**CHAPTER 13 SOLUTIONS**

**13.1.**

1. One-way ANOVA is used to compare unrelated means of an at least interval leveled variable for two or more independent groups. In this case, we are comparing the population mean math achievement for *four* different groups based on type of high school program. Assuming that the normality and homogeneity of variances assumptions are tenable or that the test is robust to their violations (evaluated in later parts of this exercise), one-way ANOVA is an appropriate inferential test.
2. Based on the boxplot, the normality assumption appears to be tenable for all but possibly the some vocational program, because with the exception of one outlier, the distributions are fairly symmetric. The homogeneity of variance assumption appears tenable as well because the interquartile ranges are similar. Finally, the results of the ANOVA are likely to mirror the pattern of the medians: the median eighth grade math achievement for rigorous academic is the highest, followed by academic. The respective medians for some vocational and other program types appear to be approximately the same and lower than the other two program types.



1. A test of the skewness must be performed to evaluate the normality assumption because not all of the groups have at least 30 students. To do that, we split the file and use the SPSS Descriptives procedure to calculate the skewness and the standard error of the skewness. The results appear below:

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For all types of high school program the skewness ratio (the skewness statistic divided by the standard error of the skewness) is less than 2 in magnitude, so we conclude that the normality assumption is tenable.

1. Levene’s test must be performed to test the homogeneity of variance assumption because the sample sizes are not equal for the four groups. The results of Levene’s test indicate that the homogeneity of variance assumption is tenable, *F*(3, 496) = .78, *p* = .51.

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1. H0: *RA**A***SV**Oand H1: Not H0
2. *P* < .0005.



1. We reject the null hypothesis in favor of the alternative.
2. According to the results of the one-way analysis of variance, the average math achievement in eighth grade of college-bound students who have always been at grade level does vary as a function of their type of high school program, *F*(3, 496) = 8.23, *p* < .0005.
3. Approximately 4.74 percent of the variance in eighth grade math achievement can be explained by type of high school program, a relatively small effect, according to Cohen’s rule of thumb guidelines.
4. Post-hoc tests are necessary, in this case, because the results of the ANOVA are statistically significant and more than two independent means are being compared. The following tables provide the summary statistics and the results of the two post-hoc tests. The results of the post-hoc analysis differ depending on the test selected.

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According to the results of the LSD Test, the students in the rigorous academic programs perform significantly better than those in all of the other programs. The students in the academic program perform significantly better than those in the vocational and other programs. There was no statistically significant difference between those in the vocational and those in the other programs. According to the results of the Tukey HSD Test, the students in the rigorous academic program outperform those in the vocational and those in the other programs, but not those in the academic programs. The students in the academic program perform significantly better than those in the other programs, but not those in the vocational programs. Note that as indicated in the chapter, the results based on the HSD test are more conservative than those based on the LSD test.

1. In this case, we perform a one-way ANOVA over an independent samples *t*-test because there are three independent levels of urbanicity.
2. Based on the boxplot, the normality assumption appears to be tenable for those from an urban environment. Although those from a suburban and rural environment have one outlier each, the sample size in each of these groups is large enough to prevent these outliers from having an impact on the validity of the test results. The homogeneity of variance assumption appears tenable as well because the interquartile ranges appear similar. Finally, the results of the ANOVA are likely to indicate that there are no statistically significant differences between the group means because the medians are all similar.



1. There are 123 students in the urban group, 215 in the suburban group, and 162 students in the rural group. Because all of these sample sizes are larger than 30, the ANOVA is robust to violations of the normality assumption, if it is violated. Levene’s test must be performed to test the homogeneity of variances assumption because the sample sizes are not equal for the three types of environments. The results of Levene’s test indicate that the homogeneity of variances assumption is tenable, *F*(2, 497) = .45, *p* = .64.
2. The results of the one-way ANOVA are summarized in the following table.

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According to the results of the one-way analysis of variance, the average self-concept score in eighth grade of college-bound students who have always been at grade level does vary as a function of the type of setting in which these are from, *F*(2, 497) = 3.79, *p* = .02.

