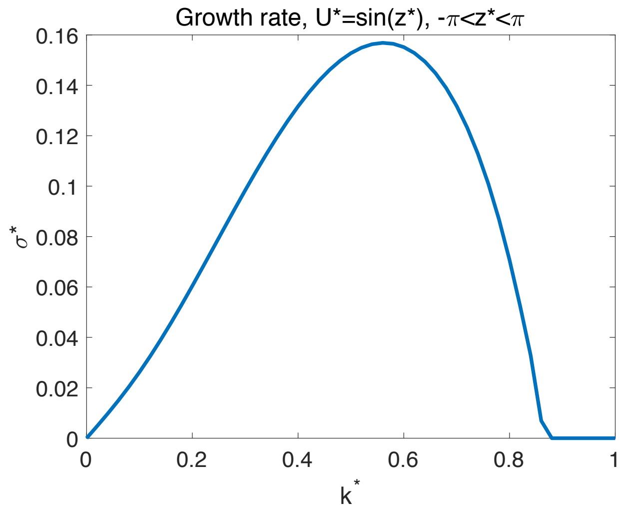

Figure 1: Growth rate for the sine profile with boundaries at $z* = \pm H^*$ and $H^* = \pi$.

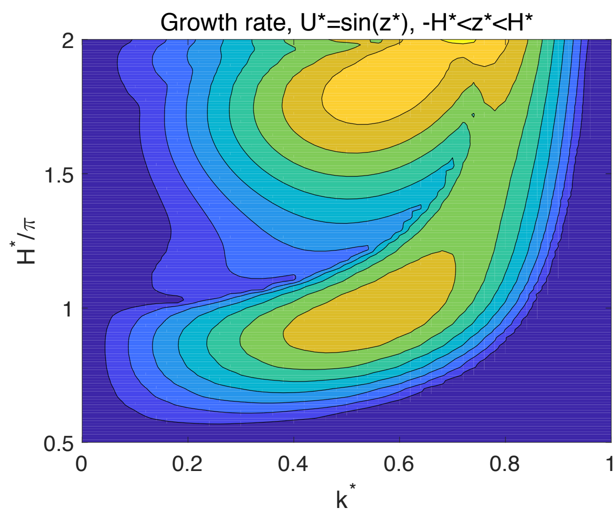

Figure 2: Growth rate for the sine profile with boundaries at $z* = \pm H^*$.

12: Sinusoidal flow

Figure 1 shows the growth rate for the sine profile $U^\star = \sin(z^\star)$ with boundaries $z^* = \pm\pi$. The result is qualitatively similar to the hyperbolic tangent shear layer. As the domain height is reduced (figure 2), the growth rate changes in complicated ways. There is no growth when the boundaries are at $z^* = \pm\pi/2$ or less.

This illustrates the fact that the existence of an inflection point is a necessary, but not a sufficient, condition for instability.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Project 12: Sinusoidal flow
clear
close all

% plotting parameters
fs=22;
```

```
ms=16;
lw=3;

dz=.1;
ks=[0:.02:1.0];

%% (a) H=pi
ztop=pi;
zbot=-ztop;
z=[zbot+dz:dz:ztop-dz]';

% velocity profile
U=sin(z);

for i=1:length(ks)
    k=ks(i);
    [sig(i)]=Ray(z,U,k);
end

figure
plot(ks,real(sig),'linewidth',lw)

xlabel('k^*','fontsize',fs)
ylabel('\sigma*','fontsize',fs)
title('Growth rate, U*=sin(z*), -\pi<z*<\pi','fontweight','normal')
set(gca,'fontsize',fs-4)


%% (b) range of H

Hs=[2:-.01:.5];

for j=1:length(Hs);
    j

    % z range
    clear z
    ztop=pi*Hs(j);
    zbot=-ztop;
    z=[zbot+dz:dz:ztop-dz]';

    % velocity profile
    U=sin(z);

    % wave number range

    for i=1:length(ks)
        k=ks(i);
```

```
        [sig1(j,i),w1]=Ray(z,U,k);
    end
end


%
figure
%pcolor(ks,Hs,real(sig1));shading interp

contourf(ks,Hs,smooth(real(sig1)));

xlabel('k^*','fontsize',fs)
ylabel('H^*/\pi','fontsize',fs)
title('Growth rate, U*=sin(z*), -H*<z*<H*','fontweight','normal')
set(gca,'fontsize',fs-4)
```