8.4. Elastic Reflectivity Functions

8.4.4 P to SV reflectivity

page 184, equation (8.44): $\boldsymbol{x_r}$ should be bold faced.

$$\mathcal{R}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) = \frac{|\sin\sigma|}{4\cos\gamma_S}\,\frac{\alpha_0}{c_e(\sigma)}\left(a_\rho\left(1+2\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)+2\,a_\beta\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)\delta\left(\hat{\mathbf{m}}\cdot(\mathbf{x}-\mathbf{x_r})\right), \tag{8.44}$$

page 184, changes to equation (8.45):

$$\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) = -\rho_0\left|\sin\sigma\right|\left(a_\rho\left(1\;+2\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)+2\,a_\beta\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)\Theta\left(\hat{\mathbf{m}}\cdot(\mathbf{x}-\mathbf{x_r})\right). \tag{8.45}$$

The normal derivative of the scattering potential is

page 184, changes to equation (8.46):

$$\frac{\partial}{\partial m}\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) = -\rho_0\left|\sin\sigma\right|\left(a_\rho\left(1\;+2\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)+2\,a_\beta\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)\delta\left(\hat{\mathbf{m}}\cdot(\mathbf{x}-\mathbf{x_r})\right), \tag{8.46}$$

page 185, changes to equations (8.47) and (8.49)

so that

$$\mathcal{R}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) \simeq -\frac{\alpha_0}{4\,c_e(\sigma)\,\rho_0\cos\gamma_S}\,\frac{\partial}{\partial m}\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma). \tag{8.47}$$

We also have the relation (8.9), or

$$\frac{\partial}{\partial m}\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) = -\frac{2\,i\,\omega\,c_e(\sigma)}{\alpha_0\,\beta_0}\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma), \tag{8.48}$$

from which it follows

$$\mathcal{R}_{\mathrm{SVP}}(\mathbf{x},\,\sigma) \simeq \frac{i\,\omega}{2\,\rho_0\,\beta_0\cos\gamma_S}\mathbb{V}_{\mathrm{SVP}}(\mathbf{x},\,\sigma). \tag{8.49}$$

8.4.5 SV to P reflectivity

page 186, changes to equations (8.52) (8.53) and (8.54)

$$\mathbb{V}_{\mathrm{PSV}}(\mathbf{x},\,\omega,\,\sigma) = +\rho_0\left|\sin\sigma\right|\left(a_\rho\left(1+2\,\frac{\beta_0}{\alpha_0}\cos\sigma\right)+2\,a_\beta\,\frac{\beta_0}{\alpha_0}\cos\sigma\right).. \tag{8.52}$$

Comparing reflectivity to the SV to P potential gradient, we find

$$\mathcal{R}_{\mathrm{PSV}}(\boldsymbol{x},\sigma) = -\frac{\beta_0}{4\,\rho_0\,c_e(\sigma)\cos\gamma_{\mathrm{P}}}\,\frac{\partial\mathbb{V}_{\mathrm{PSV}}}{\partial m}(\boldsymbol{x},\sigma), \tag{8.53}$$

or, with the substitution (8.9) for the normal derivative,

$$\mathcal{R}_{\mathrm{PSV}}(\boldsymbol{x},\sigma) = \frac{i\,\omega}{2\,\rho_0\,\alpha_0\cos\gamma_{\mathrm{P}}}\,\mathbb{V}_{\mathrm{PSV}}(\boldsymbol{x},\sigma) \tag{8.54}$$

■ 8.5 A General Formula Relating Scattering Potential and Reflectivity.

For the wave equations studied here, reflectivity in the frequency domain has been found to be effectively proportional to the scattering potential. The general form for the relation has been

$$\mathcal{R}_{ri}(\boldsymbol{x},\sigma) \simeq \frac{i\,c_{\mathrm{r}}\,\omega}{2\,C_{\mathrm{r}}^{2}\cos\gamma_r}\,\mathbb{V}_{ri}(\boldsymbol{x},\sigma). \tag{8.56}$$