Mathematical appendix 2

The Earth–Sun system

This discussion develops the ideas in Section 3.3.

The Equilibrium Tide is expressed in terms of the distance, declination and hour angle of the tide-producing body. If this is the Sun, we need values for r_s, d_s and C_s. These three parameters are easier to define for the Earth–Sun system than for the Moon–Earth system, because the solar motions are always in the plane of the ecliptic, which means that the declination in ecliptic coordinates is always zero. The eccentricity of the Earth's orbit about the Sun is 0.0168.

From orbital theory the solar distance r_s may be shown to be given approximately by:

$$\frac{\bar{r}_s}{r_s} = (1 + e\cos(h - p'))$$

where $\bar{r}_s$ is the mean solar distance, h is the Sun's geocentric mean ecliptic longitude (which increases by $\omega_3 = 0.0411^\circ$ per mean solar hour; see Table 3.2) and p' is the longitude of solar perigee which completes a full cycle in 21 000 years.

The true ecliptic longitude of the Sun increases at a slightly irregular rate through the orbit according to Kepler's second law. The true longitude λ_s is given to a first approximation by:

$$\lambda_s = h + 2e\sin(h - p')$$

where the value of λ_s is expressed in radians (to facilitate the harmonic expansions of the Equilibrium Tide discussed in the associated appendix).

The right ascension is calculated from the ecliptic longitude and ecliptic latitude. Both the ecliptic longitude and the right ascension are zero when the Sun is at the First Point of Aries (♈) at the vernal equinox, and also at the autumnal equinox. They both have values of $\pi/2$ when the Sun has its maximum declination north of the equator in June, and both have values of $3\pi/2$ when the Sun has its maximum declination south of the equator in December. Between these times there are small regular differences owing to the obliquity of the orbit. The effect can be shown (W. M. Smart, *Spherical Astronomy*, Cambridge University Press, 1940) to be represented to a first approximation by:

$$A_s = \lambda_s - \tan^2(\epsilon_s/2)\sin 2\lambda_s$$

where ϵ_s is the solar declination ecliptic latitude. The difference between the right ascension of the mean Sun and the right ascension of the true Sun (sundial time) at any time is called the ‘equation of time’. During the annual cycle, differences of more than 15 minutes occur between clock-time and solar-time. In tidal analysis these differences are accounted for by a series of harmonic terms as discussed in the associated appendix.

The solar declination in equatorial coordinates is given in terms of the ecliptic longitude of the Sun:

$$\sin d_s = \sin \lambda_s \sin \epsilon_s$$

This is greatest when $\lambda_s = \pi/2$ and the maximum declination is $\epsilon_s = 23^\circ\ 27'$, the angle that defines the tropic of Cancer. When $\lambda_s = -(\pi/2)$ the Sun is overhead at the tropic of Capricorn at the time of its maximum declination $23^\circ\ 27'$ south of the equator.