Microwave Electronics

Solutions manual

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Preface

This solution manual provides the (hopefully correct) solutions to all numerical problems of the text "Microwave Electronics". Finding good problems to stimulate students and to let them check their grasp of the theory is a challenge, sometimes lost because problems turn out to be either too trivial or too complex. We have tried to balance the two extremes, sometimes providing extra developments that are outside what the text of the problem literally asks for, and always struggling to exploit realistic parameters (as far as our knowledge goes).

Although the problems have been used for years in a number of courses on Microwave Electronics and RF Electronics we teach at Politecnico, the numerical results provided, although mostly reasonable, certainly include some errors and oversights. I would be glad to correct them if somebody points them out, e.g. by email (addresses below).

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1 A system introduction to microwave electronics

Problem 1 An antenna is working at a frequency of 100 kHz. Assuming the antenna length equal to $L = \lambda_0/100$, evaluate L.
Solution The free space wavelength at 100 kHz is:

$$\lambda_0 = \frac{c_0}{f} = \frac{3 \times 10^8}{100 \times 10^3} = 3000 \text{ m} = 3 \text{ km}$$

thus:

$$L = \frac{\lambda_0}{100} = 30 \text{ m}$$

not a pocket-size antenna.

Problem 2 A dielectric medium has $\epsilon_r = 9$. Evaluate the free-space wavelength at 10 GHz and the wavelength in the dielectric medium.
Solution The free space wavelength at 10 GHz is:

$$\lambda_0 = \frac{c_0}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m} = 3 \text{ cm}$$

while in the dielectric medium the velocity of light is $c_0/\sqrt{\epsilon_r}$, so that the wavelength scales accordingly as:

$$\lambda = \frac{c_0}{f\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{10 \times 10^9 \cdot \sqrt{9}} = 0.01 \text{ m} = 1 \text{ cm}.$$

Problem 3 Estimate the typical size of a distributed element operating at 100 MHz and at 50 GHz. Assume as the centerband dimension a quarter of the wavelength and as the wavelength $\lambda = \lambda_0/n$ where λ_0 is the free space wavelength and $n = 2.5$ is the effective refractive index.
Solution The effective (guided) wavelength ($n = 2.5$) at 100 MHz is:

$$\lambda_{g1} = \frac{c_0}{nf} = \frac{3 \times 10^8}{2.5 \cdot 100 \times 10^6} = 1.2 \text{ m}$$

while at 50 GHz:

$$\lambda_{g2} = \frac{c_0}{nf} = \frac{3 \times 10^8}{2.5 \cdot 50 \times 10^9} = 2.4 \text{ mm}$$

thus the device length at centerband will be, in the two cases:

$$L_1 = \frac{\lambda_{g1}}{4} = 30 \text{ cm}$$
$$L_2 = \frac{\lambda_{g2}}{4} = 0.6 \text{ mm}.$$

2 Passive elements and circuit layout

Problem 1 A lossless quasi-TEM line has a 50 Ω impedance and an effective permittivity $\epsilon_{\text{eff}} = 2$. Evaluate the per-unit-length parameters $\mathcal{L}$, $\mathcal{C}$. Compute the guided wavelength at 10 GHz.

Solution We have:

$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$$
$$v_f = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{c_0}{\sqrt{\epsilon_{\text{eff}}}};$$

thus:

$$\frac{1}{\mathcal{C}} = \frac{Z_0 c_0}{\sqrt{\epsilon_{\text{eff}}}} \rightarrow \mathcal{C} = \frac{\sqrt{\epsilon_{\text{eff}}}}{Z_0 c_0} = \frac{\sqrt{2}}{50 \cdot 3 \times 10^8} = 9.43 \times 10^{-11} \text{ F/m} = 94.3 \text{ pF/m}$$
$$\mathcal{L} = \frac{Z_0 \sqrt{\epsilon_{\text{eff}}}}{c_0} = \frac{50 \cdot \sqrt{2}}{3 \times 10^8} = 2.36 \times 10^{-7} \text{ H/m} = 236 \text{ nH/m}.$$

The guided wavelength at 10 GHz is:

$$\lambda_g = \frac{c_0}{f\sqrt{\epsilon_{\text{eff}}}} = \frac{3 \times 10^8}{10 \times 10^9 \cdot \sqrt{2}} = 2.12 \text{ cm}.$$

Problem 2 A lossy quasi-TEM line has a 50 Ω impedance. The dielectric attenuation is 0.1 dB/cm while the conductor attenuation is 1 dB/cm at 1 GHz. Evaluate the per-unit-length parameters $\mathcal{R}$, $\mathcal{G}$. Estimate their values and the resulting dielectric and conductor attenuation at 10 GHz. Assuming an effective permittivity $\epsilon_{\text{eff}} = 7$, evaluate the total loss over one guided wavelength at 10 GHz.

Solution The attenuations in natural units are:

$$\alpha_d = \frac{0.1 \cdot 100}{8.6859} = 1.15 \text{ Np/m}$$
$$\alpha_c = \frac{1 \cdot 100}{8.6859} = 11.5 \text{ Np/m}.$$

Then:

$$\alpha_d = \frac{\mathcal{G}}{2Y_0} \to \mathcal{G} = 2Y_0\alpha_d = 2 \cdot 50^{-1} \cdot 1.15 = 0.046 \text{ S/m}$$
$$\alpha_c = \frac{\mathcal{R}}{2Z_0} \to \mathcal{R} = 2Z_0\alpha_c = 2 \cdot 50 \cdot 11.5 = 1150 \ \Omega\text{/m}.$$

At 10 GHz the dielectric attenuation scales linearly and the conductor attenuation scales with the square root of frequency. Thus:

$$\alpha_d(10) = \alpha_d(1) \cdot \frac{10}{1} = 0.1 \cdot \frac{10}{1} = 1 \text{ dB/cm}$$
$$\alpha_c(10) = \alpha_c(1) \cdot \frac{\sqrt{10}}{1} = 1 \cdot \frac{\sqrt{10}}{1} = 3.16 \text{ dB/cm}$$

At 10 GHz the guided wavelength is:

$$\lambda_g = \frac{3}{\sqrt{7}} \text{ cm} = 1.13 \text{ cm}$$

therefore the total loss over λ_g is:

$$(\alpha_c + \alpha_d)\,\lambda_g = (1 + 3.16) \cdot 1.13 = 4.7 \text{ dB}.$$

To verify the correctness of the high-frequency approximation we need to evaluate all the per-unit-length parameters of the line. Since the characteristic impedance is assumed as real, we cannot extract exactly the per-unit-length parameters but we have to rely on the high-frequency approximation to be verified a posteriori. From the high-frequency approximation we have:

$$\mathcal{C} = \frac{\sqrt{\epsilon_{\text{eff}}}}{Z_0 c_0} = \frac{\sqrt{7}}{50 \cdot 3 \times 10^8} = 176.4 \text{ pF/m}$$
$$\mathcal{L} = \frac{Z_0\sqrt{\epsilon_{\text{eff}}}}{c_0} = \frac{50 \cdot \sqrt{7}}{3 \times 10^8} = 2.36 \times 10^{-7} \text{ H/m} = 441 \text{ nH/m}.$$

Then, let us evaluate the per-unit-length impedance and admittance at 1 GHz. We obtain:

$$\mathcal{Y} = \mathrm{j}\omega\mathcal{C} + \mathcal{G} = \mathrm{j} \cdot 2\pi \cdot 1 \times 10^9 \cdot 176.4 \times 10^{-12} + 0.046 = 1.11\mathrm{j} + 4.6 \times 10^{-2}$$
$$\mathcal{Z} = \mathrm{j}\omega\mathcal{L} + \mathcal{R} = \mathrm{j} \cdot 2\pi \cdot 1 \times 10^9 \cdot 441 \times 10^{-9} + 1150 = 2.77 \times 10^3\mathrm{j} + 1.15 \times 10^3.$$

The high-frequency approximation is therefore well verified for the parallel parameters but only marginally so for the series ones at 1 GHz. However, if we derive back the impedance and attenuation from the p.u.l. parameters we obtain:

$$Z_0 = \sqrt{\frac{\mathcal{Z}}{\mathcal{Y}}} = \sqrt{\frac{2.77 \times 10^3\mathrm{j} + 1.15 \times 10^3}{1.11\mathrm{j} + 4.6 \times 10^{-2}}} = 51.1 - \mathrm{j}9.1 \ \Omega$$
$$\gamma = \sqrt{\mathcal{Z}\mathcal{Y}} = \sqrt{(2.77 \times 10^3\mathrm{j} + 1.15 \times 10^3) \cdot (1.11\mathrm{j} + 4.6 \times 10^{-2})} =$$
$$= 12.45 + 56.34\mathrm{j} \text{ m}^{-1}$$

The attenuation obtained is similar to the one proposed:

$$\alpha_d + \alpha_c = 12.65 \text{ Np/m}$$

and the effective permittivity is:

$$\frac{2\pi}{\lambda_0}\sqrt{\epsilon_{\text{eff}}} = 56.34 \rightarrow \epsilon_{\text{eff}} = \left(\frac{56.34 \cdot 0.3}{2\pi}\right)^2 = 7.24$$

slightly different from the one proposed. However the imaginary part of the impedance is far from being negligible. We conclude that with the parameters given the high-frequency approximation is not entirely justified.

If we reduce losses in the initial data the approximation improves. Imagine in fact to have at 1 GHz:

$$\alpha_d = 0.115 \text{ Np/m}$$
$$\alpha_c = 1.15 \text{ Np/m.}$$

The dissipative parameters are reduced by a factor of 10:

$$\mathcal{G} = 0.0046 \text{ S/m}$$
$$\mathcal{R} = 115\ \Omega\text{/m.}$$

and therefore:

$$\mathcal{Y} = \mathrm{j}\omega\mathcal{C} + \mathcal{G} = 1.11\mathrm{j} + 4.6 \times 10^{-3}$$
$$\mathcal{Z} = \mathrm{j}\omega\mathcal{L} + \mathcal{R} = 2.77 \times 10^3\mathrm{j} + 1.15 \times 10^2.$$

Then:

$$Z_0 = \sqrt{\frac{\mathcal{Z}}{\mathcal{Y}}} = \sqrt{\frac{2.77 \times 10^3\mathrm{j} + 1.15 \times 10^2}{1.11\mathrm{j} + 4.6 \times 10^{-3}}} = 49,9 - \mathrm{j}0.93\ \Omega$$
$$\gamma = \sqrt{\mathcal{Z}\mathcal{Y}} = \sqrt{(2.77 \times 10^3\mathrm{j} + 1.15 \times 10^2) \cdot (1.11\mathrm{j} + 4.6 \times 10^{-3})} = $$
$$= 1.27 + 55.4 \times 10^1\mathrm{j}\ \mathrm{m}^{-1}$$

where the imaginary part of the impedance is now negligible, while in the high-frequency approximation:

$$\gamma_{HF} = \alpha + \mathrm{j}\beta = \alpha_d + \alpha_c + \mathrm{j}\frac{2\pi}{\lambda_0}\sqrt{\epsilon_{\text{eff}}} =$$
$$= 0.115 + 1.15 + \mathrm{j}\frac{2\pi}{0.3}\sqrt{7} = 1.265 + 55.41\mathrm{j}\ \mathrm{m}^{-1} \approx \gamma.$$

Problem 3 The conductivity of a 2 μm thick conductor is $\sigma = 1 \times 10^5$ S/m. Evaluate the frequency at which the skin-effect penetration depth is equal to the conductor thickness.

Solution Imposing that the skin penetration depth be $\delta = t$ where $t = 2\ \mu$m we obtain, from the definition of δ:

$$\delta = \sqrt{\frac{1}{\pi\mu\sigma f}} = t$$

from which:

$$f = \frac{1}{\pi\mu\sigma t^2} = \frac{1}{\pi \cdot 4\pi \times 10^{-7} \cdot 1 \times 10^5 \cdot (2 \times 10^{-6})^2} = 633.32 \text{ GHz} .$$

Problem 4 We suppose that the metal is non-magnetic; thus, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m. For a good conductor (gold, copper...) we can assume $\sigma \approx 1 \times 10^7$ S/m and in this case:

$$f = \frac{1}{\pi \cdot 4\pi \times 10^{-7} \cdot 4 \times 10^7 \cdot (2 \times 10^{-6})^2} = 1.58 \text{ GHz}.$$

Problem 5 A lossless transmission line with 50 Ω characteristic impedance and 5 mm guided wavelength is closed on $Z_L = 50 + \mathrm{j}50$ Ω. Compute the input impedance for a 2.5 and 1.25 mm long line.

