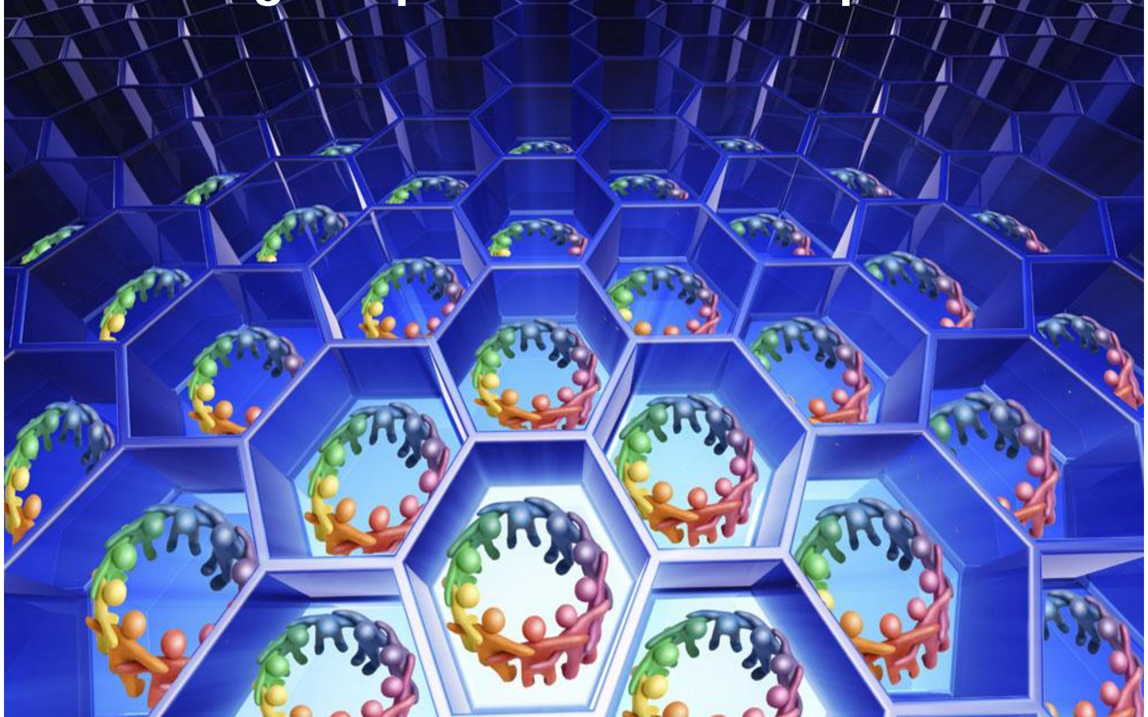
Cooperative Communications and Networking

Chapter 16

Coverage Expansion with Cooperation



Outline

- Goals
- System Model
- Relay Assignment Algorithms
- Performance Analysis
- Simulations
- Summary

Goal

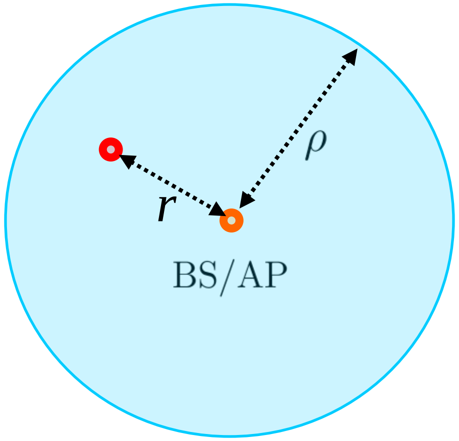
- Develop protocols by which nodes are assigned to cooperate with each other
- Relay: additional cheap access points that the service provider can deploy in the network
- Compare these techniques by simulation

System Model

N users uniformly distributed:

$$q(r) = \frac{2r}{\rho^2}, \qquad 0 \leq r \leq \rho$$

uniform angle between $[0, 2\pi)$



Communication schemes:

- Non-cooperative: direct transmission to the destination
- Cooperative: employ a relay to forward data

