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Solutions to Chapter Problems

Solutions to Chapter 2 Problems

PROBLEM 2.1

(a) $\mathbf{f} = \mathbf{x} - \mathbf{Z}\mathbf{i} = \begin{bmatrix} 150 \\ 120 \end{bmatrix}$. (b)i $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .5 & .4375 \\ .32 & .45 \end{bmatrix}$, $\mathbf{f}^{new} = \begin{bmatrix} 200 \\ 100 \end{bmatrix}$,

$\tilde{\mathbf{x}}^{new} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^4)\mathbf{f}^{new} = \begin{bmatrix} 650.81 \\ 453.98 \end{bmatrix}$. (b)ii $\mathbf{x}^{new} = \mathbf{L}\mathbf{f}^{new} = \begin{bmatrix} 1138.90 \\ 844.40 \end{bmatrix}$ where

$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 4.07 & 3.24 \\ 2.37 & 3.7 \end{bmatrix}$. In this case, the power series converges very slowly. For example,

$\tilde{\mathbf{x}}^{new} = (\mathbf{I} + \mathbf{A} + \cdots + \mathbf{A}^{25})\mathbf{f}^{new} = \begin{bmatrix} 1122.80 \\ 831.60 \end{bmatrix}$.

PROBLEM 2.2

(a) $\mathbf{A}^t = \begin{bmatrix} .35 & 0 & 0 \\ .05 & .5 & .15 \\ .2 & .3 & .55 \end{bmatrix}$, $\mathbf{L}^t = \begin{bmatrix} 1.538 & 0 & 0 \\ .449 & 2.5 & .833 \\ .983 & 1.667 & 2.778 \end{bmatrix}$. (b) $\mathbf{x}^{t+1} = \mathbf{L}^t\mathbf{f}^{t+1} = \begin{bmatrix} 2000 \\ 1000 \\ 2000 \end{bmatrix}$ for

$\mathbf{f}^{t+1} = \begin{bmatrix} 1300 \\ 100 \\ 200 \end{bmatrix}$; (c) $\mathbf{f}^t = \mathbf{x}^t - \mathbf{Z}^t\mathbf{i} = \begin{bmatrix} 650 \\ 50 \\ 100 \end{bmatrix}$ for $\mathbf{x}^t = \begin{bmatrix} 1000 \\ 500 \\ 1000 \end{bmatrix}$. This illustrates linearity, e.g, since

$\mathbf{f}^{t+1} = 2\mathbf{f}^t$ it follows that $\mathbf{x}^{t+1} = 2\mathbf{x}^t$ because $\mathbf{x}^{t+1} = \mathbf{L}^t\mathbf{f}^{t+1} = 2\mathbf{L}^t\mathbf{f}^t = 2\mathbf{x}^t$.

PROBLEM 2.3

Problem 2.3 as printed on page 63 in the text contains a typo and an implicit error in logic. The \$1000 should be \$100, otherwise comparison with the results for Problem 2.1 make little sense. But in addition it is incorrect to assume that the $\Delta f_1 = 50$ and $\Delta f_2 = -20$ should be applied to the *original* $f_1 = 150$ and $f_2 = 120$, since in the closed model the consumption (household) component of final demand will be endogenized. Finally, it should have been stated explicitly that the changes are assumed to occur in the non-household part of final demand, e.g., in exports. (See "Errata" on the website for revised statement of Problem 2.3.)

With those clarifications, we have (using superscripts "*c*" for the closed model and "*o*" for the open model)

$$\mathbf{Z}^c = \begin{bmatrix} 500 & 350 & 90 \\ 320 & 360 & 50 \\ 100 & 60 & 40 \end{bmatrix},\ \mathbf{x}^c = \begin{bmatrix} 1000 \\ 800 \\ 300 \end{bmatrix},\ \mathbf{A}^c = \begin{bmatrix} .5 & .438 & .3 \\ .32 & .45 & .167 \\ .1 & .075 & .133 \end{bmatrix},\ \mathbf{L}^c = \begin{bmatrix} 5.820 & 5.036 & 2.983 \\ 3.686 & 5.057 & 2.248 \\ 0.990 & 1.019 & 1.693 \end{bmatrix}.$$

The final demand vector that remains after households are made endogenous is

$\mathbf{f}^c = \mathbf{x}^c - \mathbf{i}'\mathbf{Z}^c = \begin{bmatrix} 60 \\ 70 \\ 100 \end{bmatrix}$. These are final demands from the remaining exogenous sectors, including exports. The vector of final demand changes for this closed model is $\Delta\mathbf{f}^c = \begin{bmatrix} 50 \\ -20 \\ 0 \end{bmatrix}$, and so the new final demand vector, reflecting these changes in export demand for sectors 1 and 2, is

$\mathbf{f}^{c,new} = \begin{bmatrix} 110 \\ 50 \\ 100 \end{bmatrix}$, giving (rounded to whole numbers)

$$\mathbf{x}^{c,new} = \mathbf{L}^c\mathbf{f}^{c,new} = \begin{bmatrix} 5.820 & 5.036 & 2.983 \\ 3.686 & 5.057 & 2.248 \\ 0.990 & 1.019 & 1.693 \end{bmatrix}\begin{bmatrix} 110 \\ 50 \\ 100 \end{bmatrix} = \begin{bmatrix} 1190 \\ 883 \\ 329 \end{bmatrix}$$

In part b of Problem 2.1 we found $\mathbf{x}^{new} = \begin{bmatrix} 1139 \\ 844 \end{bmatrix}$. Despite the fact that export demand for sector 2 has decreased, there are increases in outputs for both sectors 1 and 2, reflecting increased production due to the inclusion of households in the model.

This result can also be obtained using the model in its "delta" form ("changes in"):

$$\Delta\mathbf{x}^c = \mathbf{L}^c(\Delta\mathbf{f}^c) = \begin{bmatrix} 5.820 & 5.036 & 2.983 \\ 3.686 & 5.057 & 2.248 \\ 0.990 & 1.019 & 1.693 \end{bmatrix}\begin{bmatrix} 50 \\ -20 \\ 0 \end{bmatrix} = \begin{bmatrix} 190.28 \\ 83.16 \\ 29.12 \end{bmatrix}$$

and (rounded to whole numbers)

$$\mathbf{x}^{c,new} = \mathbf{x}^c + \Delta\mathbf{x} = \begin{bmatrix} 1000 \\ 800 \\ 300 \end{bmatrix} + \begin{bmatrix} 190 \\ 83 \\ 29 \end{bmatrix} = \begin{bmatrix} 1190 \\ 883 \\ 329 \end{bmatrix}$$

PROBLEM 2.4

(a) $\mathbf{Z} = \begin{bmatrix} 2.5 & 10.0 & 2.5 \\ 2.5 & 5.0 & 2.5 \\ 30.0 & 30.0 & 15.0 \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} 35 \\ 40 \\ 25 \end{bmatrix}$ so $\mathbf{x} = \mathbf{Zi} + \mathbf{f} = \begin{bmatrix} 50 \\ 50 \\ 100 \end{bmatrix}$ and $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .05 & .2 & .025 \\ .05 & .1 & .025 \\ .6 & .6 & .15 \end{bmatrix}$. The Hawkins-Simon conditions require positivity of all principal minors of

$$(\mathbf{I}-\mathbf{A}) = \begin{bmatrix} .95 & -.2 & -.025 \\ -.05 & .9 & -.025 \\ -.6 & -.6 & .85 \end{bmatrix}$$. Here the three first-order principal minors are 0.95, 0.9 and 0.85 (main diagonal elements); the three second-order principal minors are 0.845, 0.75 and 0.793 and the third-order principal minor is just the determinant, $|\mathbf{I}-\mathbf{A}| = 0.687$. (b)

$$\mathbf{L} = (\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} 1.092 & 0.269 & 0.040 \\ 0.084 & 1.154 & 0.036 \\ 0.830 & 1.005 & 1.230 \end{bmatrix}$$. (c) $\mathbf{x}^{new} = \begin{bmatrix} 39.317 \\ 44.606 \\ 87.181 \end{bmatrix}$ for $\mathbf{f}^{new} = \begin{bmatrix} (.75)f_1 \\ (.90)f_2 \\ (.95)f_3 \end{bmatrix} = \begin{bmatrix} 26.25 \\ 36.00 \\ 23.75 \end{bmatrix}$. (d)

$$\mathbf{Z}^{new} = \mathbf{A}(\hat{\mathbf{x}}^{new}) = \begin{bmatrix} 1.97 & 8.92 & 2.18 \\ 1.97 & 4.46 & 2.18 \\ 23.59 & 26.76 & 13.08 \end{bmatrix}$$, so $(\mathbf{v}^{new})' = (\mathbf{x}^{new})' - \mathbf{i}'(\mathbf{Z}^{new}) = \begin{bmatrix} 11.795 & 4.461 & 69.744 \end{bmatrix}$

and $\mathbf{u}^{new} = (\mathbf{Z}^{new})\mathbf{i} = \begin{bmatrix} 13.067 \\ 8.606 \\ 63.431 \end{bmatrix}$.

PROBLEM 2.5

(a) $\mathbf{Z} = \begin{bmatrix} 2 & 8 \\ 6 & 4 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$ so $\mathbf{x} = \mathbf{f} + \mathbf{Z}\mathbf{i} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}$. (b) $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .067 & .267 \\ .2 & .133 \end{bmatrix}$ so $(1-a_{11}) = 0.993$, $(1-a_{22}) = 0.867$ and $|\mathbf{I}-\mathbf{A}| = 0.756$. (c) $\mathbf{L} = (\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} 1.147 & .353 \\ .265 & 1.235 \end{bmatrix}$ and $\mathbf{x}^{new} = \mathbf{L}\mathbf{f}^{new} = \begin{bmatrix} 23.559 \\ 26.206 \end{bmatrix}$ for $\mathbf{f}^{new} = \begin{bmatrix} 15 \\ 18 \end{bmatrix}$ and $\mathbf{Z}^{new} = \mathbf{A}\hat{\mathbf{x}}^{new} = \begin{bmatrix} 1.571 & 6.988 \\ 4.712 & 3.494 \end{bmatrix}$.

PROBLEM 2.6

(a) $\mathbf{v}' = \begin{bmatrix} 10 & 11 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 12 \\ 9 \end{bmatrix}$. $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .3 & .133 \\ .2 & .133 \end{bmatrix}$, $(1-a_{11}) = 0.7$, $(1-a_{22}) = 0.867$ and $|\mathbf{I}-\mathbf{A}| = 0.58$. (b) $\mathbf{x} = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$ and $\tilde{\mathbf{x}} = \sum_{i=0}^{r} \mathbf{A}^i\mathbf{f}$ so $x_j - \tilde{x}_j < 0.05$ for $j = 1,2$ at $r = 6$:

r	1	2	3	4	5	**6**	7	8	9	10
$\tilde{x}_1$	16.800	18.720	19.488	19.795	19.918	**19.967**	19.987	19.995	19.998	19.999
$\tilde{x}_2$	12.600	14.040	14.616	14.846	14.939	**14.975**	14.990	14.996	14.998	14.999

(c) $C_r = c_1 r + c_2(r-1.5)$ so $C_r = 0.5c_2 r + c_2 r - 1.5c_2 = (0.5r + r - 1.5)c_2 = (1.5r - 1.5)c_2$. Hence, for $x_j - \tilde{x}_j < 0.2$ then $r = 5$ so $C_r = 6c_2$; hence, for one **f**, $C_r = 6c_2 < C_e + C_f = 21c_2$, so use round-by-round. (d) For 5 **f** vectors. $C_r = 5(6c_2) = 30c_2 > C_e + 4C_f = 20c_2 + 4c_2 = 24c_2$, so use the exact method. (e) For 4 **f** vectors. $C_r = 4(6c_2) = 24c_2 = C_e + 4C_f = 20c_2 + 4c_2 = 24c_2$, the costs of both methods are the same.

PROBLEM 2.7

(a) Here $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .136 & .444 & .194 \\ .091 & .222 & .161 \\ .318 & .167 & .29 \end{bmatrix}$. The three first-order principal minors of $(\mathbf{I} - \mathbf{A})$ are 0.864, 0.778 and 0.710, the second-order principal minors are 0.631, 0.551 and 0.525, and $|\mathbf{I} - \mathbf{A}| = 0.35$. The power series $\tilde{\mathbf{x}} = \sum_{i=0}^{r} \mathbf{A}^i \mathbf{f}$ is shown below; so $x_j - \tilde{x}_j \le 0.1$ for $j = 1, 2$ and 3 at $r = 12$:

x	r	0	1	2	3	4	5	6	7	8	9	10	11	**12**	13
22	$\tilde{x}_1$	5	11.1	14.9	17.4	19.0	20.0	20.7	21.2	21.4	21.6	21.8	21.8	**21.9**	21.9
18	$\tilde{x}_2$	7	10.9	13.4	15.0	16.0	16.7	17.2	17.5	17.6	17.8	17.8	17.9	**17.9**	18.0
31	$\tilde{x}_3$	12	18.2	22.7	25.6	27.4	28.7	29.5	30.0	30.4	30.6	30.7	30.8	**30.9**	30.9

PROBLEM 2.8

(a) $$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .228 & .179 & .191 & .074 & .005 & .025 & .004 & .022 \\ .040 & .155 & .149 & .086 & .016 & .033 & .004 & .018 \\ .003 & .001 & .000 & .010 & .027 & .001 & .007 & .002 \\ .027 & .023 & .003 & .223 & .021 & .020 & .004 & .014 \\ .116 & .099 & .180 & .066 & .142 & .155 & .103 & .092 \\ .057 & .001 & .014 & .030 & .001 & .073 & .002 & .042 \\ .013 & 0 & .004 & 0 & .011 & 0 & .323 & .165 \\ .141 & .042 & .065 & .026 & .067 & .039 & .109 & .288 \end{bmatrix}$$ and

$$\mathbf{L} = \begin{bmatrix} 1.339 & .296 & .312 & .172 & .034 & .058 & .030 & .067 \\ .089 & 1.214 & .209 & .153 & .038 & .057 & .025 & .051 \\ .013 & .009 & 1.011 & .019 & .034 & .008 & .018 & .013 \\ .065 & .056 & .034 & 1.306 & .038 & .041 & .021 & .041 \\ .265 & .215 & .320 & .174 & 1.207 & .230 & .229 & .240 \\ .100 & .029 & .045 & .059 & .011 & 1.089 & .018 & .074 \\ .109 & .049 & .068 & .035 & .054 & .030 & 1.547 & .372 \\ .321 & .162 & .210 & .117 & .135 & .103 & .269 & 1.506 \end{bmatrix}.$$

(b) From $\mathbf{f}=\mathbf{x}-\mathbf{A}\mathbf{x}$, $\mathbf{f}=\begin{bmatrix} 3{,}994 \\ 19{,}269 \\ 39{,}348 \\ 22{,}625 \\ 137{,}571 \\ \text{-}653 \\ 8{,}327 \\ 82{,}996 \end{bmatrix}$ so $\mathbf{x}^{new}=\mathbf{L}\mathbf{f}^{new}=\begin{bmatrix} 39{,}998 \\ 51{,}181 \\ 45{,}455 \\ 40{,}404 \\ 177{,}756 \\ 11{,}182 \\ 54{,}929 \\ 158{,}687 \end{bmatrix}$ for $\mathbf{f}^{new}=\begin{bmatrix} 5{,}192 \\ 25{,}050 \\ 39{,}348 \\ 22{,}625 \\ 110{,}057 \\ -653 \\ 8{,}327 \\ 82{,}996 \end{bmatrix}$.

PROBLEM 2.9

$\mathbf{A}=\mathbf{Z}(\hat{\mathbf{x}})^{-1}=\begin{bmatrix} 10 & 40 \\ 30 & 25 \end{bmatrix}\begin{bmatrix} 1/100 & 0 \\ 0 & 1/140 \end{bmatrix}=\begin{bmatrix} .1 & .286 \\ .3 & .179 \end{bmatrix}$ so $\mathbf{A}'=\begin{bmatrix} .1 & .3 \\ .286 & .179 \end{bmatrix}$ and hence

$\mathbf{L}'=(\mathbf{I}-\mathbf{A}')^{-1}=\begin{bmatrix} 1.257 & .459 \\ .437 & 1.377 \end{bmatrix}$. $\mathbf{v}_c^0=\begin{bmatrix} 60/100 \\ 75/140 \end{bmatrix}=\begin{bmatrix} .6 \\ .536 \end{bmatrix}$ so the normalized prices are found by $\tilde{\mathbf{p}}^0=\mathbf{L}'\mathbf{v}_c^0=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The new value-added coefficients are $\mathbf{v}_c^1=\begin{bmatrix} 1.25 & 0 \\ 0 & .75 \end{bmatrix}\mathbf{v}_c^0=\begin{bmatrix} .75 \\ .402 \end{bmatrix}$, so the new normalized prices are found as $\tilde{\mathbf{p}}^1=\mathbf{L}'\mathbf{v}_c^1=\begin{bmatrix} 1.127 \\ .881 \end{bmatrix}$.

PROBLEM 2.10

$$\mathbf{L}'=(\mathbf{I}-\mathbf{A}')^{-1}=\begin{bmatrix} 1.262 & 0.009 & 0.008 & 0.229 & 0.149 & 0.238 & 0.024 \\ 0.006 & 1.075 & 0.003 & 0.119 & 0.085 & 0.293 & 0.024 \\ 0.013 & 0.012 & 1.005 & 0.262 & 0.137 & 0.270 & 0.023 \\ 0.057 & 0.034 & 0.006 & 1.342 & 0.156 & 0.292 & 0.037 \\ 0.004 & 0.019 & 0.007 & 0.069 & 1.089 & 0.271 & 0.028 \\ 0.007 & 0.003 & 0.011 & 0.086 & 0.060 & 1.412 & 0.030 \\ 0.007 & 0.007 & 0.025 & 0.126 & 0.085 & 0.314 & 1.034 \end{bmatrix}.$$

The value-added coefficients are found as $\mathbf{v}_c^0=\mathbf{i}-\mathbf{i}'\mathbf{A}=\begin{bmatrix} .486 & .633 & .580 & .470 & .699 & .629 & .640 \end{bmatrix}'$, so that $\tilde{\mathbf{p}}^0=\mathbf{L}'\mathbf{v}_c^0=\mathbf{i}$. We define the vector of value-added growth factors as

$\mathbf{d} = [1.1\ \ 1.1\ \ 1.15\ \ 1.15\ \ 1.2\ \ 1.2\ \ 1.2]'$ and we can find the new vector of value-added coefficients by $\mathbf{v}_c^1 = \hat{\mathbf{d}}\mathbf{v}_c^0 = \begin{bmatrix} 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.2 \end{bmatrix} \begin{bmatrix} .486 \\ .633 \\ .58 \\ .47 \\ .699 \\ .629 \\ .64 \end{bmatrix} = \begin{bmatrix} .534 \\ .696 \\ .667 \\ .540 \\ .839 \\ .754 \\ .768 \end{bmatrix}$. Hence the new normalized prices are found as $\tilde{\mathbf{p}}^1 = \mathbf{L}'\mathbf{v}_c^1 = \begin{bmatrix} 1.133 \\ 1.129 \\ 1.163 \\ 1.163 \\ 1.197 \\ 1.197 \\ 1.195 \end{bmatrix}$.

PROBLEM 2.11

$\mathbf{A} = \begin{bmatrix} .4 & 1 & .5 \\ .2 & 0 & 0 \\ .4 & 0 & .5 \end{bmatrix}$, so $|\mathbf{I} - \mathbf{A}| = 0$. Hence, $(\mathbf{I} - \mathbf{A})$ is singular and $\mathbf{L}$ does not exist. The domestic transactions matrix is found by "opening" the economy to imports. If this is a "U.S. style" input-output table, then since competitive imports are included in the transactions matrix a corresponding negative entry for imports is included in final demand so, in order to "scrub" the transactions table of imports we need to subtract the value of imports from the first and third entries in the personal computers column and add those amounts to the first and third entries of final demand.

Thus $\mathbf{Z} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ becomes $\mathbf{D} = \begin{bmatrix} 2 & 2 & 1-1 \\ 1 & 0 & 0 \\ 2 & 0 & 1-1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ and $\mathbf{f}$ is $\mathbf{g} = \begin{bmatrix} 0+1 \\ 1 \\ -1+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Here $\mathbf{D}$ is the matrix of domestic transactions where the values of competitive imports are subtracted to remove them from the transactions matrix and $\mathbf{g}$ is new vector of final demands where the values of competitive imports are added to remove imports from final demand (recall that they were included originally in final demand as negative values). The vector of total outputs, $\mathbf{x}$, is unchanged, but the new vector of total value added is $\overline{\mathbf{v}}' = \mathbf{x}' - \mathbf{iD} = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}$ and, hence, gross domestic product is $\overline{\mathbf{v}}'\mathbf{i} = \mathbf{i}'\mathbf{g} = 2$. We then can compute the matrix of direct requirements as $\bar{\mathbf{A}} = \begin{bmatrix} .4 & 1 & 0 \\ .2 & 0 & 0 \\ .4 & 0 & 0 \end{bmatrix}$, for which $(\mathbf{I} - \bar{\mathbf{A}})$ is non-singular and the matrix of total requirements is $\bar{\mathbf{L}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} = \begin{bmatrix} 2.5 & 2.5 & 0 \\ .5 & 1.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

Solutions to Chapter 3 Problems

PROBLEM 3.1

Here $\mathbf{p} = \begin{bmatrix} 0.60 \\ 0.90 \\ 0.75 \end{bmatrix}$; using the $\mathbf{A}$ matrix from problem 2.2, we find that $\mathbf{A}^R = \hat{\mathbf{p}}\mathbf{A} =$

$\begin{bmatrix} .210 & 0 & 0 \\ .045 & .450 & .135 \\ .150 & .225 & .413 \end{bmatrix}$. Therefore $(\mathbf{I} - \mathbf{A}^R)^{-1} = \begin{bmatrix} 1.266 & 0 & 0 \\ .202 & 2.007 & .461 \\ .401 & .759 & 1.879 \end{bmatrix}$ and, hence,

$\mathbf{x}^{new} = (\mathbf{I} - \mathbf{A}^R)^{-1}\mathbf{f}^{new} = \begin{bmatrix} 1645.570 \\ 555.346 \\ 973.257 \end{bmatrix}$ for $\mathbf{f}^{new} = \begin{bmatrix} 1300 \\ 100 \\ 200 \end{bmatrix}$.