Solution We have:

$$\lambda_g = 5 \times 10^{-3} \text{ m} \rightarrow \beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{5 \times 10^{-3}} = 1256.7 \text{ m}^{-1}.$$

The input impedance can be evaluated from:

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

We have for $l_1 = 2.5$ mm:

$$\begin{aligned} Z_i(l_1) = Z_0 &\frac{Z_L + jZ_0 \tan(\beta l_1)}{Z_0 + jZ_L \tan(\beta l_1)} = \\ &= 50 \cdot \frac{50 + j50 + j50 \cdot \tan\left(1256.7 \cdot 2.5 \times 10^{-3}\right)}{50 + j(50 + j50) \tan(1256.7 \cdot 2.5 \times 10^{-3})} = \\ &= 50 + j50 \ \Omega \end{aligned}$$

that is not surprising, since the line is a half wavelength and therefore the input impedance is the same as the load impedance. Then for $l_2 = 1.25$ mm:

$$\begin{aligned} Z_i(l_2) = Z_0 &\frac{Z_L + jZ_0 \tan(\beta l_2)}{Z_0 + jZ_L \tan(\beta l_2)} = \\ &= 50 \cdot \frac{50 + j50 + j50 \cdot \tan\left(1256.7 \cdot 1.25 \times 10^{-3}\right)}{50 + j(50 + j50) \tan(1256.7 \cdot 1.25 \times 10^{-3})} = \\ &= 25 - j25 \ \Omega \end{aligned}$$

The line length is a quarter wavelength and therefore we should have:

$$Z_i = \frac{Z_0^2}{Z_L} = \frac{50^2}{50 + j50} = 25 - j25 \ \Omega.$$

Problem 6 In a MIM capacitor the dielectric is 100 nm thick, width permittivity equal to 2. What is the capacitance per mm^2 area?

Solution We have:

$$C = \frac{\epsilon\epsilon_0 A}{t} \rightarrow \frac{C}{A} = \frac{\epsilon\epsilon_0}{t} = \frac{2 \cdot 8.86 \times 10^{-12}}{100 \times 10^{-9}} = 1.772 \times 10^{-4}\ \mathrm{F/m^2} =$$

$$= \frac{1.772 \times 10^{-4}}{1000^2} 10^{12} = 177\ \mathrm{pF/mm^2}$$

3 CAD techniques

Problem 1 A resistive two-port has the following impedance matrix:

$$\mathbf{Z} = R \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Sketch a possible structure (implementing the above impedance matrix and evaluate the scattering matrix (assume the normalization impedance $R_0 = R$).

Solution A possible implementation of the impedance matrix is through a T tripole with R on the three arms (input output and towards ground). Concerning the scattering matrix, the structure is symmetric and reciprocal, so that $S_{11} = S_{22}$ and $S_{21} = S_{12}$. We can evaluate the S matrix directly rather than through conversion formulae. We load port 2 with R so that $V_2 = -RI_2$ and then we have:

$$\begin{aligned}
&V_1 = 2RI_1 + RI_2 \\
&V_2 = -RI_2 = RI_1 + 2RI_2 \to I_2 = -\frac{1}{3}I_1 \to \\
&V_1 = 2RI_1 - \frac{1}{3}RI_1 = \frac{5}{3}RI_1 \to R_{in} = \frac{V_1}{I_1} = \frac{5}{3}R \\
&V_2 = -RI_2 = \frac{1}{3}RI_1 \to \frac{V_2}{V_1} = \frac{\frac{1}{3}RI_1}{\frac{5}{3}RI_1} = \frac{1}{5}
\end{aligned}$$

Assuming that port 1 is loaded by a generator with open circuit voltage V_{01} and internal resistance R we further have:

$$V_1 = V_{01}\frac{\frac{5}{3}R}{R + \frac{5}{3}R} = \frac{5}{8}V_{01} \to \frac{V_2}{V_{01}} = \frac{V_2}{V_1}\frac{V_1}{V_{01}} = \frac{1}{5}\cdot\frac{5}{8} = \frac{1}{8}$$

From the input resistance we have:

$$S_{11} = S_{22} = \frac{\frac{5}{3}R - R}{\frac{5}{3}R + R} = \frac{1}{4}$$

and:

$$S_{21} = S_{12} = 2\frac{V_2}{V_{01}} = \frac{1}{4}$$

since the normalization resistances at the two ports are the same. The same computations could have been carried out by exploiting the T equivalent circuit. To verify we can exploit conversion formulae:

$$S_{11} = S_{22} = \frac{(z_{11}-1)(z_{22}+1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} = \frac{(2-1)(2+1)-1}{(2+1)(2+1)-1} = \frac{1}{4}$$
$$S_{21} = S_{12} = \frac{2z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} = \frac{2\cdot 1}{(2+1)(2+1)-1} = \frac{1}{4}.$$

Instead of using conversion formulae in this case the direct inversion etc. can be implemented. In fact:

$$\begin{aligned} S &= (Z - RI)\,(Z + RI)^{-1} = \\ &= \left[R\begin{pmatrix}2\,1\\1\,2\end{pmatrix} - R\begin{pmatrix}1\,0\\0\,1\end{pmatrix}\right]\left[R\begin{pmatrix}2\,1\\1\,2\end{pmatrix} + R\begin{pmatrix}1\,0\\0\,1\end{pmatrix}\right]^{-1} = \\ &= \begin{pmatrix}1\,1\\1\,1\end{pmatrix}\begin{pmatrix}3\,1\\1\,3\end{pmatrix}^{-1} = \begin{pmatrix}1\,1\\1\,1\end{pmatrix}\begin{pmatrix}3/8 & -1/8\\-1/8 & 3/8\end{pmatrix} = \begin{pmatrix}1/4\,1/4\\1/4\,1/4\end{pmatrix}\end{aligned}$$

Problem 2 A reactive two-port has the following impedance matrix:

$$\mathbf{Z} = \mathrm{j}X\begin{pmatrix}2\,1\\1\,2\end{pmatrix}$$

Evaluate the scattering matrix assuming $R_0 = X$ and check that the properties of the S-matrix of a lossless two-port are verified.

Solution A possible implementation of the impedance matrix is through a T tripole with jX on the three arms (input output and towards ground). Concerning the scattering matrix, the structure is symmetric and reciprocal, so that $S_{11} = S_{22}$ and $S_{21} = S_{12}$. We can evaluate the S matrix directly rather than through conversion formulae. We load port 2 with $R_0 = X$ so that $V_2 = -XI_2$ and then we have:

$$V_1 = 2\mathrm{j}XI_1 + \mathrm{j}XI_2$$
$$V_2 = -XI_2 = \mathrm{j}XI_1 + 2\mathrm{j}XI_2 \rightarrow I_2 = -\left(\frac{2}{5}+\frac{1}{5}\mathrm{j}\right)I_1 \rightarrow$$
$$V_1 = 2\mathrm{j}XI_1 - \mathrm{j}X\left(\frac{2}{5}+\frac{1}{5}\mathrm{j}\right)I_1 = \left(\frac{1}{5}+\frac{8}{5}\mathrm{j}\right)XI_1 \rightarrow R_{in} = \frac{V_1}{I_1} = \left(\frac{1}{5}+\frac{8}{5}\mathrm{j}\right)X$$
$$V_2 = -XI_2 = \left(\frac{2}{5}+\frac{1}{5}\mathrm{j}\right)XI_1 \rightarrow \frac{V_2}{V_1} = \frac{\left(\frac{2}{5}+\frac{1}{5}\mathrm{j}\right)XI_1}{\left(\frac{1}{5}+\frac{8}{5}\mathrm{j}\right)XI_1} = \frac{2}{13} - \frac{3}{13}\mathrm{j}$$

Assuming that port 1 is loaded by a generator with open circuit voltage V_{01} and internal resistance X we further have:

$$V_1 = V_{01} \frac{\left(\frac{1}{5} + \frac{8}{5}\mathrm{j}\right) X}{X + \left(\frac{1}{5} + \frac{8}{5}\mathrm{j}\right) X} = \left(\frac{7}{10} + \frac{2}{5}\mathrm{j}\right) V_{01} \rightarrow$$

$$\frac{V_2}{V_{01}} = \frac{V_2}{V_1}\frac{V_1}{V_{01}} = \left(\frac{2}{13} - \frac{3}{13}\mathrm{j}\right) \cdot \left(\frac{7}{10} + \frac{2}{5}\mathrm{j}\right) = \frac{1}{5} - \frac{1}{10}\mathrm{j}$$

From the input resistance we have:

$$S_{11} = S_{22} = \frac{\left(\frac{1}{5} + \frac{8}{5}\mathrm{j}\right) X - X}{\left(\frac{1}{5} + \frac{8}{5}\mathrm{j}\right) X + X} = \frac{2}{5} + \frac{4}{5}\mathrm{j}$$

and:

$$S_{21} = S_{12} = 2\frac{V_2}{V_{01}} = \frac{2}{5} - \frac{1}{5}\mathrm{j}$$

since the normalization resistances at the two ports are the same. The same computations could have been carried out by exploiting the T equivalent circuit. To verify we can exploit conversion formulae:

$$S_{11} = S_{22} = \frac{(z_{11} - 1)(z_{22} + 1) - z_{12}z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}} = \frac{(2\mathrm{j} - 1)(2\mathrm{j} + 1) + 1}{(2\mathrm{j} + 1)(2\mathrm{j} + 1) + 1} = \frac{2}{5} + \frac{4}{5}\mathrm{j}$$

$$S_{21} = S_{12} = \frac{2z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12}z_{21}} = \frac{2 \cdot \mathrm{j}}{(2\mathrm{j} + 1)(2\mathrm{j} + 1) + 1} = \frac{2}{5} - \frac{1}{5}\mathrm{j}.$$

The S matrix is therefore:

$$S = \begin{pmatrix} \frac{2}{5} + \frac{4}{5}\mathrm{j} & \frac{2}{5} - \frac{1}{5}\mathrm{j} \\ \frac{2}{5} - \frac{1}{5}\mathrm{j} & \frac{2}{5} + \frac{4}{5}\mathrm{j} \end{pmatrix}$$

thus:

$$S^{-1} = \begin{pmatrix} \frac{2}{5} + \frac{4}{5}\mathrm{j} & \frac{2}{5} - \frac{1}{5}\mathrm{j} \\ \frac{2}{5} - \frac{1}{5}\mathrm{j} & \frac{2}{5} + \frac{4}{5}\mathrm{j} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{2}{5} - \frac{4}{5}\mathrm{j} & \frac{2}{5} + \frac{1}{5}\mathrm{j} \\ \frac{2}{5} + \frac{1}{5}\mathrm{j} & \frac{2}{5} - \frac{4}{5}\mathrm{j} \end{pmatrix} = S^{\dagger}.$$

Problem 3 A real generator has internal impedance $Z_G = 50{-}\mathrm{j}50\ \Omega$ and open circuit voltage $V_0 = 10$ V. Assuming $R_0 = 50\ \Omega$ derive the power wave equivalent circuit (Γ_G and b_0).

Solution We have:

$$\Gamma_G = \frac{Z_G - R_0}{Z_G + R_0} = \frac{50 - \mathrm{j}50 - 50}{50 - \mathrm{j}50 + 50} = \frac{1}{5} - \frac{2}{5}\mathrm{j}$$
$$b_0 = \frac{\sqrt{R_0}}{Z_G + R_0} V_0 = \frac{\sqrt{50}}{50 - \mathrm{j}50 + 50} \cdot 10 = \sqrt{2}\left(\frac{2}{5} + \frac{1}{5}\mathrm{j}\right)$$

Problem 4 A one-port has the power wave model:

$$b = \Gamma a + b_0.$$

Exploiting the coupled current-voltage generator model for the power wave generator b_0, show that the power-wave model is equivalent to the series representation:

$$V = ZI + V_0.$$

Assume R_0 as the normalization resistance; Γ is the reflection coefficient of Z with respect to R_0.

Solution We have that the equivalent power wave generator equivalent circuit is made of an impedance with reflection coefficient Γ in parallel with a current generator $b_0/\sqrt{R_0}$, in series to a voltage generator $b_0\sqrt{R_0}$. It follows that the input impedance of the equivalent circuit is Z (obviously) and that the open-circuit voltage is:

$$V_0 = \frac{b_0}{\sqrt{R_0}} Z + b_0\sqrt{R_0} = b_0 \frac{Z + R_0}{\sqrt{R_0}}.$$

Alternatively, we can replace a and b with the corresponding voltage and current representations:

$$a = \frac{V + R_0 I}{2\sqrt{R_0}}$$
$$b = \frac{V - R_0 I}{2\sqrt{R_0}}$$

we obtain:

$$\frac{V - R_0 I}{2\sqrt{R_0}} = \Gamma \frac{V + R_0 I}{2\sqrt{R_0}} + b_0$$
$$V - R_0 I = \Gamma V + \Gamma R_0 I + 2\sqrt{R_0} b_0$$
$$V(1 - \Gamma) = R_0 I (1 + \Gamma) + 2\sqrt{R_0} b_0$$
$$V = R_0 \frac{1 + \Gamma}{1 - \Gamma} I + \frac{2\sqrt{R_0} b_0}{1 - \Gamma},$$

i.e., since:

$$
\begin{aligned}
R_0 \frac{1+\Gamma}{1-\Gamma} &= Z \\
\frac{1}{1-\Gamma} &= \frac{1}{1-\dfrac{Z-R_0}{Z+R_0}} = \frac{Z+R_0}{2R_0}
\end{aligned}
$$

we finally have:

$$V = ZI + \frac{Z+R_0}{\sqrt{R_0}} b_0.$$

Problem 5 A two-port has scattering matrix:

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \end{pmatrix}.$$

Discuss whether the two-port is (1) reciprocal; (2) reactive. Derive the impedance matrix of the two-port with $R_0 = 50\ \Omega$.

