Mathematical appendix 5

Nodal factors

This discussion relates to Sections 2.4.3 and 3.3, especially pp. 59 and 60.

Certain lunar constituents, notably $\mathbf{L_2}$, are affected by the 8.85-year cycle, and all the lunar constituents are affected by the 18.61-year nodal cycle; these modulations, which cannot be separately determined from a year of data, must also be represented in some way. In the full harmonic expansion they appear as terms separated from the main term by angular speeds (see Equation (3.3)):

$$i_d\omega_4;\, i_e\omega_5$$

The terms that are separated by ω_0 may be considered constant for all practical purposes. The modulations that cannot be resolved as independent constituents by analysis of a year of data are represented in harmonic expansion by small adjustment factors f and u. Each constituent is written:

$$H_n f_n \cos[\sigma_n t - g_n + (V_n + u_n)]$$

where V_n is the phase angle at the time zero. The nodal terms are:

f_n the nodal factor
u_n the nodal angle

both of which are functions of N sometimes p. The nodal factor and the nodal angle are 1.0 and 0.0 for solar constituents.

The application of nodal terms can be illustrated by the variations of $\mathbf{M_2}$. If yearly analyses are made throughout a nodal period of 18.61 years the value of the $\mathbf{M_2}$ amplitude will increase and decrease from the mean value by about 4 per cent. The full harmonic expansion shows that in addition to the main $\mathbf{M_2}$ term ($2\omega_1$) there are several other nearby terms; the only significant one has angular speed ($2\omega_1 + \omega_5$) and a relative amplitude of –0.0373, the negative coefficient signifying that in the full expansion this term appears as $-\cos[(2\omega_1 + \omega_5)t]$.

Writing the total constituent as:

$$H_{\mathrm{M}_2} \cos(2\omega_1 t) - \alpha H_{\mathrm{M}_2} \cos(2\omega_1 t + \omega_5 t)$$

we have:

$$
\begin{aligned}
&H_{\mathrm{M}_2}\,[(1-\alpha\cos\omega_5 t)\cos 2\omega_1 t+\alpha\sin\omega_5 t\sin 2\omega_1 t]\\
&\quad = H_{\mathrm{M}_2}\,[f\cos u\cos 2\omega_1 t - f\sin u\sin 2\omega_1 t]\\
&\quad = H_{\mathrm{M}_2}\,[f\cos(2\omega_1 t+u)]
\end{aligned}
$$

where

$$
\begin{aligned}
f\cos u &= 1-\alpha\cos\omega_5 t\\
f\sin u &= -\alpha\sin\omega_5 t
\end{aligned}
$$

and hence for small values of α:

$$
\begin{aligned}
f^2 &= 1-2\alpha\cos\omega_5 t+\cancel{\alpha}^2\\
f &\approx 1-\alpha\cos\omega_5 t = 1-0.0373\cos\omega_5 t
\end{aligned}
$$

and

$$
\tan u = -\frac{\alpha\sin\omega_5 t}{1-\alpha\cos\omega_5 t}
$$

(where α is in radians)

$$
u \approx -\alpha\sin\omega_5 t = -2.1^\circ\sin\omega_5 t
$$

The phase of the nodal angle $\omega_5 t$ is measured from the time when the ascending lunar node, at which the moon crosses the ecliptic from south to north, is at the First Point of Aries (♈). Around this time the excursions of the Moon north and south of the equator reach maximum declinations and the f value for $\mathbf{M_2}$ has a minimum value of 0.963. After an interval of 9.3 years when the Moon has minimum declinations, the f value reaches a maximum of 1.037. If the nodal adjustments were not made, analyses of a year of data made at the time of maximum declinations would give values of H_{M_2}, 7.5 per cent lower than would be obtained 9.3 years later.