PROBLEM 3.2

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \hline \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{array}\right] = \left[\begin{array}{cc|cc} 0.110 & 0.130 & 0.056 & 0.070 \\ 0.164 & 0.026 & 0.130 & 0.070 \\ \hline 0.137 & 0.156 & 0.093 & 0.125 \\ 0.192 & 0.182 & 0.093 & 0.078 \end{array}\right] \text{ so that}$$

$$\mathbf{L} = \left[\begin{array}{cc|cc} 1.205 & 0.202 & 0.115 & 0.123 \\ 0.263 & 1.116 & 0.189 & 0.131 \\ \hline 0.273 & 0.262 & 1.177 & 0.200 \\ 0.330 & 0.289 & 0.179 & 1.156 \end{array}\right] \text{ and } \Delta\mathbf{x} = \mathbf{L}\Delta\mathbf{f} = \begin{bmatrix} 409.981 \\ 475.665 \\ 170.619 \\ 196.240 \end{bmatrix} \text{ for } \Delta\mathbf{f} = \begin{bmatrix} 280 \\ 360 \\ 0 \\ 0 \end{bmatrix}.$$

PROBLEM 3.3

We begin with $\mathbf{Z} = \left[\begin{array}{cc|cc} 40 & 50 & 0 & 0 \\ 60 & 10 & 0 & 0 \\ \hline 0 & 0 & 30 & 45 \\ 0 & 0 & 70 & 45 \end{array}\right]$. If we configure the shipments of Goods 1 and 2 in a matrix defined as $\mathbf{Q}=\left[\begin{array}{cc|cc} z_1^{rr} & 0 & z_1^{rs} & 0 \\ 0 & z_2^{rr} & 0 & z_2^{rs} \\ \hline z_1^{sr} & 0 & z_1^{ss} & 0 \\ 0 & z_2^{sr} & 0 & z_2^{ss} \end{array}\right]=\left[\begin{array}{cc|cc} 50 & 0 & 60 & 0 \\ 0 & 50 & 0 & 80 \\ \hline 70 & 0 & 70 & 0 \\ 0 & 50 & 0 & 50 \end{array}\right]$ so the row sums of **Q**,

$\mathbf{x}=\mathbf{Qi}=\left[\begin{array}{c} 110 \\ 130 \\ \hline 140 \\ 100 \end{array}\right]$ is the vector of total deliveries of commodities of each type for each region and the column sums of **Q**, $\mathbf{T}=\mathbf{i}'\mathbf{Q}=\left[\begin{array}{cc|cc} 120 & 100 & 130 & 130 \end{array}\right]$, is the vector of total availability of each commodity in each region. Hence, $\mathbf{A}=\mathbf{Z}\hat{\mathbf{x}}^{-1}=\left[\begin{array}{c|c} \mathbf{A}^r & 0 \\ \hline 0 & \mathbf{A}^s \end{array}\right]=\left[\begin{array}{cc|cc} .364 & .385 & 0 & 0 \\ .545 & .077 & 0 & 0 \\ \hline 0 & 0 & .214 & .45 \\ 0 & 0 & .5 & .45 \end{array}\right]$ and

$\mathbf{C}=\mathbf{Q}\hat{\mathbf{T}}^{-1}=\left[\begin{array}{c|c} \hat{\mathbf{C}}^{rr} & \hat{\mathbf{C}}^{rs} \\ \hline \hat{\mathbf{C}}^{sr} & \hat{\mathbf{C}}^{ss} \end{array}\right]=\left[\begin{array}{cc|cc} .417 & 0 & .462 & 0 \\ 0 & .5 & 0 & .615 \\ \hline .583 & 0 & .538 & 0 \\ 0 & .5 & 0 & .385 \end{array}\right]$. Now we compute

$(\mathbf{I}-\mathbf{CA})^{-1}\mathbf{C}=\left[\begin{array}{cc|cc} 0.971 & 0.556 & 1.024 & 0.524 \\ 0.882 & 1.197 & 0.889 & 1.251 \\ \hline 1.297 & 0.714 & 1.264 & 0.677 \\ 0.663 & 1.010 & 0.673 & 0.854 \end{array}\right]$, so that $\mathbf{x}^{new}=(\mathbf{I}-\mathbf{CA})^{-1}\mathbf{C}\mathbf{f}^{new}=\left[\begin{array}{c} 148.778 \\ 214.539 \\ \hline 191.718 \\ 161.772 \end{array}\right]$ for

$\mathbf{f}^{new}=\left[\begin{array}{c} \mathbf{f}^r \\ \hline \mathbf{f}^s \end{array}\right]=\left[\begin{array}{c} 50 \\ 50 \\ \hline 40 \\ 60 \end{array}\right]$

PROBLEM 3.4

From the table we have $\mathbf{Z}^A=\begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix}$, $\mathbf{Z}^B=\begin{bmatrix} 700 & 400 \\ 100 & 200 \end{bmatrix}$ and $\mathbf{Z}^C=\begin{bmatrix} 100 & 0 \\ 50 & 0 \end{bmatrix}$.

(a) We can construct regional technical coefficients tables as $\mathbf{A}^r=\mathbf{Z}^r(\hat{\mathbf{x}}^r)^{-1}$ for regions $r = A, B$ and C as $\mathbf{A}^A=\begin{bmatrix} 0.333 & 0.333 \\ 0.167 & 0.333 \end{bmatrix}$, $\mathbf{A}^B=\begin{bmatrix} 0.583 & 0.571 \\ 0.083 & 0.286 \end{bmatrix}$, and $\mathbf{A}^C=\begin{bmatrix} 0.500 & 0 \\ 0.250 & 0 \end{bmatrix}$; (b) Yes, we can construct the national transactions table as the sum of the regional transactions matrices, $\mathbf{Z}^N=\mathbf{Z}^A+\mathbf{Z}^B+\mathbf{Z}^C=\begin{bmatrix} 1000 & 500 \\ 250 & 300 \end{bmatrix}$, and the vector of national total outputs as the sum of regional total output vectors, $\mathbf{x}^N = \mathbf{x}^A + \mathbf{x}^B + \mathbf{x}^C = \begin{bmatrix} 2,000 \\ 1,000 \end{bmatrix}$. Hence, the national technical coefficients matrix is found by $\mathbf{A}^N = \mathbf{Z}^N (\hat{\mathbf{x}}^N)^{-1} = \begin{bmatrix} .500 & .500 \\ .125 & .300 \end{bmatrix}$. (c) We do not have any origin-destination data on shipments of each good. (d) Using $\mathbf{A}^N$, we compute $(\mathbf{I} - \mathbf{A}^N)^{-1} = \begin{bmatrix} 2.435 & 1.739 \\ 0.435 & 1.739 \end{bmatrix}$. With $\mathbf{f}^{new} = \begin{bmatrix} 5,000 \\ 4,500 \end{bmatrix}$, we find that $\mathbf{x}^{new} = \begin{bmatrix} 20,000 \\ 10,000 \end{bmatrix}$. (e) The original national gross output vector was $\mathbf{x}^N = \begin{bmatrix} 2,000 \\ 1,000 \end{bmatrix}$; the corresponding national final demand vector is $\mathbf{f}^N = \begin{bmatrix} 500 \\ 450 \end{bmatrix}$, found as $\mathbf{f}^N = \mathbf{x}^N - \mathbf{Z}^N \mathbf{i}$ or as $\mathbf{f}^N = \mathbf{f}^A + \mathbf{f}^B + \mathbf{f}^C$ where $\mathbf{f}^r = \mathbf{x}^r - \mathbf{Z}^r \mathbf{i}$ for regions $r =$ *A*, *B* and *C*. This illustrates the linearity of the input-output model, since $\mathbf{x}^{new} = 10\mathbf{x}^N$ follows directly from $\mathbf{f}^{new} = 10\mathbf{f}^N$. (See also the solution to problem 2.2.)

PROBLEM 3.5

(a) The total regional final demand vector is found as $\mathbf{f} = \mathbf{x} - \mathbf{Z}\mathbf{i} = \begin{bmatrix} \mathbf{f}^N \\ \hline \mathbf{f}^S \end{bmatrix} = \begin{bmatrix} 1,453,353 \\ 111,595 \\ 2,186,205 \\ \hline 1,663,741 \\ 76,675 \\ 3,612,485 \end{bmatrix}$. Hence the regional technical coefficients are $\mathbf{A}^{NN} = \mathbf{Z}^{NN} (\hat{\mathbf{x}}^N)^{-1} = \begin{bmatrix} 0.076 & 0.005 & 0.157 \\ 0.000 & 0.003 & 0.055 \\ 0.094 & 0.053 & 0.619 \end{bmatrix}$ and $\mathbf{A}^{SS} = \mathbf{Z}^{SS} (\hat{\mathbf{x}}^S)^{-1} = \begin{bmatrix} 0.069 & 0.005 & 0.116 \\ 0.000 & 0.005 & 0.046 \\ 0.086 & 0.048 & 0.580 \end{bmatrix}$; the interregional trade coefficients are found as $\mathbf{A}^{SN} = \mathbf{Z}^{SN} (\hat{\mathbf{x}}^N)^{-1} = \begin{bmatrix} 0.002 & 0.000 & 0.009 \\ 0.000 & 0.000 & 0.001 \\ 0.020 & 0.011 & 0.185 \end{bmatrix}$ and $\mathbf{A}^{NS} = \mathbf{Z}^{NS} (\hat{\mathbf{x}}^S)^{-1} = \begin{bmatrix} 0.002 & 0.000 & 0.012 \\ 0.000 & 0.000 & 0.002 \\ 0.012 & 0.005 & 0.107 \end{bmatrix}$.

(b) For this two-region interregional model, $\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}^{NN} & \mathbf{A}^{NS} \\ \hline \mathbf{A}^{SN} & \mathbf{A}^{SS} \end{array}\right]$. The new constrained total outputs vector can be computed as $\mathbf{x}^{new} = \hat{\mathbf{r}}\mathbf{x} = \begin{bmatrix} 3{,}633{,}382 \\ 743{,}965 \\ 10{,}384{,}473 \\ \hline 3{,}697{,}202 \\ 766{,}751 \\ 13{,}004{,}947 \end{bmatrix}$ where the vector $\mathbf{r}$ is defined as $\mathbf{r} = [1 \quad 1 \quad .95 \mid 1 \quad 1 \quad .9]$, reflecting the specified reduced total outputs for construction and manufacturing in the two regions. The corresponding new vector of final demands is found as $\mathbf{f}^{new} = \mathbf{x}^{new} - \mathbf{A}\mathbf{x}^{new} = \begin{bmatrix} 1{,}556{,}842 \\ 144{,}617 \\ 2{,}132{,}819 \\ \hline 1{,}835{,}571 \\ 144{,}444 \\ 3{,}107{,}061 \end{bmatrix}$. (c) The modified final demand vector can be computed as $\mathbf{f}^{new} = \hat{\mathbf{r}}\mathbf{f} = \begin{bmatrix} 1{,}453{,}353 \\ 111{,}595 \\ 1{,}858{,}274 \\ \hline 1{,}663{,}741 \\ 76{,}675 \\ 3{,}612{,}485 \end{bmatrix}$ where $\mathbf{r} = [1 \quad 1 \quad .85 \mid 1 \quad 1 \quad 1]$ which reflects the specified reduction in final demand for construction and from the North region. The corresponding impact on total outputs is found as $\mathbf{x}^{new} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}^{new}$

$$= \left[\begin{array}{ccc|ccc} 1.145 & 0.038 & 0.567 & 0.028 & 0.012 & 0.188 \\ 0.020 & 1.014 & 0.180 & 0.007 & 0.004 & 0.054 \\ 0.348 & 0.183 & 3.218 & 0.124 & 0.058 & 0.875 \\ \hline 0.033 & 0.016 & 0.219 & 1.111 & 0.024 & 0.365 \\ 0.011 & 0.006 & 0.075 & 0.014 & 1.012 & 0.135 \\ 0.215 & 0.112 & 1.500 & 0.284 & 0.147 & 2.868 \end{array}\right] \begin{bmatrix} 1{,}453{,}353 \\ 111{,}595 \\ 1{,}858{,}274 \\ \hline 1{,}663{,}741 \\ 76{,}675 \\ 3{,}612{,}485 \end{bmatrix} = \begin{bmatrix} 3{,}447{,}445 \\ 684{,}913 \\ 9{,}875{,}755 \\ \hline 3{,}625{,}440 \\ 742{,}265 \\ 13{,}957{,}983 \end{bmatrix}$$

and for the North region, in particular, $\mathbf{x}^{N,new} = \begin{bmatrix} 3{,}447{,}445 \\ 684{,}913 \\ 9{,}875{,}755 \end{bmatrix}$. (d) Using $\mathbf{A}^{NN}$ from part (a), we find $(\mathbf{I} - \mathbf{A}^{NN})^{-1} = \begin{bmatrix} 1.131 & 0.031 & 0.468 \\ 0.016 & 1.011 & 0.152 \\ 0.282 & 0.149 & 2.760 \end{bmatrix}$. Hence, in conjunction with $\mathbf{f}^{N,new} = \begin{bmatrix} 1{,}453{,}353 \\ 111{,}595 \\ 1{,}858{,}274 \end{bmatrix}$ we find $\mathbf{x}^{N,new} = (\mathbf{I} - \mathbf{A}^{NN})^{-1}\mathbf{f}^{N,new} = \begin{bmatrix} 2{,}517{,}159 \\ 417{,}336 \\ 5{,}554{,}462 \end{bmatrix}$. Compared with the results in part (c)**,** we can conclude that interregional linkages are very important in this economy since the outputs found using region *N* alone are 27, 39 and 44 percent below their corresponding values for each industry respectively using the full two-region interregional model.

PROBLEM 3.6

If all inputs to the North from the South came instead from the Rest of China, the transactions table would be:

China 2000		North			South			Rest of China		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North	Natural Resources	1,724	6,312	406	188	1,206	86	14	49	4
	Manuf. & Const.	2,381	18,458	2,987	301	3,331	460	39	234	57
	Services	709	3,883	1,811	64	432	138	5	23	5
South	Natural Resources	0	0	0	3,564	8,828	806	103	178	15
	Manuf. & Const.	0	0	0	3,757	34,931	5,186	202	1,140	268
	Services	0	0	0	1,099	6,613	2,969	31	163	62
ROC	Natural Resources	158	707	46	33	254	18	1,581	3,154	293
	Manuf. & Const.	494	4,106	613	123	1,062	170	1,225	6,704	1,733
	Services	53	321	105	25	168	47	425	2,145	1,000
Total Output		16,651	49,563	15,011	27,866	81,253	23,667	11,661	21,107	8,910

The revised direct requirements table becomes:

China 2000		North			South			Rest of China		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North	Natural Resources	0.1035	0.1273	0.0270	0.0067	0.0148	0.0036	0.0012	0.0023	0.0005
	Manuf. & Const.	0.1430	0.3724	0.1990	0.0108	0.0410	0.0194	0.0034	0.0111	0.0064
	Services	0.0426	0.0783	0.1206	0.0023	0.0053	0.0058	0.0004	0.0011	0.0006
South	Natural Resources	0.0000	0.0000	0.0000	0.1279	0.1087	0.0340	0.0089	0.0084	0.0017
	Manuf. & Const.	0.0000	0.0000	0.0000	0.1348	0.4299	0.2191	0.0173	0.0540	0.0301
	Services	0.0000	0.0000	0.0000	0.0394	0.0814	0.1255	0.0026	0.0077	0.0070
ROC	Natural Resources	0.0095	0.0143	0.0030	0.0012	0.0031	0.0008	0.1356	0.1494	0.0329
	Manuf. & Const.	0.0297	0.0828	0.0408	0.0044	0.0131	0.0072	0.1050	0.3176	0.1945
	Services	0.0032	0.0065	0.0070	0.0009	0.0021	0.0020	0.0364	0.1016	0.1122

The revised total requirements table becomes:

China 2003		North			South			Rest of China		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North	Natural Resources	1.1603	0.2494	0.0929	0.0225	0.0575	0.0264	0.0064	0.0159	0.0084
	Manuf. & Const.	0.2938	1.7104	0.3988	0.0530	0.1579	0.0840	0.0189	0.0523	0.0311
	Services	0.0826	0.1651	1.1775	0.0114	0.0303	0.0200	0.0034	0.0092	0.0054
South	Natural Resources	0.0032	0.0077	0.0041	1.1897	0.2441	0.1081	0.0237	0.0438	0.0220
	Manuf. & Const.	0.0137	0.0332	0.0180	0.3165	1.8924	0.4892	0.0710	0.1923	0.1136
	Services	0.0026	0.0063	0.0034	0.0834	0.1879	1.1943	0.0137	0.0362	0.0244
ROC	Natural Resources	0.0365	0.0775	0.0367	0.0087	0.0236	0.0120	1.1966	0.2816	0.1075
	Manuf. & Const.	0.1028	0.2545	0.1374	0.0258	0.0714	0.0399	0.2096	1.5757	0.3577
	Services	0.0202	0.0471	0.0298	0.0060	0.0158	0.0098	0.0735	0.1930	1.1724

For the final demand $(\Delta\mathbf{f}^N)' = \begin{bmatrix} 0 & 100 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix}$, the corresponding vector of total outputs is $(\Delta\mathbf{x})' = (\mathbf{L}\Delta\mathbf{f}^N)' = \begin{bmatrix} 24.9 & 171.0 & 16.5 & 0.8 & 3.3 & 0.6 & 7.7 & 25.4 & 4.7 \end{bmatrix}$. If we recast $\Delta\mathbf{x}$ in the format of Table 3.10 we have the following table which shows the changes in production by region and sector generated by the shift in the location of inputs to production from the South to the Rest of China:

	Produced in the North (original)			**Produced in the North (revised)**		
Sector	North	South	ROC	North	South	ROC
Nat. Res.	25.6	6.8	0.8	24.9	0.8	7.7
Mfg. &Const.	172.8	29.4	2.5	171	3.3	25.4
Services	16.9	4.5	0.5	16.5	0.6	4.7
Total	215.3	40.7	3.8	212.4	4.7	37.8

PROBLEM 3.7

With $\left[\Delta\mathbf{f}\right]' = \left[(\Delta\mathbf{f}^E)' \;\vdots\; (\Delta\mathbf{f}^C)' \;\vdots\; (\Delta\mathbf{f}^W)'\right] = \left[0\ 0\ 0\ 0\ 0 \;\vdots\; 0\ 0\ 0\ 0\ 0 \;\vdots\; 0\ 0\ 100\ 50\ 25\right]$, we find

$\left[\Delta\mathbf{x}\right]' = \left[0.75\ 1.125\ 23.3\ 13.2\ 8.225 \;\vdots\; 3.525\ 5.375\ 38.175\ 20.475\ 13.4 \;\vdots\; 4.825\ 4.65\ 96.45\ 68.125\ 25.6\right]$.

PROBLEM 3.8

With the same final demand vector used in problem 3.6, but used for the 3-region, 5-sector Japanese IRIO economy where the regions are Central, North, and South, we have $\left[\Delta\mathbf{f}\right]' = \left[(\Delta\mathbf{f}^C)' \;\vdots\; (\Delta\mathbf{f}^N)' \;\vdots\; (\Delta\mathbf{f}^S)'\right] = \left[0\ 0\ 0\ 0\ 0 \;\vdots\; 0\ 0\ 0\ 0\ 0 \;\vdots\; 0\ 0\ 100\ 50\ 25\right]$ for which the corresponding vector of total outputs is given by

$$\left[\Delta\mathbf{x}\right]' = \left[0.386\ 0.024\ 13.061\ 0.892\ 3.024 \;\vdots\; 0.145\ 0.021\ 3.669\ 1.376\ 0.339 \;\vdots\; 3.634\ 0.475\ 181.630\ 56.029\ 42.904\right]$$

PROBLEM 3.9

The table of interregional transactions (**Z**) and vector of total outputs (**x**) are :

2000		United States			Japan			China			Rest of Asia		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
U.S.	Nat. Res.	75,382	296,016	17,829	351	4,764	473	174	403	17	103	2,740	83
	Manuf. & Const.	68,424	1,667,042	960,671	160	21,902	3,775	587	8,863	1,710	383	45,066	4,391
	Services	95,115	1,148,999	3,094,357	118	6,695	807	160	1,466	296	197	7,393	953
Japan	Nat. Res.	7	52	53	8,721	78,936	11,206	13	66	2	14	180	27
	Manuf. & Const.	859	41,484	11,337	28,088	1,414,078	484,802	764	20,145	2,809	462	72,258	4,108
	Services	97	4,390	1,424	24,901	662,488	1,001,832	107	2,763	335	270	7,816	1,189
China	Nat. Res.	72	343	147	50	2,316	229	49,496	183,509	15,138	102	2,430	99
	Manuf. & Const.	331	15,657	6,442	93	10,199	1,989	89,384	892,227	181,932	157	15,093	1,237
	Services	38	2,218	1,099	17	1,780	280	25,391	210,469	136,961	23	2,078	132
ROA	Nat. Res.	322	1,068	203	64	11,906	266	64	1,475	14	12,153	92,647	6,402
	Manuf. & Const.	503	56,287	18,129	278	35,418	3,562	1,141	41,496	4,685	23,022	566,274	144,417
	Services	152	4,578	1,921	41	3,982	447	138	3,669	422	15,163	213,470	239,053
TOTAL OUTPUT		468,403	5,866,935	11,609,307	140,622	3,883,455	4,658,191	408,153	2,000,741	702,248	173,080	1,727,367	1,225,460

The corresponding table of direct requirements, $\mathbf{A} = \mathbf{Z}\mathbf{x}^{-1}$ is:

2000		United States			Japan			China			Rest of Asia		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
U.S.	Nat. Res.	0.1609	0.0505	0.0015	0.0025	0.0012	0.0001	0.0004	0.0002	0.0000	0.0006	0.0016	0.0001
	Manuf. & Const.	0.1461	0.2841	0.0828	0.0011	0.0056	0.0008	0.0014	0.0044	0.0024	0.0022	0.0261	0.0036
	Services	0.2031	0.1958	0.2665	0.0008	0.0017	0.0002	0.0004	0.0007	0.0004	0.0011	0.0043	0.0008
Japan	Nat. Res.	0.0000	0.0000	0.0000	0.0620	0.0203	0.0024	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000
	Manuf. & Const.	0.0018	0.0071	0.0010	0.1997	0.3641	0.1041	0.0019	0.0101	0.0040	0.0027	0.0418	0.0034
	Services	0.0002	0.0007	0.0001	0.1771	0.1706	0.2151	0.0003	0.0014	0.0005	0.0016	0.0045	0.0010
China	Nat. Res.	0.0002	0.0001	0.0000	0.0004	0.0006	0.0000	0.1213	0.0917	0.0216	0.0006	0.0014	0.0001
	Manuf. & Const.	0.0007	0.0027	0.0006	0.0007	0.0026	0.0004	0.2190	0.4459	0.2591	0.0009	0.0087	0.0010
	Services	0.0001	0.0004	0.0001	0.0001	0.0005	0.0001	0.0622	0.1052	0.1950	0.0001	0.0012	0.0001
ROA	Nat. Res.	0.0007	0.0002	0.0000	0.0005	0.0031	0.0001	0.0002	0.0007	0.0000	0.0702	0.0536	0.0052
	Manuf. & Const.	0.0011	0.0096	0.0016	0.0020	0.0091	0.0008	0.0028	0.0207	0.0067	0.1330	0.3278	0.1178
	Services	0.0003	0.0008	0.0002	0.0003	0.0010	0.0001	0.0003	0.0018	0.0006	0.0876	0.1236	0.1951

The table of total requirements, $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is :

2000		United States			Japan			China			Rest of Asia		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
U.S.	Nat. Res.	1.2103	0.0889	0.0126	0.0043	0.0036	0.0008	0.0013	0.0019	0.0010	0.0022	0.0071	0.0016
	Manuf. & Const.	0.2953	1.4643	0.1660	0.0071	0.0164	0.0039	0.0082	0.0183	0.0113	0.0147	0.0641	0.0163
	Services	0.4140	0.4158	1.4113	0.0054	0.0096	0.0021	0.0040	0.0083	0.0051	0.0079	0.0289	0.0076
Japan	Nat. Res.	0.0002	0.0005	0.0001	1.0755	0.0366	0.0082	0.0004	0.0010	0.0006	0.0007	0.0027	0.0006
	Manuf. & Const.	0.0085	0.0201	0.0048	0.3924	1.6463	0.2196	0.0161	0.0420	0.0233	0.0235	0.1117	0.0238
	Services	0.0026	0.0061	0.0015	0.3280	0.3663	1.3236	0.0052	0.0136	0.0074	0.0090	0.0345	0.0083
China	Nat. Res.	0.0007	0.0012	0.0004	0.0013	0.0024	0.0005	1.1997	0.2184	0.1025	0.0020	0.0061	0.0013
	Manuf. & Const.	0.0043	0.0096	0.0028	0.0046	0.0108	0.0027	0.5518	2.0243	0.6666	0.0077	0.0317	0.0075
	Services	0.0009	0.0021	0.0007	0.0011	0.0026	0.0006	0.1649	0.2816	1.3374	0.0018	0.0071	0.0016
ROA	Nat. Res.	0.0015	0.0018	0.0005	0.0024	0.0070	0.0011	0.0022	0.0060	0.0028	1.0906	0.0914	0.0205
	Manuf. & Const.	0.0081	0.0239	0.0062	0.0101	0.0261	0.0051	0.0255	0.0711	0.0368	0.2440	1.5530	0.2293
	Services	0.0023	0.0055	0.0015	0.0028	0.0070	0.0014	0.0061	0.0166	0.0086	0.1562	0.2487	1.2798