Solution Since $\mathbf{S} = \mathbf{S}^T$ the two-port is reciprocal. Moreover, we have:

$$\mathbf{S}\mathbf{S}^{*T} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{j} \\ -\mathrm{j} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

i.e. the two-port is reactive. To evaluate the impedance matrix we can exploit the conversion formulae:

$$
\begin{aligned}
\mathbf{Z} = & \, \mathbf{R}^{1/2} (\mathbf{I} - \mathbf{S})^{-1} (\mathbf{I} + \mathbf{S}) \mathbf{R}^{1/2} = \\
= & \begin{pmatrix} \sqrt{R_0} & 0 \\ 0 & \sqrt{R_0} \end{pmatrix} \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \end{pmatrix} \right]^{-1} \\
& \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \end{pmatrix} \right] \begin{pmatrix} \sqrt{R_0} & 0 \\ 0 & \sqrt{R_0} \end{pmatrix} = \\
= & \, R_0 \begin{pmatrix} 0 & \mathrm{j}\left(\sqrt{2}+1\right) \\ \mathrm{j}\left(\sqrt{2}+1\right) & 0 \end{pmatrix}.
\end{aligned}
$$

We can verify from:

$$
\begin{aligned}
S_{11} = S_{22} &= \frac{(z_{11}-1)(z_{22}+1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} = \\
&= \frac{(0-1)(0+1) - \mathrm{j}\left(\sqrt{2}+1\right)\mathrm{j}\left(\sqrt{2}+1\right)}{(0+1)(0+1) - \mathrm{j}\left(\sqrt{2}+1\right)\mathrm{j}\left(\sqrt{2}+1\right)} = \frac{1}{\sqrt{2}} \\
S_{21} = S_{12} &= \frac{2z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} = \\
&= \frac{2 \cdot \mathrm{j}\left(\sqrt{2}+1\right)}{(0+1)(0+1) - \mathrm{j}\left(\sqrt{2}+1\right)\mathrm{j}\left(\sqrt{2}+1\right)} = \frac{1}{\sqrt{2}}\mathrm{j}.
\end{aligned}
$$

The structure can be realized with a reactive T having an inductor in the common branch with reactance j$R_0\left(\sqrt{2}+1\right)$ and a capacitor in the input and output branches with reactance -j$R_0\left(\sqrt{2}+1\right)$.

Problem 6 Consider a quadratic nonlinearity $y = x^2$ excited by two tones f_1 and f_2. Discuss the output spectrum when including harmonics up to the order 5 (a)using a box truncation approach; (b)using a diamond truncation approach. For this component only is the inclusion of odd-order harmonics and intermodulation products indispensable? Explain.

Solution Truncation to the order 5 implies that we have to keep all tones $mf_1 + nf_2$ such as $|m| \leq 5$, $|n| \leq 5$ (box truncation) or $|m| + |n| \leq 5$ (diamond truncation). The allowed absolute values $(|m|, |n|)$ are therefore:

Problem 7 for the box scheme: $(0,0)$, $(0,1)$, $(0,2)$, $(0,3)$, $(0,4)$, $(0,5)$; $(1,0)$, $(1,1)$, $(1,2)$, $(1,3)$, $(1,4)$, $(1,5)$; $(2,0)$, $(2,1)$, $(2,2)$, $(2,3)$, $(2,4)$, $(2,5)$; $(3,0)$, $(3,1)$, $(3,2)$, $(3,3)$, $(3,4)$, $(3,5)$; $(4,0)$, $(4,1)$, $(4,2)$, $(4,3)$, $(4,4)$, $(4,5)$; $(5,0)$, $(5,1)$, $(5,2)$, $(5,3)$, $(5,4)$, $(5,5)$;

Problem 8 for the diamond scheme: $(0,0)$, $(0,1)$, $(0,2)$, $(0,3)$, $(0,4)$, $(0,5)$; $(1,0)$, $(1,1)$, $(1,2)$, $(1,3)$, $(1,4)$; $(2,0)$, $(2,1)$, $(2,2)$, $(2,3)$; $(3,0)$, $(3,1)$, $(3,2)$; $(4,0)$, $(4,1)$; $(5,0)$.

If, however, we consider only the specific quadratic nonlinearity as an input-output system, such a nonlinearity will only generate second harmonics and second-order intermodulation products, corresponding to the $(|m|, |n|)$ pairs $(0,0)$, $(0,2)$, $(2,0)$, $(1,1)$. Of course if the nonlinearity is inserted in a circuit providing feedback also odd-order harmonics and IMPs will be generated.

Problem 9 Consider a quadratic nonlinearity $y = x^2$ excited by two sine tones f_1 and f_2. Confining the spectrum to the second harmonics, DC and second-order intermodulation products, evaluate the output spectrum directly and by remapping the frequencies on the artificial spectrum nf_0, $n = 0 \ldots N$. Use a box truncation with a proper order.

Solution Let us first evaluate the output spectrum in the natural spectrum. Supposing for simplicity all initial phases to be zero we have:

$$x(t) = X_1 \cos\omega_1 t + X_2 \cos\omega_2 t$$

thus:

$$\begin{aligned} y(t) = x^2(t) &= \left(X_1 \cos\omega_1 t + X_2 \cos\omega_2 t\right)^2 = \\ &= \frac{1}{2}X_1^2 + \frac{1}{2}X_2^2 + \frac{1}{2}X_1^2 \cos 2\omega_1 t + \frac{1}{2}X_2^2 \cos 2\omega_2 t + \\ &+ X_1 X_2 \cos\left(\omega_1 + \omega_2\right) t + X_1 X_2 \cos\left(\omega_1 - \omega_2\right) t. \end{aligned}$$

Since we are working with trigonometric function we explicitly remap only in the positive frequency axis. Using a box scheme of second order we have:

$$\begin{array}{ccc}
0 \to 0 & f_1 \to 5f_0 & 2f_1 \to 10f_0 \\
f_2 \to f_0 & f_1 + f_2 \to 6f_0 & 2f_1 + f_2 \to 10f_0 \\
2f_2 \to 2f_0 & f_1 + 2f_2 \to 7f_0 & 2f_1 + 2f_2 \to 11f_0 \\
f_1 - 2f_2 \to 3f_0 & 2f_1 - 2f_2 \to 8f_0 & \\
f_1 - f_2 \to 4f_0 & 2f_1 - f_2 \to 9f_0 &
\end{array}$$

Of course many of those frequencies will not be present in the input spectrum or generated in the output spectrum. With the mapping we have:

$$x(t) = X_1 \cos 5\omega_0 t + X_2 \cos \omega_0 t$$

thus:

$$\begin{aligned}
y(t) = x^2(t) &= \left[X_1 \cos(5\omega_0 t) + X_2 \cos(\omega_0 t)\right]^2 = \\
&= \frac{1}{2}X_1^2 + \frac{1}{2}X_2^2 + \frac{1}{2}X_1^2 \cos 2\omega_1 t + \frac{1}{2}X_2^2 \cos 2\omega_2 t + \\
&+ X_1 X_2 \cos 4\omega_0 t + X_1 X_2 \cos 6\omega_0 t
\end{aligned}$$

i.e. applying the inverse mapping:

$$\begin{aligned}
y(t) = &\frac{1}{2}X_1^2 + \frac{1}{2}X_2^2 + \frac{1}{2}X_1^2 \cos 10\omega_0 t + \frac{1}{2}X_2^2 \cos 2\omega_0 t + \\
&+ X_1 X_2 \cos(\omega_1 - \omega_2) t + X_1 X_2 \cos(\omega_1 + \omega_2) t
\end{aligned}$$

coinciding with the previous result.

Problem 10 Consider two tones $f_1 = \pi f_0$ and $f_2 = 2f_0$. Are they commensurate? Suppose now to represent them in a finite-precision arithmetic as $f_1 = 3.1415 f_0$ and $f_2 = 2.0000 f_0$. Are they commensurate now? what would be the period of the resulting two tone excitation?

Solution The two tones f_1 and f_2 are commensurate if:

$$\begin{aligned}
D_1 f_1 &= N_1 f_0 \\
D_2 f_2 &= N_2 f_0
\end{aligned}$$

where the integers (N_1, D_1) and (N_2, D_2) are mutually prime (i.e. they have 1 as only common divisor). In fact, going to the periods this implies:

$$\begin{aligned}
N_1 T_1 &= D_1 T_0 \\
N_2 T_2 &= D_2 T_0
\end{aligned}$$

i.e.:

$$T_3 = D_2 N_1 T_1 = D_1 N_2 T_2 = D_2 D_1 T_0$$

is a common period to both. If $f_1 = \pi f_0$ and $f_2 = 2f_0$ we have:

$$\begin{aligned}\pi T_1 &= T_0\\ 2T_2 &= T_0\end{aligned}$$

which means that no integer multiple of T_1 will be ever equal to an integer multiple of T_2. Consider now $f_1 = 3.1415 f_0$ and $f_2 = 2.0000 f_0$; we now obtain:

$$\begin{aligned}3.1415T_1 &= T_0\\ 2.0000T_2 &= T_0\end{aligned}$$

i.e.:

$$\begin{aligned}31415 \cdot T_1 &= 10000 \cdot T_0\\ 20000 \cdot T_2 &= 10000 \cdot T_0\end{aligned}$$

i.e. expanding into factors:

$$\begin{aligned}(5 \times 61 \times 103) \cdot T_1 &= 2^4 5^4 \cdot T_0\\ 2^5 5^4 \cdot T_2 &= 2^4 5^4 \cdot T_0\end{aligned}$$

or, simplifying:

$$\begin{aligned}(61 \times 103) \cdot T_1 &= 2^4 5^3 \cdot T_0 \to 6283T_1 = 2000T_0\\ 2T_2 &= T_0\end{aligned}$$

i.e.:

$$\begin{aligned}6283T_1 &= 2000T_0\\ 4000T_2 &= 2000T_0.\end{aligned}$$

The two frequencies are therefore commensurate and the common period is $T_3 = 2000T_0$. Starting from:

$$\begin{aligned}31415 \cdot T_1 &= 10000 \cdot T_0\\ 20000 \cdot T_2 &= 10000 \cdot T_0\end{aligned}$$

we could have applied the formula:

$$T_3 = T_0 \frac{\mathrm{mcm}\,(10000, 10000)}{\mathrm{MCD}\,(20000, 31415)} = T_0 \frac{10000}{5} = 2000T_0.$$

4 Directional couplers and power dividers

Problem 1 Imagine that an ideal 3dB, 90° coupler is fed with a 100 mW signal. What is the power on the coupled and the transmission port, respectively? What is the power on the insulated port? What is the phase difference between the coupled and transmission ports?

Solution The coupled power is one half of the input power, i.e. 50 mW. Ideally no power is on the isolated port and the phase difference between the coupled and transmission ports is $\pi/2$.

Problem 2 A Wilkinson divider on 50 Ω loads operates at 10 GHz. Assuming $\epsilon_{\text{eff}} = 4$ evaluate the lengths and characteristic impedance of the divider arms.

Solution The arm impedances are equal and given by:

$$Z_{01} = Z_0\sqrt{2} = 50 \cdot \sqrt{2} = 70.71\ \Omega$$

while the parallel resistance is $R = 2Z_0 = 100\ \Omega$. The arm length is $\lambda_g/4$ at 10 GHz, i.e.:

$$l = \frac{\lambda_g}{4} = \frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff}}}} = \frac{0.03}{4\sqrt{4}} = 3.75\ \text{mm}.$$

Problem 3 Design a 10 dB coupler on two-conductor coupled microstrips; the substrate is GaAs (permittivity 13) with thickness 0.3 mm; the centerband frequency is 10 GHz and the closing impedance 50 Ω.