1. Approximately 1.5 percent of the variance in eighth grade self-concept is explained by urbanicity, a small effect.
2. Post-hoc tests are necessary, in this case, because the results of the ANOVA are statistically significant and more than two independent means are being compared. The following tables provide the summary statistics and the results of the two post-hoc tests.

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According to both the LSD and Tukey HSD post-hoc tests, the students from urban environments have statistically significantly higher eighth grade self-concept scores than the students from rural environments. No other statistically significant differences are detected between the different environments.

1. We conduct a series of three independent samples *t*-tests, each with significance level . Comparing eighth grade self-concept between those from urban and suburban environments, there is no statistically significant difference between the mean self-concept of those from an urban (*M* = 21.99, *SD* = 5.70) and those from a suburban environment (*M* = 21.26, *SD* = 5.80), *t*(336) = 1.12, *p* = .26. Comparing eighth grade self-concept between those from suburban and rural environments, there is no statistically significant difference between the mean self-concept of those from a suburban environment (*M* = 21.26, *SD* = 5.80) and those from a rural environment (*M* = 20.09, *SD* = 6.28), *t*(375) = 1.87, *p* = .06. Comparing eighth grade self-concept between those from urban and rural environments, the mean eighth grade self-concept score of those from an urban environment (*M* = 21.99, *SD* = 5.70) is statistically significantly higher than that of those from a rural environment (*M* = 20.09, *SD* = 6.28), *t*(283) = 2.63, *p* = .009. The results based on the *t*-test with the Bonferroni adjustment agree with those from the ANOVA.
2. Based on the boxplot, the normality assumption appears to be tenable because both distributions are reasonably symmetric. The homogeneity of variance assumption appears tenable as well because the interquartile ranges are similar. Finally, based on the medians of the two groups, the results of the ANOVA are likely to indicate that the twelfth grade math achievement of those who never smoked cigarettes is higher.



1. The sample sizes are large enough (*n* = 429 for those who have never smoked cigarettes and *n* = 71 for those who tried cigarettes at least once) to suggest that any deviations from normality that occur in the population for these two groups will not compromise the validity of the results of the analysis of variance. The results of Levene’s test indicate that the homogeneity of variance assumption is tenable, *F*(1, 498) = .001, *p* = .98.
2. According to the results of the one-way analysis of variance, the average twelfth grade math achievement score of college-bound students who have always been at grade level does differ depending on whether or not they smoked cigarettes in high school, *F*(1, 498) = 18.53, *p* < .0005.
3. According to the ratio of the sums of squares, approximately 3.59 percent of the variance in twelfth grade math achievement scores can be explained by cigarette use, a relatively small effect.
4. It is not necessary to perform post-hoc testing in this case. Because there are only two groups, we may use the sample means to determine the nature of the mean difference. Students who never smoked cigarettes (*M* = 57.51, *SD* = 7.71) performed statistically significantly better, on average than those who did (*M* = 53.24, *SD* = 8.00).
5. According to the results of the independent samples *t*-test, students who never smoked cigarettes performed statistically significantly better, on average, than those who did, *t*(498) = 4.31, *p* < .0005. As expected, when there are only two groups, the results of the one-way ANOVA are consistent with those of the independent samples *t*-test.
6. Yes, but the *p*-value from the ANOVA must be divided in half because the one associated with the ANOVA is two-tailed and the question is one-tailed. The *p*-value associated with the one-tailed question is *p* < .00025.
7. Based on the boxplot, the normality assumption appears to be tenable for those who expect to attain a Bachelor’s degree or less. Although the distributions for those who expect to attain more than a Bachelor’s degree have several outliers each, the samples are large enough in each of these distributions that these outliers are unlikely to impact the validity of the test results. The homogeneity of variance assumption appears tenable as well because the interquartile ranges are similar. Finally, based on the medians, the results of the ANOVA are likely to indicate that those who expect to attain less than a college degree take statistically significantly fewer units of math than the other groups.