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System Model

transmitted signal power

transmitted data

$$y_{sd} = \sqrt{PKr_{sd}^{-\alpha}}h_{sd}x + n_{sd}$$

received signal at the destination

channel fading gain

- K : antenna design parameter
- α : path loss exponent

$$\text{SNR}(r_{sd}) = \frac{|\, h_{sd} \,|^2 \, Kr_{sd}^{-\alpha}P}{N_o}$$

$$\mathcal{P}_{nc} = \mathcal{P}(\text{SNR}(r) \leq \gamma_{nc})$$

γ_{nc} determined based on application and transmitter/receiver structure

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Protocols and Analysis

- Direct Transmission

$$\mathcal{P}_{OD}(r_{sd}) = \mathcal{P}\left(\mathrm{SNR}(r_{sd}) \le \gamma_{nc}\right)$$
$$= 1 - \exp\left(-\frac{N_o \gamma_{nc} r_{sd}^{\alpha}}{KP}\right) \simeq \frac{N_o \gamma_{nc} r_{sd}^{\alpha}}{KP}$$

$$\mathcal{P}_{OD} = \int_0^{\rho} \mathcal{P}_{OD}(r_{sd}) q(r_{sd}) dr_{sd} = \int_0^{\rho} \frac{2r_{sd}}{\rho^2}\left(1 - \exp\left(-\frac{N_o \gamma_{nc} r_{sd}^{\alpha}}{KP}\right)\right) dr_{sd}$$
$$= 1 - \frac{2}{\alpha\rho^2}\left(\frac{KP}{N_o\gamma_{nc}}\right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{N_o\gamma_{nc}\rho^{\alpha}}{KP}\right) \simeq \frac{2\gamma_{nc}\rho^{\alpha} N_o}{KP(\alpha+2)}$$

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Protocols and Analysis

- Cooperative Transmission

$$\mathrm{SNR}(r_{sd}) = \frac{|\, h_{sd} \,|^2 \, K r_{sd}^{-\alpha} P}{N_o} \qquad \mathrm{SNR}(r_{sl}) = \frac{|\, h_{sl} \,|^2 \, K r_{sl}^{-\alpha} P}{N_o}$$

$|\, h_{sd} \,|^2$ and $|\, h_{sl} \,|^2$ are mutually independent exponential random variables with unit mean.

$$\mathcal{P}_{OC} = \Pr(\mathrm{Outage}|SNR_{sd} \leq \gamma_c)\Pr(SNR_{sd} \leq \gamma_c)$$

$$\Pr(\mathrm{Outage}|SNR_{sd} \leq \gamma_c) = \Pr(SNR_{sl} \leq \gamma_c) + \Pr(SNR_{sl} > \gamma_c)\underbrace{\Pr(SNR_d \leq \gamma_c|SNR_{sd} \leq \gamma_c)}_{\Pr(SNR_d \leq \gamma_c)}$$

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Relay Assignment Algorithms

- Genie-Aided Algorithm:
  - Lower bound on the outage probability for any assignment protocol
  - For any source node put a relay at the optimal position on the line joining the source and the destination (BS/AP)

Outage probability of the cooperative transmission (conditioned on fixed topology):

$$\mathcal{P}_{OC}(r_{sd}, r_{sl}, r_{ld}) = \mathcal{P}\left((\text{SNR}(r_{sd}) \leq \gamma) \cap (\text{SNR}(r_{sl}) \leq \gamma)\right)$$
$$+ \mathcal{P}\left((\text{SNR}(r_{sd}) \leq \gamma) \cap (\text{SNR}(r_{ld}) \leq \gamma) \cap (\text{SNR}(r_{sl}) > \gamma)\right)$$

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Relay Assignment Algorithms (GA)

$$\mathcal{P}_{OC}(r_{sd}, r_{sl}, r_{ld}) = \left(1 - \exp(-\frac{N_o \gamma r_{sd}^{\alpha}}{KP})\right)\left(1 - \exp(-\frac{N_o \gamma r_{sl}^{\alpha}}{KP})\right)$$
$$+ \left(1 - \exp(-\frac{N_o \gamma r_{sd}^{\alpha}}{KP})\right)\left(1 - \exp(-\frac{N_o \gamma r_{ld}^{\alpha}}{KP})\right)\exp(-\frac{N_o \gamma r_{sl}^{\alpha}}{KP})$$
$$= \left(1 - \exp(-\frac{N_o \gamma r_{sd}^{\alpha}}{KP})\right) \times \left(1 - \exp(-\frac{N_o \gamma \left(r_{sl}^{\alpha} + r_{ld}^{\alpha}\right)}{KP})\right)$$

