List of Errata
for Manifolds, Tensors, and Forms

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1. p. ix In the second epigram 'bersetzen' should read 'übersetzen'.
2. p. 2 The first sentence in Footnote 1 should be rewritten as follows for clarity: 'By definition, W is a **translate** of a subspace $U \subset V$, if for some fixed $v \in V$ with $v \notin U$, $W = \{u + v : u \in U\}$.'
3. p. 3 The second line following Exercise 1.5 should read '...induces a linear map $T : \mathrm{span}(U) \to W$...'.
4. p. 4 The first line of Exercise 1.9 should read 'A linear endomorphism T is **idempotent** if...'.
5. p. 4 The seventh line into Section 1.3: add the word 'is' after the word 'it'.
6. p. 4 Exercise 1.12. The phrase 'is equivalent to' should be replaced by the word 'implies'.
7. p. 14 One instance of the word 'condition' should be deleted from the line beginning 'However, if $\mathbb{F}$ is a real field...'.
8. p. 16 Line 12 from the top: '...as the next result shows. For simplicity we consider only the case $\mathbb{F} = \mathbb{R}$.'
9. p. 16 Theorem 1.5 should read '*Every real inner product space has an orthonormal basis.*'
10. p. 16 The proof of Theorem 1.5 needs a little revision. For instance, the sentence 'We use induction on $k = \dim V$.' should be changed to 'We use induction on the number of orthonormal vectors in the space.'. The rest of the proof works, but should be rewritten slightly to accommodate this change.
11. p. 16 Equation (1.44) should read:

$$\pi(v) = \sum_{i=1}^{k-1} g(e_i, e_i) g(e_i, v) e_i. \tag{1.44}$$

12. p. 17 In the first line of the second paragraph the phrase 'which is valid only for real fields' should be deleted. The sentence would then read 'Below we offer a second proof of Theorem 1.5 in order to define....'.
13. p. 17 The line following (1.47) should read 'If $\det(g_{ij}) = 0$ then...'.
14. p. 23 The line following (1.61) should read '...where $\widetilde{A}_{ij}$ is the cofactor...'.
15. p. 27 Exercise 1.48 should read: 'A theorem due to Schur guarantees that any matrix $\boldsymbol{A}$ is similar (by a unitary transformation) to a sum of a diagonal matrix $\boldsymbol{D}$...'.
16. p. 43 In the second line of the equation just below Exercise 2.8 the summation should be over the indices $j_1 \cdots j_p$ rather than $j_1 \cdots j_n$.

17. p. 50 Problem 2.18. The last line should read 'where $d = (n-t)/2$ and t is the signature of the metric.'.

18. p. 52 Exercise 2.21d. The second equation should read:

$$\frac{1}{\det(I - zA)} = \sum_{k=0}^{\infty} \operatorname{tr}(\operatorname{Sym}^k A) z^k.$$

19. p. 64 The discussion of orientations is not correct. The paragraph beginning 'The sign of this determinant is important' should continue: 'A manifold is said to be **orientable** if it is possible to choose an ordering of the local coordinates so that the Jacobian determinants of the transition functions are positive on every pair of overlapping neighborhoods. If this is possible, then the manifold has two opposite **orientations**, obtained by applying an even or odd permutation simultaneously to every set of local coordinates. For the most part...'.

20. p. 66 Footnote 14 should read 'In this example there is only one coordinate overlap, so the sign of the Jacobian is immaterial. We could have chosen the sign to be positive simply by reversing the order of the coordinates on V.

21. p. 66 Example 3.7, lines 6 and 7 should read 'Mathematically, the Möbius strip can be viewed as the quotient space $(\mathbb{R} \times [-1,1])/\sim$, where we have $(x,y) \sim (x+1,-y)$. If you substitute $\mathbb{R}$ for the closed interval $[-1,1]$ in the definition...'.

22. p. 67 Example 3.7, five lines above the start of Exercise 3.16. The phrase 'sign mismatch' should be 'negative sign'.

23. p. 67 Example 3.7, two lines above the start of Exercise 3.16. The parenthetical phrase beginning '(Basically the reason ...' and ending '...sign.)' should be deleted.

24. p. 78 Footnote 21 should read '...especially when one of the vectors in the frame is the tangent vector of a curve.'

25. p. 79 The line just above Equation (3.32) should read '...would be to write..'.

26. p. 81 The definition given at that start of the second paragraph is not quite consistent with the discussion later in that paragraph. It should read 'We say that f **vanishes to first order at** p if $f(p) = 0$ and $\partial f/\partial x^i$ vanishes at p for all i.' (It might have better to start the paragraph with the definition of a function vanishing to k^{th} order, and then to have shown that the notion is well-defined.)