1. There are 48 students who expect to attain less than a college degree, 159 expect a Bachelor’s degree, 190 a Master’s degree, and 103 a terminal degree. Because all of these sample sizes are larger than 30, the ANOVA is robust to violations of the normality assumption, if it is violated. Levene’s test must be performed to test the homogeneity of variance assumption because the sample sizes are not equal for the four expected levels of education. The results of Levene’s test indicate that the homogeneity of variance assumption is tenable, *F*(3, 496) = .37, *p* = .77.
2. According to the results of the one-way analysis of variance, the average number of years of mathematics taken by college-bound students who have always been at grade level does vary as a function of their expected educational attainment, *F*(3, 496) = 15.70, *p* < .0005.
3. Approximately 8.67 percent of the variance in units of math taken can be explained by the highest degree the student anticipates earning, a moderate effect.
4. Post-hoc tests are necessary, in this case, because the results of the ANOVA are statistically significant and more than two independent means are being compared. The following tables provide the summary statistics and the results of the two post-hoc tests.

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According to both the LSD and HSD post-hoc tests, students who anticipate earning less than a college degree take statistically significantly fewer units of math than any other degree type. Also, students who anticipate earning a BA take statistically significantly fewer units of math than those in the PhD group. According to the LSD, but not the HSD, students who anticipate earning a BA take statistically significantly fewer units of math than those who anticipate earning a Master’s degree. No other statistically significant differences are detected.

1. The statistically significant differences detected with the bootstrap approach are the same as for the LSD test.



1. The related Syntax file should contain:

GET

 FILE='…NELS.sav'.

DATASET NAME DataSet1 WINDOW=FRONT.

DATASET ACTIVATE DataSet1.

ONEWAY unitmath BY edexpect

 /STATISTICS DESCRIPTIVES HOMOGENEITY

 /MISSING ANALYSIS

 /POSTHOC=TUKEY LSD ALPHA(0.05).

BOOTSTRAP

 /SAMPLING METHOD=SIMPLE

 /VARIABLES TARGET=unitmath INPUT=edexpect

 /CRITERIA CILEVEL=95 CITYPE=PERCENTILE NSAMPLES=2000

 /MISSING USERMISSING=EXCLUDE.

ONEWAY unitmath BY edexpect

 /STATISTICS DESCRIPTIVES HOMOGENEITY

 /MISSING ANALYSIS

 /POSTHOC=TUKEY LSD ALPHA(0.05).

1. Based on the boxplot, the normality assumption does not appear to be tenable. However, the samples are large enough that these outliers are unlikely to impact the validity of the test results. The homogeneity of variance assumption appears tenable as well because the interquartile ranges are similar. Finally, based on the medians, the results of the ANOVA are likely to indicate that those who never smoked cigarettes attended schools with a similar school attendance rate than those who smoked cigarettes at least once.



1. There are 359 students that reported that they have never smoked cigarettes while 58 reported that they had at least once. Because both of these sample sizes are larger than 30, the ANOVA is robust to any violation of the normality assumption. Levene’s test must be performed to test the homogeneity of variance assumption because the sample sizes are not equal for the two groups. The results of Levene’s test indicate that the homogeneity of variance assumption is tenable, *F*(1, 415) = 2.30, *p* = .13.
2. According to the results of the one-way analysis of variance, among college-bound students who have always been at grade level, the average school attendance rate of those who ever smoked (*M* = 94.12, *SD* = 2.43) does not differ statistically significantly from that of those who never smoked (*M* = 93.57, *SD* = 4.96), *F*(1, 415) = .68, *p* = .41.
3. Post-hoc tests are not necessary, in this case, because the results of the ANOVA are not statistically significant.
4. No. Because the *p-*value at .41/2 = .21 still exceeds .05.
5. According to the boxplots, the distributions are appropriate for use in one-way ANOVA. Further, for the sample, the median socio-economic status is higher for those who attended nursery school, confirming that the hypothesized direction of the difference is correct. Because there are more than 30 students who owned a computer (*n* = 281) and who did not own a computer (*n* = 139), the ANOVA is robust to violations of the normality assumption. The results of Levene’s test indicate that the homogeneity of variances assumption is tenable, *F*(1, 418) = .40, *p* = .53. According to the results of the one-way ANOVA, among college bound students who are always at grade level, socio economic status does vary as a function of computer ownership, *F*(1, 418) = 72.43, *p* < .00025 (we halve the *p* value because the question was directional). According to the sample means, the mean socio economic status of students who owned a computer (*M* = 20.74, *SD* = 6.51) is statistically significantly higher than that of students who did not own a computer (*M* = 15.07, *SD* = 6.24). According to the value of R2, approximately 14.77 percent of the variance in socio economic status can be explained by computer ownership, a moderate effect according to Cohen’s rule of thumb guidelines. Finally, because the alternative hypothesis is non-directional, although there are only two groups, the ANOVA is appropriate to use in this case. The ANOVA is mathematically equivalent to a non-directional *t*-test, and would, therefore, yield the same results as the *t*-test.



