Introduction: 8 March 2019

$\hbar\omega \cdot \lambda = hc = 1239.842\ \text{eV nm}$

$\Im = (5.034\text{x}10^{15} ph/sec)P[W]\lambda[nm]$

$I/I_0 = \exp(-\rho\mu x)$

$a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 0.5290\ \text{Å}$

$N_A = 6.022 \text{ x } 10^{23}$ molecules/mole

$N_L = 2.686 \text{ x } 10^{19}$ molecules/cc @ 0°C

Electric Fields, Intensity, Scattering:

$r_e = e^2/4\pi\epsilon_0 mc^2$; $\sigma_e = 8\pi r_e^2/3$

$\sigma = \left[8\pi r_e^2/3\right]\left[\omega^4/\left(\left(\omega^2 - \omega_s^2\right)^2 + (\gamma\omega)^2\right)\right]$

$f(\Delta\mathbf{k}, \omega) = \sum_{s=1}^{Z}\left[\omega^2 e^{-i\Delta\mathbf{k}\cdot\Delta\mathbf{r}_s}/\left(\omega^2 - \omega_s^2 + i\gamma\omega\right)\right]$

$\sigma = \left[8\pi r_e^2/3\right]|f|^2$

$f^0(\omega) = \sum_{k}^{Z}\left[g_{ik}\omega^2/\left(\omega^2 - \omega_{ik}^2 + i\gamma\omega\right)\right]$

$\bar{I} = 1.327x10^{-3} E_0^2 [\text{V/ cm}] W/cm^2$

$E_0 = 2.745x10^{-7}\sqrt{\bar{I}\left[\text{W/ cm}^2\right]}\ \text{V/ }\overset{0}{\text{A}}$

Refractive Effects:

$n(\omega) = 1 - \left[n_a r_e \lambda^2/2\pi\right]\left(f_1^0 - i\, f_2^0\right)$

$n(\omega) = 1 - \delta + i\beta$

$l_{\text{abs}} = \lambda/4\pi\beta$

$\Delta\phi = [2\pi\delta/\lambda]\Delta r$

$\theta_c = \sqrt{2\delta}$

$R_{s,\perp} \simeq \left[\left(\delta^2 + \beta^2\right)/4\right]$

$\phi_B \simeq \frac{\pi}{4} - \frac{\delta}{2}$

$\bar{I} = (1/2)(1-\delta)\sqrt{\epsilon_0/\mu_0}\ |\mathbf{E}|^2$

$m\lambda = 2d\sin\theta$ (Bragg’s Law)

Coherence:

$\Delta\tau\cdot\Delta\omega \geq ½$; $\Delta E_{FWHM} \cdot \Delta\tau_{FWHM} \geq 1.825$ eV·fsec

$\Delta\mathbf{x}\cdot\Delta\mathbf{p} \geq \hbar/2$; $\Delta x\cdot\Delta k \geq ½$

$l_{coh} = \lambda^2 / 2\Delta\lambda$

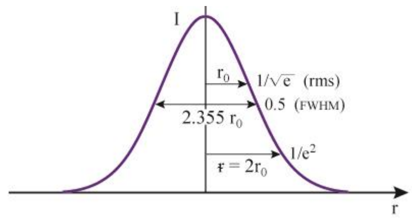
$d\cdot \theta = \lambda/2\pi$ (Gaussian)

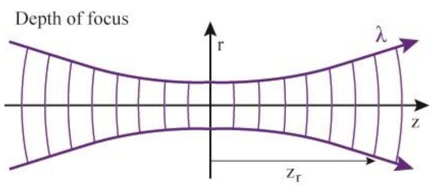
$d_{FWHM} \cdot 2\theta_{FWHM} \cong \lambda/2$ (Gaussian)

$d\cdot\theta_{null} = 1.22\lambda$ (Sharp edge)

Gaussian:

$I/I_0 = \exp(-r^2/2r_0^2)$





$Z_R = \pi ɍ^2/\lambda$, DOF = $2Z_R$

(Rayleigh range; Gaussian area doubles)

