Financial Market Microstructure Theory

Based on de Jong and Rindi, Chapters 2–5

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Determinants of the bid-ask spread

The early literature focused on empirically finding determinants of the bid-ask spread. Typical regression model:

$$s_i = \beta_0 + \beta_1 \ln(M_i) + \beta_2 (1/p_i) + \beta_3 \sigma_i + \beta_4 \ln(V_i) + u_i$$

with

- s_i: average (percentage) bid-ask spread for firm i
- M_i: market capitalization (-)
- p_i: stock price level (-)
- σ_i: volatility of stock price $(+)$
- V$_i$: trading volume (-)

Theories of the bid-ask spread

Market Microstructure literature has identified three reasons for the existence of a bid-ask spread and other implicit transaction costs:

1. Order processing costs
2. Inventory Control
3. Asymmetric Information

See Madhavan (2000), section 2 for a good review

Textbook treatments of this material

- O'Hara (1995), Chapters 2, 3 and 4
- Lyons (2001), Chapter 4
- De Jong and Rindi, Chapter 2, 3 and 4

Inventory models

Inventory control

Important role of market makers: provide opportunity to trade at all times ("immediacy")

Market makers absorb temporary imbalances in order flow

- will hold inventory of assets
- inventory may deviate from desired inventory position
- risk of price fluctuations

Market maker requires compensation for service of providing "immediacy"

Inventory control (2)

Modeling market makers' inventory control problem

- avoid bankruptcy ("ruin problem")
  - Garman (1976)
- price risk on inventory
  - Stoll (1978), Ho and Stoll (1981,1983)
  - de Jong and Rindi, Section TBA

The Ho and Stoll (1981) model

- one monopolistic, passive specialist
- specialist sets bid and ask prices as a markup on "true" price: $ask = p^* + a$, $bid = p^* - b$
- random arrival of buy and sell orders
  - Poisson process with arrival rates λ_a and λ_b
- number of orders is declining in markup (elastic demand/supply)
- specialist maximizes expected utility of final wealth $E[U(W_T)]$

Ho and Stoll specify complicated dynamics for prices, inventory and wealth

Bid and ask prices in the Ho-Stoll model:

$$ask = p^* + a = p^* - \beta I + (A + \lambda q)$$

$$bid = p^* - b = p^* - \beta I - (A + \lambda q)$$

with

$$\lambda = \frac{1}{2}\sigma^2 ZT$$

In words, quote markups a and b depend on

- fixed component (A), reflecting monopoly power of specialist
- component proportional to trade size (λq), depending on volatility of stock price (σ^2), risk aversion (Z), and time horizon (T)
- inventory level I

The bid ask spread $S = a + b$ is independent of the inventory level

$$S = a + b = 2\left[A + \lambda q\right]$$

Location of the midpoint of bid and ask quotes (m) does depend on inventory level

$$m = \frac{ask + bid}{2} = p^{*} - \beta I$$

We may write

$$ask = m + S/2, \qquad bid = m - S/2$$

where m moves with p and I

Information based models

Asymmetric information

Traders on financial markets typically have differential information

Usual distinction in microstructure literature: informed (I) and uninformed (U) traders

- U has only publicly available information
- I has public and some private information

This has important implications for price formation: trading with potentially better informed party leads to **adverse selection**

Information based models

Several types of information based models

I. Rational Expectations models [de Jong and Rindi, Chapter 2]
  - focus on market equilibrium information content of prices
  - trading mechanism not specified

II. Strategic trader models [de Jong and Rindi, Chapter 3]
  - informed traders exploit information during trading process
  - trading in batch auctions

III. Sequential trade models [de Jong and Rindi, Chapter 4]
  - explicit trading mechanism (market maker)
  - transaction costs may arise from differences in information among traders