Valid for the cooperation transmission with any relay-assignment algorithm

Next step: finding the optimal relay position

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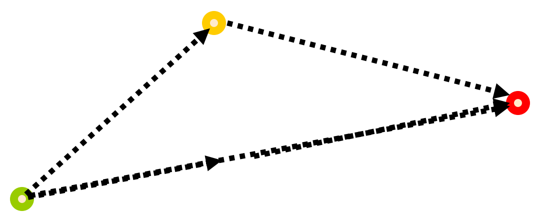
Relay Assignment Algorithms (GA)



1 Optimal relay position should be on the line joining the source and the destination

$$r_{ld} = r_{sd} - r_{sl}$$

Optimal relay position is found by solving

$$r_{sl}^{*} = \arg\min_{r_{sl}} \mathcal{P}_{OC}(r_{sd}, r_{sl}),$$
$$\text{subject to} \quad 0 \leq r_{sl} \leq r_{sd}.$$

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Relay Assignment Algorithms (GA)

② $r^*_{sl} = \frac{r_{sd}}{2}$ for $\alpha > 1$

Genie-aided algorithm outage probability: $\mathcal{P}_{OG}(r_{sd}) = \mathcal{P}_{OC}(r_{sd}, r^*_{sl})$

$$\mathcal{P}_{OG} = \int_0^{\rho} q(r_{sd}) \mathcal{P}_{OG}(r_{sd}) dr_{sd}$$

$$= 1 + \frac{2}{\alpha\rho^2} \left(\frac{kP}{N_o\gamma_c\left(1+2^{1-\alpha}\right)} \right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{N_o\gamma_c\left(1+2^{1-\alpha}\right)\rho^{\alpha}}{kP} \right)$$

$$- \frac{2}{\alpha\rho^2} \left(\frac{kP}{N_o\gamma_c} \right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{N_o\gamma_c\rho^{\alpha}}{kP} \right) - \frac{2}{\alpha\rho^2} \left(\frac{kP}{N_o\gamma_c 2^{1-\alpha}} \right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{N_o\gamma_c 2^{1-\alpha}\rho^{\alpha}}{kP} \right)$$

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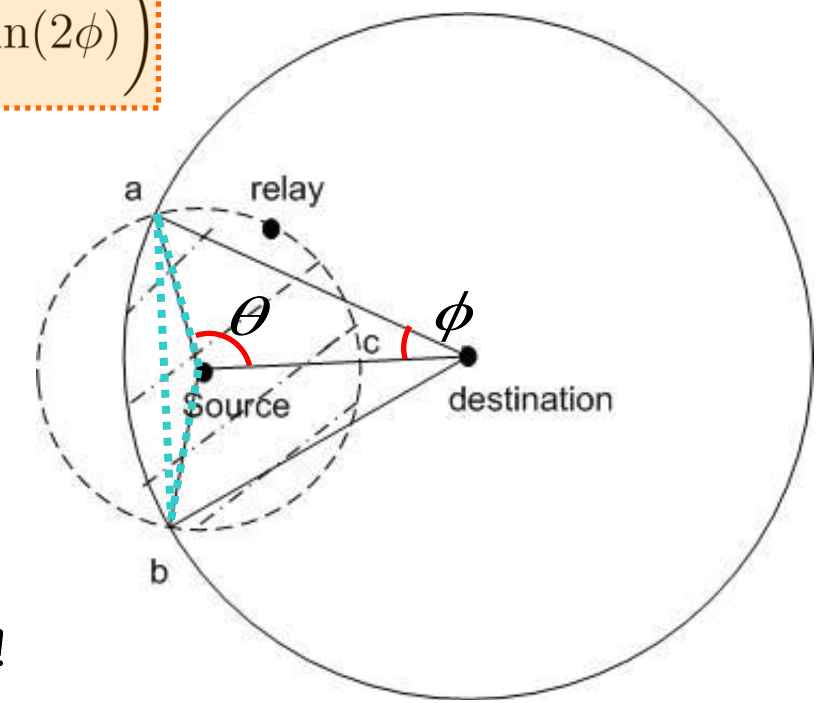
Relay Assignment Algorithms (NN)