DOF = $\pm\, \pi ɍ^2/\lambda = \pm\, 4\pi r_0^2/\lambda$

Synchrotron Radiation

Bending Magnet:

$\hbar\omega_c = 3e\hbar B\gamma^2/2m$

Wiggler:

$\hbar\omega_c = 3e\hbar B\gamma^2/2m$

$n_c = (3K/4)\left[1+\left(K^2/2\right)\right]$

$\bar{P}_T = \left[\pi e K^2 \gamma^2 \bar{I}\, N/3\epsilon_0\lambda_u\right]$

Undulator:

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$ =1.306nm λ_u[cm](1+K^2/2)/E_e^2[GeV]

$$K = \left[eB_0\lambda_u/2\pi mc\right]$$ = 0.9337B_0[T]λ_u[T]

$$\theta_{\text{cen}} = \left[1/\gamma * \sqrt{N}\right]\ ;\ (\Delta\lambda/\lambda)_{cen} = (1/N)$$

$$\bar{P}_{\text{cen}} = \left[\pi e\gamma^2 \bar{I}/\epsilon_0\lambda_u\right]\left[K^2[JJ]^2/\left(1+K^2/2\right)^2\right]$$

$$\bar{P}_{\text{coh},N} = \left[(\lambda/2\pi)/d_x\theta_x\right]\left[(\lambda/2\pi)/d_y\theta_y\right]\bar{P}_{\text{cen}}$$

$$\bar{P}_{\text{coh},\lambda/\Delta\lambda} = \eta\left[(\lambda/2\pi)^2/(d_x\theta_{Tx})(d_y,\theta_{Ty})\right](\Delta\lambda/\lambda)\,N\bar{P}_{\text{cen}}$$

FELS:

$$\rho_{FEL} = \left[\hat{K}^2 r_e n_e \lambda_u^2/32\pi\gamma^3\right]^{1/3}$$

$$\hat{K} = K[JJ] = K\left[J_0(x) - J_1(x)\right]$$

$$x = K^2/\left(4+2K^2\right)$$

$$\hat{I}/I = \hat{P}/P_0 = e^{z/L_g}\ ;\ L_G = \lambda_u/4\pi\sqrt{3}\rho_{FEL}$$

$$\hat{P}_{sat} \simeq \rho_{FEL}\cdot\gamma mc^2\cdot\hat{I}/e$$

$$(\Delta\omega/\omega)_{FWHM} = 2\sqrt{3}\rho_{FEL}$$

$$\sigma\sigma' \simeq \lambda/4\pi$$

HHG:

Return energy

$\hbar\omega_{max}$ = 3.17 Up+ Ip

Up ≡ $\overline{KE}$ = $e^2E_0^2/4m\omega^2$ = $r_eI\lambda^2/2\pi c$

Up = 9.337x10^{-14}I[W/cm^2]λ^2[μm] eV

$$x_{\max} = eE_0/m\omega^2$$

$$x_{\max} = 1.95\,\text{nm}\,\lambda_L^2[800nm]\,I^{1/2}\left[5x10^{14}W/cm^2\right]$$

$$\bar{I} = 1.327x10^{-3}E_0^2[\text{V/ cm}]W/cm^2$$

$$E_0 = 2.745x10^{-7}\sqrt{\bar{I}\left[\text{W/ cm}^2\right]}\ \text{V/ }\overset{0}{\text{A}}$$

Bohr atom (n = 1)

$a_B = a_0 = 0.5292\,\text{Å}$

$\tau_B = 2\pi a_0 / \alpha\,\text{c} = 152.0\,\text{asec}$

$E_B = e / 4\pi\varepsilon_0 a_0^2 = 51.42\,\text{V}/\text{Å}$

$\bar{I}_C = 0.5\sqrt{\epsilon_0 / \mu_0}\,E_B^2 = 3.509\,\text{x}\,10^{16} W / cm^2$