Information based models I: Rational expectations models

Rational Expectations Equilibrium

Model assumptions

- one risky asset and a riskless asset
- risky asset: random payoff v
- zero interest rate, no borrowing constraints
- two traders, U and I, risk averse and with fixed endowments of the risky asset
- U is uninformed, I (informed) recieves signal about payoff v
- period 1: trading (exchange of risky asset)

  period 2: payoff v realized

Steps in the analysis of this model

1. derive distribution of asset's payoff conditional on public and private information

2. find demand schedule of traders by utility maximization

3. find equilibrium price by equating aggregate supply and demand

Important result: equilibrium price reveals some of I's private information

If U is smart (rational), he will take this information into account in his decisions

Rational Expectations Equilibrium (REE) price

- price consistent with rational behaviour of U and I

- market clears

Information structure

Informed and uninformed traders receive information about the value v of the security

U: public information $v = \bar{v} + \epsilon_v$, so that $v \sim N(\bar{v},\ \sigma_v^2)$

I: additional private signal $s = v + \epsilon_s$, or $s|v \sim N(v,\ \sigma_s^2)$

Combining public information and private signal, the informed trader's distribution of the value is

$$v|s \sim N\left(\beta s + (1-\beta)\bar{v},\ (1-\beta)\sigma_v^2\right), \quad \beta = \frac{1/\sigma_s^2}{1/\sigma_s^2 + 1/\sigma_v^2}$$

- mean is weighted average of public information and signal
- informed variance is smaller than uninformed variance

Trading behavior

Traders generate wealth by trading the risky asset

$$W = d(v - p)$$

where d is demand for asset, v is value (payoff) and p is price

Wealth is stochastic, maximize expected utility

Assumptions: CARA utility function, normal distribution for wealth

$$\mathsf{E}\left[U(W)\right] = \mathsf{E}\left[-\exp(-aW)\right] = -\exp(-a\mathsf{E}[W] + \tfrac{1}{2}a^2\mathrm{Var}(W))$$

Maximization of $\mathsf{E}\left[U(W)\right]$ w.r.t. asset demand d gives

$$d = \frac{\mathsf{E}[v] - p}{a\mathrm{Var}(v)}$$

Aggregate demand

Demand by uninformed trader

$$d_U = \frac{\mathsf{E}[v] - p}{a\mathrm{Var}(v)} = \frac{\bar{v} - p}{a\sigma_v^2}$$

and demand by informed trader

$$d_I = \frac{\mathsf{E}[v|s] - p}{a\mathrm{Var}(v|s)} = \frac{\beta s + (1-\beta)\bar{v} - p}{a(1-\beta)\sigma_v^2}$$

Aggregate demand:

$$D \equiv d_U + d_I = \frac{1}{a\sigma_v^2}\left(2\bar{v} + \frac{\beta}{1-\beta}s - \frac{1}{1-\beta}p\right)$$

Walrasian equilibrium

Aggregete supply X is exogenous

Solve for equilibrium price from market clearing condition $D = X$

$$p = \alpha s + (1-\alpha)\bar{v} - \tfrac{1}{2}a(1-\alpha)\sigma_v^2 X, \quad \alpha = \frac{1/\sigma_s^2}{1/\sigma_s^2 + 2/\sigma_v^2}$$

Price is weighted average of public mean $\bar{v}$ and private signal s, minus a compensation for risk aversion

Notice that equilibrium price depends on private signal s

If aggregate supply X is fixed, market price reveals private signal!

Rational Expectations Equilibrium

Uninformed investors can back out signal s from equilibrium price p

- observing market price is as good as observing private signal
- price is **fully revealing**

Effectively, the uninformed also become informed traders!

$$d_U = d_I = \frac{\mathsf{E}[v|p] - p}{a\mathrm{Var}(v|p)}$$

REE price p^* satisfies this equation and clears the market

$$p^* = \beta s + (1-\beta)\bar{v} - \tfrac{1}{2}a(1-\beta)\sigma_v^2 X$$

Same form as before, but with higher weight on private signal ($\beta > \alpha$)