- Nearest-Neighbor Protocol:

$$A(r_{sd}, r_{sl}) = r_{sl}^2\theta + \frac{1}{2}r_{sl}^2\sin(2\theta) + \left(\rho^2\phi - \frac{1}{2}\rho^2\sin(2\phi)\right)$$

$$p_{r_{sl}}(x) = \frac{\partial}{\partial x}\left(1 - \left(1 - \frac{A(r_{sd}, r_{sl})}{\pi\rho^2}\right)^{N-1}\right)$$



• For each source, the relay is chosen to be its nearest neighbor

• Relay's location distribution is not uniform!

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Relay Assignment Algorithms (NN)

- Conditioned Probability

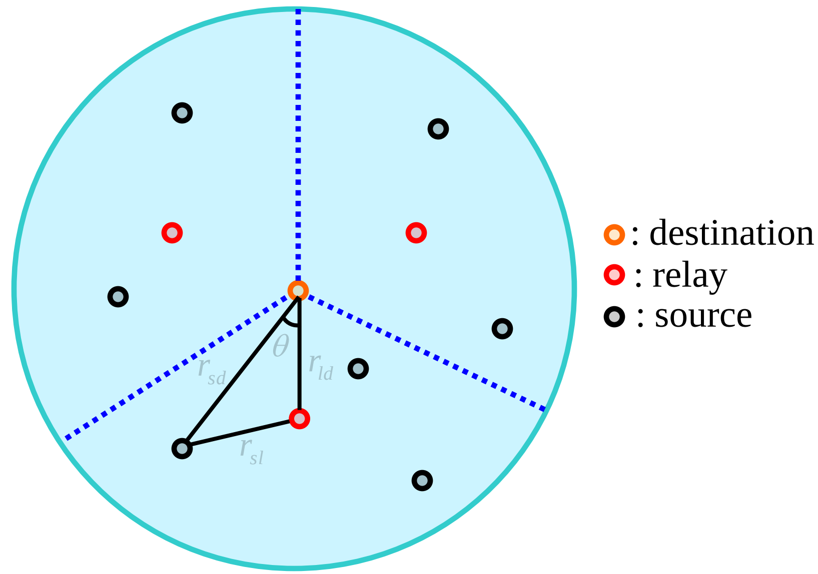
$$\mathcal{P}_{ONN}(r_{sd}, r_{sl}, r_{ld}) = \left(1 - \exp(-\frac{N_o \gamma r_{sd}^{\alpha}}{KP})\right) \times \left(1 - \exp(-\frac{N_o \gamma \left(r_{sl}^{\alpha} + r_{ld}^{\alpha}\right)}{KP})\right)$$

- Average outage probability

$$\begin{aligned}\mathcal{P}_{ONN} &= \int_0^{\rho} \int \mathcal{P}_{ONN}(r_{sd}, r_{sl}, r_{ld}) \, \mathcal{P}_{rn}(r_{sl}) \, q(r_{sd}) \, dr_{sl} \\ &\simeq 1 - \frac{4}{\alpha \rho^2} \left(\frac{KP}{N_o \gamma_c}\right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{N_o \gamma_c \rho^{\alpha}}{KP}\right) + \frac{2}{\alpha \rho^2} \left(\frac{KP}{2N_o \gamma_c}\right)^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{2N_o \gamma_c \rho^{\alpha}}{KP}\right)\end{aligned}$$

Relay Assignment Algorithms (FR)

- Fixed Relays Strategy:
  - Deploy fixed nodes to act as relays
  - Reduce the overhead of communications to pair for cooperation



Optimum relay position:

Divide the cell into m equal sectors

(m = number of fixed relays)

$$r_{ld}^{*} = \arg\min \mathcal{P}_{OC}(r_{ld}),$$
$$\text{s.t. } 0 < r_{ld} < \rho$$

$$r_{sl}(\theta) = \sqrt{r_{sd}^2 + r_{ld}^2 - 2r_{sd}r_{ld}\cos(\theta)}$$

where $\mathcal{P}_{OC} = \int_0^{\rho} \frac{2l_{sd}}{\rho^2} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} \mathcal{P}_{OC}(r_{sd}, r_{sl}(\theta), r_{ld}) \frac{m}{2\pi} d\theta dl_{sd}$

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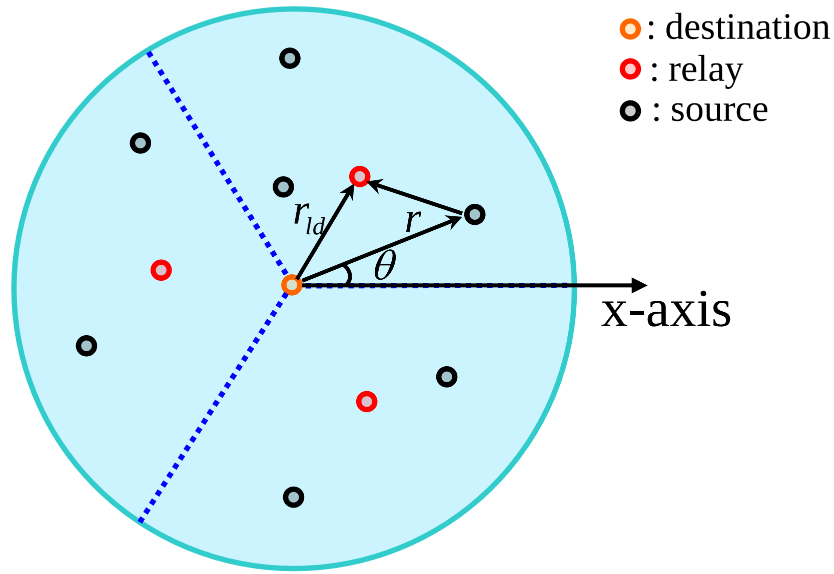
Relay Assignment Algorithms (FR)

– Solving the optimization problem is hard!
– Consider the following heuristic:



Let $q(r_{ld}) = \mathrm{E}\left(\|re^{j\theta} - r_{ld}\|^2\right)$

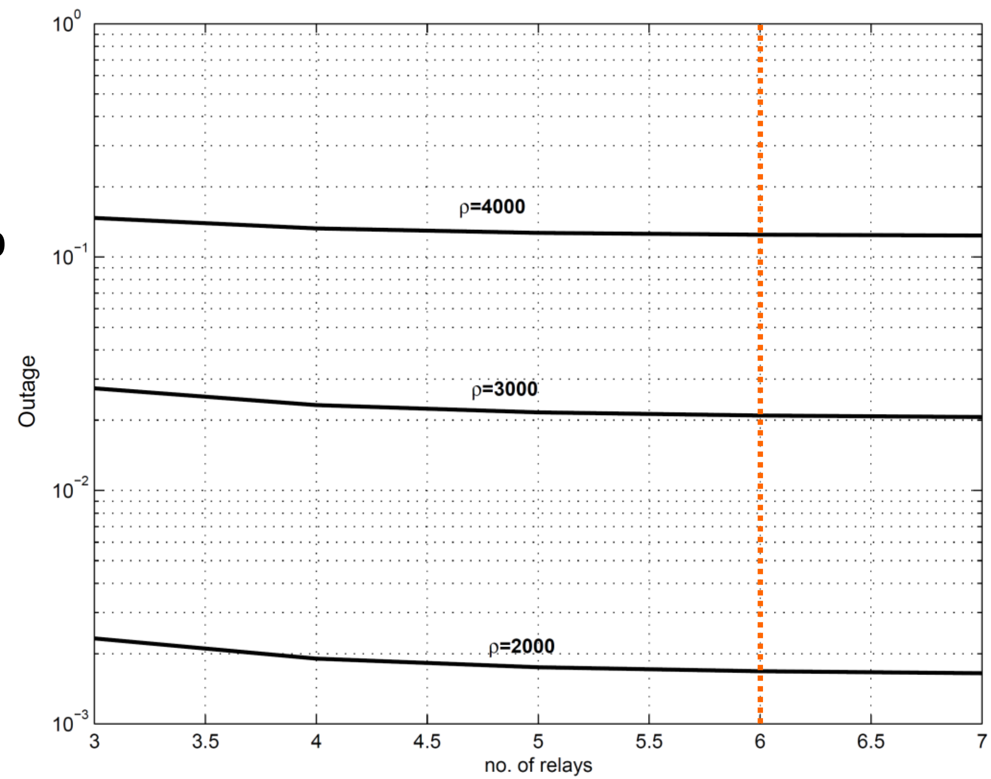
$$r_{ld}^* = \arg\,\min \mathrm{E}\left(\|re^{j\theta} - r_{ld}\|^2\right)$$