Bandwidth limited pulse

$\Delta E_{FWHM} \cdot \Delta t_{FWHM} \geq 1.825\text{eV}\cdot\text{fsec}$

Tunneling

$\gamma_K = \sqrt{I_p / 2U_p}$

Plasmas:

$\omega_p^2 = \left(e^2 n_e / \epsilon_0 m\right)$

$\lambda_D = \sqrt{\epsilon_0 \kappa T / e^2 n_e}$

$\omega_c = \left(eB/m\right)$; $r_L = m\,\text{v}/\,\text{eB}$; [3.7 μm for 9 Tesla, 100 eV e^-]

$\text{v}_{\exp} = \sqrt{Z\gamma\kappa T_e / M}$ = (0.28 nm/fsec)[for Z=10, 1keV, $10m_p$]

$n_c = \left(\epsilon_0 m\omega^2 / e^2\right)$ = $1.11\text{x}10^{21}$e/cc/λ^2[μm]

$\left(\text{v}_{\text{os}} / \text{v}_{\text{e}}\right)^2 = \left[\left(I / c\right) / n_c \kappa T\right]$

$I = \sigma T^4$ [10^{13} W/cm$^2 \rightarrow 100$eV]

$\hbar\omega|_{\max} = 2.82\kappa T$

EUV/SXR Lasers:

$I / I_0 = e^{GL} = e^{z/Lg}$

$G = n_u \sigma_{\text{stim}} F$

$\sigma_{\text{stim}} = \left[\pi\lambda r_e / \left(\Delta\lambda/\lambda\right)\right]\left(g_l / g_l\right) f_{lu}$

$\left(\Delta\lambda/\lambda\right)_{\text{FWHM}} = \left(2\sqrt{2\ln 2} / c\right)\sqrt{\kappa T / M}$

$P/A = 16\pi^2 c^2 \hbar \left(\Delta\lambda/\lambda\right) GL / \lambda^4$

Laser lines:

Ne like: 3p → 3s, Ar 46.86 nm (26.46 eV)

Ni like: 4d → 4p , Mo 18.9 nm, Ag 13.9 nm, Cd 13.2 nm

Ne Kα: 1.46 nm (849 eV)

Cu Kα: 1.541 Å (8.048 eV), 1.544 Å (8.028 eV)

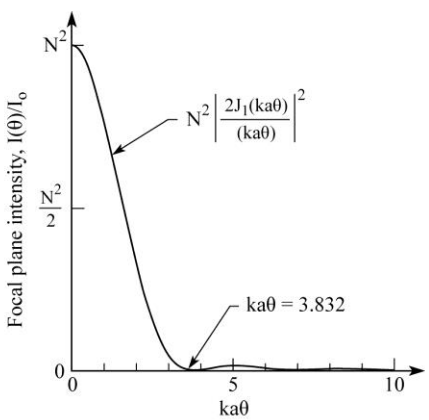
Optics:

$\theta_c = \sqrt{2\delta}$

$sin\theta = \lambda/d$ (grating equation)

$m\,\lambda = 2d\,sin\theta$ (Bragg's equation)

Airy Pattern (Resolution):



$$\Delta r_{Rayl} = r_{null} = 0.610\lambda/NA$$

$$\Delta z_{DOF} = \pm\frac{1}{2}\left[\lambda/(\text{NA})^2\right]$$

$D_{FWHM,\ Airy} = 0.515\ \lambda/NA$

$z \gg d^2/\lambda$ (Fraunhofer region)

$NA = \sin\theta = \lambda/2\Delta r$ (zp)

$D = 4N\Delta r$ (zp)

$f = 4N(\Delta r)^2/\lambda$ (zp)

$N < \lambda / \Delta \lambda$ (zp)

$\Delta r_{Rayl} = 1.220 \Delta r$ (zp)

$\Delta z_{DOF} = \pm 2 (\Delta r)^2 / \lambda$ (zp)

Imaging:

Rayleigh dip = 26.5% (15.3 % contrast)

$\sigma = \mathrm{NA}_{\mathrm{cond}} / \mathrm{NA}_{\mathrm{obj}}$

$f = R / 2N\delta$ (CRL)