The original vector of final demands can be computed by $\mathbf{f} = \mathbf{x} - \mathbf{Z}\mathbf{i}$, so

$$\mathbf{f}' = \left[70{,}067 \;\; 3{,}083{,}962 \;\; 7{,}252{,}751 \;\middle|\; 41{,}344 \;\; 1{,}802{,}261 \;\; 2{,}950{,}579 \;\middle|\; 154{,}222 \;\; 786{,}002 \;\; 321{,}762 \;\middle|\; 46{,}495 \;\; 832{,}154 \;\; 742{,}423 \right]$$

For growth in China at 8%, growth in the U.S. and Japan at 4%, and growth in the rest of Asia at 3%, the final demand in the next year is found by multiplying the first three elements of $\mathbf{f}$ (U.S. final demand) by 1.04, the next three (Japanese final demand) by 1.04, the next three (Chinese final demand) by 1.08, and the last three (final demand for the other nations in Asia) by 1.03, to yield

$$(\mathbf{f}^{new})' = \left[72{,}869 \;\; 3{,}207{,}321 \;\; 7{,}542{,}861 \;\middle|\; 42{,}998 \;\; 1{,}874{,}352 \;\; 3{,}068{,}602 \;\middle|\; 166{,}560 \;\; 848{,}883 \;\; 347{,}503 \;\middle|\; 47{,}890 \;\; 857{,}119 \;\; 764{,}696 \right]$$

The corresponding vector of total outputs is then found as $\mathbf{x}^{new} = \mathbf{L}\mathbf{f}^{new}$:

$$(\mathbf{x}^{new})' = \left[487{,}149 \;\; 6{,}101{,}723 \;\; 12{,}073{,}729 \;\middle|\; 146{,}262 \;\; 4{,}039{,}397 \;\; 4{,}844{,}723 \;\middle|\; 440{,}002 \;\; 2{,}156{,}077 \;\; 757{,}348 \;\middle|\; 178{,}822 \;\; 1{,}784{,}590 \;\; 1{,}263{,}502 \right]$$

Finally, the vector of the percentage growth in total output for each and all regions and sectors is then found as

$$100 \times \frac{(\mathbf{x}^{new} - \mathbf{x})}{\mathbf{x}} = \left[4.002 \;\; 4.002 \;\; 4.000 \;\middle|\; 4.011 \;\; 4.016 \;\; 4.004 \;\middle|\; 7.803 \;\; 7.764 \;\; 7.846 \;\middle|\; 3.317 \;\; 3.313 \;\; 3.104 \right].$$

PROBLEM 3.10

(a) Partition $(\mathbf{I}-\mathbf{A})$ as $(\mathbf{I}-\mathbf{A}) = \left[\begin{array}{cc|ccc} 1.0 & -0.1 & -0.3 & -0.2 & -0.2 \\ -0.1 & 0.9 & -0.1 & 0.0 & 0.0 \\ \hline -0.2 & 0.0 & 0.9 & -0.3 & -0.1 \\ -0.3 & 0.0 & 0.0 & 0.9 & -0.3 \\ -0.3 & -0.2 & -0.1 & -0.1 & 0.8 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{E} & \mathbf{F} \\ \hline \mathbf{G} & \mathbf{H} \end{array}\right]$ and further partition $\mathbf{H}$ as $\mathbf{H} = \left[\begin{array}{cc|c} 0.9 & -0.3 & -0.1 \\ 0 & 0.9 & -0.3 \\ \hline -0.1 & -0.1 & 0.8 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H}_1 & \mathbf{H}_2 \\ \hline \mathbf{H}_3 & \mathbf{H}_4 \end{array}\right]$. We define $(\mathbf{I}-\mathbf{A})^{-1} = \left[\begin{array}{c|c} \mathbf{S} & \mathbf{T} \\ \hline \mathbf{U} & \mathbf{V} \end{array}\right]$ where matrices in similar positions in $(\mathbf{I}-\mathbf{A})$ and $(\mathbf{I}-\mathbf{A})^{-1}$ have the same dimensions. From the results on the inverse of a partitioned inverse (Appendix A), we find that we need $\mathbf{E}^{-1}$ and $\mathbf{H}^{-1}$, the inverses of a 2×2 and a 3×3 matrix. Therefore, to find $\mathbf{H}^{-1}$ we again use the results on the inverse of a partitioned matrix, where $\mathbf{H}$ is partitioned as above. This requires that $\mathbf{H}_1^{-1}$ and $\mathbf{H}_4^{-1}$ be found; since these are 2×2 and 1×1 matrices, respectively, this is easily done. Hence, we have $\mathbf{H}^{-1} = \left[\begin{array}{cc|c} 1.144 & 0.415 & 0.299 \\ 0.050 & 1.177 & 0.448 \\ \hline 0.149 & 0.199 & 1.343 \end{array}\right]$. This in conjunction with $\mathbf{E}^{-1}$, $\mathbf{F}$ and $\mathbf{G}$ allows us to find $\mathbf{S}$, $\mathbf{T}$, $\mathbf{U}$ and $\mathbf{V}$ which comprise $(\mathbf{I}-\mathbf{A})^{-1}$: $\mathbf{S} = \begin{bmatrix} 1.566 & 0.332 \\ 0.253 & 1.172 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 0.638 & 0.640 & 0.711 \\ 0.231 & 0.150 & 0.148 \end{bmatrix}$,

$\mathbf{U} = \begin{bmatrix} 0.708 & 0.217 \\ 0.802 & 0.270 \\ 0.839 & 0.478 \end{bmatrix}$ and $\mathbf{V} = \begin{bmatrix} 1.441 & 0.707 & 0.622 \\ 0.388 & 1.509 & 0.815 \\ 0.525 & 0.554 & 1.733 \end{bmatrix}$. Therefore

$(\mathbf{I}-\mathbf{A})^{-1} = \left[\begin{array}{cc|ccc} 1.566 & 0.332 & 0.638 & 0.640 & 0.711 \\ 0.253 & 1.172 & 0.231 & 0.150 & 0.148 \\ \hline 0.708 & 0.217 & 1.441 & 0.707 & 0.622 \\ 0.802 & 0.270 & 0.388 & 1.509 & 0.815 \\ 0.839 & 0.478 & 0.525 & 0.554 & 1.733 \end{array}\right]$. (b) In this problem we used the method of partitioning repeatedly (sometimes called recursive application of the method) on subpartitions of the original four partitions of $(\mathbf{I}-\mathbf{A})$. We can in theory invert an infinitely large matrix by recursively partitioning into smaller and smaller submatrices.

Solutions to Chapter 4 Problems

PROBLEM 4.1

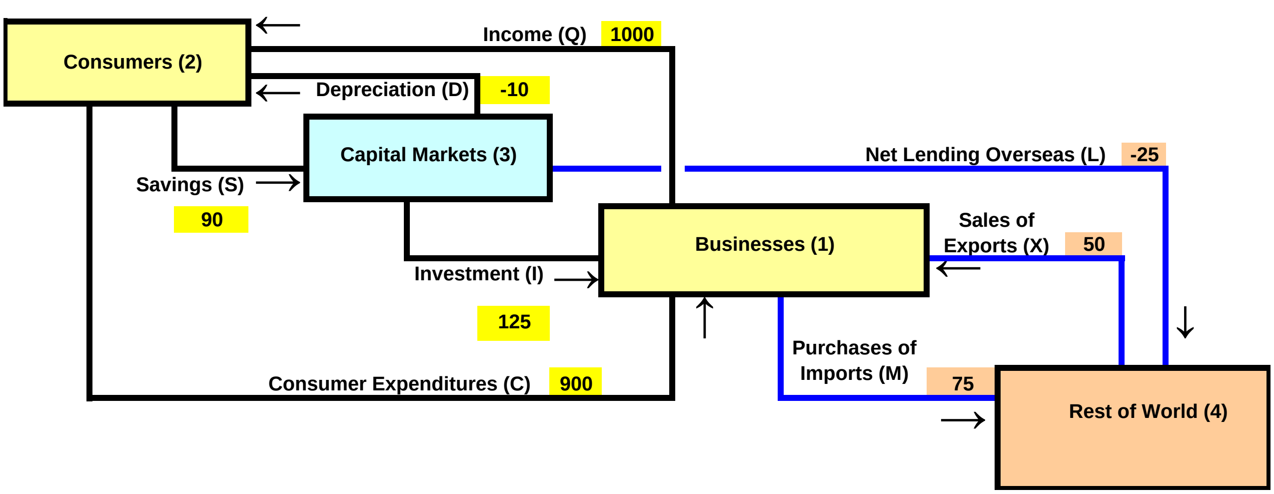
Production (Domestic Product Account)			
Debits		**Credits**	
Income (*Q*)	1000	Sales of consumption goods (*C*)	900
		Sales of capital goods (*I*)	100
Total	1000	Total	1000

Consumption (Income and Outlay Account)			
Debits		**Credits**	
Purchases of consumption goods (*C*)	900	Income (*Q*)	1000
Savings (*S*)	90	Depreciation (*D*)	-10
Total	990	Total	990

Accumulation (Capital Transactions Account)			
Debits		**Credits**	
Purchase of capital goods (*I*)	100	Savings (*S*)	90
Depreciation (*D*)	-10		
Total	90	Total	90

$$Q = 1,000 = C + I = 900 + 100 = 1,000$$
$$C + S = 900 + 90 = Q + D = 1000 - 10 = 990$$
$$I + D = 100 - 10 = S = 90$$

PROBLEM 4.2



Production (Domestic Product Account)			
Debits		**Credits**	
Income (Q)	1000	Sales of consumption goods (C)	900
Imports (W)	75	Sales of capital goods (I)	125
		Exports (X)	50
Total	1075	Total	1075

Consumption (Income and Outlay Account)			
Debits		**Credits**	
Purchases of consumption goods (C)	900	Income (Q)	1000
Savings (S)	90	Depreciation (D)	-10
Total	990	Total	990

Accumulation (Capital Transactions Account)			
Debits		**Credits**	
Purchase of capital goods (I)	125	Savings (S)	90
Depreciation (D)	-10		
Net Lending Overseas (L)	-25		
Total	90	Total	90

Rest of World (Balance of Payments Account)			
Debits		**Credits**	
Sales of exports (X)	50	Purchases of Imports (W)	75
		Net Overseas Lending (L)	-25
Total	50	Total	50

Production: $Q+M=1,000+75=C+I+X=900+125+50=1,075$
Consumption: $C+S=900+90=Q+D=1000-10=990$
Capital Accumulation: $I+D+L=100-10-25=S=90$
Rest of World: $X=50=L+M=-25+75=50$

PROBLEM 4.3

(a) The corresponding balance equations are:

Domestic Product Account: $Q+M=C+I+X+G$
Income and Outlay Account: $C+S+T=Q+D$
Capital Transactions Account: $I+D+B=S$
Balance of Payments Account: $X=M$
Government Account: $G=T+B$

(b)

	Prod.	Cons.	Cap.	ROW	Govt.	Total
Production		475	54	46	25	600
Consumption	554		-29			525
Capital Accum.		30				30
Rest of World	46					46
Govt.		20	5			25
Total	600	525	30	46	25	

PROBLEM 4.4

The vector of price indices is $\mathbf{p} = \begin{bmatrix} 2/5 & 3/6 & 5/9 & 7/12 \end{bmatrix} = \begin{bmatrix} 0.400 & 0.500 & 0.556 & 0.583 \end{bmatrix}$. This vector is comprised of the ratios of the year 2000 prices to the year 2005 prices. Hence, $\mathbf{Z}^{2000}$, $\mathbf{A}^{2000}$ and $\mathbf{x}^{2000}$ can be computed as

$$\mathbf{Z}^{2000} = \hat{\mathbf{p}}\mathbf{Z}^{2005} = \begin{bmatrix} .4 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .556 & 0 \\ 0 & 0 & 0 & .583 \end{bmatrix} \begin{bmatrix} 24 & 86 & 56 & 64 \\ 32 & 15 & 78 & 78 \\ 104 & 49 & 62 & 94 \\ 14 & 16 & 63 & 78 \end{bmatrix} = \begin{bmatrix} 9.6 & 34.4 & 22.4 & 25.6 \\ 16 & 7.5 & 39 & 39 \\ 57.78 & 27.22 & 34.44 & 52.22 \\ 8.17 & 9.33 & 36.75 & 45.5 \end{bmatrix}$$

and $\mathbf{A}^{2000} = \mathbf{Z}^{2000}(\hat{\mathbf{x}}^{2000})^{-1} = \hat{\mathbf{p}}\mathbf{Z}^{2005}(\hat{\mathbf{p}}\hat{\mathbf{x}}^{2005})^{-1} = \hat{\mathbf{p}}\mathbf{Z}^{2005}(\hat{\mathbf{x}}^{2005})^{-1}\hat{\mathbf{p}}^{-1} = \hat{\mathbf{p}}\mathbf{A}^{2005}\hat{\mathbf{p}}^{-1} =$

$$\begin{bmatrix} .0603 & .2191 & .0860 & .0967 \\ .1005 & .0478 & .1497 & .1473 \\ .3629 & .1734 & .1322 & .1972 \\ .0513 & .0594 & .1410 & .1718 \end{bmatrix} \text{ and } \mathbf{x}^{2000} = \hat{\mathbf{p}}\mathbf{x}^{2005} = \begin{bmatrix} 159.1 \\ 157 \\ 260.56 \\ 264.83 \end{bmatrix}.$$

PROBLEM 4.5

The impact of the sum of total outputs is indicated in the following table at each level of aggregation:

Aggregation Level	x´i	Agregated Sector Total Output							
		1	2	3	4	5	6	7	8
8	16.26	2.31	1.84	1.12	1.60	2.88	1.43	2.26	2.82
7	16.56	2.48	3.33	1.13	1.61	2.87	2.28	2.87	
6	15.54	4.85	3.14	1.15	1.58	2.20	2.62		
5	15.62	4.78	3.11	3.86	1.58	2.29			
4	15.72	4.73	5.51	3.91	1.57				
3	15.53	6.15	5.44	3.94					

PROBLEM 4.6

First, compute $\mathbf{Z} = \mathbf{A}\hat{\mathbf{x}} = \begin{bmatrix} 26370 & 9 & 465 & 41257 & 377 & 2768 & 193 \\ 160 & 1647 & 1511 & 22531 & 6038 & 104 & 322 \\ 579 & 857 & 50 & 3273 & 5887 & 13734 & 2676 \\ 12056 & 2865 & 58464 & 287046 & 15360 & 46582 & 1257 \\ 5172 & 1462 & 17314 & 59830 & 36984 & 23082 & 3256 \\ 7262 & 4470 & 11387 & 44987 & 43664 & 84651 & 1693 \\ 193 & 191 & 697 & 8906 & 4453 & 5013 & 532 \end{bmatrix}$. The aggregation matrix is $\mathbf{S} = \begin{bmatrix} 1\ 1\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 1\ 1\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 1\ 1\ 1 \end{bmatrix}$ so $\mathbf{Z}^* = \mathbf{SZS}' = \begin{bmatrix} 28186 & 65763 & 9804 \\ 16358 & 348834 & 85496 \\ 18750 & 143121 & 203328 \end{bmatrix}$,

$\mathbf{x}^* = \mathbf{Sx} = \begin{bmatrix} 114341 \\ 927192 \\ 1060811 \end{bmatrix}$ and $\mathbf{A}^* = \mathbf{Z}^*(\mathbf{x}^*)^{-1} = \begin{bmatrix} .247 & .071 & .009 \\ .143 & .376 & .081 \\ .164 & .154 & .192 \end{bmatrix}$. Hence,

$\mathbf{L}^* = (\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{bmatrix} 1.365 & 0.163 & 0.032 \\ 0.358 & 1.686 & 0.172 \\ 0.345 & 0.355 & 1.276 \end{bmatrix}$; $\tilde{\mathbf{x}}^* = \begin{bmatrix} 315.2 \\ 460.5 \\ 523.0 \end{bmatrix}$ and $\tilde{\mathbf{x}} = (\mathbf{I} - \mathbf{A})^{-1}\Delta\mathbf{f} = \begin{bmatrix} 174.5 \\ 120.9 \\ 116.5 \\ 329.7 \\ 184.6 \\ 218.3 \\ 110.1 \end{bmatrix}$. The first-order bias is found by $\boldsymbol{\varphi} = (\mathbf{A}^*\mathbf{S} - \mathbf{SA})\Delta\mathbf{f} = \begin{bmatrix} 17.065 \\ 7.721 \\ 5.790 \end{bmatrix}$ and $\mathbf{i}'\boldsymbol{\varphi} = 30.576$, and the total aggregation bias is found by $\boldsymbol{\tau} = \tilde{\mathbf{x}}^* - \mathbf{S}\tilde{\mathbf{x}} = \begin{bmatrix} 19.739 \\ 14.317 \\ 10.020 \end{bmatrix}$ and $\mathbf{i}'\boldsymbol{\tau} = 44.076$.

PROBLEM 4.7

Compute the missing quantities: $C = Q + D - S - T$, $I = -D + S - L - B$, $X = L + M$, and $G = T + B$ summarized in the following table:

	Prod.	Cons.	Cap.	ROW	Govt.	Total
Prod.		**C=410**	**I=80**	**X=55**	**G=30**	575
Cons.	Q=500		D=-10			490
Cap.		S=60				60
ROW	M=75		L=-20			55
Govt.		T=20	B=10			30
Total	575	490	60	55	30	

PROBLEM 4.8

		Commodities Agric.	Mining	Manuf.	Serv.	Industries Nat. Res.	Manuf.	Serv.	Final Demand	Total Output
Comm.	Agriculture					20	12	18	57	107
	Mining					5	30	12	102	149
	Manufacturing					10	13	11	115	149
	Services					12	17	40	101	170
Ind.	Natural Resources	99			10					109
	Manufacturing	8	143	137	10					298
	Services		6	12	150					168
	Value Added					62	226	87	375	
	Total Output	107	149	149	170	109	298	168		575

PROBLEM 4.9

(a) First we compute $\mathbf{f} = \mathbf{x} - \mathbf{Zi} = \begin{bmatrix} 500 \\ 250 \\ 100 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 500 \\ 500 \\ 900 \end{bmatrix}$. The import similarity scaling factors are

$r_i = \dfrac{m_i}{u_i + f_i}$, or $\mathbf{r} = \begin{bmatrix} .15 \\ .14 \\ .21 \end{bmatrix}$. We can then compute $\bar{\mathbf{D}} = \mathbf{Z} - \hat{\mathbf{r}}\mathbf{Z} = \begin{bmatrix} 425 & 0 & 0 \\ 43 & 258 & 129 \\ 158 & 118.5 & 434.5 \end{bmatrix}$,

$\bar{\mathbf{M}} = \hat{\mathbf{r}}\mathbf{Z} = \begin{bmatrix} 75 & 0 & 0 \\ 7 & 42 & 21 \\ 42 & 31.5 & 115.5 \end{bmatrix}$, $\mathbf{h} = \hat{\mathbf{r}}\mathbf{f} = \begin{bmatrix} 75 \\ 35 \\ 21 \end{bmatrix}$ and $\bar{\mathbf{g}} = \mathbf{g} - \hat{\mathbf{r}}\mathbf{f} = \begin{bmatrix} 575 \\ 320 \\ 289 \end{bmatrix}$. Note that $\mathbf{x} = \bar{\mathbf{D}}\mathbf{i} + \bar{\mathbf{g}}$ still holds, but this balance equation now accounts for only domestic transactions with interindustry imports reassigned to total value added. The new total value added vector is $\bar{\mathbf{v}}' = \mathbf{x}' - \mathbf{i}'\bar{\mathbf{D}} = \begin{bmatrix} 374 & 373.5 & 436.5 \end{bmatrix}$, which inflates the original vector of total valued added, $\mathbf{v}' = \begin{bmatrix} 250 & 300 & 300 \end{bmatrix}$ by interindustry imports to each industry, i.e., $\bar{\mathbf{m}}' = \begin{bmatrix} 124 & 73.5 & 136.5 \end{bmatrix}$, excluding the value of imports consumed directly in final demand.

(b) Since $\mathbf{M} = \begin{bmatrix} 150 & 0 & 0 \\ 25 & 50 & 30 \\ 35 & 75 & 100 \end{bmatrix}$, we can compute $\mathbf{D} = \mathbf{Z} - \mathbf{M} = \begin{bmatrix} 350 & 0 & 0 \\ 25 & 250 & 120 \\ 165 & 75 & 450 \end{bmatrix}$ and $\mathbf{m} = \mathbf{Mi} = \begin{bmatrix} 150 \\ 105 \\ 210 \end{bmatrix}$, and $\mathbf{g} = \mathbf{f} + \mathbf{m} = \begin{bmatrix} 650 \\ 355 \\ 310 \end{bmatrix}$ where $\mathbf{f} = \mathbf{x} - \mathbf{Zi} = \begin{bmatrix} 500 \\ 250 \\ 100 \end{bmatrix}$. Note that the balance equation $\mathbf{x} = \mathbf{Di} + \mathbf{g}$ holds. Then the new total value added vector, $\tilde{\mathbf{v}}' = \mathbf{x}' - \mathbf{i}'\mathbf{D} = \begin{bmatrix} 460 & 425 & 430 \end{bmatrix}$, inflates the original vector of total valued added, $\mathbf{v}' = \mathbf{x}' - \mathbf{i}'\mathbf{Z} = \begin{bmatrix} 250 & 300 & 300 \end{bmatrix}$, by the total value of all imports to each industry, $\tilde{\mathbf{m}}' = \begin{bmatrix} 210 & 125 & 130 \end{bmatrix}$, i.e. $\tilde{\mathbf{m}}' = \tilde{\mathbf{v}}' - \mathbf{v}'$. (c) We compute the Leontief inverse for (a) as

$$\mathbf{L}^{I} = (\mathbf{I} - \mathbf{A}^{I}) = \begin{bmatrix} 1.739 & 0 & 0 \\ .222 & 1.613 & .368 \\ .548 & .451 & 1.871 \end{bmatrix} \text{ for } \mathbf{A}^{I} = \bar{\mathbf{D}}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .425 & 0 & 0 \\ .043 & .344 & .129 \\ .158 & .158 & .435 \end{bmatrix}$$

and

$$\mathbf{L}^{II} = (\mathbf{I} - \mathbf{A}^{II}) = \begin{bmatrix} 1.538 & 0 & 0 \\ .146 & 1.551 & .338 \\ .488 & .282 & 1.88 \end{bmatrix} \text{ for } \mathbf{A}^{II} = \mathbf{D}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .35 & 0 & 0 \\ .025 & .333 & .12 \\ .165 & .1 & .45 \end{bmatrix}$$

. The mean absolute deviation between $\mathbf{L}^{I}$ and $\mathbf{L}^{II}$ is found to be $\text{MAD} = (1/9)\sum_{i=1}^{3}\sum_{j=1}^{3}\left|l_{ij}^{I} - l_{ij}^{II}\right| = .067$.