Solution The coupling is $C^2 = 0.1$ i.e. $C = \sqrt{0.1} = 0.31623$. Thus, the even and odd mode impedances will be:

$$Z_{0e} = 50 \cdot \sqrt{\frac{1+C}{1-C}} = 50 \cdot \sqrt{\frac{1+0.31623}{1-0.31623}} = 69.371\ \Omega$$
$$Z_{0o} = 50 \cdot \sqrt{\frac{1-C}{1+C}} = 50 \cdot \sqrt{\frac{1-0.31623}{1+0.31623}} = 36.038\ \Omega.$$

From the graph in Fig. 4.16 (repeated here in Fig. 4.1) we approximately obtain:

$$S/h \approx 2.5 \times 10^{-1} \to S = 2.5 \times 10^{-1} \cdot 300 = 75 \quad \mu\text{m}$$
$$W/h \approx 6 \times 10^{-1} \to W = 6 \times 10^{-1} \cdot 300 = 180 \quad \mu\text{m},$$

while from the graph in Fig. 4.8 (repeated here in Fig. 4.2) we can obtain an approximation of the even and odd effective permittivities; we have:

$$\epsilon_{\mathrm{eff},o} \approx 7.2$$
$$\epsilon_{\mathrm{eff},e} \approx 8.8.$$

Averaging the even and odd mode quarter-wave lengths we obtain for the coupler length (centerband at 10 GHz) the value:

$$l = \frac{1}{2}\left(\frac{\lambda_0}{4\sqrt{\epsilon_{\mathrm{eff},e}}} + \frac{\lambda_0}{4\sqrt{\epsilon_{\mathrm{eff},o}}}\right) = \frac{1}{2}\left(\frac{0.03}{4\sqrt{8.8}} + \frac{0.03}{4\sqrt{7.2}}\right) = 2.66 \text{ mm}$$

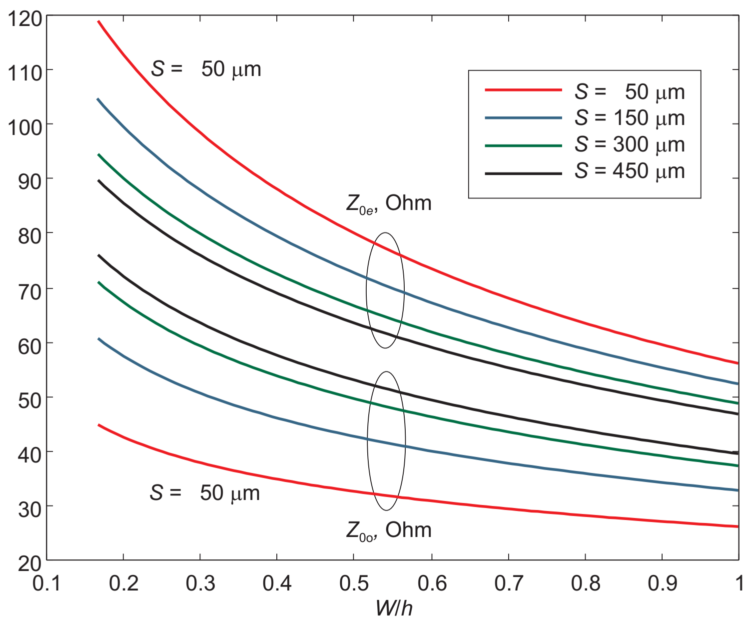

Figure 4.1 Even (Z_{0e}) and odd (Z_{0o}) mode microstrip impedances (two symmetric coupled lines) on GaAs substrate.

Problem 4 Design a four conductor Lange 3 dB coupler; the substrate is GaAs (permittivity 13) with thickness 0.3 mm; the centerband frequency is 10 GHz and the closing impedance 50 Ω.

Solution From the computation in Example 4.2 we obtain the even and odd-mode impedances of the equivalent two-conductor coupler as:

$$Z_{0e} = 176.2\ \Omega$$
$$Z_{0o} = 52.6\ \Omega.$$

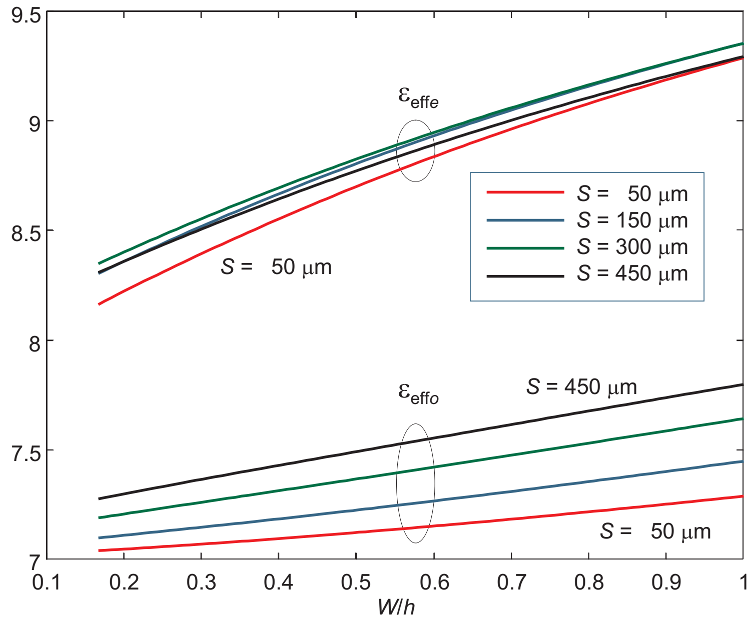

Figure 4.2 Even ($\epsilon_{\text{eff}e}$) and odd ($\epsilon_{\text{eff}o}$) mode effective permittivity of coupled microstrips on a 300 μm GaAs substrate.

From the graph in Fig. 4.16 we approximately obtain:

$$S/h \approx 5 \times 10^{-2} \rightarrow S = 5 \times 10^{-2} \cdot 300 = 15 \quad \mu\text{m}$$
$$W/h \approx 5 \times 10^{-2} \rightarrow W = 5 \times 10^{-2} \cdot 300 = 15 \quad \mu\text{m}.$$

More accurate values can be obtained from TX calculators in CAD suites as:

$$S = 30 \quad \mu\text{m}$$
$$W = 25 \quad \mu\text{m}.$$

An approximation of the even and odd effective permittivities is:

$$\epsilon_{\text{eff},o} \approx 7$$
$$\epsilon_{\text{eff},e} \approx 8.$$

Averaging the even and odd mode quarter-wave lengths we obtain for the coupler length (centerband at 10 GHz) the value:

$$l = \frac{1}{2}\left(\frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff},e}}} + \frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff},o}}}\right) = \frac{1}{2}\left(\frac{0.03}{4\sqrt{8}} + \frac{0.03}{4\sqrt{7}}\right) = 2.74 \text{ mm}.$$

Problem 5 Design (dimensions and impedances) a hybrid ring with 3 dB coupling on 50 Ω at 5 GHz. Assume that the line effective permittivity is 5.

Solution The ring impedance is given by:

$$Z_{01} = Z_{02} = Z_0\sqrt{2} = 50\sqrt{2} = 70.71 \ \Omega$$

The arm length is $\lambda_g/4$ at 5 GHz, i.e.:

$$l = \frac{\lambda_g}{4} = \frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff}}}} = \frac{0.06}{4\sqrt{5}} = 6.70 \text{ mm}.$$

Problem 6 Design (dimensions and impedances) a branch-line coupler with 3 dB coupling on 100 Ω at 20 GHz. Assume that the line effective permittivity is 5.

Solution The arm impedances are given by:

$$Z_{01} = Z_0/\sqrt{2} = 100/\sqrt{2} = 70.71\ \Omega$$
$$Z_{02} = 100\ \Omega.$$

The arm length is $\lambda_g/4$ at 10 GHz, i.e.:

$$l = \frac{\lambda_g}{4} = \frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff}}}} = \frac{0.03}{4\sqrt{5}} = 3.35 \text{ mm}.$$

Problem 7 Design (dimensions and impedances) a Wilkinson divider on 70 Ω at 30 GHz. Assume that the line effective permittivity is 2.

Solution The arm impedances are given by:

$$Z_{01} = Z_0\sqrt{2} = 70 \cdot \sqrt{2} = 99\ \Omega$$

while the parallel resistance is $R = 2Z_0 = 140\ \Omega$. The arm length is $\lambda_g/4$ at 10 GHz, i.e.:

$$l = \frac{\lambda_g}{4} = \frac{\lambda_0}{4\sqrt{\epsilon_{\text{eff}}}} = \frac{0.01}{4\sqrt{2}} = 1.77 \text{ mm}.$$

5 Active RF and microwave semiconductor devices

Problem 1 A HEMT has gate length of 50 nm, thickness of the supply layer $d + \Delta d = 10$ nm, gate width 100 μm. The relative dielectric constant of the supply layer is $\epsilon_r = 13$. Evaluate the maximum device transconductance and cutoff frequency assuming an equivalent electron saturation velocity $v_n = 2 \times 10^7$ cm/s.

Solution We have:

$$g_m \approx W v_{n,\text{sat}} \frac{\epsilon}{d + \Delta d} = 100 \times 10^{-6} \cdot 2 \times 10^5 \cdot \frac{13 \cdot 8.86 \times 10^{-12}}{10 \times 10^{-9}} = 230 \text{ mS}$$

while:

$$f_T \approx \frac{v_{n,\text{sat}}}{2\pi L_g} = \frac{2 \times 10^5}{2\pi \cdot 50 \times 10^{-9}} = 637 \text{ GHz}.$$

Problem 2 The cutoff frequency of a HEMT is 400 GHz while the maximum oscillation frequency is 900 GHz. The gate periphery is $W = 200$ μm while the transconductance per unit length is 800 mS/mm. The gate and intrinsic resistances are $R_G = 5$ Ω and $R_I = 4$ Ω. Estimate the gate-source capacitance and the output resistance R_{DS}.

Solution The transconductance is:

$$g_m = 800 \cdot 0.2 = 160 \text{ mS}$$

and since:

$$f_T = \frac{g_m}{2\pi C_{GS}}$$

we have:

$$C_{GS} = \frac{g_m}{2\pi f_T} = \frac{160 \times 10^{-3}}{2\pi \cdot 400 \times 10^9} = 63.67 \text{ fF}.$$

The maximum frequency of oscillation is:

$$f_{\max} = \frac{f_T}{2} \sqrt{\frac{R_{DS}}{R_G + R_I}}$$

thus:

$$R_{DS} = (R_G + R_I)\left(\frac{2f_{\max}}{f_T}\right)^2 = 9 \cdot \frac{4 \cdot 900^2}{400^2} = 182.25\ \Omega.$$

Problem 3 A heterojunction bipolar transistor has a base to emitter capacitance $C_{BE} = 5$ pF. The DC collector current is $I_C = 100$ mA. Estimate the ideal cutoff frequency.

Solution Ideally the cutoff frequency is:

$$f_T = \frac{g_m}{2\pi C_{BE}} = \frac{I_C}{2\pi C_{BE} V_T} = \frac{100 \times 10^{-3}}{2\pi \cdot 5 \times 10^{-12} \cdot 26 \times 10^{-3}} = 122\ \text{GHz}$$

where $V_T = k_B T/q = 26$ mV at ambient temperature, and $g_m = I_C/V_T$.

Problem 4 A FET has $R_{DS} \to \infty$, $V_{T0} = -2$ V, drain current at $v_{GS} = 0$ equal to $I_{DSS} = 100$ mA, output conductance $\partial I_D/\partial V_{DS} = 100$ mS for $v_{GS} = 0$, $v_{DS} \to 0$. Evaluate the values of the parameters of the quadratic Curtice model β, α, V_{T0}, λ. (Neglect the difference between intrinsic and extrinsic voltages.)

Solution The parameters of the quadratic Curtice model are β, V_{T0}, λ, α according to the above-threshold formula:

$$i_D = \beta\left(v_{GS} - V_{T0}\right)^2\left(1 + \lambda v_{DS}\right)\tanh\left(\alpha v_{DS}\right).$$

Taking into account that $R_{DS} = \left(\partial I_D/\partial V_{DS}\right)^{-1}$ for large v_{DS} and that (for $v_{DS} \to \infty$):

$$R_{DS}^{-1} = \partial I_D/\partial V_{DS} = \beta\left(v_{GS} - V_{T0}\right)^2 \lambda$$

we have that R_{DS}^{-1} is (neglecting parasitics) proportional to λ; thus we immediately get $\lambda = 0$. $V_{T0} = -2$ V as given. To evaluate β we have that the saturation current (large v_{DS}) is, for $v_{GS} = 0$:

$$I_{DSS} = \beta V_{T0}^2 \to \beta = \frac{I_{DSS}}{V_{T0}^2} = \frac{0.1}{(-2)^2} = 2.5 \times 10^{-2}\ \text{A/V}^2.$$

Then, with $\lambda = 0$:

$$\frac{\partial I_D}{\partial V_{DS}} = \alpha\beta\left(v_{GS} - V_{T0}\right)^2\left(1 - \tanh^2\left(\alpha v_{DS}\right)\right)$$

and for $v_{DS} \to 0$ and $v_{GS} = 0$:

$$\frac{\partial I_D}{\partial V_{DS}} = \alpha\beta\left(-V_{T0}\right)^2 \to \alpha = \frac{1}{\beta\left(-V_{T0}\right)^2}\frac{\partial I_D}{\partial V_{DS}} = \frac{1}{I_{DSS}}\frac{\partial I_D}{\partial V_{DS}} = \frac{1}{0.1}0.1 = 1\ \text{A/V}.$$

Thus finally the model reads, above threshold:

$$i_D = 2.5 \times 10^{-2} \cdot \left(v_{GS} + 2\right)^2 \tanh\left(v_{DS}\right).$$

Problem 5 Consider a simplified small-signal equivalent circuit of a bipolar transistor in the common emitter configuration, where only the intrinsic circuit is considered, the input includes the base-emitter capacitance C_{BE} and the base-emitter resistance R_{BE}, while the output has the current generator βI_B in parallel with the output resistance R_{CE}. Evaluate the maximum available power gain of the stage and the optimum input and output matching condition.