27. pp. 83-84 In Equations (3.49), (3.50), and (3.52) the subscript i_i should be i_1.

28. p. 89 The first line on the page should read '...with nine coordinate patches, U_i, $i = 1, \ldots, 9$.
29. p. 92 Third line of Equation (3.67). $dt \wedge dx \wedge dy$ should be $dt \wedge dz \wedge dx$.
30. p. 99 In the paragraph following the proof of Theorem 3.7 make the following substitutions. (i) $X_i \in TN$ should be replaced by $X_i \in \Gamma(TN)$, (ii) $\omega^i \in T^*N$ should be replaced by $\omega^i \in \Gamma(T^*N)$, (iii) $Y_i \in TM$ should be replaced by $Y_i \in \Gamma(TM)$, and (iv) $\mu^i \in T^*M$ should be replaced by $\mu^i \in \Gamma(T^*M)$.
31. p. 101 Seventh line from the top: replace the semicolon by a comma so it reads '...to get the curve, hence the name...'.
32. p. 112 Exercise 3.55a needs another hint. For the simple induction to work, one must show that Cartan's formula holds for functions and one-forms. For the latter, one needs the fact that the Lie derivative and exterior derivatives commute. (This follows from the definition of the Lie derivative, because the exterior derivative commutes with pullbacks.)
33. p. 118 The second line in Exercise 4.1 should read '...if it is...'.
34. p. 124 Line 5 should begin 'where $s_0, s_1 : M \to I \times M$ are the inclusion maps...'.
35. pp. 133-134 Example 4.6 needs to be rewritten. The first line should read 'We can use the result of this last exercise to prove...'. On p. 134 the two paragraphs following 'We first show by contradiction...' should be replaced by the following two paragraphs (to fix an index issue and to specify the correct retraction).

    'We first show by contradiction that there is no retraction $r : B^n \to S^{n-1}$ of the n-ball onto its boundary. Suppose that there were. Then, by pulling back everything we would have the composition

    $$H^{n-1}_{\mathrm{dR}}(S^{n-1}) \xrightarrow{r^*} H^{n-1}_{\mathrm{dR}}(B^n) \xrightarrow{\iota^*} H^{n-1}_{\mathrm{dR}}(S^{n-1}), \tag{4.26}$$

    where $\iota^* \circ r^* = 1$. But the ball is contractible, so $H^{n-1}_{\mathrm{dR}}(B^n)$ vanishes whereas $H^{n-1}_{\mathrm{dR}}(S^{n-1})$ does not, a contradiction.

    Now suppose that $f : B^n \to B^n$ has no fixed point. Then $f(x) \neq x$ for all $x \in B^n$. Let $g : B^n \to S^{n-1}$ be the map sending x to the point where the ray from $f(x)$ to x meets S^{n-1}. Explicitly,

    $$g(x) = x + \lambda_x(x - f(x)), \tag{4.27}$$

    where $\lambda_x \in \mathbb{R}^+$ is chosen so that $\|g(x)\| = 1$. But this is a retraction of the ball onto its boundary, a contradiction.'
36. p. 139 The line just above Exercise 5.1 is missing a word. It should read 'A (convex) **polytope** (Figure 5.3) is the convex hull...'.

37. p. 139 Footnote 1 should read: 'For a friendly and comprehensive tour of the world of convex polytopes, see [92]. The reader is advised that there exist more general (i.e., nonconvex) polytopes, but we do not consider them here.'
38. p. 141 Seven lines up from the bottom: '$(n-1)$' should be '$(d-1)$'.
39. p. 144 The last line should begin '$\bar{\sigma} = \langle p_0, \ldots, p_{n+1} \rangle$ is a triangulation...'.
40. p. 147 Line 4. 'S_{n-1}' should be replaced by 'S_{n+1}'.
41. p. 147 Example 5.4. Seven lines up from the bottom. The sentence should read 'Since K is the boundary of a triangle, say $\partial\langle p_0, p_1, p_2 \rangle$, every 0-chain...'.
42. p. 148 Example 5.4. The line beginning 'There are no 2-cycles in K...', should read instead 'There are no 2-chains in K...'.
43. p. 150 Seven lines up from the bottom. The sentence should read 'The simplicial complex K induces a sequence of chain spaces...'.
44. p. 158 Eight lines up from the bottom. The sentence should read '...U is an open neighborhood of $\bar{\sigma}$ in $\operatorname{Aff} \bar{\sigma}$...'.
45. p. 162 Theorem 6.3. In the phrase '...and c a k-chain c on M' the second instance of 'c' should be deleted.
46. pp. 168-169 In the second part of the proof the two occurrences of
$$\int_{\mathbb{H}^n} a_n(x^1, \ldots, x^{n-1}, 0)\, dx^1 \cdots dx^{n-1}$$
should be replaced by
$$\int_{\mathbb{R}^{n-1}} a_n(x^1, \ldots, x^{n-1}, 0)\, dx^1 \cdots dx^{n-1}.$$
47. p. 170 The last part of the sentence on the third and fourth lines from the bottom of the page should read '...where $C^\bullet(M)$ is the complex of smooth singular cochains.'
48. p. 172 Theorem 6.9. The first sentence should read 'For any finite-dimensional complex C and...'.
49. p. 172 The third line from the bottom should be replaced by 'Fix a projection map $\pi : C_\ell \to Z_\ell$. Composing...'.
50. p. 178 The paragraph following Example 7.1 should be modified to read 'A (smooth) **basis of sections** over a subset $U \subseteq M$ is a collection $\{e_1, \ldots, e_m\}$ of m sections that is linearly independent at each point of U. When $E = TM$ such a smooth basis of sections is just what we have called a frame field. As E is locally a product...'.
51. p. 178 In the last line n should be replaced by m.
52. p. 179 The second line in Exercise 7.3 should read '...in Chapter 3.)'.