PROBLEM 4.10

First, to aggregate the North and South regions and leave the Rest of China region unaggregated, we construct the aggregation matrix

$$\mathbf{S} = \left[\begin{array}{ccc|ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

and compute $\mathbf{Z}^{(a)} = \mathbf{SZS}'$ and $\mathbf{x}^{(a)} = \mathbf{Sx}$, which are shown in the following table:

China 2003		North and South Nat Res	North and South Manuf & Const	North and South Services	Rest of China Nat Res	Rest of China Manuf & Const	Rest of China Services
North & South	Natural Resources	5,625	17,001	1,339	117	227	19
	Manuf. & Const.	6,902	60,554	9,203	241	1,374	325
	Services	1,920	11,225	5,017	35	186	68
ROC	Natural Resources	43	305	22	1,581	3,154	293
	Manuf. & Const.	155	1,334	212	1,225	6,704	1,733
	Services	29	193	53	425	2,145	1,000
Total Output		44,517	130,816	38,678	11,661	21,107	8,910

The corresponding technical coefficients matrix and Leontief inverse are

$$\mathbf{A}^{(a)} = \left[\begin{array}{ccc|ccc} .126 & .13 & .035 & .01 & .011 & .002 \\ .155 & .463 & .238 & .021 & .065 & .037 \\ .043 & .086 & .13 & .003 & .009 & .008 \\ \hline .001 & .002 & .001 & .136 & .149 & .033 \\ .003 & .01 & .005 & .105 & .318 & .194 \\ .001 & .001 & .001 & .036 & .102 & .112 \end{array}\right]$$ and

$$\mathbf{L}^{(a)} = \left[\begin{array}{ccc|ccc} 1.207 & .315 & .135 & .031 & .062 & .032 \\ .395 & 2.055 & .580 & .093 & .252 & .149 \\ .099 & .219 & 1.213 & .018 & .046 & .03 \\ \hline .005 & .013 & .006 & 1.196 & .279 & .106 \\ .015 & .039 & .022 & .207 & 1.567 & .353 \\ .004 & .009 & .006 & .073 & .191 & 1.171 \end{array}\right]$$. For the reference final demands

$\tilde{\mathbf{f}} = [100 \quad 100 \quad 100 \mid 100 \quad 100 \quad 100 \mid 100 \quad 100 \quad 100]'$ we can write

$\tilde{\mathbf{f}}^{(a)} = \mathbf{S}\tilde{\mathbf{f}} = [200 \quad 200 \quad 200 \mid 100 \quad 100 \quad 100]'$. We can now compute

$\tilde{\mathbf{x}} = \mathbf{L}\tilde{\mathbf{f}} = [165 \quad 284 \quad 151 \mid 178 \quad 371 \quad 164 \mid 163 \quad 227 \quad 147]'$ and

$\tilde{\mathbf{x}}^{(a)} = \mathbf{L}^{(a)}\tilde{\mathbf{f}}^{(a)} = [344 \quad 655 \quad 316 \mid 163 \quad 228 \quad 147]'$. Hence, we can compute the aggregation bias as $100 \times \dfrac{\left|\mathbf{S}\tilde{\mathbf{x}} - \tilde{\mathbf{x}}^{(a)}\right|\mathbf{i}}{\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}} = .114$ percent.

Solutions to Chapter 5 Problems

PROBLEM 5.1

(a) $\mathbf{x} = \mathbf{V}\mathbf{i} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}$, $\mathbf{q} = (\mathbf{i}'\mathbf{V})' = \begin{bmatrix} 20 \\ 30 \\ 10 \end{bmatrix}$, $\mathbf{v}' = \mathbf{x}' - \mathbf{i}'\mathbf{U} = \begin{bmatrix} 23 & 15 \end{bmatrix}$, $\mathbf{e} = \mathbf{x} - \mathbf{U}\mathbf{i} = \begin{bmatrix} 12 \\ 21 \\ 5 \end{bmatrix}$,

and $\mathbf{B} = \mathbf{U}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .1 & .167 \\ .067 & .233 \\ .067 & .1 \end{bmatrix}$. (b) $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1} = \begin{bmatrix} .75 & 0.167 & 1 \\ .25 & 0.833 & 0 \end{bmatrix}$ so we can compute

$(\mathbf{I} - \mathbf{DB})^{-1}\mathbf{D} = \begin{bmatrix} 1.021 & .555 & 1.22 \\ .435 & 1.149 & 0.129 \end{bmatrix}$.

PROBLEM 5.2

(a) $\mathbf{B} = \mathbf{U}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .083 & .25 \\ .25 & .5 \end{bmatrix}$. (b) $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1} = \begin{bmatrix} 1.0 & .2 \\ 0 & .8 \end{bmatrix}$, $\mathbf{C} = \mathbf{V}'\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .833 & 0 \\ .167 & 1 \end{bmatrix}$. Therefore,

$(\mathbf{I} - \mathbf{DB})^{-1}\mathbf{D} = \begin{bmatrix} 1.333 & 0.889 \\ .444 & 1.63 \end{bmatrix}$ and $(\mathbf{I} - \mathbf{C}^{-1}\mathbf{B})^{-1}\mathbf{C}^{-1} = \begin{bmatrix} 1.412 & .706 \\ .235 & 2.118 \end{bmatrix}$. (c) Under an industry-based technology assumption, $\Delta\mathbf{x} = \begin{bmatrix} 12.44 \\ 10.81 \end{bmatrix}$. Under a commodity-based technology assumption, $\Delta\mathbf{x} = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$. These are different because the accounting of secondary production is different in the two assumptions.

PROBLEM 5.3

Since $\mathbf{V}$ is nonsquare, $\mathbf{C}$ will be nonsquare and hence no unique $\mathbf{C}^{-1}$ exists. We therefore cannot use the mixed-technology assumption requiring computation of $\mathbf{C}^{-1}$; that is, we cannot find $(\mathbf{I} - \mathbf{C}^{-1}\mathbf{B})^{-1}\mathbf{C}^{-1}$ or $\mathbf{R}(\mathbf{I} - \mathbf{BR})^{-1}$ where $\mathbf{R} = [\mathbf{C}_1^{-1}(\mathbf{I} - \langle\mathbf{D}_2'\rangle) + \mathbf{D}_2]$. We can use the industry-based technology assumption with $\mathbf{T} = \begin{bmatrix} .333 & .333 & 1.333 \\ .667 & .667 & -.333 \end{bmatrix}$ and $\mathbf{T}(\mathbf{I} - \mathbf{BT})^{-1} = \begin{bmatrix} .685 & .685 & 1.476 \\ .949 & .949 & -.264 \end{bmatrix}$.

PROBLEM 5.4

For $\mathbf{V}_1 = \begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix}$ and $\mathbf{V}_2 = \begin{bmatrix} 0 & 2 \\ 0 & 6 \end{bmatrix}$, $\mathbf{x}_1 = \mathbf{V}_1\mathbf{i} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$, $\mathbf{q} = (\mathbf{i}'\mathbf{V})' = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$,

$\mathbf{C}_1 = \mathbf{V}_1'(\hat{\mathbf{x}}_1)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{D}_2 = \mathbf{V}_2(\hat{\mathbf{q}})^{-1} = \begin{bmatrix} 0 & .2 \\ 0 & .6 \end{bmatrix}$, $\mathbf{C}_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$\mathbf{R} = \mathbf{C}_1^{-1}(\mathbf{I} - \langle \mathbf{D}_2'\mathbf{i} \rangle) + \mathbf{D}_2 = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}$, $\mathbf{R}(\mathbf{I} - \mathbf{BR})^{-1} = \begin{bmatrix} 1.333 & .889 \\ .444 & 1.630 \end{bmatrix}$, $\mathbf{q}_1 = \mathbf{i}'\mathbf{V}_1 = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$,

$\mathbf{D}_1 = \mathbf{V}_1(\hat{\mathbf{q}}_1)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{C}_2 = \mathbf{V}_2'(\hat{\mathbf{x}})^{-1} = \begin{bmatrix} 0 & 0 \\ 0.167 & 0.750 \end{bmatrix}$,

$\mathbf{T} = (\mathbf{I} + \mathbf{D}_1\mathbf{C}_2 - \langle \mathbf{C}_2'\mathbf{i} \rangle)^{-1}\mathbf{D}_1 = \begin{bmatrix} 1.2 & 0 \\ -0.2 & 1 \end{bmatrix}$ and $\mathbf{T}(\mathbf{I} - \mathbf{BT})^{-1} = \begin{bmatrix} 1.412 & .706 \\ .235 & 2.118 \end{bmatrix}$. There are alternative partitions of the **V** matrix into its $\mathbf{V}_1$ and $\mathbf{V}_2$ components. The requirement, of course, is that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}$.

PROBLEM 5.5

(a) From table 5.5, the commodity by industry total requirements matrix with the assumption of industry-based technology is $\mathbf{D}^{-1}(\mathbf{I} - \mathbf{BD})^{-1}$. Since there are more commodities than industries, **D** is non-square and $\mathbf{D}^{-1}$ does not exist so it is impossible to compute $\mathbf{D}^{-1}(\mathbf{I} - \mathbf{BD})^{-1}$. (b) For industry-by-commodity total requirements using the assumption of industry based technology, we have $\mathbf{D}(\mathbf{I} - \mathbf{BD})^{-1} = \begin{bmatrix} 1.164 & .077 & .082 & .25 \\ .321 & 1.159 & 1.122 & .321 \\ .182 & .148 & .197 & .1.187 \end{bmatrix}$. (c) $\mathbf{V}_1 = \mathbf{V} - \mathbf{V}_2 = \begin{bmatrix} 99 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 30 \end{bmatrix}$,

$\mathbf{q} = \mathbf{V}'\mathbf{i} \begin{bmatrix} 256 \\ 149 \\ 170 \end{bmatrix}$, $\mathbf{x} = \mathbf{Vi} = \begin{bmatrix} 109 \\ 298 \\ 168 \end{bmatrix}$ and $\mathbf{x}_1 = \mathbf{V}_1\mathbf{i} = \begin{bmatrix} 99 \\ 10 \\ 30 \end{bmatrix}$. Since $\mathbf{V}_1$ is diagonal, $\mathbf{x}_1 = \mathbf{V}_1\mathbf{i} = \mathbf{q}_1 = \mathbf{V}_1'\mathbf{i}$

and, $\mathbf{C}_1 = \mathbf{V}_1'\hat{\mathbf{x}}_1^{-1} = \mathbf{C}_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Next $\mathbf{D}_2 = \mathbf{V}_2(\hat{\mathbf{q}})^{-1} = \begin{bmatrix} 0 & 0 & .059 \\ .59 & .852 & .059 \\ .023 & .081 & .706 \end{bmatrix}$ so that

$\mathbf{R} = \mathbf{C}_1^{-1}\left[\mathbf{I} - \left\langle \mathbf{D}_2'\mathbf{i} \right\rangle\right] + \mathbf{D}_2 = \begin{bmatrix} .387 & 0 & .059 \\ .59 & .925 & .059 \\ .023 & .081 & .882 \end{bmatrix}$ and $\mathbf{R}^{-1} = \begin{bmatrix} 2.573 & .015 & -.173 \\ -1.656 & 1.084 & .038 \\ .083 & -.099 & 1.134 \end{bmatrix}$ (note the negative elements in $\mathbf{R}^{-1}$). We can now compute the family of total requirements matrices as

$(\mathbf{I} - \mathbf{BR})^{-1} = \begin{bmatrix} 1.260 & .213 & .308 \\ .093 & 1.070 & .111 \\ .141 & .121 & 1.324 \end{bmatrix}$, $(\mathbf{I} - \mathbf{BR})^{-1}\mathbf{R}^{-1} = \begin{bmatrix} 2.916 & .22 & .14 \\ -1.524 & 1.15 & .15 \\ .272 & .001 & 1.483 \end{bmatrix}$,

$\mathbf{R}(\mathbf{I} - \mathbf{BR})^{-1} = \begin{bmatrix} .496 & .09 & .197 \\ .837 & 1.117 & .362 \\ .161 & .198 & 1.185 \end{bmatrix}$ and $(\mathbf{I} - \mathbf{RB})^{-1} = \begin{bmatrix} 1.144 & .085 & .141 \\ .334 & 1.187 & .309 \\ .186 & .099 & 1.324 \end{bmatrix}$.

PROBLEM 5.6

We start with $\mathbf{C} = \mathbf{V}'\hat{\mathbf{x}}^{-1}$. The column sums of C are found by premultiplying by $\mathbf{i}'$, so $\mathbf{i}'\mathbf{C} = \mathbf{i}'\mathbf{V}'\hat{\mathbf{x}}^{-1}$. Since $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$, we rewrite this as $\mathbf{i}'\mathbf{C} = (\mathbf{Vi})'\hat{\mathbf{x}}^{-1}$ and substituting $\mathbf{x} = \mathbf{Vi}$ yields $\mathbf{i}'\mathbf{C} = \mathbf{x}'\hat{\mathbf{x}}^{-1} = \mathbf{i}'$. Similarly, if we start with $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$, the column sums of **D** are found with $\mathbf{i}'\mathbf{D} = \mathbf{i}'\mathbf{V}\hat{\mathbf{q}}^{-1}$, using the same property of the transpose of a product of matrices yields $\mathbf{i}'\mathbf{D} = (\mathbf{V}'\mathbf{i})'\hat{\mathbf{q}}^{-1}$ and substituting $\mathbf{q} = \mathbf{V}'\mathbf{i}$ yields $\mathbf{i}'\mathbf{D} = \mathbf{q}'\hat{\mathbf{q}}^{-1} = \mathbf{i}'$.

PROBLEM 5.7

For the problem specified we need the total industry-by-commodity requirements using an industry-based technology: $\mathbf{D}(\mathbf{I}-\mathbf{BD})^{-1} = \begin{bmatrix} 1.164 & .078 & .082 & .25 \\ .321 & 1.160 & 1.122 & .321 \\ .182 & .148 & .197 & 1.187 \end{bmatrix}$. For the final demand of 100 for manufactured products, we have $\Delta\mathbf{x} = \mathbf{D}(\mathbf{I}-\mathbf{BD})^{-1}\Delta\mathbf{f} = \begin{bmatrix} 25.02 \\ 32.08 \\ 118.65 \end{bmatrix}$ for

$\Delta\mathbf{f} = \begin{bmatrix} 0 & 0 & 0 & 100 \end{bmatrix}'$.

PROBLEM 5.8

We can derive $\mathbf{D} = \mathbf{V}(\hat{\mathbf{q}})^{-1}$ where $\mathbf{q} = \mathbf{i}'\mathbf{V}$. The standard calculation for producing the commodity-by-commodity transactions matrix is $\mathbf{Z} = \mathbf{U}(\mathbf{D}')^{-1}$, so for this case,

$\mathbf{D} = \begin{bmatrix} .75 & 0 & 0 \\ .25 & .667 & .189 \\ 0 & .333 & .811 \end{bmatrix}$ so $\mathbf{Z}_C = \mathbf{U}(\mathbf{D}')^{-1} = \begin{bmatrix} 26.667 & 7.019 & 19.315 \\ 6.667 & 43.359 & -3.025 \\ 13.333 & 17.151 & 6.516 \end{bmatrix}$. Applying the Almon purifying algorithm (Appendix 5.2) to yield a non-negative transactions matrix results in

$$\tilde{\mathbf{Z}}_C = \mathbf{U}(\mathbf{D}')^{-1} = \begin{bmatrix} 26.667 & 7.019 & 19.314 \\ 6.667 & 40.333 & 0 \\ 13.333 & 17.151 & 6.516 \end{bmatrix}.$$

PROBLEM 5.9

If the net exports are reduced by a rise in imports of \$1trillion, then final demand is reduced by the same amount and, all other values remaining constant, GDP is also reduced by the same amount. The corresponding changes in total outputs are found as

$$\Delta\mathbf{x} = \mathbf{D}(\mathbf{I}-\mathbf{BD})^{-1}\Delta\mathbf{f} = -\begin{bmatrix} 75840 & 91880 & 7742 & 1550525 & 184877 & 355257 & 48461 \end{bmatrix}'.$$

Solutions to Chapter 6 Problems

PROBLEM 6.1

In each case, we want the column sums of the appropriate Leontief inverse:

Problem	Output Multipliers							
2.1	6.444	6.944						
2.2	2.970	4.167	3.611					
2.3	6.444	6.944						
2.4	2.006	2.428	1.307					
2.5	1.412	1.588						
2.6	1.839	1.437						
2.7	2.573	3.373	2.877					
2.8	2.301	2.031	2.209	2.035	1.551	1.616	2.156	2.364
2.9	1.716	1.814						
2.10	1.919	1.605	1.722	1.925	1.487	1.608	1.599	
2.11	4.000	5.000	1.000					

PROBLEM 6.2

Using problem 2.2 as an example, the row vector of output multipliers is $\mathbf{m}(o)$ = [2.970 4.167 3.611]. In conjunction with the final-demand vector used in that problem, namely $\mathbf{f}^{t+1} = \begin{bmatrix} 1300 \\ 100 \\ 200 \end{bmatrix}$,

we find $\mathbf{m}(o)\mathbf{f}^{t+1}$ = 5000. In the solution to problem 2.2, we had $\mathbf{x}^{t+1} = \begin{bmatrix} 2000 \\ 1000 \\ 2000 \end{bmatrix}$, and the sum of these elements is 5000; that is, $\mathbf{i}'\mathbf{x}^{t+1}$ = 5000.

PROBLEM 6.3

Output multipliers for the three-sector model, closed with respect to households, are

$\mathbf{m}(o)$ = [10.496 11.112 6.924]. The type I income multipliers require that we have the labor-input coefficients, which are a_{31} = 0.100 and a_{32} = 0.075, along with the Leontief inverse of the model that is open with respect to households. This was $(\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} 4.074 & 3.241 \\ 2.370 & 3.704 \end{bmatrix}$. Then $m(h)_1 =$ (0.1)(4.074) +(0.075)(2.370) = 0.5852 and $m(h)_2 =$ (0.1)(3.241) + (0.075)(3.704) = 0.6019; $m(h)_1^I =$ 0.5852/0.1 = 5.852 and $m(h)_2^I =$ 0.6019/0.075 = 8.025. The total household income multipliers can be found as the first two elements in the bottom row of the Leontief inverse of the model closed with respect to households. From problem 2.3, these are $\overline{m}(h)_1 =$ 0.990 and $\overline{m}(h)_2 =$ 1.019, so the type II income multipliers are therefore

$m(h)_1^{II} = 0.990/0.1 = 9.90$ and $m(h)_2^{II} = 1.019/0.075 = 13.59$. For both sectors, the ratio of the type II to the type I income multiplier is 1.69.

PROBLEM 6.4

The larger effect is in sector 1. The difference, for $100 in exogenous demand, is $29.50.

PROBLEM 6.5

(a) Concentrate on stimulating export demand for the product of sector 1; it has the larger output multiplier. (b) Knowing $a_{31} = 0.1$ and $a_{32} = 0.18$, we can find $m(h)_1 = 0.4451$ and $m(h)_2 = 0.3534$. Thus, converting output effects to income earned per dollar of new final demand for each of the sectors does not change the ranking; stimulation of export demand for the output of sector 1 is still more beneficial.

PROBLEM 6.6

(a) $\mathbf{m}(o)^{rr} = [1.468 \;\; 1.318]$ and $\mathbf{m}(o)^{ss} = [1.356 \;\; 1.356]$. (b) $\mathbf{m}(o)^{r} = [2.071 \;\; 1.869]$ and $\mathbf{m}(o)^{s} = [1.660 \;\; 1.610]$. (c) $\mathbf{m}(o)^{\bullet r} = [1.478 \;\; 0.593 \;\; 0.464 \;\; 1.405]$ and $\mathbf{m}(o)^{\bullet s} = [1.292 \;\; 0.368 \;\; 0.323 \;\; 1.287]$.

PROBLEM 6.7

(a) Sector 1 in region *r*; output multipliers for sectors 1 and 2 in region *s* are equal. (b) Sector 1 in region *r*. (c) In region *r*. (d) Still region *r*.

PROBLEM 6.8

(a) $\mathbf{m}(o)^{rr} = [1.853 \;\; 1.753]$ and $\mathbf{m}(o)^{ss} = [1.937 \;\; 1.531]$. (b) $\mathbf{m}(o)^{r} = [3.813 \;\; 3.477]$ and $\mathbf{m}(o)^{s} = [3.850 \;\; 3.306]$. (c) $\mathbf{m}(o)^{\bullet r} = [2.268 \;\; 1.545 \;\; 1.270 \;\; 2.207]$ and $\mathbf{m}(o)^{\bullet s} = [2.288 \;\; 1.562 \;\; 1.201 \;\; 2.105]$. (d) Sector 1 in region *r*; sector 1 in region *s*. (e) Sector 1 in region *s*. (f) In region *r*. (f) Still in region *r*.

PROBLEM 6.9

We now want the Leontief inverse matrices for each of the three regions. These are found to be:

$$(\mathbf{I}-\mathbf{A}^A)^{-1} = \begin{bmatrix} 1.714 & 0.857 \\ 0.429 & 1.714 \end{bmatrix}, \; (\mathbf{I}-\mathbf{A}^B)^{-1} = \begin{bmatrix} 1.667 & 2.286 \\ 0.333 & 2.857 \end{bmatrix}, \; (\mathbf{I}-\mathbf{A}^C)^{-1} = \begin{bmatrix} 1.0 & 0 \\ 0.5 & 2.0 \end{bmatrix}$$

The largest column sum is associated with sector 2 in region *B*; that is, with walnuts.

PROBLEM 6.10

$|(\mathbf{I}-\bar{\mathbf{A}})| = .5879$ and $|(\mathbf{I}-\mathbf{A})| = .7575$ so $|(\mathbf{I}-\bar{\mathbf{A}})| / |(\mathbf{I}-\mathbf{A})| = .7761$ and $\mathbf{g} = (1-h) - \mathbf{h}_R \mathbf{L} \mathbf{h}_C = .7761$ and $(1/g) = 1.29$.