Solution Connect to the transistor input a generator with internal admittance Y_g and short-circuit current A_g. Since the internal feedback is neglected the circuit is one-directional and in this case unconditionally stable. Power matching at the input implies:

$$Y_{go} = G_{go} + \mathrm{j}B_{go} = Y_{in}^* = \frac{1}{R_{BE}} - \mathrm{j}\omega C_{BE};$$

in such conditions $I_B = A_g/2$. Output matching implies:

$$Z_{Lo} = R_{CE};$$

thus the load current is:

$$I_L = \frac{\beta I_B}{2} = \frac{\beta A_g}{4}$$

while the load power (coinciding with the load available power since the load is power matched) is:

$$P_{\mathrm{av},L} = R_{Lo} |I_L|^2 = \frac{1}{16}\beta^2 R_{CE} |A_g|^2$$

while the input available power is:

$$P_{\mathrm{av},g} = \frac{1}{4} R_{BE} |A_g|^2$$

therefore the maximum available gain of the stage (i.e. the power gain of the stage with input and output matching) is:

$$\mathrm{MAG} = \frac{P_{\mathrm{av},L}}{P_{\mathrm{av},g}} = \frac{\frac{1}{16}\beta^2 R_{CE} |A_g|^2}{\frac{1}{4} R_{BE} |A_g|^2} = \frac{R_{CE}}{4R_{BE}}\beta^2.$$

Problem 6 Consider a simplified small-signal equivalent circuit of a FET in the common source configuration, where only the intrinsic circuit is considered, the input includes the gate resistance R_G and the gate-source capacitance C_{GS}, while the output has the transconductance generator $g_m V^*$ where V^* is the voltage across C_{GS} in parallel with the output resistance R_{DS}. Evaluate the maximum available power gain of the stage and the optimum input and output matching condition.

Solution Connect to the transistor input a generator with internal impedance Z_g and open-circuit voltage E_g. Since the internal feedback is neglected the circuit is one-directional and in this case unconditionally stable. Power matching at the input implies:

$$Z_{go} = R_{go} + \mathrm{j}X_{go} = Z_{in}^* = R_G - \frac{1}{\mathrm{j}\omega C_{GS}};$$

in such conditions $I_G = E_g/2R_{go} = E_g/2R_G$ and:

$$V^* = \frac{I_G}{\mathrm{j}\omega C_{GS}} = \frac{E_g}{2R_G}\frac{1}{\mathrm{j}\omega C_{GS}}$$

Output matching implies:

$$Z_{Lo} = R_{DS};$$

thus the load current is:

$$I_L = \frac{g_m V^*}{2} = \frac{g_m}{\mathrm{j}\omega C_{GS}}\frac{E_g}{4R_G}$$

while the load power (coinciding with the load available power since the load is power matched) is:

$$P_{\mathrm{av},L} = R_{Lo}\,|I_L|^2 = R_{DS}\frac{g_m^2}{\omega^2 C_{GS}^2}\frac{|E_g|^2}{16R_G^2}$$

while the input available power is:

$$P_{\mathrm{av},g} = \frac{|E_g|^2}{4R_G};$$

therefore the maximum available gain of the stage (i.e. the power gain of the stage with input and output matching) is:

$$\mathrm{MAG} = \frac{P_{\mathrm{av},L}}{P_{\mathrm{av},g}} = \frac{R_{DS}\dfrac{g_m^2}{\omega^2 C_{GS}^2}\dfrac{|E_g|^2}{16R_G^2}}{\dfrac{|E_g|^2}{4R_G}} = \frac{g_m^2}{\omega^2 C_{GS}^2}\frac{R_{DS}}{4R_G}.$$

Notice that the MAG decreases with the square of the frequency and is $\mathrm{MAG} = 1$ at the (angular) maximum oscillation frequency:

$$\omega_{\max} = \frac{g_m}{2C_{GS}}\sqrt{\frac{R_{DS}}{R_G}} = \frac{\omega_T}{2}\sqrt{\frac{R_{DS}}{R_G}}$$

where ω_T is the angular cutoff frequency.

6 Microwave linear amplifiers

Problem 1 A real generator with $\Gamma_G = 0.2$ and $b_0 = 1\ \mathrm{W}^{1/2}$ is connected to a load with $\Gamma_L = 0.5$. Evaluate the power delivered to the load and the maximum available power of the generator.

Solution We have:

$$P_L = |b_0|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L \Gamma_G|^2} = 1^2 \cdot \frac{1 - 0.5^2}{|1 - 0.5 \cdot 0.2|^2} = 0.925\ \mathrm{W}$$
$$P_{av} = |b_0|^2 \frac{1}{1 - |\Gamma_G|^2} = 1^2 \cdot \frac{1}{1 - 0.2^2} = 1.042\ \mathrm{W}.$$

Alternatively, we could evaluate the corresponding series equivalent circuit. We have:

$$Z_G = R_0 \frac{1 + \Gamma_G}{1 - \Gamma_G} = R_0 \frac{1 + 0.2}{1 - 0.2} = 1.5 R_0$$
$$Z_L = R_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} = R_0 \frac{1 + 0.5}{1 - 0.5} = 3 R_0$$
$$V_0 = b_0 \frac{Z_G + R_0}{\sqrt{R_0}} = \frac{1.5 R_0 + R_0}{\sqrt{R_0}} = 2.5 \sqrt{R_0}$$
$$P_{av} = \frac{V_0^2}{4 R_G} = \frac{2.5^2 R_0}{4 \cdot 1.5 R_0} = 1.042\ \mathrm{W}$$
$$P_L = \frac{V_0^2}{(R_L + R_G)^2} R_L = \frac{\left(2.5 \sqrt{R_0}\right)^2 \cdot 3 R_0}{(3 R_0 + 1.5 R_0)^2} = 0.925\ \mathrm{W}.$$

Problem 2 A loaded two-port has the following characteristics: $P_{in} = 10$ mW; $P_{\mathrm{av},in} = 20$ mW; $P_L = 100$ mW; $P_{\mathrm{av},out} = 300$ mW. Evaluate the two-port gains G_{op}, G_{av}, G_{t}.

Solution We have:

$$G_{\mathrm{op}} = \frac{P_L}{P_{in}} = \frac{100}{10} = 10$$
$$G_{\mathrm{av}} = \frac{P_{\mathrm{av},out}}{P_{\mathrm{av},in}} = \frac{300}{20} = 15$$
$$G_{\mathrm{t}} = \frac{P_L}{P_{\mathrm{av},in}} = \frac{100}{20} = 5.$$

Problem 3 A two-port has the following scattering matrix ($R_0 = 50\ \Omega$):

$$S = \begin{pmatrix} 0 & 0 \\ 10 & 0 \end{pmatrix}$$

Evaluate the two-port MAG. Is the two-port unilateral?

Solution The two-port is unilateral since $S_{12} = 0$. We can therefore apply the MUG definition:

$$\mathrm{MAG} = \mathrm{MUG} = \frac{|S_{21}|^2}{\left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right)} = 100.$$

On the other hand since the device is power matched when closed on the normalization resistances and the device is unilateral the maximum gain is $|S_{21}|^2$.

Problem 4 A two-port has the following scattering matrix ($R_0 = 50\ \Omega$):

$$S = \begin{pmatrix} 0.1 & 0.01 \\ 10 & 0.1 \end{pmatrix}$$

Compute the input and output reflection coefficients when the two-port is loaded on 100 Ω.

Solution We have:

$$\Gamma_G = \Gamma_L = \frac{100 - 50}{100 + 50} = \frac{1}{3}.$$

Then:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.1 + \frac{10 \cdot 0.01 \cdot \frac{1}{3}}{1 - 0.1 \cdot \frac{1}{3}} = 0.134$$
$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1 - S_{11}\Gamma_G} = 0.1 + \frac{10 \cdot 0.01 \cdot \frac{1}{3}}{1 - 0.1 \cdot \frac{1}{3}} = 0.134.$$

Problem 5 A two-port has $K = 2$, $S_{21} = 15(1 + \mathrm{j})$ and $S_{12} = 0.1$. Evaluate the two-port MAG and MSG. Assume the two-port is unconditionally stable.

Solution We have:

$$\mathrm{MAG} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right) = \left|\frac{15(1+\mathrm{j})}{0.1}\right|\left(2 - \sqrt{2^2 - 1}\right) = 56.84$$
$$\mathrm{MSG} = \frac{|S_{21}|}{|S_{12}|} = \left|\frac{15(1+\mathrm{j})}{0.1}\right| = 212.$$

Problem 6 Consider the parameter $|S_{21}|^2$. To what power gain (and in which loading conditions) does it correspond?

Solution We have, by definition, that:

$$|S_{21}|^2 = \frac{|b_2|^2}{|a_1|^2}$$

when the two-port is loaded on the corresponding normalization resistances. Since $P_L = |b_2|^2 - |a_2|^2$ and $a_2 = 0$ we have that in this case $P_L = |b_2|^2$. Moreover, the available power of the generator connected to port 1 is:

$$P_{\mathrm{av},in} = |a_1|^2 \frac{1}{1 - |\Gamma_G|^2} \equiv |a_1|^2$$

if the generator internal impedance coincides with the normalization resistance. We conclude that:

$$|S_{21}|^2 = \frac{|b_2|^2}{|a_1|^2} = \frac{P_L}{P_{\mathrm{av},in}} = G_{\mathrm{t}}$$

i.e. the parameter is the two-port transducer gain when the two-port is closed on its normalization resistances. The same result can be found by directly using the definition of the transducer gain.

Problem 7 Discuss the stability (according to the one- and two-parameter criteria) of the two-port with scattering matrix:

$$S = \begin{pmatrix} \mathrm{j}0.1 & 10 \\ 0.1 & 0.1 \end{pmatrix}.$$

Suppose now to exchange ports 1 and 2, the new scattering matrix becomes:

$$S' = \begin{pmatrix} 0.1 & 0.1 \\ 10 & \mathrm{j}0.1 \end{pmatrix}.$$

Does the 2-port stability change?

Solution We have for the stability parameters (two-parameter criterion):

$$\begin{aligned}
\Delta_S &= S_{11}S_{22} - S_{12}S_{21} = \mathrm{j}0.1 \cdot 0.1 - 10 \cdot 0.1 = -1.0 + 1.0 \times 10^{-2}\mathrm{j} \\
K &= \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta_S|^2}{2\,|S_{12}S_{21}|} = \\
&= \frac{1 - |\mathrm{j}0.1|^2 - |0.1|^2 + \left|-1.0 + 1.0 \times 10^{-2}\mathrm{j}\right|^2}{2\,|10 \cdot 0.1|} = 0.99
\end{aligned}$$

thus the two-port is potentially unstable, both criteria are violated. Since both stability parameters are invariant upon exchange of port 1 with port 2 the situation does not change when interchanging the ports. The one-parameter stability criterion yields e.g. (a similar result can be obtained from μ_2):

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta_S| + |S_{12} S_{21}|} =$$
$$= \frac{1 - |\mathrm{j}0.1|^2}{|0.1 + \mathrm{j}0.1 \cdot (-1.0 + 1.0 \times 10^{-2}\mathrm{j})| + |10 \cdot 0.1|} = 0.867 < 1$$

while it should be > 1 for unconditional stability. If we interchange the ports we obtain:

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta_S| + |S_{12} S_{21}|} =$$
$$= \frac{1 - |0.1|^2}{|\mathrm{j}0.1 - 0.1 \cdot (-1.0 + 1.0 \times 10^{-2}\mathrm{j})| + |0.1 \cdot 10|} = 0.867 < 1.$$

Problem 8 Discuss the stability (according to the one- and two-parameter criteria) of the unilateral two-port with scattering matrix:

$$S = \begin{pmatrix} \mathrm{j}1.1 & 0 \\ 5 & 0.1 \end{pmatrix}.$$

Solution We have for the stability parameters (two-parameter criterion):

$$\Delta_S = S_{11} S_{22} - S_{12} S_{21} = \mathrm{j}1.1 \cdot 0.1 - 5 \cdot 0 = 0.11\mathrm{j} \to |\Delta_S| = 0.11 < 1$$
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta_S|^2}{2\,|S_{12} S_{21}|} = \frac{1 - |\mathrm{j}1.1|^2 - |0.1|^2 + |0.11|^2}{2\,|5 \cdot 0|} = -\infty < 1$$

thus the two-port not unconditionally stable, in fact beware, $|S_{11}| > 1$ (a practically very uncommon occurrence) and therefore unconditional stability is lost anyway.

The one-parameter stability criterion yields that a two-port is unconditionally stable is $\mu_1 > 1$ or $\mu_2 > 1$. In this case we have:

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta_S| + |S_{12} S_{21}|} = \frac{1 - |\mathrm{j}1.1|^2}{|0.1 + \mathrm{j}1.1 \cdot (0.11\mathrm{j})| + |5 \cdot 0|} = -10 < 1$$

thus potential instability is immediately found. Moreover:

$$\mu_2 = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^* \Delta_S| + |S_{12} S_{21}|} = \frac{1 - |0.1|^2}{|\mathrm{j}1.1 - 0.1 \cdot (0.11\mathrm{j})| + |5 \cdot 0|} = 0.909 < 1$$

Thus also with the second criterion (that should be equivalent to the first) the two-port turns out to be potentially unstable.

Problem 9 We want to design a 10 dB amplifier with parallel and series feedback. What is the minimum device $|S_{21}|$?