Criticism on REE models

- how prices attain the rational expectations equilibrium (REE) solution is not specified
  - learning is the usual defense
- a fully revealing equilibrium is not stable if information is costly
  - Grossman-Stiglitz (1988): impossibility of informationally efficient markets

Information based models II: Strategic trader models

Kyle (1985) model

Batch auction market. Sequence of actions:

0. Informed traders observe signal about value of the security
1. Traders submit buy and sell orders
   - only market orders: only quantity specified
   - simultaneous order submission
   - traders are anonymous
2. Auctioneer collects all orders and fixes the price
   - Kyle (1985) assumes auctioneer sets zero-expected-profit prices
3. Auction price and net aggregate order flow are revealed to all traders

Then go to the next trading round

Assumptions of the Kyle model

- Uninformed trader gives random order of size $\mu \sim N(0,\ \sigma_\mu^2)$
  - $\mu > 0$ is a buy order, $\mu < 0$ is a sell order
- Informed trader gives order of size x, that maximizes his expected trading profits $\Pi = \mathsf{E}[x(v-p)]$
- Auctioneer observes aggregate order flow $D = x + \mu$ and sets a price p according to a zero-expected-profit rule

$$p = \mathsf{E}[v|D] = \mathsf{E}[p|x+\mu]$$

- Assume this price schedule is linear

$$p = \bar{v} + \lambda(x+\mu)$$

The informed trader's problem

For simplicity, assume the informed trader has perfect information (knows payoff v) and is risk neutral

Maximizes expected trading profits $\Pi = \mathsf{E}[x(v-p)]$ facing the price schedule $p = \bar{v} + \lambda(x+\mu)$

- a large order x will increase the price: price impact of trading!

Substituting price function in expected profit gives

$$\Pi = \mathsf{E}[x(v-p)] = \mathsf{E}[x(v-\bar{v}-\lambda(x+\mu))] = x(v-\bar{v}) - \lambda x^2$$

Maximization w.r.t. order size x gives

$$x = \frac{1}{2\lambda}(v-\bar{v}) \equiv b(v-\bar{v})$$

The auctioneer's problem

Auctioneer sets "fair" price given the order flow

$$p = \mathsf{E}[v|x+\mu] = \mathsf{E}[v] + \frac{\mathrm{Cov}(v,\, x+\mu)}{\mathrm{Var}(x+\mu)}(x+\mu)$$

The term $\mathrm{Cov}(v,\, x+\mu)$ is determined by the behaviour of the informed trader, who uses a rule $x = b(v - \bar{v})$

Substituting this rule gives

$$p = \bar{v} + \frac{b\sigma_v^2}{b^2\sigma_v^2 + \sigma_\mu^2}(x+\mu) \equiv \bar{v} + \lambda(x+\mu)$$

Equilibrium

In equilibrium, the informed trader's order strategy and the auctioneer's pricing rule must be consistent, i.e.

$$\lambda = \frac{b\sigma_v^2}{b^2\sigma_v^2 + \sigma_\mu^2}$$

and

$$b = \frac{1}{2\lambda}$$

This implies

$$\lambda = \tfrac{1}{2}\frac{\sigma_v}{\sigma_\mu}, \qquad b = \frac{\sigma_\mu}{\sigma_v}$$

Qualitative implications of Kyle model

- Steepness of price schedule is measure for implicit trading cost (**Kyle's lambda**)
- lambda increases with overall uncertainty on the security's value (σ_v)
- lambda decreases with number of uninformed ('noise') traders
  - even though more uninformed traders makes the informed traders more aggressive
- price is informative about private signal (true asset value)

$$v|p \sim \; N(\bar{v} + \lambda(x + \mu), \; \tfrac{1}{2}\sigma_v^2)$$

  - Informed trader gives away half of his information

Summary of Kyle (1985) model

- informed traders will trade strategically, i.e. they condition trades on their private information
  - maximize trading profits per trading round
- auctioneer will use an *upward-sloping price schedule* as a protection device against adverse selection
- net aggregate order flow reveals part of the private information
  - order flow is informative, prices respond to trading
- after many trading rounds, prices converge to their full information (rational expectations) value
- prices are semi-strong form efficient (but not strong form efficient)