Solutions to Chapter 7 Problems

PROBLEM 7.1

The assumption of industry-based technology for an industry-by-industry technical coefficients matrix means that, generally, $\mathbf{Z} = \mathbf{DB}\hat{\mathbf{x}}$ where $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$ and $\mathbf{B} = \mathbf{U}\hat{\mathbf{x}}^{-1}$. The matrices **U** and **V** as well as the vectors **q** and **x** for years 1997, 2003 and 2005 are provided in Appendix B, so that

Z(1997)	1	2	3	4	5	6	7
1	74,807	27	1,170	150,337	2,897	13,738	12
2	501	19,330	4,722	115,074	53,759	6,331	35
3	997	26	739	6,665	10,450	44,999	24
4	49,998	19,663	179,517	1,362,631	175,369	428,076	1,932
5	21,358	11,509	74,280	372,728	199,152	247,197	663
6	33,123	44,541	108,369	489,159	540,798	1,562,040	3,725
7	106	825	540	34,282	20,330	28,677	4
Z(2003)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	61,199	9	1,286	145,882	321	18,714	2,153
2	570	33,088	7,612	176,292	88,879	2,496	12,046
3	942	47	1,278	8,128	10,047	65,053	48,460
4	47,412	22,981	264,578	1,246,562	133,807	523,227	227,310
5	23,463	12,824	96,960	369,498	183,671	287,032	119,187
6	24,800	40,759	142,346	490,430	472,802	2,490,571	426,150
7	2,782	3,406	10,953	83,306	58,947	182,603	55,918
Z(2005)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	70,629	10	1,973	172,428	435	18,296	1,739
2	832	56,798	9,707	302,783	123,117	4,273	17,745
3	1,597	74	1,329	7,886	12,449	74,678	54,282
4	61,158	34,779	337,412	1,445,451	183,602	593,372	255,282
5	25,620	16,748	131,675	445,685	236,309	350,316	123,084
6	26,352	50,611	159,600	525,827	590,537	2,915,594	511,919
7	3,091	3,773	12,087	98,416	72,256	197,062	60,628

To generate price indices relative to the year 2005, the elements in each row of the historical price indices are divided by the last element in that row to yield the following table of relative price indices:

	1997	2003	2005
Agriculture	0.815	0.925	1
Mining	0.481	0.653	1
Construction	0.865	0.900	1
Manufactuirng	0.852	0.961	1
Trade Transport & Utilities	0.923	0.947	1
Services	0.588	0.690	1
Other	0.867	0.897	1

The constant price transactions tables expressed relative to 2005 dollars are then found as $\mathbf{Z}(1997)^{(2005)} = \hat{\mathbf{p}}_{1997}^{(2005)}\mathbf{Z}(1997)$ where $\hat{\mathbf{p}}_{1997}^{(2005)}$ is a matrix with the first column of the relative price table placed along the diagonal and zeros elsewhere. Also, $\mathbf{Z}(2003)^{(2005)}$ is computed in the same manner, i.e., $\mathbf{Z}(2003)^{(2005)} = \hat{\mathbf{p}}_{2003}^{(2005)}\mathbf{Z}(2003)$, but $\mathbf{Z}(2005)^{(2005)} = \hat{\mathbf{p}}_{2005}^{(2005)}\mathbf{Z}(2005)$ is, of course, identical to the $\mathbf{Z}(2005)$.

Z(1997)$^{(2005)}$	1	2	3	4	5	6	7
1	60,967	22	953	122,524	2,361	11,197	10
2	241	9,290	2,269	55,304	25,837	3,043	17
3	863	22	640	5,766	9,041	38,932	21
4	42,605	16,755	152,973	1,161,145	149,438	364,778	1,646
5	19,715	10,624	68,566	344,057	183,832	228,182	612
6	19,485	26,202	63,749	287,754	318,131	918,889	2,191
7	92	715	469	29,737	17,635	24,874	4

Z(2003)$^{(2005)}$	1	2	3	4	5	6	7
1	56,611	8	1,190	134,944	297	17,311	1,991
2	372	21,614	4,973	115,160	58,059	1,631	7,869
3	847	42	1,150	7,315	9,042	58,544	43,612
4	45,568	22,088	254,292	1,198,098	128,605	502,885	218,472
5	22,231	12,150	91,869	350,096	174,027	271,960	112,928
6	17,105	28,112	98,179	338,258	326,100	1,717,792	293,923
7	2,494	3,053	9,820	74,687	52,848	163,709	50,132

PROBLEM 7.2

First we must compute the technical coefficients matrices $\mathbf{A}(2005)$, $\mathbf{A}(1997)^{(2005)}$ and $\mathbf{A}(1997)$:

A(2005)	1	2	3	4	5	6	7
1	0.2258	0.0000	0.0015	0.0384	0.0001	0.0017	0.0007
2	0.0027	0.1432	0.0075	0.0675	0.0367	0.0004	0.0070
3	0.0051	0.0002	0.0010	0.0018	0.0037	0.0071	0.0215
4	0.1955	0.0877	0.2591	0.3222	0.0547	0.0566	0.1010
5	0.0819	0.0422	0.1011	0.0994	0.0704	0.0334	0.0487
6	0.0843	0.1276	0.1225	0.1172	0.1760	0.2783	0.2026
7	0.0099	0.0095	0.0093	0.0219	0.0215	0.0188	0.0240

A(1997)	1	2	3	4	5	6	7
1	0.2611	0.0002	0.0017	0.0397	0.0012	0.0021	0.0000
2	0.0017	0.1146	0.0070	0.0304	0.0226	0.0010	0.0000
3	0.0035	0.0002	0.0011	0.0018	0.0044	0.0068	0.0000
4	0.1745	0.1166	0.2679	0.3600	0.0737	0.0647	0.0020
5	0.0745	0.0682	0.1108	0.0985	0.0836	0.0374	0.0007
6	0.1156	0.2641	0.1617	0.1292	0.2272	0.2363	0.0039
7	0.0004	0.0049	0.0008	0.0091	0.0085	0.0043	0.0000

$\mathbf{A}(1997)^{(2005)}$	1	2	3	4	5	6	7
1	0.2611	0.0003	0.0016	0.0380	0.0011	0.0029	0.0000
2	0.0010	0.1146	0.0039	0.0171	0.0118	0.0008	0.0000
3	0.0037	0.0003	0.0011	0.0018	0.0041	0.0100	0.0000
4	0.1824	0.2067	0.2638	0.3600	0.0680	0.0938	0.0020
5	0.0844	0.1311	0.1182	0.1067	0.0836	0.0587	0.0007
6	0.0834	0.3233	0.1099	0.0892	0.1448	0.2363	0.0026
7	0.0004	0.0088	0.0008	0.0092	0.0080	0.0064	0.0000

Then, for the constant price tables,

$$\frac{1}{7}[\mathbf{i}'|\mathbf{A}(2005)-\mathbf{A}(1997)^{(2005)}|] = \begin{bmatrix}.009 & .062 & .007 & .02 & .014 & .017 & .057\end{bmatrix}.$$

The most changed sectors in decreasing order are 2, 7 and 4. For the current price tables,

$$\frac{1}{7}[\mathbf{i}'|\mathbf{A}(2005)-\mathbf{A}(1997)|] = \begin{bmatrix}.015 & .032 & .01 & .015 & .016 & .01 & .057\end{bmatrix}.$$

The most changed sectors in decreasing order are 7, 2 and 5. Differences in rates of inflation explain the differences in rankings.

PROBLEM 7.3

The marginal input coefficients table, computed as $\left[\mathbf{Z}(2005)-\mathbf{Z}(1997)\right]\left[\langle \mathbf{x}(2005)-\mathbf{x}(1997)\rangle\right]^{-1}$ is:

	1	2	3	4	5	6	7
1	-0.1594	-0.0001	0.0013	0.0315	-0.0025	0.0012	0.0011
2	0.0126	0.1644	0.0079	0.2678	0.0711	-0.0005	0.0113
3	0.0229	0.0002	0.0009	0.0017	0.0020	0.0077	0.0346
4	0.4257	0.0663	0.2498	0.1182	0.0084	0.0428	0.1618
5	0.1626	0.0230	0.0908	0.1041	0.0381	0.0267	0.0782
6	-0.2583	0.0266	0.0810	0.0523	0.0510	0.3501	0.3245
7	0.1139	0.0129	0.0183	0.0915	0.0532	0.0436	0.0387

Note that marginal coefficients deal with changes, so negative entries can appear and do in this case.

PROBLEM 7.4

From the transactions matrix, $\mathbf{Z}(0)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, and vector of total outputs, $\mathbf{x}(0)=\begin{bmatrix}10\\ 10\end{bmatrix}$, we can compute $\mathbf{A}(0)=\mathbf{Z}(0)\hat{\mathbf{x}}(0)^{-1}=\begin{bmatrix}.1 & .2\\ .3 & .4\end{bmatrix}$. We can compute $\mathbf{u}(1)=\mathbf{x}(1)-\mathbf{f}(1)=\begin{bmatrix}25\\ 20\end{bmatrix}-\begin{bmatrix}12\\ 6\end{bmatrix}=\begin{bmatrix}13\\ 14\end{bmatrix}$ and $\mathbf{v}(1)=\mathbf{x}(1)-\mathbf{va}(1)=\begin{bmatrix}25\\ 20\end{bmatrix}-\begin{bmatrix}10\\ 8\end{bmatrix}=\begin{bmatrix}15\\ 12\end{bmatrix}$. [We use $\mathbf{va}(1)$ for the value added vector in 2010 to differentiate it from $\mathbf{v}(1)$, the total intermediate inputs vector in 2010.] Performing the RAS procedure using $\mathbf{A}(0)$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}(1)$, yields $\tilde{\mathbf{A}}(1) = \begin{bmatrix} .262 & .323 \\ .338 & .277 \end{bmatrix}$.

PROBLEM 7.5

For the 1997 input-output table, $\mathbf{A}(1997) = \mathbf{Z}(1997)\hat{\mathbf{x}}(1997)^{-1}$ is shown in the answer to problem 7.2. From Appendix B, we can retrieve $\mathbf{x}(2005)$ and easily compute $\mathbf{u}(2005) = \mathbf{Z}(2005)\mathbf{i}$ and $\mathbf{v}(2005)' = \mathbf{i}'\mathbf{Z}(2005)$ given in the table below

	1	2	3	4	5	6	7
u(2005)′	265,510	515,254	152,295	2,911,056	1,329,436	4,780,440	447,314
v(2005)′	189,279	162,793	653,783	2,998,476	1,218,705	4,153,590	1,024,680
x(2005)′	312,754	396,563	1,302,388	4,485,529	3,355,944	10,477,640	2,526,325

Performing the RAS procedure using $\mathbf{A}(1997)$, $\mathbf{u}(2005)$, $\mathbf{v}(2005)$ and $\mathbf{x}(2005)$, yields $\tilde{\mathbf{A}}(2005)$, given in the table below

$\tilde{\mathbf{A}}$(2005)	1	2	3	4	5	6	7
1	0.2448	0.0001	0.0015	0.0357	0.0009	0.0021	0.0007
2	0.0037	0.1423	0.0140	0.0622	0.0373	0.0022	0.0048
3	0.0052	0.0001	0.0015	0.0025	0.0050	0.0109	0.0023
4	0.1592	0.0618	0.2274	0.3148	0.0519	0.0638	0.1129
5	0.0737	0.0392	0.1020	0.0933	0.0639	0.0400	0.0420
6	0.1172	0.1557	0.1525	0.1256	0.1780	0.2588	0.2419
7	0.0015	0.0112	0.0030	0.0343	0.0261	0.0185	0.0011

For the 2005 "real" input-output table, $\mathbf{A}(2005) = \mathbf{Z}(2005)\hat{\mathbf{x}}(2005)^{-1}$ (also derived in the solution to problem 7.2), the MAPE is computed as

$$(\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}100\times\left[\frac{\left|\tilde{a}_{ij}(2005)-a_{ij}(2005)\right|}{a_{ij}(2005)}\right] = 49.028 \quad \text{[for } a_{ij}(2005)\neq 0 \text{ and 0 otherwise].}$$

Note that in this case the RAS estimate is very weak since the average error is nearly 50%.

PROBLEM 7.6

$\tilde{\mathbf{Z}}^Z(1)$ computed via RAS, using $\mathbf{Z}(0)$, $\mathbf{v}(1)$ and $\mathbf{u}(1)$ yields $\tilde{\mathbf{Z}}^Z(1) = \begin{bmatrix} 167.5 & 104.5 & 53 \\ 61.2 & 104.1 & 69.7 \\ 36.3 & 16.5 & 202.2 \end{bmatrix}$.

Knowing $\mathbf{x}(0)$, we can compute $\mathbf{A}(0) = \mathbf{Z}(0)\hat{\mathbf{x}}(0)^{-1} = \begin{bmatrix} .133 & .11 & .025 \\ .067 & .15 & .045 \\ .033 & .02 & .11 \end{bmatrix}$. Performing the RAS procedure with $\mathbf{A}(0)$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}(1)$, yields $\tilde{\mathbf{A}}^A(1) = \begin{bmatrix} .168 & .139 & .035 \\ .061 & .139 & .047 \\ .036 & .022 & .135 \end{bmatrix}$. Then

$\tilde{\mathbf{A}}^Z(1) = \tilde{\mathbf{Z}}^Z(1)\hat{\mathbf{x}}(1)^{-1} = \begin{bmatrix} .168 & .139 & .035 \\ .061 & .139 & .047 \\ .036 & .022 & .135 \end{bmatrix}$, the same as $\tilde{\mathbf{A}}^A(1)$. The explanation for why this is true is discussed in section 7.4.3.

PROBLEM 7.7

We modify $\mathbf{A}(0)$ by setting the entry for the known coefficient, $a(1)_{32} = 0$. We define the modified matrix as $\bar{\mathbf{A}}(0) = \begin{bmatrix} .133 & .11 & .025 \\ .067 & .15 & .045 \\ .033 & 0 & .11 \end{bmatrix}$. Thus, we can define the matrix of known coefficients as $\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & .033 & 0 \end{bmatrix}$. The "intermediate estimate" $\tilde{\mathbf{A}}^*(1)$ is then found by adding $\mathbf{K}$ to the result of applying the RAS procedure using $\bar{\mathbf{A}}(0)$, $\bar{\mathbf{u}}(1)$, $\bar{\mathbf{v}}(1)$ and $\bar{\mathbf{x}}(1)$, to yield $\tilde{\mathbf{A}}^*(1)^{(case1)} = \begin{bmatrix} .164 & .151 & .032 \\ .06 & .149 & .042 \\ .041 & .033 & .142 \end{bmatrix}$. The MAPE for the RAS estimate, $\tilde{\mathbf{A}}(1)$, compared with the known $\mathbf{A}(1)$ is 24.05. The MAPE for the modified RAS estimate, $\tilde{\mathbf{A}}^*(1)^{(case1)}$ (including the known coefficient), is 23.83, so the estimate with the additional information is better.

For the second case, we modify $\mathbf{A}(0)$ by setting the entry for the new known coefficient, $a(1)_{11} = 0$ and, as before, define the modified matrix as $\bar{\mathbf{A}}(0) = \begin{bmatrix} 0 & .11 & .025 \\ .067 & .15 & .045 \\ .033 & .02 & .11 \end{bmatrix}$. Thus, we can define the new matrix of known coefficients as $\mathbf{K} = \begin{bmatrix} .2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, so the "intermediate estimate" $\tilde{\mathbf{A}}^*(1)$ is then found by adding $\mathbf{K}$ to the result of applying the RAS procedure using $\bar{\mathbf{A}}(0)$, $\bar{\mathbf{u}}(1)$, $\bar{\mathbf{v}}(1)$ and $\bar{\mathbf{x}}(1)$, to yield $\tilde{\mathbf{A}}^*(1)^{(case2)} = \begin{bmatrix} .2 & .24 & .097 \\ .156 & .051 & .027 \\ .109 & .009 & .093 \end{bmatrix}$. The MAPE for the modified RAS estimate, $\tilde{\mathbf{A}}^*(1)^{(case2)}$ (including the new known coefficient), is 128.49, so the estimate without the additional information is better in this case.

PROBLEM 7.8

Note that $\mathbf{u}(1)$ and $\mathbf{v}(1)$ are identical to $\mathbf{u}(0)$ and $\mathbf{v}(0)$, respectively, so an RAS procedure will converge immediately and is unnecessary. By reducing $v_1(1)$ to substantially below the existing value, without any other changes, it is unlikely that both row and column constraints in the RAS procedure can be satisfied simultaneously, so the procedure will not converge.

PROBLEM 7.9

The matrices $\mathbf{A}(1997)$, $\mathbf{A}(2005)$ and $\tilde{\mathbf{A}}(2005)$ by using RAS with $\mathbf{A}(1997)$, $\mathbf{u}(2005)$, $\mathbf{v}(2005)$ and $\mathbf{x}(2005)$, were computed in problems 7.1 and 7.5. The MAPE for $\tilde{\mathbf{A}}(2005)$ compared with $\mathbf{A}(2005)$ is 49.03. The matrices $\mathbf{L}(2005)$ and $\tilde{\mathbf{L}}(2005) = [\mathbf{I} - \tilde{\mathbf{A}}(2005)]^{-1}$ are computed as:

$\mathbf{L}(2005)$	1	2	3	4	5	6	7
1	1.3139	0.0102	0.0247	0.0789	0.0076	0.0103	0.0122
2	0.0462	1.1863	0.0515	0.1331	0.0584	0.0152	0.0296
3	0.0109	0.0034	1.0054	0.0075	0.0074	0.0116	0.0257
4	0.4324	0.1907	0.4421	1.5707	0.1332	0.1404	0.2098
5	0.1773	0.0865	0.1737	0.1969	1.1072	0.0714	0.0950
6	0.2861	0.2701	0.3053	0.3508	0.3136	1.4409	0.3600
7	0.0330	0.0231	0.0300	0.0486	0.0342	0.0329	1.0390

$\tilde{\mathbf{L}}(2005)$	1	2	3	4	5	6	7
1	1.3426	0.0081	0.0218	0.0746	0.0084	0.0114	0.0126
2	0.0407	1.1812	0.0534	0.1223	0.0585	0.0188	0.0266
3	0.0124	0.0044	1.0073	0.0094	0.0095	0.0164	0.0078
4	0.3670	0.1485	0.3931	1.5503	0.1297	0.1533	0.2197
5	0.1607	0.0795	0.1686	0.1853	1.1006	0.0807	0.0876
6	0.3324	0.3030	0.3381	0.3682	0.3148	1.4147	0.3999
7	0.0254	0.0261	0.0278	0.0665	0.0398	0.0339	1.0187

The MAPE for $\tilde{\mathbf{L}}(2005)$ compared with $\mathbf{L}(2005)$ is 12.33.

Solutions to Chapter 8 Problems

PROBLEM 8.1

(a) The table of value added, intermediate inputs, final demands and intermediate outputs of the economies are given below:

	Lilliput (L)	Brobdingnag (B)	Houyhnhnm (H)
Value Added ($\mathbf{v}'$)	[15 7]	[27 6]	[77.14 46.03]
Intermediate Inputs ($\mathbf{i}'\mathbf{Z}$)	[5 8]	[8 9]	[22.86 68.97]
Final Demands ($\mathbf{f}$)	$\begin{bmatrix} 13 \\ 9 \end{bmatrix}$	$\begin{bmatrix} 24 \\ 9 \end{bmatrix}$	$\begin{bmatrix} 49.33 \\ 73.84 \end{bmatrix}$
Intermediate Outputs ($\mathbf{u}$)	$\begin{bmatrix} 7 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 11 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 50.67 \\ 41.16 \end{bmatrix}$

(b) The true technical coefficients matrices are $\mathbf{A}^L = \begin{bmatrix} .050 & .400 \\ .200 & .133 \end{bmatrix}$ and $\mathbf{A}^B = \begin{bmatrix} .200 & .267 \\ .029 & .333 \end{bmatrix}$. The L estimate of the $\mathbf{A}^B$ matrix is ${}^L\mathbf{A}^B = \begin{bmatrix} .088 & .529 \\ .141 & .071 \end{bmatrix}$; the mean absolute deviation (MAD) between ${}^L\mathbf{A}^B$ and $\mathbf{A}^B$ is 0.187. The B estimate of $\mathbf{A}^L$ is ${}^B\mathbf{A}^L = \begin{bmatrix} .207 & .190 \\ .043 & .343 \end{bmatrix}$, with a MAD of 0.183. Therefore the Brobdingnagian economist does slightly better. (c) The two Houyhnhnm estimates are ${}^H\mathbf{A}^L = \begin{bmatrix} .207 & .190 \\ .043 & .343 \end{bmatrix}$ and ${}^H\mathbf{A}^B = \begin{bmatrix} .200 & .267 \\ .029 & .333 \end{bmatrix}$, respectively (note that $\mathbf{A}^H = \mathbf{A}^B = {}^H\mathbf{A}^B = \begin{bmatrix} .200 & .267 \\ .029 & .333 \end{bmatrix}$). The error, as measured by MAD, is 0.183 in the first case and, of course, zero in the second case since $\mathbf{A}^H = \mathbf{A}^B$. (d) The true impact is found from $\Delta\mathbf{x}^L = \mathbf{L}^L\Delta\mathbf{f} = \begin{bmatrix} 197.3 \\ 218.6 \end{bmatrix}$ for $\Delta\mathbf{f} = \begin{bmatrix} 100 \\ 150 \end{bmatrix}$ and $\mathbf{L}^L = (\mathbf{I} - \mathbf{A}^L)^{-1} = \begin{bmatrix} 1.166 & .538 \\ .269 & 1.278 \end{bmatrix}$. Using the same final demand vector with $(\mathbf{I} - {}^H\mathbf{A}^L)^{-1} = \begin{bmatrix} 1.281 & .371 \\ .083 & 1.546 \end{bmatrix}$ yields $\Delta[{}^H\mathbf{x}^L] = \begin{bmatrix} 183.8 \\ 240.3 \end{bmatrix}$. The mean absolute deviation between these two vectors is 17.6.

PROBLEM 8.2

(a) The table of value added, intermediate inputs, final demands, and intermediate outputs of the economies are given in the following table:

	Lilliput (L)	Brobdingnag (B)	Houyhnhnm (H)
Value Added ($\mathbf{v}'$)	$[11\ \ 6\ \ 4]$	$[21\ \ 4\ \ 14]$	$[60.5\ \ 33.0\ \ 22.0]$
Intermediate Inputs ($\mathbf{i}'\mathbf{Z}$)	$[9\ \ 9\ \ 8]$	$[14\ \ 11\ \ 16]$	$[49.5\ \ 49.5\ \ 44.0]$
Final Demands ($\mathbf{f}$)	$\begin{bmatrix} 7 \\ 8 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$	$\begin{bmatrix} 38.5 \\ 44.0 \\ 33.0 \end{bmatrix}$
Intermediate Outputs ($\mathbf{u}$)	$\begin{bmatrix} 13 \\ 7 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 19 \\ 7 \\ 15 \end{bmatrix}$	$\begin{bmatrix} 71.5 \\ 38.5 \\ 33.0 \end{bmatrix}$

(b) For these three-sector economies we find $\mathbf{A}^L = \begin{bmatrix} .050 & .400 & .500 \\ .200 & .133 & .083 \\ .200 & .067 & .083 \end{bmatrix}$ and

$\mathbf{A}^B = \begin{bmatrix} .200 & .267 & .267 \\ .029 & .333 & .033 \\ .171 & .133 & .233 \end{bmatrix}$. The RAS estimates are ${}^B\mathbf{A}^L = \begin{bmatrix} .264 & .199 & .395 \\ .052 & .343 & .068 \\ .134 & .059 & .204 \end{bmatrix}$ and

${}^L\mathbf{A}^B = \begin{bmatrix} .033 & .460 & .365 \\ .106 & .123 & .049 \\ .261 & .151 & .120 \end{bmatrix}$. The mean absolute deviation for the L estimate of B is 0.109, while the MAD for the B estimate of L is 0.121. (c) The Houyhnhnm estimates are

${}^H\mathbf{A}^L = \begin{bmatrix} .050 & .400 & .500 \\ .200 & .133 & .083 \\ .200 & .067 & .083 \end{bmatrix}$ and ${}^H\mathbf{A}^B = \begin{bmatrix} .033 & .460 & .365 \\ .106 & .123 & .049 \\ .261 & .151 & .120 \end{bmatrix}$. Note in this case that $\mathbf{A}^H = \mathbf{A}^L$.

The error, as measured by MAD, is 0.0 in the first case and 0.109 in the second case.

PROBLEM 8.3

(a) The cost of (1), using the exact inverse, is $c_1 + c_2$; with $c_2 = 10c_1$, this cost is $11c_1$. With $m = 4$, the cost of using the round-by-round approximation, method (2), is $4c_1$. Thus use method (2), the round-by-round approximation, in this case. (b) Since the RAS procedure converges to within a tolerance of 0.01 in 2 iterations, the cost of the RAS estimate of region 2's coefficients matrix is $5c_1$. Then utilizing it in a round-by-round application, with $m = 4$, gives a total cost of $6c_1$. (c) The summary of cost calculations at different levels of tolerance are given by the table below.