53. p. 186 The second line in Section 7.6 should read '...by taking interior products:'.
54. p. 189 The fourth line in Exercise 7.9 (c) should read 'In the notation of Exercise 7.8 let...'.
55. p. 205 Eleven lines up from the bottom the sentence should begin 'By torsion-freeness (e.g. $\nabla_Y Z = \nabla_Z Y$) the terms...'.
56. p. 206 Exercise 8.13. Remove the word 'independent'.
57. p. 208 Footnote 8. The phrase '...simply connected and homologically trivial manifold is a sphere' should instead read '...simply connected manifold with the same homology as a sphere is a sphere'.
58. p. 218 Line 13. The inequalities are reversed. The statement '$t_1 \leq t_2 \leq \cdots \leq t_n$' should read '$t_1 \geq t_2 \geq \cdots \geq t_n$'.
59. p. 223 Exercise 8.28 is missing a word. The last sentence should read '...together with the fact...'.
60. p. 223 Footnote 34 should read 'See Exercises 8.56 and 8.58.'.
61. p. 227 The line preceding Equation (8.110) should read 'Show that X is a Killing field if and only if, for all $Y, Z \in \Gamma(TM)$,...'.
62. p. 239 In the final sentence of Exercise 8.52 (b) the word should be 'pseudotensor'.
63. p. 250 The last paragraph should begin 'Now let $f : M \to N$ be a nonconstant smooth map between...'.
64. p. 252 In Exercise 9.1 'G' should be replaced by 'g'.
65. p. 258 Four lines up from the bottom. The sentence should start 'Let M and N be two disjoint oriented compact....'.
66. p. 260 Example 9.1. The answer to (c) is $+1$ not -1.
67. p. 264 The fourth line in Example A.1 should read '...is easily seen to be...'.
68. p. 270 In the definition of a vector space, the third line should have been '...for every $a, b \in \mathbb{F}$ and $v, w \in V$ we have:'.
69. p. 271 The start of the fourth line from the top of the page should be changed to '...a positive definite inner product ....'.
70. p. 278 Middle of the page. 'Newton's second law' should be 'Newton's first law'.
71. p. 291 The first line in the second paragraph of Section F.3 should read '...the rod is...'.
72. p. 296 The final sentence of Section G.1 should read '...so if the law...'.
73. p. 300 Bottom of the page. The proof that

$$B_0(G) = \{\textstyle\sum_i a_i p_i : \sum_i a_i = 0\}$$

has some unfortunate sign errors. Basically a bunch of plus signs should be minus signs. The equations following the expression '...with $c_{ij} = -c_{ji}$ and...' should instead read

$$\begin{aligned} b = \partial c &= \textstyle\sum_{ij} c_{ij}(p_j - p_i) = \sum_i (\sum_j c_{ij} p_j) - \sum_j (\sum_i c_{ij} p_i) \\ &= \textstyle\sum_i (\sum_j c_{ij} p_j) - \sum_i (\sum_j c_{ji} p_j) = \sum_{ij} (c_{ij} - c_{ji}) p_j \\ &= \displaystyle\sum_j a_j p_j, \end{aligned}$$

where $a_j := \sum_i (c_{ij} - c_{ji}) = 2 \sum_i c_{ij}$. Thus

$$\textstyle\sum_j a_j = 2 \sum_{ij} c_{ij} = 0.$$

74. p. 301 Equation (G.8). The symbol i_j should be I_j instead.
75. p. 311 Exercise H.3. Replace the word 'orthonormality' by 'orthogonality'.
76. p. 313 The second line following Equation (H.16) is missing a parenthesis. It should read '...a partial derivative.)'.