we get $$r_{ld}^* = \frac{2m}{3\pi} \sin\left(\frac{\pi}{m}\right) \rho$$

Relay Assignment Algorithms (FR)

$$k = 1, \alpha = 3, P = 0.05,$$
$$R = 1, N_0 = 10^{-12}$$

Conclusion: dividing the cell into 6 sectors with a relay deployed in each sector provides good enough performance.



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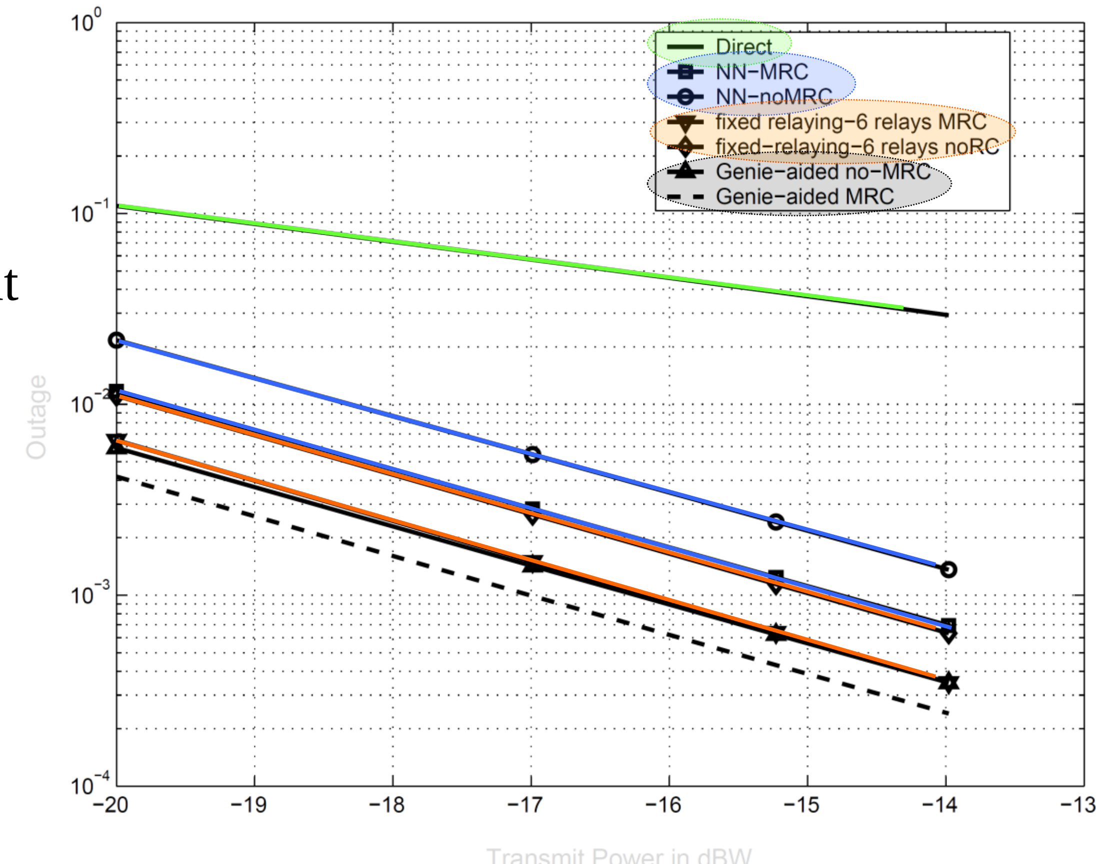
Simulation Results