RAS Tolerance	Number of Iterations	RAS Cost	Impact Analysis Cost	Total Cost
.01	2	$2c_1$	$4c_1$	$6c_1$
.001	3	$3c_1$	$4c_1$	$7c_1$
.0001	3	$3c_1$	$4c_1$	$7c_1$
.00001	3	$3c_1$	$4c_1$	$7c_1$
.000001	4	$4c_1$	$4c_1$	$8c_1$

Therefore, the maximum affordable tolerance is .00001.

PROBLEM 8.4

Values for $\lambda = \{\log_2[1+(x^r/x^n)]\}^\delta$ for various x^r/x^n and δ are shown in the table below

x^r/x^n	0.01	0.1	0.25	0.5	0.75	1.0
$\log_2[1+(x^r/x^n)]$	0.0144	0.1375	0.3219	0.5850	0.8074	1
$\{\log_2[1+(x^r/x^n)]\}^0$	1	1	1	1	1	1
$\{\log_2[1+(x^r/x^n)]\}^{0.1}$	0.6542	0.8200	0.8928	0.9478	0.9788	1
$\{\log_2[1+(x^r/x^n)]\}^{0.3}$	0.2800	0.5514	0.7118	0.8514	0.9378	1
$\{\log_2[1+(x^r/x^n)]\}^{0.5}$	0.1198	0.3708	0.5647	0.7648	0.8985	1
$\{\log_2[1+(x^r/x^n)]\}^1$	0.0144	0.1375	0.32	0.5850	0.8074	1

PROBLEM 8.5

The matrix of simple location quotients (SLQ) and corresponding estimate of the regional technical coefficients matrix using SLQ are $\mathbf{SLQ} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ .8607 & .8607 & .8607 \end{bmatrix}$ and

$$\mathbf{A}^{(SLQ)} = \begin{bmatrix} .1830 & .0668 & .0087 \\ .1377 & .3070 & .0707 \\ .1794 & .2074 & .2581 \end{bmatrix}.$$

PROBLEM 8.6

The matrix of cross–industry location quotients (CIQ) and corresponding estimate of the regional technical coefficients matrix using CIQ are $\mathbf{CIQ} = \begin{bmatrix} 1 & .8274 & 1 \\ 1 & 1 & 1 \\ .7508 & .6212 & 1 \end{bmatrix}$ and

$$\mathbf{A}^{(CIQ)} = \begin{bmatrix} .1830 & .0553 & .0087 \\ .1377 & .3070 & .0707 \\ .1565 & .1497 & .2999 \end{bmatrix}.$$

PROBLEM 8.7

The intermediate inputs and outputs for the regional economy are given by
$\mathbf{u}^{(R)} = \mathbf{A}^{(R)}\mathbf{x}^{(R)} = [4{,}615.3 \quad 15{,}877.7 \quad 52{,}584.2]'$ and
$\mathbf{v}^{(R)} = (\mathbf{i}'\mathbf{A}^{(R)}\hat{\mathbf{x}}^{(R)})' = [2{,}969.5 \quad 22{,}368.8 \quad 47{,}738.9]'$.
The corresponding estimate of the matrix of regional technical coefficients is

$$\mathbf{A}^{(RAS)} = \begin{bmatrix} .1241 & .0270 & .0059 \\ .0712 & .0945 & .0367 \\ .1640 & .1129 & .2370 \end{bmatrix}.$$

PROBLEM 8.8

$MAD^{(SLQ)} = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(SLQ)} - a_{ij}^{(R)}\right| = .0606$; $MAD^{(CIQ)} = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(CIQ)} - a_{ij}^{(R)}\right| = .0558$;
$MAD^{(RAS)} = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(RAS)} - a_{ij}^{(R)}\right| = .0073$. The RAS technique produces the most accurate estimate in these examples.

PROBLEM 8.9

Results for Region 2 (South China) using 2000 Chinese MRIO data.

	Intraregional Input Coefficients	**Leontief Inverse**
Survey	$\begin{bmatrix} 0.1279 & 0.1086 & 0.0340 \\ 0.1348 & 0.4299 & 0.2191 \\ 0.0394 & 0.0814 & 0.1255 \end{bmatrix}$	$\begin{bmatrix} 1.1889 & 0.2418 & 0.1069 \\ 0.3130 & 1.8828 & 0.4839 \\ 0.0827 & 0.1861 & 1.1933 \end{bmatrix}$
	Using $\mathbf{A}^n$	
LQ	$\begin{bmatrix} 0.1252 & 0.1301 & 0.0336 \\ 0.1517 & 0.4605 & 0.2411 \\ 0.0411 & 0.0867 & 0.1235 \end{bmatrix}$	$\begin{bmatrix} 1.2033 & 0.3113 & 0.1317 \\ 0.3804 & 2.0378 & 0.5751 \\ 0.0940 & 0.2161 & 1.2039 \end{bmatrix}$
CIQ	$\begin{bmatrix} 0.1252 & 0.1263 & 0.0351 \\ 0.1517 & 0.4605 & 0.2411 \\ 0.0429 & 0.0842 & 0.1235 \end{bmatrix}$	$\begin{bmatrix} 1.2019 & 0.3018 & 0.1311 \\ 0.3806 & 2.0324 & 0.5743 \\ 0.0953 & 0.2099 & 1.2024 \end{bmatrix}$
FLQ	$\begin{bmatrix} 0.1076 & 0.1085 & 0.0301 \\ 0.1406 & 0.4076 & 0.2228 \\ 0.0369 & 0.0723 & 0.1061 \end{bmatrix}$	$\begin{bmatrix} 1.1598 & 0.2241 & 0.0950 \\ 0.3025 & 1.7994 & 0.4587 \\ 0.0724 & 0.1548 & 1.1598 \end{bmatrix}$

FLQA	$\begin{bmatrix} 0.1076 & 0.1109 & 0.0301 \\ 0.1406 & 0.4163 & 0.2228 \\ 0.0369 & 0.0739 & 0.1061 \end{bmatrix}$	$\begin{bmatrix} 1.1613 & 0.2328 & 0.0972 \\ 0.3077 & 1.8307 & 0.4667 \\ 0.0734 & 0.1609 & 1.1613 \end{bmatrix}$
RPC	$\begin{bmatrix} 0.1155 & 0.1199 & 0.0310 \\ 0.1350 & 0.4097 & 0.2145 \\ 0.0396 & 0.0837 & 0.1192 \end{bmatrix}$	$\begin{bmatrix} 1.1738 & 0.2531 & 0.1029 \\ 0.2978 & 1.8187 & 0.4533 \\ 0.0811 & 0.1841 & 1.1830 \end{bmatrix}$
RAS	$\begin{bmatrix} 0.1151 & 0.1141 & 0.0303 \\ 0.1462 & 0.4235 & 0.2278 \\ 0.0409 & 0.0823 & 0.1205 \end{bmatrix}$	$\begin{bmatrix} 1.1759 & 0.2477 & 0.1046 \\ 0.3320 & 1.8711 & 0.4961 \\ 0.0857 & 0.1867 & 1.1884 \end{bmatrix}$
Using Round's $\mathbf{A}^r = \mathbf{A}^n \hat{\rho}^r$		
LQ	$\begin{bmatrix} 0.1263 & 0.1324 & 0.0346 \\ 0.1530 & 0.4687 & 0.2483 \\ 0.0414 & 0.0882 & 0.1272 \end{bmatrix}$	$\begin{bmatrix} 1.2080 & 0.3242 & 0.1401 \\ 0.3933 & 2.0810 & 0.6077 \\ 0.0971 & 0.2257 & 1.2138 \end{bmatrix}$
CIQ	$\begin{bmatrix} 0.1263 & 0.1285 & 0.0361 \\ 0.1530 & 0.4687 & 0.2483 \\ 0.0433 & 0.0856 & 0.1272 \end{bmatrix}$	$\begin{bmatrix} 1.2066 & 0.3143 & 0.1394 \\ 0.3935 & 2.0751 & 0.6068 \\ 0.0984 & 0.2192 & 1.2122 \end{bmatrix}$
FLQ	$\begin{bmatrix} 0.1086 & 0.1105 & 0.0310 \\ 0.1418 & 0.4148 & 0.2295 \\ 0.0373 & 0.0736 & 0.1093 \end{bmatrix}$	$\begin{bmatrix} 1.1629 & 0.2321 & 0.1003 \\ 0.3110 & 1.8281 & 0.4819 \\ 0.0744 & 0.1608 & 1.1668 \end{bmatrix}$
FLQA	$\begin{bmatrix} 0.1086 & 0.1128 & 0.0310 \\ 0.1418 & 0.4237 & 0.2295 \\ 0.0373 & 0.0752 & 0.1093 \end{bmatrix}$	$\begin{bmatrix} 1.1645 & 0.2413 & 0.1028 \\ 0.3166 & 1.8611 & 0.4906 \\ 0.0754 & 0.1672 & 1.1685 \end{bmatrix}$
RPC	$\begin{bmatrix} 0.1165 & 0.1221 & 0.0319 \\ 0.1361 & 0.4169 & 0.2209 \\ 0.0400 & 0.0851 & 0.1228 \end{bmatrix}$	$\begin{bmatrix} 1.1772 & 0.2623 & 0.1089 \\ 0.3064 & 1.8488 & 0.4768 \\ 0.0834 & 0.1914 & 1.1912 \end{bmatrix}$
RAS	$\begin{bmatrix} 0.1151 & 0.1141 & 0.0303 \\ 0.1462 & 0.4235 & 0.2278 \\ 0.0409 & 0.0823 & 0.1205 \end{bmatrix}$	$\begin{bmatrix} 1.1759 & 0.2477 & 0.1046 \\ 0.3320 & 1.8711 & 0.4961 \\ 0.0857 & 0.1867 & 1.1884 \end{bmatrix}$

	Total Intraregional Intermediate Inputs	Percentage Differences[a]	Average Percentage Difference[b]	MAPE[c]
Survey	**0.3022 0.6199 0.3786**			
Using $\mathbf{A}^n$				
LQ	0.3180 0.6773 0.3982	5.24 9.25 5.17	6.55	7.22
CIQ	0.3198 0.6710 0.3997	5.84 8.23 5.56	6.55	7.19
FLQ	0.2852 0.5884 0.3591	-5.63 -5.08 -5.15	-5.29	7.93
FLQA	0.2852 0.6010 0.3591	-5.62 -3.05 -5.15	-4.61	7.71
RPC	0.2901 0.6133 0.3646	-4.00 -1.08 -3.69	-2.92	4.93
RAS	0.3022 0.6199 0.3786	0 0 0	0	5.41
Using Round's $\mathbf{A}^r = \mathbf{A}^n \hat{\rho}^r$				
LQ	0.3208 0.6892 0.4102	6.15 11.18 8.34	8.56	8.38
CIQ	0.3226 0.6829 0.4117	6.76 10.15 8.74	8.55	8.65
FLQ	0.2876 0.5989 0.3699	-4.81 -3.40 -2.30	-3.50	7.43
FLQA	0.2876 0.6117 0.3699	-4.81 -1.34 -2.30	-2.82	7.23
RPC	0.2926 0.6241 0.3756	-3.17 0.67 -0.79	1.54[d]	4.50
RAS	0.3022 0.6199 0.3786	0 0 0	0	5.41

[a] This is $\{[(\mathbf{i}'\tilde{\mathbf{A}} - \mathbf{i}'\mathbf{A}) \oslash \mathbf{i}'\mathbf{A}] \times 100\}$, where "$\oslash$" indicates element-by-element division.

[b] This is a simple, unweighted average. Various kinds of weightings (e.g., using some measure of the size of each sector) are frequently used.

[c] Calculated as $(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|a_{ij} - \tilde{a}_{ij}|}{a_{ij}}) \times 100$

[d] This is the average of the absolute values of the differences, so that the negatives and positives do not cancel out.

	Intraregional Output Multipliers	Percentage Differences[e]	Average Percentage Difference	MAPE[f]
Survey	**1.5846 2.3108 1.7841**			
Using $\mathbf{A}^n$				
LQ	1.6778 2.5651 1.9108	5.88 11.01 7.10	8.00	14.71
CIQ	1.6779 2.5441 1.9078	5.89 10.10 6.94	7.64	13.96
FLQ	1.5347 2.1784 1.7135	-3.15 -5.73 -3.96	-4.28	7.33
FLQA	1.5425 2.2245 1.7252	-2.66 -3.73 -3.30	-3.23	5.62
RPC	1.5527 2.2559 1.7392	-2.01 -2.37 -2.51	-2.30	3.12
RAS	1.5936 2.3055 1.7891	0.57 -0.23 0.28	0.36	2.13
Using Round's $\mathbf{A}^r = \mathbf{A}^n \hat{\rho}^r$				
LQ	1.6984 2.6309 1.9617	7.18 13.85 9.96	10.33	18.77
CIQ	1.6985 2.6087 1.9584	7.18 12.89 9.77	9.95	17.95
FLQ	1.5482 2.2210 1.7419	-2.30 -3.88 -1.96	-2.71	4.69
FLQA	1.5565 2.2696 1.7619	-1.78 -1.78 -1.24	-1.60	3.43
RPC	1.5670 2.3025 1.7769	-1.11 -0.36 -0.40	-0.62	2.28
RAS	1.5936 2.3055 1.7891	0.57 -0.23 0.28	0.36	2.13

[e] Calculated as $\{[(\mathbf{i}'\tilde{\mathbf{L}} - \mathbf{i}'\mathbf{L}) \oslash \mathbf{i}'\mathbf{L}] \times 100\}$.

[f] Calculated as $(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|l_{ij} - \tilde{l}_{ij}|}{l_{ij}}) \times 100$

Results for Region 3 (Rest of China) using 2000 Chinese MRIO data.

	Intraregional Input Coefficients	**Leontief Inverse**
Survey	$\begin{bmatrix} 0.1356 & 0.1494 & 0.0329 \\ 0.1050 & 0.3176 & 0.1945 \\ 0.0364 & 0.1016 & 0.1122 \end{bmatrix}$	$\begin{bmatrix} 1.1950 & 0.2773 & 0.1050 \\ 0.2046 & 1.5624 & 0.3498 \\ 0.0725 & 0.1902 & 1.1707 \end{bmatrix}$
Using $\mathbf{A}^n$		
LQ	$\begin{bmatrix} 0.1311 & 0.1362 & 0.0352 \\ 0.1293 & 0.3925 & 0.2055 \\ 0.0429 & 0.0905 & 0.1290 \end{bmatrix}$	$\begin{bmatrix} 1.1992 & 0.2861 & 0.1159 \\ 0.2853 & 1.7742 & 0.4301 \\ 0.0887 & 0.1984 & 1.1984 \end{bmatrix}$
CIQ	$\begin{bmatrix} 0.1311 & 0.1362 & 0.0352 \\ 0.1015 & 0.3925 & 0.1789 \\ 0.0387 & 0.0905 & 0.1290 \end{bmatrix}$	$\begin{bmatrix} 1.1886 & 0.2822 & 0.1059 \\ 0.2210 & 1.7506 & 0.3684 \\ 0.0757 & 0.1944 & 1.1910 \end{bmatrix}$
FLQ	$\begin{bmatrix} 0.1057 & 0.1288 & 0.0247 \\ 0.0643 & 0.2485 & 0.1133 \\ 0.0245 & 0.0772 & 0.0938 \end{bmatrix}$	$\begin{bmatrix} 1.1341 & 0.2001 & 0.0559 \\ 0.1030 & 1.3661 & 0.1735 \\ 0.0394 & 0.1218 & 1.1198 \end{bmatrix}$
FLQA	$\begin{bmatrix} 0.1252 & 0.1288 & 0.0272 \\ 0.0762 & 0.2485 & 0.1250 \\ 0.0290 & 0.0772 & 0.1035 \end{bmatrix}$	$\begin{bmatrix} 1.1632 & 0.2059 & 0.0640 \\ 0.1260 & 1.3723 & 0.1951 \\ 0.0485 & 0.1248 & 1.1343 \end{bmatrix}$
RPC	$\begin{bmatrix} 0.1223 & 0.1270 & 0.0328 \\ 0.1263 & 0.3835 & 0.2008 \\ 0.0397 & 0.0837 & 0.1193 \end{bmatrix}$	$\begin{bmatrix} 1.1810 & 0.2572 & 0.1026 \\ 0.2676 & 1.7322 & 0.4048 \\ 0.0786 & 0.1762 & 1.1786 \end{bmatrix}$
RAS	$\begin{bmatrix} 0.1342 & 0.1480 & 0.0381 \\ 0.1020 & 0.3290 & 0.1715 \\ 0.0409 & 0.0916 & 0.1300 \end{bmatrix}$	$\begin{bmatrix} 1.1927 & 0.2777 & 0.1069 \\ 0.2011 & 1.5784 & 0.3199 \\ 0.0772 & 0.1793 & 1.1882 \end{bmatrix}$
Using Round's $\mathbf{A}^r = \mathbf{A}^n\hat{\rho}^r$		
LQ	$\begin{bmatrix} 0.1251 & 0.1295 & 0.0335 \\ 0.1234 & 0.3732 & 0.1957 \\ 0.0409 & 0.0860 & 0.1228 \end{bmatrix}$	$\begin{bmatrix} 1.1844 & 0.2587 & 0.1029 \\ 0.2583 & 1.7022 & 0.3895 \\ 0.0806 & 0.1790 & 1.1830 \end{bmatrix}$

CIQ	$\begin{bmatrix} 0.1251 & 0.1295 & 0.0335 \\ 0.0969 & 0.3732 & 0.1703 \\ 0.0369 & 0.0860 & 0.1228 \end{bmatrix}$	$\begin{bmatrix} 1.1754 & 0.2557 & 0.0945 \\ 0.2005 & 1.6827 & 0.3344 \\ 0.0691 & 0.1758 & 1.1768 \end{bmatrix}$
FLQ	$\begin{bmatrix} 0.1009 & 0.1224 & 0.0235 \\ 0.0614 & 0.2363 & 0.1078 \\ 0.0234 & 0.0734 & 0.0893 \end{bmatrix}$	$\begin{bmatrix} 1.1262 & 0.1855 & 0.0510 \\ 0.0956 & 1.3402 & 0.1612 \\ 0.0366 & 0.1128 & 1.1123 \end{bmatrix}$
FLQA	$\begin{bmatrix} 0.1195 & 0.1224 & 0.0259 \\ 0.0727 & 0.2363 & 0.1190 \\ 0.0277 & 0.0734 & 0.0985 \end{bmatrix}$	$\begin{bmatrix} 1.1533 & 0.1905 & 0.0583 \\ 0.1168 & 1.3455 & 0.1809 \\ 0.0449 & 0.1154 & 1.1258 \end{bmatrix}$
RPC	$\begin{bmatrix} 0.1167 & 0.1207 & 0.0312 \\ 0.1206 & 0.3646 & 0.1912 \\ 0.0379 & 0.0796 & 0.1136 \end{bmatrix}$	$\begin{bmatrix} 1.1679 & 0.2334 & 0.0915 \\ 0.2432 & 1.6661 & 0.3679 \\ 0.0717 & 0.1596 & 1.1651 \end{bmatrix}$
RAS	$\begin{bmatrix} 0.1342 & 0.1480 & 0.0381 \\ 0.1020 & 0.3290 & 0.1715 \\ 0.0409 & 0.0916 & 0.1300 \end{bmatrix}$	$\begin{bmatrix} 1.1927 & 0.2777 & 0.1069 \\ 0.2011 & 1.5784 & 0.3199 \\ 0.0772 & 0.1793 & 1.1882 \end{bmatrix}$

	Total Intraregional Intermediate Inputs	**Percentage Differences**[a]	**Average Percentage Difference**[b]	**MAPE**[c]
Survey	**0.2771 0.5687 0.3396**			
	Using $\mathbf{A}^n$			
LQ	0.3033 0.6192 0.3696	9.47 8.88 8.85	9.07	12.79
CIQ	0.2713 0.6192 0.3430	-2.07 8.88 1.02	3.99[d]	9.57
FLQ	0.1945 0.4545 0.2317	-29.81 -20.08 -31.76	-27.22	26.26
FLQA	0.2304 0.4545 0.2557	-16.84 -20.08 -24.70	-20.54	19.53
RPC	0.2883 0.5942 0.3529	4.05 4.49 3.92	4.15	11.35
RAS	0.2771 0.5687 0.3396	0 0 0	0	8.22
	Using Round's $\mathbf{A}^r = \mathbf{A}^n \hat{\rho}^r$			
LQ	0.2895 0.5887 0.3519	4.47 3.52 3.64	3.88	10.63
CIQ	0.2590 0.5887 0.3266	-6.54 3.52 -3.81	4.62[d]	9.63
FLQ	0.1856 0.4321 0.2206	-33.02 -24.02 -35.03	-30.69	29.78
FLQA	0.2199 0.4321 0.2434	-20.63 -24.02 -28.31	-24.32	23.36
RPC	0.2751 0.5649 0.3360	-0.70 -0.66 -1.05	-0.81	20.69
RAS	0.2771 0.5687 0.3396	0 0 0	0	8.22

	Intraregional Output Multipliers	Percentage Differences[e]	Average Percentage Difference	MAPE[f]
Survey	**1.4721 2.0299 1.6256**			
Using $\mathbf{A}^n$				
LQ	1.5732 2.2587 1.7444	6.87 11.27 7.31	8.48	13.21
CIQ	1.4853 2.2272 1.6654	0.90 9.72 2.45	4.36	4,12
FLQ	1.2765 1.6880 1.3492	-13.29 -16.84 -17.00	-15.71	30.92
FLQA	1.3377 1.7031 1.3934	-9.13 -16.10 -14.28	-13.17	25.87
RPC	1.5272 2.1656 1.6860	3.75 6.68 3.72	4.72	9.40
RAS	1.4711 2.0354 1.6150	-0.07 0.27 -0.65	0.33[d]	3.03
Using Round's $\mathbf{A}^r = \mathbf{A}^n \hat{\rho}^r$				
LQ	1.5233 2.1400 1.6754	3.48 5.42 3.07	3.99	8.25
CIQ	1.4450 2.1142 1.6057	-1.84 4.15 -1.22	2.41[d]	5.13
FLQ	1.2584 1.6384 1.3245	-14.51 -19.29 -18.52	-17.44	34.10
FLQA	1.3150 1.6514 1.3651	-10.67 -18.65 -16.02	-15.11	29.50
RPC	1.4828 2.0591 1.6422	0.73 1.44 -0.07	0.75[d]	8.81
RAS	1.4711 2.0354 1.6150	-0.07 0.27 -0.65	0.33[d]	3.03

[a] This is $\{[(\mathbf{i}'\tilde{\mathbf{A}} - \mathbf{i}'\mathbf{A}) \oslash \mathbf{i}'\mathbf{A}] \times 100\}$, where "$\oslash$" indicates element-by-element division.

[b] This is a simple, unweighted average. Various kinds of weightings (e.g., using some measure of the size of each sector) are frequently used.

[c] Calculated as $(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|a_{ij} - \tilde{a}_{ij}|}{a_{ij}}) \times 100$

[d] This is the average of the absolute values of the differences, so that the negatives and positives do not cancel out.

[e] Calculated as $\{[(\mathbf{i}'\tilde{\mathbf{L}} - \mathbf{i}'\mathbf{L}) \oslash \mathbf{i}'\mathbf{L}] \times 100\}$.