Solution The low-frequency ideal gain of the feedback amplifier is equal to $10 \log_{10} |S_{21f}|^2 = 10$ dB, i.e. $|S_{21f}|^2 = 10$, $|S_{21f}| = \sqrt{10} = 3.16$.The limitation on the open-loop S_{21} is therefore:

$$|S_{21}| \geq 2\,(1 + |S_{21f}|) = 2 \cdot (1 + 3.16) = 8.32.$$

7 Low-noise amplifier design

Problem 1 A noisy tripole has the following admittance matrix:

$$\mathbf{Y} = \begin{pmatrix} \mathrm{j}\omega C & -\mathrm{j}\omega C \\ -\mathrm{j}\omega C & \dfrac{1}{R} + \mathrm{j}\omega C \end{pmatrix}$$

Derive an equivalent circuit of the tripole and evaluate the short-circuit noise current correlation matrix.

Solution The tripole is a passive reciprocal structure that can be easily implemented though a π circuit with input arm admittance $Y_1 = 0$, common arm admittance $Y_2 = \mathrm{j}\omega C$ and output arm admittance $Y_3 = \dfrac{1}{R}$. The resistor R is the only noisy elements. We can apply the generalized Nyquist formula where:

$$\mathbf{S} = 4k_B T_0 \operatorname{Re} \mathbf{Y} = 4k_B T_0 \begin{pmatrix} 0 & 0 \\ 0 & 1/R \end{pmatrix}.$$

Alternatively we can compute the short-circuit input and output currents induced by the resistor noise current generator A_n; we find that $I_2 = A_n$ and $I_1 = 0$; thus $S_{i_1 i_1} = 0$, $S_{i_1 i_2} = 0$, $S_{i_2 i_2} = 4k_B T_0 R^{-1}$.

Problem 2 A voltage noise source has a power spectrum of 1 $(\mathrm{nV})^2/\mathrm{Hz}$[1]. Assuming a bandwidth of 500 MHz, evaluate the mean square value of the noise voltage and the noise available power on 50 Ω. Evaluate the noise available power spectral density of the generator.

Solution Let us call the voltage noise source spectrum S_{v_n}. The noise available power can be expressed as:

$$P_{av} = \int_B \frac{S_{v_n}}{4R} df = \int_B p_n df = \frac{B S_{v_n}}{4R} = \frac{v_{n,rms}^2}{4R}$$

[1] Spectral units like V^2/Hz, A^2/Hz etc., when the V or A unit is associated to a multiplier, like in $\mu\mathrm{V}^2/\mathrm{Hz}$, must be interpreted as $(\mu\mathrm{V})^2/\mathrm{Hz}$, that is $10^{-12}\mathrm{V}/\mathrm{Hz}$. This meaning has been explicited in the text whenever possible.

where the noise available power spectral density is:

$$p_n = \frac{S_{v_n}}{4R} = \frac{1 \times 10^{-18}}{4 \times 50} = 5.0 \times 10^{-21} \text{ W/Hz}$$

we then have

$$v_{n,rms} = \sqrt{BS_{v_n}} = \sqrt{500 \times 10^6 \cdot 1 \times 10^{-18}} = 22.36\ \mu\text{V}.$$
$$P_{av} = \frac{BS_{v_n}}{4R} = \frac{500 \times 10^6 \cdot 1 \times 10^{-18}}{4 \cdot 50} = 2.5 \text{ pW}.$$

We can also check that:

$$P_{av} = Bp_n = 500 \times 10^6 \cdot 5.0 \times 10^{-21} = 2.5 \text{ pW}.$$

If the noise was the thermal noise of the resistor, what would the resistor temperature be? we have ($T_0 = 300$ K):

$$p_n = k_B T_0 \frac{T}{T_0} = 1.69 \times 10^{-19} \cdot 26 \times 10^{-3} \frac{T}{T_0} = 5.0 \times 10^{-21} \rightarrow$$
$$T = \frac{300 \cdot 5.0 \times 10^{-21}}{1.69 \times 10^{-19} \cdot 26 \times 10^{-3}} = 341 \text{ K}.$$

Problem 3 A resistor with $R = 1$ kΩ operates with a bandwidth of 5 GHz. Evaluate the power spectral density at 300 K. Evaluate the spectral density of the resistor noise voltage and the r.m.s. noise voltage value over the specified bandwidth.

Solution The power spectral noise density p_n is $k_B T$ independent on the resistor value. Taking into account that $k_B T = 26$ meV at ambient temperature (300 K) we have:

$$p_n = q \cdot k_B T|_{\text{eV}} = 1.69 \times 10^{-19} \cdot 26 \times 10^{-3} = 4.394 \times 10^{-21} \text{ W/Hz}.$$

The power spectrum of the noise voltage is:

$$S_{v_n} = 4k_B TR = 4 \cdot 4.394 \times 10^{-21} \cdot 1 \times 10^3 = 1.7576 \times 10^{-17} \text{ V}^2/\text{Hz}.$$

The r.m.s. noise voltage $v_{n,rms}$ is (B system bandwidth):

$$v_{n,rms} = \sqrt{S_{v_n} B} = \sqrt{4k_B TRB} =$$
$$= \sqrt{4 \cdot 1.69 \times 10^{-19} \cdot 26 \times 10^{-3} \cdot 1 \times 10^3 \cdot 5 \times 10^9} = 0.29 \text{ mV}.$$

Just for a check we can evaluate the total power as the noise available power spectral density by the bandwidth or by means of the open circuit voltage quadratic mean, thus obtaining the same value:

$$p_n B = 4.394 \times 10^{-21} \cdot 5 \times 10^9 = 2.1 \times 10^{-11} \text{ W}$$
$$\frac{v_{n,rms}^2}{4R} = \frac{\left(0.29 \times 10^{-3}\right)^2}{4 \cdot 1 \times 10^3} = 2.1 \times 10^{-11} \text{ W}.$$

Problem 4 In the circuit in Fig. 7.1 (a) assume $Z_1 = 50 + j50\ \Omega$, $Z_L = 50 - j50\ \Omega$; the two noise generators are the thermal noise (Nyquist law) generators of the two impedances, respectively (i.e. i_{n1} is associated to Z_1, e_{n2} to Z_L). Assuming 1 GHz bandwidth, evaluate at 300 K the total power on the load.

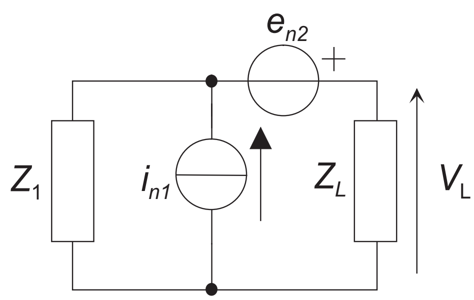

Figure 7.1 Circuit from Es.4.

Solution We evaluate the voltage on the load through phasor analysis. We have (assuming that the noise sources are uncorrelated):

$$V_L = (Z_1 I_{n1} + E_{n2}) \frac{Z_L}{Z_1 + Z_L}$$

$$\overline{V_L V_L^*} = \left| \frac{Z_L}{Z_1 + Z_L} \right|^2 \left(|Z_1|^2 \overline{I_{n1} I_{n1}^*} + \overline{E_{n2} E_{n2}^*} \right) =$$

$$= 4k_B T \left| \frac{Z_L}{Z_1 + Z_L} \right|^2 \left(|Z_1|^2 G_1 + R_L \right)$$

We have:

$$Y_1 = G_1 + jB_1 = \frac{1}{Z_1} = \frac{1}{50 + j50} = \frac{1}{100} - \frac{1}{100} j$$

$$Y_L = G_L + jB_L = \frac{1}{Z_L} = \frac{1}{50 - j50} = \frac{1}{100} + \frac{1}{100} j$$

thus:

$$\overline{V_L V_L^*} = 4k_B T \left| \frac{Z_L}{Z_1 + Z_L} \right|^2 \left(|Z_1|^2 G_1 + R_L \right) =$$

$$= 4 \cdot 1.69 \times 10^{-19} \cdot 26 \times 10^{-3} \cdot \left| \frac{50 - j50}{50 + j50 + 50 - j50} \right|^2 \cdot$$

$$\cdot \left(|50 + j50|^2 \frac{1}{100} + 50 \right) =$$

$$= 8.788 \times 10^{-19}\ \mathrm{V}^2/\mathrm{Hz}$$

The load voltage effective value will be:

$$v_{L,rms} = \sqrt{B \overline{V_L V_L^*}} = \sqrt{1 \times 10^9 \cdot 8.788 \times 10^{-19}} = 2.965 \times 10^{-5}\ \mathrm{V}$$

and the power on the load is:

$$P_L = G_L v_{L,rms}^2 = \frac{1}{100} \cdot \left(2.965 \times 10^{-5}\right)^2 = 8.79 \text{ pW}.$$

Problem 5 In the circuit in Fig. 7.1 (b) compute the minimum noise figure and optimum generator impedance of the two-port in the grey box assuming $Z_1 = 10$ Ω, $g_m = 500$ mS. The two (uncorrelated) noise generators e_{n1} and i_{n2} are white, with spectral density equal to 100 $(\text{pV})^2/\text{Hz}$ and 100 $(\text{pA})^2/\text{Hz}$, respectively. The system bandwidth is 100 MHz. (*Hint: the noise figure is a ratio of available noise powers at the output port, that is independent from the output conductance of the two-port and therefore reduces to a ratio of short-circuit noise current spectra at port 2.*)

Solution The total noise current at the short-circuited port 2 is given by:

$$I_t = g_m V^* + I_2 = \frac{g_m Z_1 \left(E_{ns} + E_1\right)}{Z_1 + Z_s} + I_2$$

and thus the power spectrum of i_t is given, via the symbolic definition, by:

$$S_{i_t} = \overline{I_t I_t^*} = \frac{g_m^2 \left|Z_1\right|^2 \left(\overline{E_{ns} E_{ns}^*} + \overline{E_1 E_1^*}\right)}{\left|Z_1 + Z_s\right|^2} + \overline{I_2 I_2^*}.$$

Let us exploit Nyquist or Nyquist-like definitions (through the noise resistance R_n and the noise conductance G_n) for the power spectra:

$$\begin{aligned} \overline{E_{ns} E_{ns}^*} &= 4 k_B T_0 R_s \\ \overline{E_1 E_1^*} &= 4 k_B T_0 R_n \\ \overline{I_2 I_2^*} &= 4 k_B T_0 G_n. \end{aligned}$$

Substituting we have:

$$S_{i_t} = 4 k_B T_0 \left[\frac{g_m^2 \left|Z_1\right|^2 \left(R_s + R_n\right)}{\left|Z_1 + Z_s\right|^2} + G_n\right].$$

The noise figure can be expressed as the ratio of the total output short-circuit current power spectrum and the same but taking only into account the noise introduced by the source noise generator:

$$\begin{aligned} \text{NF} &= \frac{\dfrac{g_m^2 \left|Z_1\right|^2 \left(R_s + R_n\right)}{\left|Z_1 + Z_s\right|^2} + G_n}{\dfrac{g_m^2 \left|Z_1\right|^2 R_s}{\left|Z_1 + Z_s\right|^2}} = \\ &= 1 + \frac{R_n}{R_s} + \frac{G_n}{R_s} \frac{\left|Z_1 + Z_s\right|^2}{g_m^2 \left|Z_1\right|^2} = \\ &= 1 + \frac{R_n}{R_s} + \frac{G_n}{R_s} \frac{\left(R_s + R_1\right)^2 + \left(X_s + X_1\right)^2}{g_m^2 \left|Z_1\right|^2}. \end{aligned}$$

The optimum source reactance is immediately found as:

$$X_{so} = -X_1.$$

Let us now optimize vs. R_s the resulting noise figure:

$$\begin{aligned}
\mathrm{NF} &= 1 + \frac{R_n}{R_s} + \frac{G_n}{R_s}\frac{(R_s + R_1)^2}{g_m^2 |Z_1|^2} = \\
&= 1 + \frac{R_n}{R_s} + \frac{G_n}{R_s}\frac{R_s^2 + 2R_sR_1 + R_1^2}{g_m^2 |Z_1|^2} = \\
&= 1 + \frac{R_n}{R_s} + \frac{G_nR_s}{g_m^2 |Z_1|^2} + \frac{2G_nR_1}{g_m^2 |Z_1|^2} + \frac{G_nR_1^2}{R_sg_m^2 |Z_1|^2} = \\
&= 1 + \frac{1}{R_s}\left(R_n + \frac{G_nR_1^2}{g_m^2 |Z_1|^2}\right) + \frac{2G_nR_1}{g_m^2 |Z_1|^2} + R_s\left(\frac{G_n}{g_m^2 |Z_1|^2}\right) = \\
&= 1 + \frac{a}{R_s} + b + cR_s.
\end{aligned}$$

The minimum can be found by differentiating the noise figure vs. R_s and imposing zero derivative; the optimum value is:

$$R_{so} = \sqrt{\frac{a}{c}} = \sqrt{\frac{R_n + \dfrac{G_nR_1^2}{g_m^2 |Z_1|^2}}{\dfrac{G_n}{g_m^2 |Z_1|^2}}} = \sqrt{R_1^2 + \frac{g_m^2 |Z_1|^2 R_n}{G_n}}$$

while for the minimum noise figure we have:

$$\begin{aligned}
\mathrm{NF}_{\mathrm{min}} &= 1 + 2\sqrt{ac} + b = \\
&= 1 + 2\sqrt{\left(R_n + \frac{R_1^2G_n}{g_m^2 |Z_1|^2}\right)\frac{G_n}{g_m^2 |Z_1|^2}} + \frac{2R_1G_n}{g_m^2 |Z_1|^2}.
\end{aligned}$$

