Modular Representations of Finite Groups of Lie Type (Revisions)

2 In line 3 of 1.2, replace "quasisiplit" by "quasisplit".

5 In line -1, replace $q^3(q^2+1)(q^3-1)$ by $q^3(q^2-1)(q^3+1)$.

6 In line -4, replace $(c, c^q, 1/c^{q+1})$ by (c, c^{q-1}, c^{-q}).

7 In line 6 (first line of text after displayed matrix), replace $c+c^q+a^{q+1}=0$ by $c+c^q-a^{q+1}=0$

13 In line 19, replace $c(v', v)$ by $c\langle v', v\rangle$.

16 In the last line of 2.8, replace n twice by 2.

16 In statement of Theorem 2.9, read "nonisomorphic simple KG-modules".

18 In (3) replace the superscript V by U.

19 In line 7, replace $q^{\ell-1}$ by $(q-1)^\ell$.

19 In line 9, replace "distingished" by "distinguished".

30 In line 11 of 3.11, replace II.8.20 by II.8.22.

30 By convention, notation such as $-w \cdot 0$ is shorthand for Lusztig's more explicit $-w\rho - \rho$. Consult Scott [**363**] for a careful discussion of the notational problems here. (And line -6 should be deleted. It perpetuates an error in an earlier draft.)

38 At the end of 4.6, insert the following new text:

By now all Weyl modules having *fundamental* highest weights have been studied. Such a module $V(\varpi_i)$ with $1 \leq i \leq \ell$ is easily seen to be simple when ϖ_i is *minuscule* (minimal nonzero in the natural ordering of dominant weights): then all weights of $V(\varpi_i)$ are W-conjugate to ϖ_i. In particular, all fundamental weights are minuscule in type A_ℓ. Other modules $V(\varpi_i)$ often fail to be simple when p is *bad* (see 17.11 below), possible only for $p \in \{2, 3, 5\}$. In types $\mathrm{E}_7\, .E_8$, simplicity also sometimes fails for the larger primes $7, 13, 19$.

Wong [436, §4] uses his contravariant form methods to analyze fundamental weights in types B_ℓ, D_ℓ. Apart from exceptions when p is bad (here $p = 2$), all Weyl modules $V(\varpi_i)$ are simple. For the more complicated case of C_ℓ, see (4.5) above.

In the five exceptional Lie types, Jantzen [ja91], 4.6, gives a complete summary, based partly on the computations by Gilkey–Seitz [186] and partly on his own computations based on the Sum Formula: see (3.9) above. In the most complicated situations, involving types E_7, E_8, one sees for the first time fundamental weights where good primes lead to non-simple Weyl modules. (But in all Lie types, the exceptions turn out to be limited to primes less than the Coxeter number.)

Added reference: [ja91] J.C. Jantzen, *First cohomology groups for classical Lie algebras*, pp. 289–315, *Representation theory of finite groups and finite-dimensional algebras (Bielefeld, 1991)*, Progr. Math., 95, Birkhäuser, Basel, 1991.

46+ In sections 5.6–5.9, as well as 9.6, most results are formulated separately for Chevalley groups and twisted groups, with proofs in the twisted case left rather sketchy or omitted altogether. The inductive proofs suggested in the twisted case are out of focus, due to reliance on the imprecise notion of "length" $|\lambda|$ and the messy non-strict inequalities. In particular, the proof of 5.7(b) is unconvincing as written. Florian Herzig has suggested the more precise alternative $||\lambda|| := \sum_{\alpha>0}\langle\lambda, \alpha^\vee\rangle$. Besides being additive and satisfying $||\lambda|| = ||\widetilde{\lambda}||$, this refined length ensures that $\mu < \lambda$ implies $||\mu|| < ||\lambda||$. With this adjustment the statements in these sections remain true and the inductive arguments become rigorous.

80 Just before Theorem 9.6 and in the statement of the theorem for the twisted case, use the refined length $||\lambda||$ in place of $|\lambda|$, as indicated in the correction above for sections 5.6–5.9.

94 In line 4 of 10.7, read "Here we consider first the case ..."

94 In lines 18–19 of 10.7, read " ...which ensures for p *odd* that Ballard's lower bound for the dimension of a PIM in 9.7 involves $|W\mu|$." (For a minuscule weight, exactness of Ballard's bound requires $p - 1 \neq 1$).

109 Add a clause to the second paragraph on B_2: " ...in the case $p = 7$, while Ye and Yu [] treat $p = 5$." [Reference added below.]

140 In the first line, replace "nonprojective" by "nonsimple"..

140 In the last line, replace "do achieve" by "approach". In Table 6 the diagrams for 10^+ and 25^+ must be replaced by corrected versions given in Benson's errata posted on his preprint page.

147 In the statement of Theorem 14.5, replace $f \geq f(ct\lambda)$ by $f \geq f(c(t\lambda))$.

162 In line 5 of 16.5, replace 11.12 by 11.13.

168 At the end of the third line of 16.12, replace 11.12 by 11.13.

199 In lines -10 to -8, replace ϖ four times by ϖ_1, and replace $\mathrm{SL}(2, K)$ twice by $\mathrm{SL}(n, K)$.

(April 2015) Many of these were observed by Florian Herzig.

Updated references

41 Replace "New York" by "Berlin".

42 C.P. Bendel, D.K. Nakano, and C. Pillen, *Second cohomology groups for Frobenius kernels and related structures*, Adv. Math. **209** (2007), 162–197.

43 C.P. Bendel, D.K. Nakano, and C. Pillen, *Extensions for finite groups of Lie type II: Filtering the truncated induction functor*, pp. 1–23, *Representations of Algebraic Groups, Quantum Groups, and Lie Algebras*, Contemp. Math., 413, Amer. Math. Soc., Providence, RI, 2006.

84 J.F. Carlson, Z. Lin, and D.K. Nakano, *Support varieties for modules over Chevalley groups and classical Lie algebras*, Trans. Amer. Math. Soc. **360** (2008), 1879–1906.

85 J.F. Carlson, N. Mazza, and D.K. Nakano, *Endotrivial modules for finite groups of Lie type*, J. Reine Angew. Math. **595** (2006), 93–119.

207 T. Holm and W. Willems, *A local conjecture on Brauer character degrees of finite groups*, Trans. Amer. Math. Soc. **359** (2006), 591–603.

332 B.J. Parshall and L.L. Scott, *Extensions, Levi subgroups and character formulas*, J. Algebra **319** (2008), 680–701.

338 C. Pillen, *Self-extensions for finite symplectic groups via algebraic groups*, pp. 173–184, *Representations of Algebraic Groups, Quantum Groups, and Lie Algebras*, Contemp. Math., 413, Amer. Math. Soc., Providence, RI, 2006.

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Added references

S. Foulle, *Characters of the irreducible representations with fundamental highest weight for the symplectic group in characteristic* p, arXiv:math.RT/0512312.

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Jia-chen Ye and Gui-hai Yu, *The first Cartan invariant of* $\mathrm{Sp}(4, 5^n)$ [Chinese], Tongji Daxue Xuebao Ziran Kexue Ban **32** (2004), 1682–1687.