b) Because there are only 15 students in the *NELS* data set with a non-English only background, the ANOVA is not robust to violations of the normality assumption and so this assumption needs to be tested. Because the skewness ratio of reading achievement in twelfth grade is less than negative 2 (skewness ratio = -3.69), the normality assumption is not tenable.

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 We may proceed by trying to use a nonlinear transformation to normalize the data prior to conducting the ANOVA or by using a non-parametric test described in Chapter 16, such as the Mann-Whitney U Test. In this case, we will attempt to find an appropriate nonlinear transformation.

 ACHRDG12 is severely negatively skewed. Following the methods of Chapter 4, we reflect the variable, translate it so that it takes on positive values and then take the log and square root transformations. The result is that the reflected square root transformation is most effective at reducing the skew.

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 Because the skewness ratios of twelfth grade reading achievement are less than 2 in magnitude for all four types of home language backgrounds, the normality assumption is tenable with the transformed variable.

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The results of Levene’s test indicate that the homogeneity of variance assumption is tenable for the transformed data, *F*(3, 496) = .53, *p* = .66. According to the results of the one-way ANOVA, among college bound students who are always at grade level, (the square root of) twelfth grade reading achievement does not vary as a function of home language background, *F*(3, 496) = 1.87, *p* = .13.

c) One-way ANOVA is not appropriate. The scores are dependent. A paired samples *t*-test should be used instead.

d) One-way ANOVA is not the most natural choice because COMPUTER ownership (the dependent variable) in this example is dichotomous, not continuous. One may use a contingency table to analyze the relationship between these categorical variables, but also, because in the long run, with a large enough sample size, a binomial (0/1) variable approaches normality, one may use the ANOVA. Recall that the mean of COMPUTER is equal to the proportion of students with a computer in the household. Below are solutions using both approaches.

 According to the results of the contingency table analysis, computer ownership does not vary much as a function of home language background. For all home language backgrounds, students were almost equally likely to own a computer as not. In the English only households, 46.67 percent owned a computer, in non-English dominant, 42.86 percent owned, in English dominant, 48.94 percent owned, and in English only, 47.64 percent owned.

Prior to conducting the analysis of variance, we conduct an analysis of the underlying assumptions. Because there are fewer than 30 students with a non-English only home language background, we test the tenability of the normality assumption. The skewness ratio of computer ownership is not severely skewed for any of the different home language backgrounds. The results of Levene’s test indicate that the homogeneity of variances assumption is tenable, *F*(3, 496) = 1.09, *p* = .35. According to the results of the one-way ANOVA, among college bound students who are always at grade level, computer ownership does not vary as a function of home language background, *F*(3, 496) = .12, *p* = .95.