Introducing the values given for the power spectra of e_1 and i_1 we obtain the values of G_n and R_n:

$$S_{e_1} = 4k_BTR_n = 0.1 \times 10^{-18} \to R_n = \frac{0.1 \times 10^{-18}}{4 \cdot 1.69 \times 10^{-19} \cdot 26 \times 10^{-3}} = 5.689\ \Omega$$

$$S_{i_1} = 4k_BTG_n = 0.1 \times 10^{-18} \to G_n = \frac{0.1 \times 10^{-18}}{4 \cdot 1.69 \times 10^{-19} \cdot 26 \times 10^{-3}} = 5.689\ \mathrm{S}$$

Thus:

$$\mathrm{NF}_{\mathrm{min}} = 1 + 2\sqrt{\left(5.689 + \frac{10^2 \cdot 5.689}{0.5^2 \cdot |10|^2}\right)\frac{5.689}{0.5^2 \cdot |10|^2}} + \frac{2 \cdot 10 \cdot 5.689}{0.5^2 \cdot |10|^2} = 10.63$$

$$R_{so} = \sqrt{10^2 + \frac{0.5^2 \cdot |10|^2 \cdot 5.689}{5.689}} = 11.2\ \Omega$$

$$X_{so} = -X_1 = 0\ \Omega$$

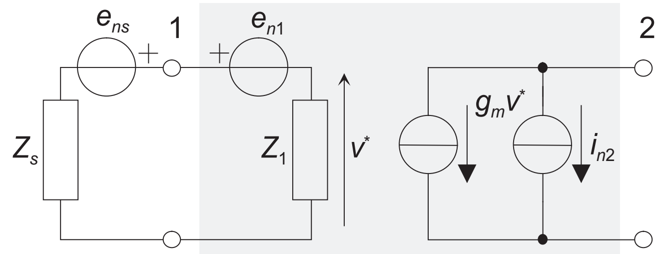

Figure 7.2 Circuit from Es. 5.

Problem 6 A noisy two-port has the optimum source impedance $Z_{Go} = 25 + j32\ \Omega$, minimum noise figure $\mathrm{NF}_{\min} = 2$ dB, and series noise conductance $g_n = 50$ mS. Supposing that the source reactance is the optimum one, estimate the variation of the noise figure when the source resistance varies between 5 and 50 Ω.

Solution We have $R_{Go} = 25\ \Omega$, $X_G = X_{Go} = 32\ \Omega$; thus:

$$\mathrm{NF} = \mathrm{NF}_{\min} + \frac{g_n}{R_G}\left[(R_G - R_{Go})^2 + (X_G - X_{Go})^2\right] = \mathrm{NF}_{\min} + \frac{g_n}{R_G}(R_G - R_{Go})^2;$$

but $\mathrm{NF}_{min} = 10^{2/10} = 1.5849$; thus, for $R_G = 5\ \Omega$ and $R_G = 50\ \Omega$ we have the extreme values:

$$\mathrm{NF} = 1.5849 + \frac{g_n}{R_G}(R_G - R_{Go})^2 = 5.58$$
$$\mathrm{NF} = 1.5849 + \frac{g_n}{R_G}(R_G - R_{Go})^2 = 2.2$$

The resulting behaviour is shown in Fig. 7.3.

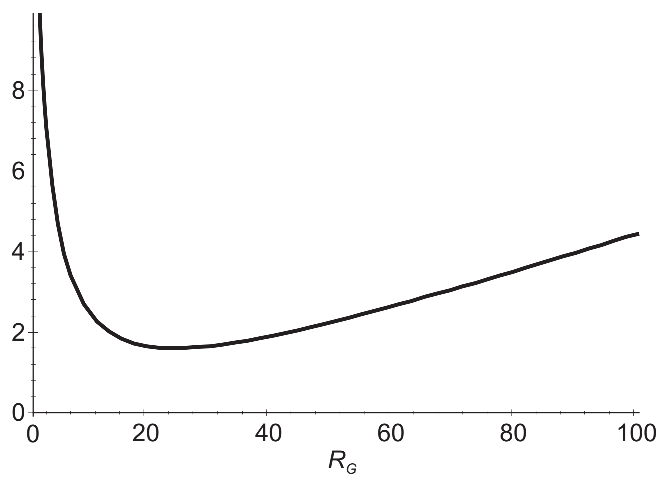

Figure 7.3 Behaviour of the noise figure as a function of R_G, Problem 6.

Problem 7 Two amplifiers are cascaded (50 Ω design) with $G_{\text{av},1} = 10$ dB, $G_{\text{av},2} = 20$ dB, $\text{NF}_1 = 1$ dB, $\text{NF}_2 = 6$ dB. Evaluate the total noise figure according to the Friis formula.

Solution The gains etc. in natural units are:

$$\begin{aligned} G_{\text{av},1} &= 10^{10/10} = 10 \\ G_{\text{av},2} &= 10^{20/10} = 100 \\ \text{NF}_1 &= 10^{1/10} = 1.26 \\ \text{NF}_2 &= 10^{6/10} = 3.98 \end{aligned}$$

From the Friis formula:

$$NF = 1 + (\text{NF}_1 - 1) + \frac{\text{NF}_2 - 1}{G_{\text{av},1}} = 1 + 1.26 - 1 + \frac{3.98 - 1}{10} = 1.558.$$

The gain of the second amplifier therefore plays no role.

Problem 8 A resistive attenuator designed on 50 Ω has 3 dB loss. What is the noise figure?

Solution For a resistive two-port the loss coincides with the noise figure, which is therefore 3 dB.

Problem 9 A common-gate LNA has to be designed on a 50 Ω generator with a device having a specific transconductance of 400 mS/mm. Evaluate the device periphery needed and the low-frequency noise figure, assuming $P = 0.9$.

Solution The LNA matching condition is, at low frequency:

$$Z_{in} = \frac{1}{g_m} \to g_m = \frac{1}{50} = 20 \text{ mS}.$$

Therefore the gate periphery needed is:

$$W = \frac{20}{400} = 0.05 \text{ mm} = 50 \ \mu\text{m}.$$

For the low-frequency noise figure we have:

$$F \approx 1 + P = 1.9.$$

Problem 10 An inductive series feedback amplifier must be designed on 50 Ω at 10 GHz. Assuming $C_{GS} = 0.2$ pF and $g_m = 200$ mS, evaluate L_S and L_G. Evaluate the noise figure, assuming $P = 0.7$ and neglecting the gate noise source (and therefore the correlation).

Solution The inductances are derived from the design formulae:

$$\begin{aligned} L_S &= \frac{C_{GS} R_0}{g_m} = \frac{0.2 \times 10^{-12} \cdot 50}{0.2} = 50 \text{ pH} \\ L_G &= \frac{1}{\omega^2 C_{GS}} - L_S = \frac{1}{(2\pi \cdot 10 \times 10^9)^2 \cdot 0.2 \times 10^{-12}} - 5.0 \times 10^{-11} = 1.21 \text{ nH}. \end{aligned}$$

The noise figure is given, assuming $R = 0$, by:

$$F = 1 + \frac{\omega}{\omega_T} \frac{P}{Q_L} = 1 + \frac{g_m R_0 \omega^2 P}{\omega_T^2} = 1 + \frac{\omega^2 C_{GS}^2 R_0 P}{g_m}.$$

Thus, we have:

$$F = 1 + \frac{\left(2\pi \cdot 10^{10}\right)^2 \cdot \left(0.2 \times 10^{-12}\right)^2 \cdot 50 \cdot 0.7}{0.2} = 1.028.$$

8 Power amplifiers

Problem 1 In a class A power amplifier the gain in linearity is 20 dB and the 1 dB compression corresponds to an input power of 0 dBm. What is the output power at the 1 dB compression point?

Solution The output power in linearity at 0 dBm input power will be 20 dBm taking into account the linear 20 dB gain of the amplifier. However, the output power is compressed by 1 dB. Thus, the output power will be 19 dBm.

Problem 2 A class A power amplifier, with a single-tone input of 100 μW, has a second harmonic output power 10 nW. The 1 dB compression point is at 1 mW input power. What is the second harmonic output power for an input of 200 μW?

Solution Since the 1 dB compression point occurs at an input power much larger than 200 μW, we can approximate the harmonic distortion according to the low-power model where the second harmonic is a quadratic function of the input power at the fundamental. Thus, doubling the input power makes the second harmonic power increase by a factor of 4, i.e. to 40 nW.

Problem 3 A class A power amplifier, with a two-tone input of 100 μW, has a CIMR_3 of 60 dB. The 1 dB compression point is at 1 mW input power. What is the CIMR_3 for an input of 200 μW?

Solution Since the 1 dB compression point occurs at an input power much larger than 200 μW, we can evaluate the CIMR_3 according to the low-power model where the third-order intermodulation products increase with the cube of the input power while the output power at the fundamental increases linearly as a function of the input power at the fundamentals. Thus, increasing the input power by 3 dB the output power at the fundamentals increases by 2 dB while the third-order intermodulation product by 9 dB. The CIMR_3 thus decreases by 6 dB becoming 54 dB.

Problem 4 A HEMT has 16 V breakdown voltage, 0.5 V knee voltage (onset of current saturation) and maximum current of 500 mA. What is the maximum class A output power? What is the optimum load and optimum class A working point?

Solution The maximum class A output power is:

$$P_{RF,M} = \frac{(V_{DS,\mathrm{br}} - V_{DS,\mathrm{k}})I_{D,\max}}{8} = \frac{(16 - 0.5) \times 0.5}{8} = 0.968 \text{ W}.$$

The optimum DC working point is for:

$$I_D = \frac{I_{D,\max}}{2} = 250 \text{ mA}$$
$$V_{DS} = 0.5 + \frac{16 - 0.5}{2} = 7.75 \text{ V}.$$

The optimum load will have a parallel reactive part (compensating the output capacitance) that cannot be derived with the information available. The optimum load resistance is:

$$R_{Lo} = \frac{V_{DS,\mathrm{br}} - V_{DS,\mathrm{k}}}{I_{D,\max}} = \frac{16 - 0.5}{0.5} = 31 \ \Omega.$$

Problem 5 A class-A power amplifier working at 1 dB compression point has $\mathrm{CIMR}_3 = 15$ dB. The output power is 20 W and the gain in linearity is 20 dB; the efficiency is 35% in the working point. Evaluate the input backoff needed to increase the CIMR_3 up to 30 dB. Evaluate the efficiency in the initial condition and in backoff and the corresponding power-added efficiency.

Solution The output power in the 1 dB compression point is 43 dBm while the corresponding output power in linearity would be, for the same input power, 44 dBm. If we reduce by x dB the input power the output power (in linearity) will be reduced by x dB while the third-order intermodulation will decrease by $3x$ dB, leading to an increase of $2x$ dB of the CIMR_3. In other words, to increase the CIMR_3 by 15 dB as requested we have to reduce the input power by 7.5 dB, that corresponds to the required backoff. This leads to an output power of $44 - 7.5 = 36.5$ dBm (i.e. $10^{-3} \cdot 10^{(36.5/10)} = 4.47$ W) corresponding to an input power of $36.5 - 20 = 16.5$ dBm. Concerning the efficiency, in class A the DC power is constant; since in the 1 dB compression working point the efficiency is 35% we have:

$$P_{DC} = \frac{20}{0.35} = 57.14 \text{ W}.$$

In the backoff condition therefore the efficiency will be:

$$\eta = \frac{4.47}{57.14} = 7.82\%.$$

Alternatively, we could use the fact that the class A efficiency decreases as the backoff, i.e. an output backoff of $10^{(-6.5/10)} = 0.224$ corresponds to $\eta = 0.224 \cdot 35 = 7.836\%$ (take into account that while the input backoff is 7.5 dB the output is 6.5 dB only, since we start from the 1 dB compression point). Concerning the PAE, the gain in the 1 dB compression point is 19 dB or in natural units $10^{19/10} = 79.433$,while in backoff we can assume a 20 dB gain or in natural units

100. The two PAEs will therefore be:

$$\mathrm{PAE}_1 = 35 \cdot \left(1 - \frac{1}{79.433}\right) = 34.559\%$$
$$\mathrm{PAE}_2 = 7.82 \cdot \left(1 - \frac{1}{100}\right) = 7.742\%.$$

Problem 6 A receiver stage has bandwidth $B = 2$ GHz, noise figure of 4 dB, output signal over noise ratio of 20 dB. Assuming input thermal noise at 300 K, evaluate the sensitivity and noise floor of the receiver. Suppose than that the third-order intermodulation product intercept be $\mathrm{IIP}_3 = 10$ dBm; evaluate the Spurious Free Dynamic Range of the receiver.

