow M_w W$ and $L \rightarrow M_L L$. Assuming that it still operates in the subthreshold regime, compute

- a) The factor change in its input-referred thermal noise power spectral density.
- b) The factor change in its input-referred $1/f$ noise power spectral density.