Rotation of a second-order tensor

Mathematically, we can rotate a vector from unprimed to primed coordinates by multiplying it by a rotation matrix.

$\bar{\bar{x}}' = \bar{\bar{R}}\bar{\bar{x}}$,

How do we rotate a second-order tensor? We can think of a second-order tensor as a relationship between two vectors in unprimed coordinates

$\bar{\bar{y}} = \bar{\bar{A}}\bar{\bar{x}}$

and then ask how does this relationship transforms in the primed coordinates

$\bar{\bar{y}}' = \bar{\bar{A}}'\bar{\bar{x}}'$

From the last equation we have

$$\bar{\bar{R}}\bar{\bar{y}} = \bar{\bar{A}}'\bar{\bar{R}}\bar{\bar{x}} \Rightarrow \bar{\bar{y}} = \underbrace{\bar{\bar{R}}^T\bar{\bar{A}}'\bar{\bar{R}}}_{\bar{\bar{A}}}\bar{\bar{x}} \Rightarrow \bar{\bar{A}} = \bar{\bar{R}}^T\bar{\bar{A}}'\bar{\bar{R}} \Rightarrow \bar{\bar{A}}' = \bar{\bar{R}}\bar{\bar{A}}\bar{\bar{R}}^T$$

where we used the fact that rotation matrices are orthogonal, i.e.,

$\bar{\bar{R}}^{-1} = \bar{\bar{R}}^T$