Information based models III: Sequential trading models

The Glosten Milgrom (1985) model

Market structure

- quote driven market
- unit trade size
- one trade per period
- no explicit transaction costs
- trading is anonymous

- informed and uninformed traders, arrive randomly
  - uninformed have exogenous demand (buys and sells)
  - informed exploit information
- specialist market maker, sets quotes for buy (ask) and sell (bid)
  - uninformed
  - risk neutral and competitive (zero profit condition): quotes equal expected value of asset

Specialist faces an adverse selection problem: looses on trading with informed traders

In response, market maker quotes higher prices for buyer-initiated transactions (ask) and lower for seller initiated (bid)

Informativeness of trades

- Essential idea:
  - informed traders are more likely to buy when there is good news
  - trade direction (buy or sell) conveys information about true value
  - adverse selection problem for the market maker: informed traders only buy on one side of the market
- For example, buy trade will be interpreted as a 'good' signal for the asset value; market maker updates expectations

$$\mathsf{E}[v|\text{buy}] > \mathsf{E}[v]$$

- Market maker will set zero-expected profit or regret-free prices

$$ask = \mathsf{E}[v|\text{buy}], \quad bid = \mathsf{E}[v|\text{sell}]$$

Inference on value

- Stock has two possible values, high and low:

$$\begin{cases} v = v_H \textit{ with probability } \theta \\ v = v_L \textit{ with probability } 1 - \theta \end{cases}$$

  Expected value is
  $\mathsf{E}[v] = P(v = v_H)v_H + P(v = v_L)v_L = \theta v_H + (1 - \theta)v_L$

- Suppose one observes a 'buy' transaction. What is the expected value of the asset given this trade?

$$\mathsf{E}[v|\text{buy}] = P(v = v_H|\text{buy})v_H + P(v = v_L|\text{buy})v_L$$

- Bayes' rule for discrete distributions

$$P(v = v_H|\text{buy}) = \frac{P(\text{buy}|v = v_H)P(v = v_H)}{P(\text{buy})}$$

- The unconditional 'buy' probability is

$$P(\text{buy}) = P(\text{buy}|v = v_H)P(v = v_H) + P(\text{buy}|v = v_L)P(v = v_L)$$

- Probability of buy trade depends on the true value:
  - $P(\text{buy}|v = v_H) > P(\text{buy})$
  - $P(\text{buy}|v = v_L) < P(\text{buy})$

  this is the adverse selection effect

Buy/sell probabilities in the Glosten-Milgrom model

- Fraction μ of informed, $1-\mu$ of uninformed traders
  - Uninformed traders buy with probability γ, and sell with probability $1-\gamma$
  - Informed traders buy if $v = v_H$, sell if $v = v_L$
- Conditional buy/sell probabilities

$$\begin{cases} P(\textsf{buy}|v = v_H) = \mu * 1 + (1-\mu)\gamma \\ P(\textsf{buy}|v = v_L) = \mu * 0 + (1-\mu)\gamma \end{cases}$$

$$P(\textsf{sell}|v = v_H) = 1 - P(\textsf{buy}|v = v_H), \quad P(\textsf{sell}|v = v_L) = 1 - P(\textsf{buy}|v = v_L)$$

- Unconditional probability of a buy

$$P(\textsf{buy}) = (\mu * 1 + (1-\mu)\gamma)\theta + (\mu * 0 + (1-\mu)\gamma)(1-\theta)$$

- Updated probability of a high value

$$
\begin{aligned}
P(v = v_H|\textsf{buy}) &= \frac{P(\textsf{buy}|v = v_H)P(v = v_H)}{P(\textsf{buy})} \\
&= \frac{(\mu + (1-\mu)\gamma)\theta}{(\mu + (1-\mu)\gamma)\theta + (1-\mu)\gamma(1-\theta)} \\
P(v = v_L|\textsf{buy}) &= 1 - P(v = v_H|\textsf{buy})
\end{aligned}
$$