Solution The noise floor in natural units is $F = 10^{4/10} = 2.512$ while $(S/N)_L = 100$. The sensitivity $P_{in,\mathrm{S}}$ is given by:

$$\begin{aligned}P_{in,\mathrm{S}} &= F \cdot k_B T_0 B \cdot (S/N)_L = \\ &= 2.512 \cdot 26 \times 10^{-3} \cdot 1.6 \times 10^{-19} \cdot 2 \times 10^9 \cdot 100 = 2.09 \times 10^{-9} \approx 2.1 \text{ nW}\end{aligned}$$

or, in log units,

$$P_{in,\mathrm{S}}\big|_{\mathrm{dBm}} = 10 \log_{10} \left(\frac{2.09 \times 10^{-9}}{1 \times 10^{-3}}\right) = -56.8 \text{ dBm}.$$

The noise floor corresponds to $(S/N)_L = 1$; we have:

$$\begin{aligned}P_{in,\mathrm{NF}} &= F \cdot k_B T_0 B = \\ &= 2.512 \cdot 26 \times 10^{-3} \cdot 1.6 \times 10^{-19} \cdot 2 \times 10^9 = 2.09 \times 10^{-11} \approx 21 \text{ pW}\end{aligned}$$

or, in log units,

$$P_{in,\mathrm{NF}}\big|_{\mathrm{dBm}} = 10 \log_{10} \left(\frac{2.09 \times 10^{-11}}{1 \times 10^{-3}}\right) = -76.8 \text{ dBm}.$$

Concerning the SFDR we have, taking into account that $\mathrm{IIP}_3 = 10$ mW:

$$\mathrm{SFDR} = \frac{\mathrm{IIP}_3^{2/3} P_{in,\mathrm{NF}}^{-2/3}}{(S/N)_L} = \frac{\left(10 \times 10^{-3}\right)^{2/3} \cdot \left(2.09 \times 10^{-11}\right)^{-2/3}}{100} = 6117$$

or, in log units:

$$\mathrm{SFDR}\big|_{\mathrm{dB}} = 10 \log_{10} 6117 = 37.86 \text{ dBm}.$$

Problem 7 An amplifier is designed on a 50 Ω load to provide, at low frequency, small signal available gain $G_1 = 25$ dB. The 1 dB compression point is for an input power $P_{in,1} = 10$ dBm while the saturation power is $P_{sat,1} = 36$ dBm. Evaluate the corresponding performances for a balanced amplifier closed on 50 Ω in which two identical amplifiers are connected in tandem.

Solution The small-signal gain of the balanced amplifier is the same for the two stages, i.e. $G_2 = 25$ dB. However, for a total input power $P_{in,2} = 13$ dB each of the two amplifiers will receive an input power $P_{in,1} = 10$ dBm so that the total output power will be $2 \cdot \alpha \cdot G_1 \cdot P_{in,1}$ where $\alpha = 10^{-1/10} = 0.794$ corresponds to a 1 dB penalty; in log units:

$$P_2|_{\mathrm{dBm}} = (3 - 1 + 25)|_{\mathrm{dB}} + 10|_{\mathrm{dBm}} = 37 \text{ dBm}.$$

Taking into account that in linearity the output power would have been $13 + 25 = 38$ dBm, we find that the input power of 13 dBm corresponds to the 1 dB compression point. The saturation power will be however twice than for the single amplifier, i.e. it will correspond to $P_{sat,2} = 36 + 3 = 39$ dBm.

9 Microwave measurements

Problem 1 A reflectometer has the following readings when loaded by a short, an open and a matched standard:

$$a_{m1}^{\mathrm{s}} = 0.98, \quad b_{m1}^{\mathrm{s}} = -0.96,$$
$$a_{m1}^{\mathrm{o}} = 0.98, \quad b_{m1}^{\mathrm{o}} = 0.94,$$
$$a_{m1}^{R_0} = 0.99, \quad b_{m1}^{R_0} = 0.01.$$

Evaluate the error coefficients e_{12}, e_{21} and e_{22} and the reflectivity of a DUT with readings $a_{m1} = 0.98$, $b_{m1} = 0.94\mathrm{j}$. What is the apparent reflectivity measured without the correction?

Solution The system to be solved is:

$$\begin{aligned} 0.98 - 0.96 \cdot e_{12} + 0.98 \cdot e_{21} - 0.96 \cdot e_{22} &= 0 \\ 0.98 + 0.94 \cdot e_{12} - 0.98 \cdot e_{21} - 0.94 \cdot e_{22} &= 0 \\ 0.99 + 0.01 \cdot e_{12} &= 0 \end{aligned}$$

yielding: $e_{12} = -99.0$, $e_{21} = -95.948$, $e_{22} = 2.0737$. Thus:

$$\Gamma_{\mathrm{DUT}} = \frac{a_{m1} + e_{12} b_{m1}}{e_{21} a_{m1} + e_{22} b_{m1}} = \frac{0.98 - 99.0 \cdot 0.94\mathrm{j}}{-95.948 \cdot 0.98 + 2.0737 \cdot 0.94\mathrm{j}} =$$
$$= -3.0926 \times 10^{-2} + 0.98905\mathrm{j}.$$

The apparent reflectivity would have been:

$$\Gamma'_{\mathrm{DUT}} = \frac{0.94\mathrm{j}}{0.98} = 0.95918\mathrm{j}.$$

Problem 2 The Line of a TRL calibration set has a length $L = 1$ mm. The measured transmission matrices of the Line and of the Thru are:

$$\mathbf{T}_T^m = \begin{pmatrix} 0.89794 & 3.0829 \times 10^{-2} \\ 2.1044 \times 10^{-3} & 0.97875 \end{pmatrix}$$
$$\mathbf{T}_L^m = \begin{pmatrix} 0.72572 - 0.52727\mathrm{j} & 2.4916 \times 10^{-2} - 1.8103 \times 10^{-2}\mathrm{j} \\ 1.7008 \times 10^{-3} - 1.2357 \times 10^{-3}\mathrm{j} & 0.79103 - 0.57472\mathrm{j} \end{pmatrix}$$

Find the complex propagation constant of the line.

Solution We have:

$$\mathbf{R}_M = \mathbf{T}_L^m \left(\mathbf{T}_T^m\right)^{-1} =$$
$$= \begin{pmatrix} 0.80821 - 0.58720\mathrm{j} & -1.6937 \times 10^{-7} - 2.2871 \times 10^{-7}\mathrm{j} \\ 1.6463 \times 10^{-8} - 6.9072 \times 10^{-10}\mathrm{j} & 0.8082 - 0.58720\mathrm{j} \end{pmatrix}$$

whose eigenvalues are equal and satisfy the relationship:

$$\exp(-\gamma L) = 0.80821 - 0.5872\mathrm{j}$$

i.e.:

$$\gamma L = -\ln\left(0.80821 - 0.5872\mathrm{j}\right) = 9.9737 \times 10^{-4} + 0.62832\mathrm{j}$$
$$\gamma = \left(9.9737 \times 10^{-4} + 0.62832\mathrm{j}\right) 10^3 = 0.99737 + 628.32\mathrm{j}$$

Problem 3 An active load pull system exploiting the active load technique needs to implement a load of impedance $Z_L = 50+\mathrm{j}50\ \Omega$. Evaluate the loop gain and phase delay needed, supposing to use a loop coupler with 10 dB power coupling. The reference impedance is $R_0 = 50\ \Omega$.

Solution Let us call b_2 the outgoing wave from port 2. The loop wave sampled by the coupler will be $b_l = Cb_2$ where:

$$20 \log_{10} C = -10 \rightarrow C = 10^{-10/20} = 0.31623.$$

The b_s wave is then amplified with total loop amplification A_l and phase delay ϕ_l; the wave entering the loop coupler will therefore be:

$$a_l = b_l A_l \exp\left(\mathrm{j}\phi_l\right) = C A_l \exp\left(\mathrm{j}\phi_l\right) b_2$$

while the wave entering port 2 of the two port after the coupler is:

$$a_2 = -\mathrm{j} a_l \sqrt{1 - C^2} = -\mathrm{j}\sqrt{1 - C^2} C A_l \exp\left(\mathrm{j}\phi_l\right) b_2.$$

Thus:

$$\frac{a_2}{b_2} = \Gamma_L = -\mathrm{j}\sqrt{1 - C^2} C A_l \exp\left(\mathrm{j}\phi_l\right).$$

But the reflection coefficient vs. the reference impedance is:

$$\Gamma_L = \frac{Z_L - R_0}{Z_L + R_0} = \frac{50 + \mathrm{j}50 - 50}{50 + \mathrm{j}50 + 50} = 0.2 + 0.4\mathrm{j}.$$

Thus the complex amplification is:

$$A_l \exp\left(\mathrm{j}\phi_l\right) = \frac{\Gamma_L}{-\mathrm{j}\sqrt{1 - C^2} C} = \frac{0.2 + 0.4\mathrm{j}}{-\mathrm{j}\sqrt{1 - 0.31623^2} \cdot 0.31623} =$$
$$= -1.3333 + 0.66666\mathrm{j} = 1.4907 \exp\left(\mathrm{j}2.6779\right)$$

yielding also the gain and phase delay of the loop.

Problem 4 An active load pull system (active load technique) needs to implement a short circuit. Evaluate the loop gain and phase delay needed, supposing to use a loop coupler with 20 dB power coupling. The reference impedance is $R_0 = 50\ \Omega$.

Solution Let us call b_2 the outgoing wave from port 2 of the DUT. The loop wave sampled by the coupler will be $b_l = Cb_2$ where:

$$20 \log_{10} C = -20 \to C = 10^{-20/20} = 0.1.$$

The b_s wave is then amplified with total loop amplification A_l and phase delay ϕ_l; the wave entering the loop coupler will therefore be:

$$a_l = b_l A_l \exp(\mathrm{j}\phi_l) = CA_l \exp(\mathrm{j}\phi_l)\, b_2$$

while the wave entering port 2 of the two port after the coupler is:

$$a_2 = -\mathrm{j} a_l \sqrt{1 - C^2} = -\mathrm{j}\sqrt{1 - C^2} C A_l \exp(\mathrm{j}\phi_l)\, b_2.$$

Thus:

$$\frac{a_2}{b_2} = \Gamma_L = -\mathrm{j}\sqrt{1 - C^2} C A_l \exp(\mathrm{j}\phi_l).$$

But the reflection coefficient vs. the reference impedance is:

$$\Gamma_L = \frac{0 - R_0}{0 + R_0} = \frac{50 + \mathrm{j}50 - 50}{50 + \mathrm{j}50 + 50} = -1.$$

Thus the complex amplification is:

$$A_l \exp(\mathrm{j}\phi_l) = \frac{\Gamma_L}{-\mathrm{j}\sqrt{1 - C^2} C} = \frac{-1}{-\mathrm{j}\sqrt{1 - 0.1^2} \cdot 0.1} = -10.05\mathrm{j}$$

yielding also the gain and phase delay of the loop.

Problem 5 A noise source has ENR = 30 and the OFF temperature is $T_s^{\mathrm{OFF}} =$ 300 K. We apply the Y-factor technique, obtaining in the first step (noise source directly connected to the noise figure meter) $P_{n1}^{\mathrm{ON}} = 300\ \mu$W, $P_{n1}^{\mathrm{OFF}} = 15\ \mu$W. In the second step (DUT connected) we obtain $P_{n1}^{\mathrm{ON}} = 3500\ \mu$W, $P_{n1}^{\mathrm{OFF}} = 250$ μW. Find the T^{ON} of the noise source, and the DUT gain, noise figure and noise temperature.

Solution The ON temperature of the source with ENR (Excess Noise Ratio) equal to 30 is, from the ENR definition:

$$T_s^{\mathrm{ON}} = T_s^{\mathrm{OFF}} + \mathrm{ENR} \cdot T_0 = 300 + 30 \cdot 290 = 9000\ \mathrm{K}.$$

In the first step we obtain:

$$Y_1 = \frac{P_{n1}^{\mathrm{ON}}}{P_{n1}^{\mathrm{OFF}}} = \frac{300}{15} = 20$$

from which the noise temperature of the Noise Figure Meter (NFM) can be estimated as:

$$T_{\mathrm{NFM}} = \frac{T_s^{\mathrm{ON}} - Y_1 T_s^{\mathrm{OFF}}}{Y_1 - 1} = \frac{9000 - 20 \cdot 300}{20 - 1} = 157.89\ \mathrm{K}$$

In the second step we have:

$$Y_2 = \frac{P_{n2}^{\mathrm{ON}}}{P_{n2}^{\mathrm{OFF}}} = \frac{3500}{250} = 14$$

from which the total noise temperature T_2 of the cascade of the DUT and of the NFM can be estimated as:

$$T_2 = \frac{T_s^{\mathrm{ON}} - Y_2 T_s^{\mathrm{OFF}}}{Y_2 - 1} = \frac{9000 - 14 \cdot 300}{14 - 1} = 369.23K.$$

The device gain is:

$$G_{\mathrm{DUT}} = \frac{P_{n2}^{\mathrm{ON}} - P_{n2}^{\mathrm{OFF}}}{P_{n1}^{\mathrm{ON}} - P_{n1}^{\mathrm{OFF}}} = \frac{3500 - 250}{300 - 15} = 11.404$$

We can now compute the DUT noise temperature as:

$$T_{\mathrm{DUT}} = T_2 - \frac{T_{\mathrm{NFM}}}{G_{\mathrm{DUT}}} = 369.23 - \frac{157.89}{11.404} = 355.38$$

and the DUT noise figure as:

$$F_{\mathrm{DUT}} = 1 + \frac{T_{\mathrm{DUT}}}{T_0} = 1 + \frac{355.38}{290} = 2.22.$$