[f] Calculated as $(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|l_{ij} - \tilde{l}_{ij}|}{l_{ij}}) \times 100$

PROBLEM 8.10

Compute the vectors of total intermediate inputs and outputs for the real Washington State table:

$$\mathbf{u}(1) = \mathbf{Z}^{(W)}\mathbf{i} = \begin{bmatrix} 4245.9 & 369.4 & 3140.1 & 12737.6 & 12718 & 38753.8 & 1112.4 \end{bmatrix}'$$

$$\mathbf{v}(1) = (\mathbf{i}'\mathbf{Z}^{(W)})' = \begin{bmatrix} 2849.7 & 119.8 & 4423 & 17945.8 & 15384.7 & 31052.5 & 1301.7 \end{bmatrix}'$$

Apply RAS using $\mathbf{A}^{(US)}$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}^{(W)}$, the estimated matrix of technical coefficients for

Washington State is $\mathbf{A}^{(est)} = \begin{bmatrix} .2078 & .0000 & .0013 & .0299 & .0002 & .0027 & .0013 \\ .0001 & .0099 & .0005 & .0022 & .0031 & .0000 & .0005 \\ .0061 & .0004 & .0025 & .0032 & .0109 & .0177 & .0571 \\ .0526 & .0362 & .0883 & .0836 & .0246 & .0243 & .0456 \\ .0534 & .0415 & .0664 & .0508 & .0694 & .0273 & .0490 \\ .0493 & .1151 & .0852 & .0589 & .1561 & .2070 & .1531 \\ .0016 & .0028 & .0019 & .0029 & .0057 & .0044 & .0059 \end{bmatrix}$.

The mean absolute deviation between the estimated and actual Washington State matrices of technical coefficients is $MAD = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(est)} - a_{ij}^{(W)}\right| = .0098$.

PROBLEM 8.11

The exogenously specified technical coefficients are given by

$\mathbf{A}^{(exog)} = \begin{bmatrix} .1154 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1207 & 0 & 0 & .1637 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. The modified vectors of total intermediate inputs and outputs for the real Washington State table are found as: $\mathbf{u}(1) = \mathbf{x}^{(W)} - \mathbf{Z}^{(W)}\mathbf{i} - \mathbf{A}^{(exog)}\mathbf{x}^{(W)} =$

$\begin{bmatrix} 3359.7 & 369.4 & 3140.1 & 12737.6 & 12718 & 29359.6 & 1112.4 \end{bmatrix}'$

$\mathbf{v}(1) = \mathbf{x}^{(W)} - (\mathbf{i}'\mathbf{Z}^{(W)})' - (\mathbf{i}'\mathbf{A}^{(exog)}\hat{\mathbf{x}}^{(W)})' = \begin{bmatrix} 1963.5 & 49.6 & 4423 & 17945.8 & 6060.7 & 31052.5 & 1301.7 \end{bmatrix}'$

Applying RAS using $\mathbf{A}^{(US)}$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}^{(W)}$, the estimated matrix of technical coefficients for Washington State is $\mathbf{A}^{(est)} = \begin{bmatrix} .1154 & .0001 & .0017 & .0377 & .0002 & .0035 & .0017 \\ .0002 & .0096 & .0005 & .0023 & .0030 & .0000 & .0005 \\ .0096 & .0004 & .0025 & .0031 & .0100 & .0180 & .0576 \\ .0830 & .0337 & .0890 & .0809 & .0229 & .0248 & .0463 \\ .0850 & .0389 & .0675 & .0496 & .0649 & .0281 & .0502 \\ .0752 & .1207 & .0830 & .0551 & .1637 & .2044 & .1502 \\ .0026 & .0026 & .0020 & .0029 & .0053 & .0046 & .0060 \end{bmatrix}$. The mean absolute deviation between the estimated and actual Washington State matrices of technical coefficients is $MAD = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(est)} - a_{ij}^{(W)}\right| = .0066$. In this case the constrained RAS procedure improves the estimate.

PROBLEM 8.12

The new exogenously specified technical coefficients are given by

$$\mathbf{A}^{(exog)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0287 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0525 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .2224 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

. Compute the revised vectors of total intermediate inputs and outputs for the real Washington State table:

$$\mathbf{u}(1) = \mathbf{x}^{(W)} - \mathbf{Z}^{(W)}\mathbf{i} - \mathbf{A}^{(exog)}\mathbf{x}^{(W)} = \begin{bmatrix} 4245.9 & 369.4 & 3140.1 & 12720.9 & 8649.3 & 37827.6 & 1112.4 \end{bmatrix}'$$

$$\mathbf{v}(1) = \mathbf{x}^{(W)} - (\mathbf{i}'\mathbf{Z}^{(W)})' - (\mathbf{i}'\mathbf{A}^{(exog)}\hat{\mathbf{x}}^{(W)})' = \begin{bmatrix} 2849.7 & 103.1 & 4423 & 13877.1 & 15384.7 & 31052.5 & 375.5 \end{bmatrix}'$$

Apply RAS using $\mathbf{A}^{(US)}$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}^{(W)}$, the estimated matrix of technical coefficients for Washington State is

$$\mathbf{A}^{(est)} = \begin{bmatrix} .2088 & .0000 & .0013 & .0298 & .0002 & .0027 & .0007 \\ .0001 & .0104 & .0005 & .0022 & .0031 & .0000 & .0003 \\ .0063 & .0005 & .0026 & .0033 & .0113 & .0184 & .0329 \\ .0530 & .0287 & .0895 & .0834 & .0251 & .0247 & .0258 \\ .0527 & .0433 & .0659 & .0525 & .0692 & .0272 & .0271 \\ .0485 & .1200 & .0844 & .0575 & .1554 & .2059 & .2224 \\ .0016 & .0030 & .0019 & .0029 & .0058 & .0045 & .0033 \end{bmatrix}$$

. The mean absolute deviation between the estimated and actual Washington State matrices of technical coefficients is $MAD = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left| a_{ij}^{(est)} - a_{ij}^{(W)} \right| = .0077$—not as good an estimate as that obtained in problem 8.11. Finally, for the combined case, the new exogenously specified technical coefficients are given by

$$\mathbf{A}^{(exog)} = \begin{bmatrix} .1154 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0287 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0525 & 0 & 0 & 0 \\ 0 & .1207 & 0 & 0 & .1637 & 0 & .2224 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

. The vectors of total intermediate inputs and outputs for the real Washington State table are:

$\mathbf{u}(1) = \mathbf{x}^{(W)} - \mathbf{Z}^{(W)}\mathbf{i} - \mathbf{A}^{(exog)}\mathbf{x}^{(W)} = \begin{bmatrix} 3359.7 & 369.4 & 3140.1 & 12720.9 & 8649.3 & 28433.4 & 1112.4 \end{bmatrix}'$

$\mathbf{v}(1) = \mathbf{x}^{(W)} - (\mathbf{i}'\mathbf{Z}^{(W)})' - (\mathbf{i}'\mathbf{A}^{(exog)}\hat{\mathbf{x}}^{(W)})' = \begin{bmatrix} 1963.5 & 32.9 & 4423 & 13877.1 & 6060.7 & 31052.5 & 375.5 \end{bmatrix}'$

Apply RAS using $\mathbf{A}^{(US)}$, $\mathbf{u}(1)$, $\mathbf{v}(1)$ and $\mathbf{x}^{(W)}$, the estimated matrix of technical coefficients for Washington State is $\mathbf{A}^{(est)} = \begin{bmatrix} .1154 & .0001 & .0018 & .0376 & .0002 & .0036 & .0010 \\ .0002 & .0108 & .0005 & .0022 & .0031 & .0000 & .0003 \\ .0100 & .0005 & .0026 & .0031 & .0104 & .0187 & .0327 \\ .0847 & .0287 & .0908 & .0802 & .0234 & .0254 & .0259 \\ .0835 & .0423 & .0663 & .0525 & .0639 & .0277 & .0270 \\ .0746 & .1207 & .0822 & .0531 & .1637 & .2033 & .2224 \\ .0026 & .0030 & .0020 & .0028 & .0054 & .0047 & .0034 \end{bmatrix}$. The mean absolute deviation between the estimated and actual Washington State matrices of technical coefficients is $MAD = (\frac{1}{49})\sum_{i=1}^{7}\sum_{j=1}^{7}\left|a_{ij}^{(est)} - a_{ij}^{(W)}\right| = .0044$—better than either of the previous cases.

Solutions to Chapter 9 Problems

PROBLEM 9.1

(a) (1) $\mathbf{Q}=\begin{bmatrix}0 & 1 & 0\\ .5 & .5 & .5\end{bmatrix}$. (2) $\mathbf{D}=\begin{bmatrix}0 & .667 & 0\\ .05 & .033 & .05\end{bmatrix}$. (3) $\tilde{\mathbf{Q}}=\begin{bmatrix}0 & 0 & 0\\ 0 & .708 & 0\end{bmatrix}$ so

$\boldsymbol{\varepsilon}=\mathbf{D}(\mathbf{I}-\mathbf{A})^{-1}+\tilde{\mathbf{Q}}=\begin{bmatrix}.077 & .769 & .077\\ .058 & .785 & .058\end{bmatrix}$, for $\mathbf{A}=\begin{bmatrix}0 & .667 & 0\\ .10 & .067 & .1\\ 0 & 0 & 0\end{bmatrix}$ and

$\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}1.077 & .769 & .077\\ .115 & 1.154 & .115\\ 0 & 0 & 1\end{bmatrix}$. The energy conservation conditions do not hold.

(b) $\mathbf{Z}^*=\begin{bmatrix}0 & 20 & 0\\ 1 & 1 & 1\\ 0 & 0 & 0\end{bmatrix}$, $\mathbf{f}^*=\begin{bmatrix}0\\ 17\\ 20\end{bmatrix}$ and $\mathbf{x}^*=\begin{bmatrix}20\\ 20\\ 20\end{bmatrix}$ so $\mathbf{A}^*=\begin{bmatrix}0 & 1 & 0\\ .05 & .05 & .05\\ 0 & 0 & 0\end{bmatrix}$ and

$\mathbf{L}^*=(\mathbf{I}-\mathbf{A}^*)^{-1}=\begin{bmatrix}1.056 & 1.111 & 0.056\\ 0.056 & 1.111 & 0.056\\ 0 & 0 & 1\end{bmatrix}$, which is easy to see conforms to energy conservation conditions as defined in the text.

PROBLEM 9.2

(a) $\boldsymbol{\varepsilon}=\mathbf{DL}=\begin{bmatrix}.225 & .336\\ .129 & .440\end{bmatrix}$ for $\mathbf{A}=\begin{bmatrix}.02 & .04\\ .06 & .08\end{bmatrix}$ and $\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix}1.023 & .044\\ .067 & 1.09\end{bmatrix}$.

(b) $\Delta\mathbf{g}=\mathbf{DL}\Delta\mathbf{f}=\begin{bmatrix}28.5\\ 19.8\end{bmatrix}$ and $\Delta\mathbf{g}^{direct}=\mathbf{D}\Delta\mathbf{f}=\begin{bmatrix}25.4\\ 16.2\end{bmatrix}$ so we can calculate

$\Delta\mathbf{g}^{indirect}=\Delta\mathbf{g}-\Delta\mathbf{g}^{direct}=\begin{bmatrix}3.1\\ 3.6\end{bmatrix}$ for $\Delta\mathbf{f}=\mathbf{f}^{new}-\mathbf{f}=\begin{bmatrix}200\\ 100\end{bmatrix}-\begin{bmatrix}94\\ 86\end{bmatrix}=\begin{bmatrix}106\\ 14\end{bmatrix}$. (c) $\mathbf{D}=\begin{bmatrix}.2 & .3\\ .1 & .3\end{bmatrix}$

so $\mathbf{DL}=\begin{bmatrix}.225 & .336\\ .122 & .331\end{bmatrix}$ and, hence, $\Delta\mathbf{g}=\mathbf{DL}\Delta\mathbf{f}=\begin{bmatrix}28.5\\ 17.6\end{bmatrix}$ and $\Delta\mathbf{g}^{direct}=\mathbf{D}\Delta\mathbf{f}=\begin{bmatrix}25.4\\ 14.8\end{bmatrix}$ so

$\Delta\mathbf{g}^{indirect}=\Delta\mathbf{g}-\Delta\mathbf{g}^{direct}=\begin{bmatrix}3.1\\ 2.8\end{bmatrix}$. Hence, the differences in total energy requirements between parts (c) and (b) are given by $\begin{bmatrix}28.5\\ 19.8\end{bmatrix}-\begin{bmatrix}28.5\\ 17.6\end{bmatrix}=\begin{bmatrix}0\\ 2.2\end{bmatrix}$. The differences in direct energy requirements are given by $\begin{bmatrix} 25.4 \\ 16.2 \end{bmatrix} - \begin{bmatrix} 25.4 \\ 14.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}$, and the differences in indirect energy requirements are given by $\begin{bmatrix} 3.1 \\ 3.6 \end{bmatrix} - \begin{bmatrix} 3.1 \\ 2.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}$.

PROBLEM 9.3

The reference final demand vector is found by $\mathbf{f} = \mathbf{x} - \mathbf{Zi} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} - \begin{bmatrix} 9 \\ 20 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 24 \end{bmatrix}$ for $\mathbf{Z} = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 0 & 20 \\ 3 & 2 & 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$. From the table of prices we have $\tilde{\mathbf{Q}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1.292 \end{bmatrix}$ and $\mathbf{P} = \begin{bmatrix} 0 & 0 & 0.408 \\ 0.333 & 0.286 & 0.5 \end{bmatrix}$. (a) We can compute the energy flows as $\mathbf{E} = \begin{bmatrix} 0 & 0 & 49 \\ 9 & 7 & 2 \end{bmatrix}$ where $e_{kj} = z_{kj} / p_{kj}$ for $k = 1, 2$ and $j = 1, 2, 3$ and $\mathbf{q} = \tilde{\mathbf{Q}}\mathbf{f} = \begin{bmatrix} 0 \\ 31 \end{bmatrix}$ and $\mathbf{g} = \mathbf{Ei} + \mathbf{q} = \begin{bmatrix} 49 \\ 49 \end{bmatrix}$. We can express the energy flows as the energy rows in a hybrid units transactions matrix and corresponding vectors of final demands and total outputs: $\mathbf{Z}^* = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 0 & 49 \\ 9 & 7 & 2 \end{bmatrix}$, $\mathbf{f}^* = \begin{bmatrix} 1 \\ 0 \\ 31 \end{bmatrix}$ and $\mathbf{x}^* = \begin{bmatrix} 10 \\ 49 \\ 49 \end{bmatrix}$ and

$\mathbf{A}^* = \mathbf{Z}^*(\hat{\mathbf{x}}^*)^{-1} = \begin{bmatrix} .2 & .122 & .02 \\ 0 & 0 & 1 \\ .9 & .143 & .041 \end{bmatrix}$. (b) For this economy $\mathbf{G} = \begin{bmatrix} 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix}$ so the direct energy coefficients are found as $\mathbf{G}(\hat{\mathbf{x}}^*)^{-1}\mathbf{A}^* = \begin{bmatrix} 0 & 0 & 1 \\ .9 & .143 & .041 \end{bmatrix}$. (c) $(\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{bmatrix} 1.556 & 0.229 & 0.272 \\ 1.716 & 1.428 & 1.525 \\ 1.716 & 0.428 & 1.525 \end{bmatrix}$

and hence $\boldsymbol{\alpha} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{bmatrix} 1.716 & 1.428 & 1.525 \\ 1.716 & 0.428 & 1.525 \end{bmatrix}$. For $\Delta\mathbf{f}^* = \begin{bmatrix} 2 \\ 0 \\ 18 \end{bmatrix}$ we compute total energy as $\Delta\mathbf{g} = \boldsymbol{\alpha}\Delta\mathbf{f}^* = \begin{bmatrix} 30.887 \\ 30.887 \end{bmatrix}$. (d) Using the energy prices defined above for final demand we can compute $\Delta\mathbf{f} = \begin{bmatrix} 2 \\ 0 \\ 23.24 \end{bmatrix}$ and hence $\Delta\mathbf{g} = \boldsymbol{\varepsilon}\ \Delta\mathbf{f} = \begin{bmatrix} 48.23 \\ 450.86 \end{bmatrix}$ where $\mathbf{A} = \begin{bmatrix} .2 & .3 & .033 \\ 0 & 0 & .667 \\ .3 & .1 & .033 \end{bmatrix}$,

$\mathbf{L} = \begin{bmatrix} 1.385 & .451 & .359 \\ .308 & 1.174 & .821 \\ .462 & .262 & 1.231 \end{bmatrix}$ and $\boldsymbol{\varepsilon} = \mathbf{DL} + \mathbf{G} = \begin{bmatrix} .754 & .427 & 2.010 \\ 1.385 & .835 & 1.984 \end{bmatrix}$.

PROBLEM 9.4

(a) $\boldsymbol{\alpha}\hat{\mathbf{x}}^* = \boldsymbol{\alpha}\mathbf{Z}^* + \mathbf{G}$ and $\mathbf{Z}^* = \mathbf{A}^*\hat{\mathbf{x}}^*$, so $\boldsymbol{\alpha}\hat{\mathbf{x}}^* = \boldsymbol{\alpha}\mathbf{A}^*\hat{\mathbf{x}}^* + \mathbf{G}$. Rearranging, $\boldsymbol{\alpha}(\mathbf{I} - \mathbf{A}^*)\hat{\mathbf{x}}^* = \mathbf{G}$ or $\boldsymbol{\alpha} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I} - \mathbf{A}^*)^{-1}$. (b) The first matrix satisfies the energy conservation conditions, since $\boldsymbol{\alpha}_{ref.pet} + \boldsymbol{\alpha}_{elec.} = \boldsymbol{\alpha}_{crude} = \begin{bmatrix} .6 & .5 & .3 \end{bmatrix}$. The second matrix fails to satisfy the energy conservation conditions, since $\boldsymbol{\alpha}_{ref.pet} + \boldsymbol{\alpha}_{elec.} = \begin{bmatrix} .6 & .2 & .2 \end{bmatrix} \neq \boldsymbol{\alpha}_{crude}$.

PROBLEM 9.5

(a) $\mathbf{g} = \mathbf{Ei} + \mathbf{q} = \begin{bmatrix} 40 \\ 40 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & .25 & 0 \\ .5 & .13 & .25 \\ 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{L} = \begin{bmatrix} 1.17 & .33 & .08 \\ .67 & 1.33 & .33 \\ 0 & 0 & 1 \end{bmatrix}$. Then

$\boldsymbol{\varepsilon} = \mathbf{DL} + \tilde{\mathbf{Q}} = \begin{bmatrix} .67 & 1.33 & .33 \\ .67 & .94 & .83 \end{bmatrix}$ where $\mathbf{D} = \mathbf{E}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ .5 & .13 & .75 \end{bmatrix}$ and $\tilde{\mathbf{Q}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}$. (b) For the hybrid units calculations, $\mathbf{Z}^* = \begin{bmatrix} 0 & 40 & 0 \\ 5 & 5 & 15 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathbf{x}^* = \begin{bmatrix} 40 \\ 40 \\ 20 \end{bmatrix}$, $\mathbf{G} = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \end{bmatrix}$ and

$\mathbf{A}^* = \begin{bmatrix} 0 & 1 & 0 \\ .13 & .13 & .75 \\ 0 & 0 & 0 \end{bmatrix}$, so $\boldsymbol{\alpha} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{bmatrix} 1.167 & 1.333 & 1.0 \\ .167 & 1.333 & 1.0 \end{bmatrix}$.

PROBLEM 9.6

Using $\mathbf{A}^* = \mathbf{Z}^*(\hat{\mathbf{x}}^*)^{-1}$ we can compute $\mathbf{A}^{*(0)} = \begin{bmatrix} 0 & 0 & .4 & 0 \\ 0 & 0 & .6 & 0 \\ .05 & .05 & .12 & .24 \\ .375 & .333 & .3 & .20 \end{bmatrix}$ and

$$(\mathbf{I}-\mathbf{A}^{*(0)})^{-1} = \begin{bmatrix} 1.1024 & .0945 & .6299 & .1890 \\ .1535 & 1.1417 & .9449 & .2835 \\ .2559 & .2362 & 1.5748 & .4724 \\ .6767 & .6086 & 1.2795 & 1.6339 \end{bmatrix}$$. The change in final demand can be written as $(\Delta\mathbf{f}^*)' = \begin{bmatrix} 0 & 0 & 0 & 200 \end{bmatrix}$ so the corresponding change in total energy consumption can be expressed as $\Delta\mathbf{g}^{(0)} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I}-\mathbf{A}^{*(0)})^{-1}\Delta\mathbf{f}^* = \begin{bmatrix} 18.2677 \\ 27.4016 \\ 0 \end{bmatrix}$ where $\mathbf{G}(\hat{\mathbf{x}}^*)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The total primary energy intensity is $\mathbf{i}'\Delta\mathbf{g}^{(0)} = 45.6693$.

PROBLEM 9.7

Using $\mathbf{A}^* = \mathbf{Z}^*(\hat{\mathbf{x}}^*)^{-1}$, for the alternative power generation technologies we can specify the corresponding matrices of technical coefficients and total requirements as

$$\mathbf{A}^{*(I)} = \begin{bmatrix} 0 & 0 & .2 & 0 \\ 0 & 0 & .7 & 0 \\ .05 & .05 & .1 & .24 \\ .375 & .333 & .4 & .20 \end{bmatrix}, \ (\mathbf{I}-\mathbf{A}^{*(I)})^{-1} = \begin{bmatrix} 1.0506 & .0467 & .3113 & .0934 \\ .1770 & 1.1634 & 1.0895 & .3268 \\ .2529 & .2335 & 1.5564 & .4669 \\ .6927 & .6234 & 1.3781 & 1.6634 \end{bmatrix},$$

$$\mathbf{A}^{*(II)} = \begin{bmatrix} 0 & 0 & .5 & 0 \\ 0 & 0 & .4 & 0 \\ .05 & .05 & .12 & .24 \\ .375 & .333 & .4 & .2 \end{bmatrix} \text{ and } (\mathbf{I}-\mathbf{A}^{*(II)})^{-1} = \begin{bmatrix} 1.1313 & .1212 & .8081 & .2424 \\ .1051 & 1.0970 & .6465 & .1939 \\ .2626 & .2424 & 1.6162 & .4848 \\ .7054 & .6351 & 1.4562 & 1.6869 \end{bmatrix}.$$

For $\Delta\mathbf{g}^{(0)} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I}-\mathbf{A}^{*(0)})^{-1}\Delta\mathbf{f}^*$, $\Delta\mathbf{g}^{(I)} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I}-\mathbf{A}^{*(I)})^{-1}\Delta\mathbf{f}^*$, and for $\Delta\mathbf{g}^{(II)} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}(\mathbf{I}-\mathbf{A}^{*(II)})^{-1}\Delta\mathbf{f}^*$, where $\mathbf{G}(\hat{\mathbf{x}}^*)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\Delta\mathbf{f}^* = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 30 \end{bmatrix}$, we can write

$\Delta\mathbf{g} = \begin{bmatrix} \Delta\mathbf{g}^{(0)} & \Delta\mathbf{g}^{(I)} & \Delta\mathbf{g}^{(II)} \end{bmatrix} = \left[\begin{array}{c|c|c} 18.2677 & 9.0272 & 23.4343 \\ 27.4016 & 31.5953 & 18.7475 \\ 0 & 0 & 0 \end{array}\right]$ which provides the total energy of each fuel type to support $\Delta\mathbf{f}^*$. The total primary energy intensity is given by $\mathbf{i}'(\Delta\mathbf{g}) = \begin{bmatrix} 45.6693 & 40.6226 & 42.1818 \end{bmatrix}$, so technology *I* consumes 1.5592 less primary energy than technology *II*. Both new technologies *I* and *II* are more efficient than the base technology.