- Expected asset value after the trade

$$
\mathsf{E}[v|\textsf{buy}] = P(v = v_H|\textsf{buy})v_H + (1 - P(v = v_H|\textsf{buy}))v_L
$$

Numerical example

Let $v_H = 100$, $v_L = 0$, $\theta = 1/2$, $\mu = 1/4$ and $\gamma = 1/2$

Prior expectation of value: $\mathsf{E}[v] = (1/2) * 100 + (1/2) * 0 = 50$

$$\begin{cases} P(\mathsf{buy}|v = v_H) = 1/4 * 1 + 3/4 * 1/2 = 5/8 \\ P(\mathsf{buy}|v = v_L) = 1/4 * 0 + 3/4 * 1/2 = 3/8 \end{cases}$$

Unconditional probability of a buy

$$P(\mathsf{buy}) = P(\mathsf{buy}|v = v_H)\theta + P(\mathsf{buy}|v = v_H)(1-\theta) = \frac{5}{8} * \frac{1}{2} + \frac{3}{8} * \frac{1}{2} = \frac{1}{2}$$

Posterior probability of high value

$$P(v = v_H|\mathsf{buy}) = \frac{P(\mathsf{buy}|v = v_H)\theta}{P(\mathsf{buy})} = \frac{5/8 * 1/2}{1/2} = 5/8$$

Posterior expected value of the asset

$$\mathsf{E}[v|\mathsf{buy}] = (5/8) * 100 + (3/8) * 0 = 62.50$$

Sell side

After a sell, the updated probability of a high value is

$$P(v = v_H|\mathsf{sell}) = \frac{P(\mathsf{sell}|v = v_H)\theta}{P(\mathsf{sell})}$$

In numerical example this amounts to

$$P(v = v_H|\mathsf{sell}) = \frac{(1 - 5/8)1/2}{1/2} = 3/8$$

and $\mathsf{E}[v|\mathsf{sell}] = (3/8) * 100 + (5/8) * 0 = 37.50$

Bid-ask spread will be

$$S = \mathsf{E}[v|\mathsf{buy}] - \mathsf{E}[v|\mathsf{sell}] = 62.50 - 37.50 = 25$$

Bid-ask spread

Zero-profit bid and ask prices are

$$\begin{cases} \text{ask} = \mathsf{E}[v|\text{buy}] \\ \text{bid} = \mathsf{E}[v|\text{sell}] \end{cases}$$

Endogenously, a bid-ask spread will emerge

$$S = \mathsf{E}[v|\text{buy}] - \mathsf{E}[v|\text{sell}]$$

Expression for bid-ask spread is complicated, but qualitatively the spread

- increases with $v_H - v_L$ (volatility of asset)
- increases with fraction of informed traders μ

Main results of Glosten-Milgrom model

- endogenous bid-ask spread
- market is semi-strong form efficient
  - prices are martingales with respect to public information
- with many trading rounds, prices converge to full information value

The Easley and O'Hara (1987) model

Extension of the Glosten-Milgrom model

- possibility that there is no information (event uncertainty)
  - trades signal about quality of information (good or bad) but also about the *existence* of information (O'Hara 3.4)
- choice of trade size (small or large)

As in the GM model, uninformed trading is exogenous, split over small and large trade size

Informed trader faces tradeoff: large size trade means higher profit, but also sends stronger signal of information

Possible outcomes

- Separating equilibrium: if large size is large enough, informed trader will always trade large quantity
  - small trades only by U, hence no bid-ask spread for small size!
- Pooling equilibrium: I randomizes between small and large trades: hides some of his information to improve prices for large trades
  - spread for small size smaller than spread for large size

Important assumptions

- trading is anonymous
- informed traders act competitively: exploit information immediately