- Fixed relaying has the best Performance

- Cooperation yields around 7dBW savings in the transmit power with respect to direct transmission



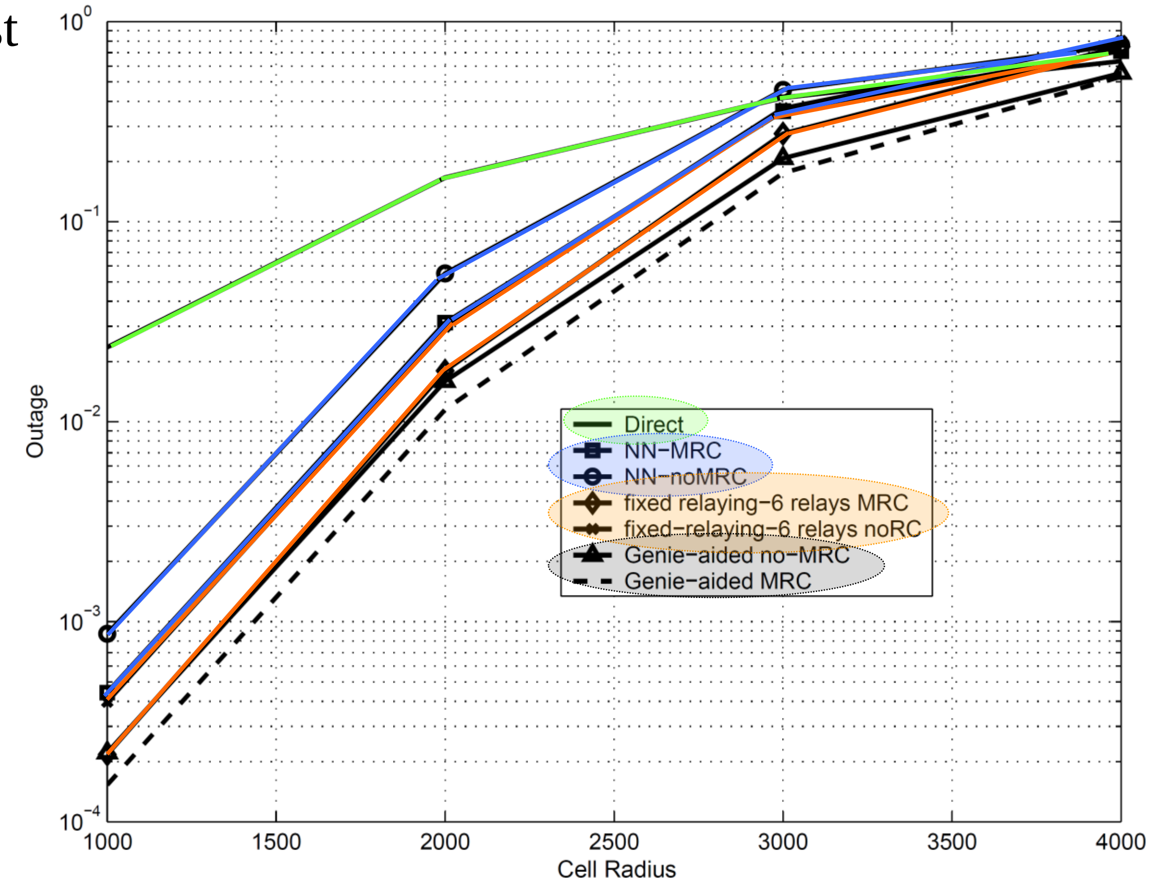
[17]

Simulation Results

- Fixed relaying has the best performance

- The gap between direct transmissions and cooperation decreases with increasing the cell size

- Direct transmission: good enough for larger cell sizes



Summary

- Address the relay assignment problem for coverage extension
- Two relay assignment protocols:
  - Nearest-Neighbor protocol
  - Fixed Relays strategy
- Provide a lower bound on performance of relay assignment protocols using a genie
- Simulation results:
  - Fixed relays strategy outperforms nearest-neighbor protocol
  - For larger cell sizes, the performance gap between direct and cooperative transmission diminishes