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1. One-way ANOVA is not appropriate because the variable ABSENT12 is ordinal. A nonparametric test as described in Chapter 16, such as the Mann-Whitney U-Test would be appropriate to use.
2. There are 106 students from the Northeast, 151 from the North Central, 150 from the South, and 93 from the West. Because these sample sizes are greater than 30, the ANOVA is robust to violations of the normality assumption. The results of Levene’s test indicate that the homogeneity of variance assumption is tenable, *F*(3, 496) = 1.52, *p* = .21. According to the results of the one-way ANOVA, among college bound students who are always at grade level, average family size does not vary by region, *F*(3, 496) = 1.67, *p* =.17.
3. Because there are more than 30 students from each level of urbanicity, 123 from an urban environment, 215 from a suburban environment, and 162 from a rural environment, the ANOVA is robust to violations of the normality assumption. The results of Levene’s test indicate that the homogeneity of variances assumption is tenable, *F*(2, 497) = .86, *p* = .42. According to the results of the one-way ANOVA, among college bound students who are always at grade level, twelfth grade self-concept does vary as a function of urbanicity, *F*(2, 497) = 5.47, *p* = .004. According to the value of R2, approximately 2.15 percent of the variance in twelfth grade self concept can be explained by urbanicity, a small effect according to Cohen’s rule of thumb guidelines. According to Tukey’s HSD post hoc test, the mean twelfth grade self concept of students from an urban area (*M* = 33.15, *SD* = 6.32) is statistically significantly higher than that of students from a rural area (*M* = 30.33, *SD* = 7.71). No other statistically significant differences are detected by the Tukey HSD test. According to the LSD post hoc test, the mean twelfth grade self concept of students from an urban area (*M* = 33.15, *SD* = 6.32) is statistically significantly higher than that of students from a rural area (*M* = 30.33, *SD* = 7.71) and higher than that of students from a suburban area (*M* = 31.39, *SD* = 7.20). No statistically significant difference between those from a suburban and rural environment is detected by the LSD test.
	1. The sample sizes are large enough to suggest that the ANOVA will be robust to violations of the normality assumption. However, the severity of the skew is a concern. The skewness ratio for males is 53.39 and for females it is 24.46. Further, according to the results of Levene’s test, the homogeneity of variances assumption is not tenable, *F*(1, 457) = 9.48, *p* = .002. Because these violations, ANOVA on the data in their current form is not appropriate. Our options are to use a non-linear transformation to normalize the data and stabilize the variances and proceed with the ANOVA, to use a non-parametric test such as the Kruskal-Wallis analysis of variance described in Chapter 17, or to use a Bootstrap approach

When taking the square root of expected income, we see that although the skew is reduced, it does not have enough of an effect to make the homogeneity of variances assumption tenable.

According to the bootstrap approach, males expect to earn statistically significantly more than females, on average



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* 1. a)3 b) 2 c) 6 d) 7 e) 5

ANOVA

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Sum of Squares | df | Mean Square | F | Sig. |
| Between Groups |  2,240 | 2 | 1120 | 5 | .001 < *p* < .01 |
| Within Groups | 26,208 | 117 | 224 |  |  |
| Total | 28448 | 119 |  |  |  |

The results of the one-way analysis of variance indicate that there is at least one statistically significant difference in serotonin levels among bulimics, recovering bulimics, and the control group, *F*(2, 117) = 5, *p* = .01.

1. The one-way ANOVA can be performed by hand. Alternatively, it can be performed using SPSS. Using SPSS, we obtain the following output:

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None of the distributions is statistically significantly skewed and the results of Levene’s test indicate that the homogeneity of variance assumption is tenable, in this case, *F*(3, 16) = .31, *p* = .82. The results of the one-way ANOVA indicate that the mean score in the population is not the same across all four groups, *F*(3, 16) = 6.97, *p* = .003.

1. A post-hoc analysis should be conducted because the results of the ANOVA are statistically significant and there are more than two groups. The Tukey or LSD can be performed by hand. Alternatively, they can be performed using SPSS. Using SPSS, we obtain the following output:

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The results of the Tukey test indicate that, on average, students in Group 4 score statistically significantly higher than students in both Group 2 and Group 3. There were no other statistically significant differences detected by the Tukey test. The results of the LSD test indicate that, on average, students in Group 4 score statistically significantly higher than students in both Group 2 and Group 3 and also that, on average, students in Group 1 score statistically significantly higher than students in Group 2. There were no other statistically significant differences detected by the LSD test.