PROBLEM 9.8

The technical coefficient matrix incorporating the new manufacturing technology is

$$\mathbf{A}^{*(new)} = \begin{bmatrix} 0 & 0 & .2 & 0 \\ 0 & 0 & .7 & 0 \\ .05 & .05 & .1 & .12 \\ .375 & .333 & .4 & .2 \end{bmatrix} \text{ and } \mathbf{L}^{*(new)} = \begin{bmatrix} 1.0580 & .0546 & .5461 & .0819 \\ .0870 & 1.0819 & .8191 & .1229 \\ .1451 & .1365 & 1.3652 & .2048 \\ .5866 & .5276 & 1.1092 & 1.4164 \end{bmatrix}$$

. So, for $\Delta\mathbf{g}^{(new)} = \mathbf{G}(\hat{\mathbf{x}}^*)^{-1}\mathbf{L}^{*(new)}\Delta\mathbf{f}^*$, we can write

$$\mathbf{i}'(\Delta\mathbf{g}) = \mathbf{i}'\left[\Delta\mathbf{g}^{(0} \mid \Delta\mathbf{g}^{(new)}\right] = \begin{bmatrix} 18.2677 & 13.3788 \\ 27.4016 & 20.0683 \\ 0.0000 & 0.0000 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 45.6693 & 33.4471 \end{bmatrix}$$

. Hence the primary energy saved by adopting the new technology is $45.6693 - 33.4471 = 12.2222$.

PROBLEM 9.9

From the original technical coefficients matrix, $\mathbf{A}^{*(0)}$, we can compute

$$(\mathbf{I} - \mathbf{A}^{*(0)}) = \begin{bmatrix} 1 & 0 & -.4 & 0 \\ 0 & 1 & -.6 & 0 \\ -.05 & -.05 & .88 & -.24 \\ -.375 & -.333 & -.3 & .8 \end{bmatrix}$$

. The GDP for the original economy can be found by

$GDP = \mathbf{i}'\tilde{\mathbf{Q}}\mathbf{f}^* = \mathbf{i}'\tilde{\mathbf{Q}}(\mathbf{I} - \mathbf{A}^{*(0)})\mathbf{x}^* = 105$ where (from Appendix 9.1) $\tilde{\mathbf{Q}} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ and

$(\mathbf{x}^*)' = \begin{bmatrix} 40 & 60 & 100 & 200 \end{bmatrix}$. For a 10% reduction in availability of oil, the vector of total outputs becomes $(\mathbf{x}^*)' = \begin{bmatrix} 36 & 60 & 100 & 200 \end{bmatrix}$. Hence we can compute the GDP as the sum of the corresponding final demand (measured in dollars) which we determine once again by $GDP = \mathbf{i}'\tilde{\mathbf{Q}}\mathbf{f}^* = \mathbf{i}'\tilde{\mathbf{Q}}(\mathbf{I} - \mathbf{A}^{*(0)})\mathbf{x}^* = 90.1$. The reduction in GDP due to the oil shortage is $105 - 90.2 = 14.9$. When the new technologies are incorporated into the technical coefficients matrix it becomes

$$\mathbf{A}^{*(new)} = \begin{bmatrix} 0 & 0 & .2 & 0 \\ 0 & 0 & .7 & 0 \\ .05 & .05 & .1 & .12 \\ .375 & .333 & .4 & .2 \end{bmatrix}$$

and

$$(\mathbf{I} - \mathbf{A}^{*(new)}) = \begin{bmatrix} 1 & 0 & -.2 & 0 \\ 0 & 1 & -.7 & 0 \\ -.05 & -.05 & .9 & -.12 \\ -.375 & -.333 & -.4 & .8 \end{bmatrix}$$

and, as before, we compute GDP by

$GDP = \mathbf{i}'\tilde{\mathbf{Q}}\mathbf{f}^* = \mathbf{i}'\tilde{\mathbf{Q}}(\mathbf{I} - \mathbf{A}^{*(new)})\mathbf{x}^* = 198.2$. This turns out to be not a reduction at all but an increase in GDP of 93.2.

PROBLEM 9.10

The transactions and total requirements matrices are found by $\mathbf{Z}^{*(80)} = \mathbf{A}^*\hat{\mathbf{x}}^{*(80)}$ and $(\mathbf{I} - \mathbf{A}^{*(80)})^{-1}$ for 1980 and similarly for 1963. Final demand is found by $\mathbf{f}^{*(80)} = \mathbf{x}^{*(80)} - \mathbf{A}^{*(80)}\mathbf{x}^{*(80)}$ and similarly for 1963. The total requirements matrices and vectors of final demand are

1980	1	2	3	4	5	6	7	8	9	Final Demand
1	1.0081	0.0059	0.016	1.718	0.0099	0.0003	0.0007	0.0002	0.0002	3,258
2	0.0164	1.0923	1.0933	1.0115	1.0301	0.0014	0.0015	0.0032	0.0006	-10,684
3	0.0115	0.0076	1.0851	0.4513	0.0106	0.001	0.0007	0.003	0.0004	10,461
4	0.0038	0.0035	0.0089	1.1062	0.006	0.0002	0.0003	0.0001	0.0001	3,155
5	0.0058	0.0578	0.098	0.6569	1.1341	0.0005	0.0008	0.0004	0.0003	4,066
6	0.7803	2.1173	3.4665	15.2493	2.6707	1.154	0.1361	0.0722	0.0559	3,596,887
7	5.0854	2.9969	8.6363	27.2464	3.696	0.5453	1.7127	0.1654	0.1614	7,804,130
8	0.479	0.3575	3.3772	9.3886	0.5332	0.0517	0.0613	1.1667	0.0288	925,557
9	3.5349	4.7573	10.3297	30.9091	6.5227	0.369	0.3197	0.2444	1.3022	15,022,410
1963										
1	1.0058	0.0026	0.0094	1.9521	0.0049	0.0004	0.0011	0.0002	0.0002	2,199
2	0.0056	1.0532	0.9444	1.0968	1.0861	0.0012	0.0021	0.0026	0.0007	-2,359
3	0.0032	0.0033	1.0732	0.2727	0.0094	0.0008	0.0008	0.0026	0.0004	8,630
4	0.0019	0.0011	0.0037	1.1145	0.0016	0.0001	0.0002	0.0001	0.0001	1,037
5	0.0025	0.0061	0.0483	0.8888	1.1087	0.0004	0.0009	0.0004	0.0003	3,540
6	0.2843	0.8465	2.0483	14.2093	1.88	1.1866	0.1853	0.0793	0.0749	2,820,771
7	1.6793	1.3446	4.2429	14.2964	2.1657	0.4915	1.7788	0.1478	0.1362	4,989,750
8	0.2025	0.1894	1.7661	7.6651	0.7549	0.0485	0.0604	1.1074	0.0233	456,425
9	0.8733	3.1616	6.1679	21.3339	5.0473	0.2267	0.2346	0.202	1.2401	6,933,979

If we denote the energy rows of $(\mathbf{I} - \mathbf{A}^{*(80)})^{-1}$ as $\boldsymbol{\alpha}^{80}$, the vector of total energy output as $\mathbf{g}^{80}$, and final demand as $\mathbf{f}^{80}$ (and now in all cases dropping the * for simplicity) with the analogous designations for 1963, we can compute the changes in energy consumption as

$$\mathbf{g}^{80} - \mathbf{g}^{63} = (\boldsymbol{\alpha}^{80} - \boldsymbol{\alpha}^{63})\mathbf{f}^{63} + \boldsymbol{\alpha}^{63}(\mathbf{f}^{80} - \mathbf{f}^{63}) + (\boldsymbol{\alpha}^{80} - \boldsymbol{\alpha}^{63})(\mathbf{f}^{80} - \mathbf{f}^{63}) = \begin{bmatrix} 6{,}121.4 \\ 6{,}457.3 \\ 11{,}337.2 \\ 4{,}698.8 \\ 6{,}049.6 \end{bmatrix}$$

where the effect caused by changing final demand is $\boldsymbol{\alpha}^{63}(\mathbf{f}^{80}-\mathbf{f}^{63})=\begin{bmatrix}10{,}467.2\\9{,}433.3\\9{,}967.9\\3{,}738.7\\8{,}021.1\end{bmatrix}$; the effect caused by changes in production functions is $(\boldsymbol{\alpha}^{80}-\boldsymbol{\alpha}^{63})\mathbf{f}^{63}=\begin{bmatrix}-2460.4\\-1257.5\\756.8\\613.3\\-502.4\end{bmatrix}$; and the effect of interaction of final demand and production function changes is $(\boldsymbol{\alpha}^{80}-\boldsymbol{\alpha}^{63})(\mathbf{f}^{80}-\mathbf{f}^{63})=\begin{bmatrix}-1885.4\\-1718.5\\612.5\\346.8\\-1469.1\end{bmatrix}$.

Solutions to Chapter 10 Problems

PROBLEM 10.1

(a) $\mathbf{A}=\begin{bmatrix} .1 & .3 \\ .5 & .1 \end{bmatrix}$ and so $\mathbf{L}=\begin{bmatrix} 1.364 & .455 \\ .758 & 1.364 \end{bmatrix}$. Forming $\mathbf{D}=\begin{bmatrix} \mathbf{D}^e \\ \hline \mathbf{D}^v \\ \hline \mathbf{D}^l \end{bmatrix}$, we find

$\mathbf{DL}=\begin{bmatrix} .288 & .318 \\ .5 & .5 \\ .652 & .773 \\ .500 & .500 \\ .652 & .773 \end{bmatrix}$. The four candidate projects are represented by the following matrix, the columns of which are the final-demand change vectors: $\Delta\mathbf{F}=\overbrace{\begin{bmatrix} 3 & 4.5 & 6 & 3 \\ 3 & 5.5 & 4 & 7 \end{bmatrix}}^{\textit{Projects}}$. Total allocated budget is given by $\mathbf{i}'[\Delta\mathbf{F}]=\begin{bmatrix} 6 & 10 & 10 & 10 \end{bmatrix}$; that is, all four candidate projects satisfy the budget constraint. Then the corresponding matrix of total outputs, where the columns are the vectors of total outputs corresponding to each scenario, is $\Delta\mathbf{X}^*=\mathbf{DL}\Delta\mathbf{F}=\begin{bmatrix} 1.8 & 3.0 & 3.0 & 3.1 \\ 3.0 & 5.0 & 5.0 & 5.0 \\ 4.3 & 7.2 & 7.0 & 7.4 \\ 3.0 & 5.0 & 5.0 & 5.0 \\ 4.3 & 7.2 & 7.0 & 7.4 \end{bmatrix}$.

Project 4, using 3.1×10^{15} Btus of oil, exceeds the 3.0 limit. (b) Project 2 should be chosen, since it maximizes employment, among the three feasible projects (from the bottom row of $\Delta\mathbf{X}^*$).

PROBLEM 10.2

Parts (a), (b) and (c) of this question can be answered simultaneously by finding $\mathbf{x}^*=\mathbf{DLf}^{new}$. Here $\mathbf{A}=\begin{bmatrix} .1 & .2 \\ .3 & .4 \end{bmatrix}$ and $\mathbf{L}=\begin{bmatrix} 1.250 & .417 \\ .625 & 1.875 \end{bmatrix}$. Following the same order in **D** as in the text in Table 10.2, we can form $\mathbf{D}=\begin{bmatrix} 1 & 0 \\ .05 & .2 \\ \hdashline 3 & 5 \\ 1 & 2 \\ \hdashline .002 & .002 \end{bmatrix}$. (It should be clear that one could list first the emissions, then the energy consumption, then employment; any other order would be equally acceptable.) In the present case, $\mathbf{x}^* = \begin{bmatrix} 29.167 \\ 12.708 \\ \hline 368.75 \\ 141.667 \\ \hline .171 \end{bmatrix}$. That is, $\mathbf{x}^{e*} = \begin{bmatrix} 29.167 \\ 12.708 \end{bmatrix}$ represents 29,167,000 tons of coal and 12,708,000 barrels of oil that will be consumed in production next year; $\mathbf{x}^{V*} = \begin{bmatrix} 368.75 \\ 141.667 \end{bmatrix}$ indicates that 368,750,000 pounds of SO_2 and 141,667,000 pounds of NO_x will be emitted because of that production; and $\mathbf{x}^{l*} = [0.171]$ represents the fact that 171,000 workers will be employed. Total output is found as $\mathbf{x}^{new} = \mathbf{L}\mathbf{f}^{new} = \begin{bmatrix} 29.167 \\ 56.25 \end{bmatrix}$; that is, $x_1^{new} = \$29{,}167{,}000$ and $x_2^{new} = \$56{,}250{,}000$.

PROBLEM 10.3

$\mathbf{L} = \begin{bmatrix} 1.247 & .598 & .839 \\ .857 & 2.494 & 1.665 \\ 0 & 0 & 1.111 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 4.2 & 7 & 9.1 \\ 7.6 & 2.6 & .5 \\ .73 & .33 & .63 \end{bmatrix}$, and $\Delta\mathbf{F} = \begin{bmatrix} 2 & 4 & 2 & 2 \\ 2 & 0 & 0 & 2 \\ 2 & 2 & 4 & 3 \end{bmatrix}$, with, as before, one column in $\Delta\mathbf{F}$ for each of the projects. (a) Project 4 contributes most to gross regional product, since $\mathbf{i}'[\Delta\mathbf{F}] = \begin{bmatrix} 6 & 6 & 6 & 7 \end{bmatrix}$. (b) $\Delta\mathbf{X}^* = \mathbf{DL}\Delta\mathbf{F} = \begin{bmatrix} 112.986 & 95.527 & 123.618 & 138.27 \\ 67.98 & 69.341 & 68.44 & 79.236 \\ 8.628 & 8.496 & 9.832 & 10.49 \end{bmatrix}$

Project 4 also consumes the most energy (79.236×10^6 bbls of oil). (c) Project 4 also contributes the most to regional employment (10.490×10^6 workers).

PROBLEM 10.4

From $\mathbf{Z}$ and $\mathbf{x}$ we compute $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .14 & .35 \\ .8 & .05 \end{bmatrix}$ and $\mathbf{L} = \begin{bmatrix} 1.769 & .652 \\ 1.49 & 1.601 \end{bmatrix}$ so that $\mathbf{D}(\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 1.276 & .996 \\ .113 & .115 \end{bmatrix}$. Therefore, for $\mathbf{f}^{new} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{x}^* = \begin{bmatrix} 1.276 \\ \hline .113 \end{bmatrix}$, meaning that for each new dollar's worth of final demand for the output of sector 1, there will be 1.276 pounds of pollutant emitted and 0.113 new workers. Similarly, with $\mathbf{f}^{new} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, we find $\mathbf{x}^* = \begin{bmatrix} .996 \\ \hline .115 \end{bmatrix}$, meaning that for each new dollar's worth of final demand for the output of sector 2, there will be 0.996 pounds of pollutant emitted and 0.115 new workers. (a) There would not be a conflict between unions and environmentalists; each dollar's worth of new demand for sector 2 generates less pollution and also generates more employment (notice that this is true despite the fact that sector 2's direct-pollution coefficient per dollar of output is larger than sector 1's direct-pollution coefficient). (b) If regulation of emissions was on only direct emissions, as is commonly the case, rather than total (direct plus indirect) emissions then there would be a conflict generated by increasing sector 2's output at the expense of sector 1, since sector 2's direct emission rate is higher.

PROBLEM 10.5

We can augment the transactions matrix with the pollution abatement and elimination data to yield $\bar{\mathbf{Z}} = \begin{bmatrix} 140 & 350 & 5 \\ 800 & 50 & 12 \\ 10 & 25 & 0 \end{bmatrix}$. Total pollution output is found by adding pollution generation in the interindustry matrix to the pollution tolerated (reflected as a negative value in final demand), so $x_p = 10 + 25 - 12 = 23$, which we can augment to the total industry outputs vector to yield $\bar{\mathbf{x}} = \begin{bmatrix} 2000 & 1850 & 23 \end{bmatrix}'$. Hence final demands are $\bar{\mathbf{f}} = \bar{\mathbf{x}} - \bar{\mathbf{Z}}\mathbf{i} = \begin{bmatrix} 1050 & 1425 & -12 \end{bmatrix}'$. For $\Delta\bar{\mathbf{f}} = \begin{bmatrix} 100 & 100 & 0 \end{bmatrix}'$ the changes in outputs and pollution are $\Delta\bar{\mathbf{x}} = \begin{bmatrix} 145.36 & 170 & 3.02 \end{bmatrix}'$ and hence the new levels of outputs and pollution are $\bar{\mathbf{x}}^{new} = \begin{bmatrix} 2145.36 & 2020 & 26.02 \end{bmatrix}'$.

PROBLEM 10.6

From the table of direct impact coefficients, $\mathbf{D} = \begin{bmatrix} 4.2 & 7 & 9.1 \\ 7.6 & 2.6 & .5 \\ 7.3 & 3.3 & 6.3 \end{bmatrix}$, so the baseline environmental, energy, and employment impacts are found by

$\mathbf{x}^{*N} = \mathbf{D}\mathbf{x}^N = \begin{bmatrix} 166{,}626{,}387.6 & 23{,}949{,}036.4 & 109{,}974{,}029.7 \end{bmatrix}'$ and

$\mathbf{x}^{*R} = \mathbf{D}\mathbf{x}^R = \begin{bmatrix} 2{,}256{,}140.7 & 396{,}313.8 & 1{,}450{,}654.2 \end{bmatrix}'$. Then

$$\Delta\bar{\mathbf{x}}^N = \begin{bmatrix} \mathbf{D}^{*N} \\ \hline (\mathbf{I}-\mathbf{A}^N)^{-1} \end{bmatrix} \Delta\mathbf{f}^N = \begin{bmatrix} 10.7832 & 16.2768 & 14.7759 \\ 10.4543 & 5.2368 & 1.3729 \\ 12.5176 & 9.4836 & 10.1120 \\ \hline 1.2516 & 0.1306 & 0.0287 \\ 0.2881 & 1.5256 & 0.1576 \\ 0.3857 & 0.5549 & 1.4892 \end{bmatrix} \begin{bmatrix} 250 \\ 3{,}000 \\ 7{,}000 \end{bmatrix} = \begin{bmatrix} 154{,}957.3 \\ 27{,}934.5 \\ 102{,}364.1 \\ \hline 906.1 \\ 5{,}752.2 \\ 12{,}185.3 \end{bmatrix}$$

$$\Delta \bar{\mathbf{x}}^R = \left[\frac{\mathbf{D}^{*R}}{(\mathbf{I}-\mathbf{A}^R)^{-1}}\right]\Delta \mathbf{f}^R = \begin{bmatrix} 7.9150 & 9.5207 & 12.4438 \\ 9.0188 & 3.2720 & 0.8721 \\ 10.2453 & 5.0627 & 8.5550 \\ \hline 1.1281 & 0.0409 & 0.0075 \\ 0.1223 & 1.1048 & 0.0601 \\ 0.2550 & 0.1775 & 1.3178 \end{bmatrix} \begin{bmatrix} 50 \\ 600 \\ 1{,}400 \end{bmatrix} = \begin{bmatrix} 23{,}529.5 \\ 3{,}635.2 \\ 15{,}527.0 \\ \hline 91.5 \\ 753.1 \\ 1{,}964.2 \end{bmatrix}$$

where $\Delta \mathbf{f}^R = .2\Delta \mathbf{f}^N$. Hence, the comparative percentage changes from $\mathbf{x}^{*(N)}$ and $\mathbf{x}^{*(R)}$ are:

	Nation	Region
Nat. Res.	0.09	1.04
Manuf.	0.12	0.92
Services	0.09	1.07
Pollution	0.17	1.11
Energy	0.12	0.79
Employ.	0.09	1.15

PROBLEM 10.7

The levels of pollution, energy consumption, and employment accompanying the baseline levels of total industry output are found by $\mathbf{x}^* = \mathbf{D}\mathbf{x} = \begin{bmatrix} 4.2 & 7 & 9.1 \\ 7.6 & 2.6 & .5 \\ 7.3 & 3.3 & 6.3 \end{bmatrix}\begin{bmatrix} 8{,}262.7 \\ 95{,}450.8 \\ 170{,}690.3 \end{bmatrix} = \begin{bmatrix} 2{,}256{,}140.7 \\ 396{,}313.8 \\ 1{,}450{,}654.2 \end{bmatrix}$. A ten percent reduction in energy availability defines

$\mathbf{x}^{*new} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .9 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2{,}256{,}140.7 \\ 396{,}313.8 \\ 1{,}450{,}654.2 \end{bmatrix} = \begin{bmatrix} 2{,}256{,}140.7 \\ 356{,}682.4 \\ 1{,}450{,}654.2 \end{bmatrix}$. Hence, this limits total industry output to

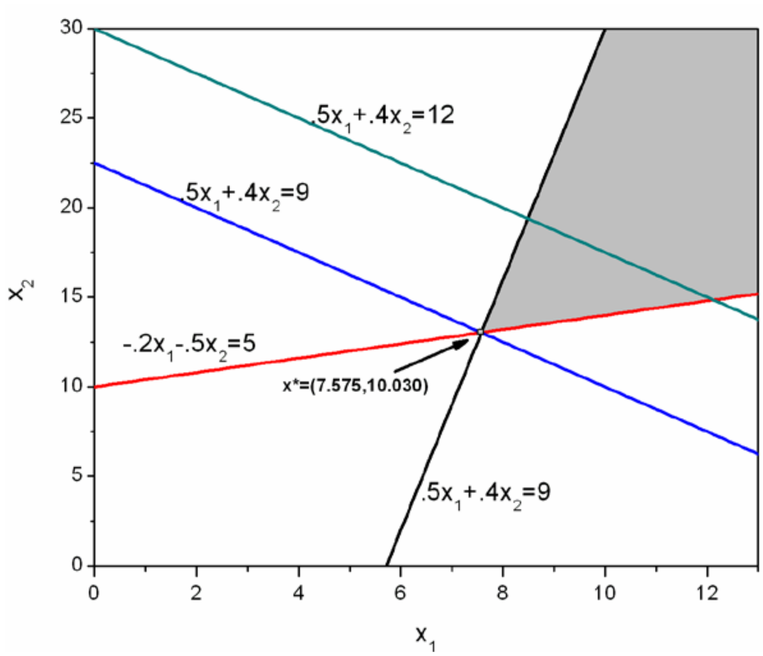
$\mathbf{x}^{new} = \mathbf{D}^{-1}\mathbf{x}^{*new} = \begin{bmatrix} -0.0766 & 0.0732 & 0.1049 \\ 0.2301 & 0.2079 & -0.3488 \\ -0.0317 & -0.1937 & 0.2199 \end{bmatrix}\begin{bmatrix} 2{,}256{,}140.7 \\ 356{,}682.4 \\ 1{,}450{,}654.2 \end{bmatrix} = \begin{bmatrix} 5{,}362.0 \\ 87{,}210.5 \\ 178{,}367.8 \end{bmatrix}$, which in turn means

$\mathbf{f}^{new} = (\mathbf{I}-\mathbf{A})\mathbf{x}^{new} = \begin{bmatrix} 0.8908 & -0.0324 & -0.0036 \\ -0.0899 & 0.9151 & -0.0412 \\ -0.1603 & -0.1170 & 0.7651 \end{bmatrix}\begin{bmatrix} 5{,}362.0 \\ 87{,}210.5 \\ 178{,}367.8 \end{bmatrix} = \begin{bmatrix} 1{,}308.7 \\ 71{,}975.5 \\ 125{,}406.0 \end{bmatrix}$. The change in GDP is $\mathbf{i}'\mathbf{f}^{new} - \mathbf{i}'\mathbf{f} = -2{,}637.7$ or a 1.31% reduction in GDP, where the original final demand is

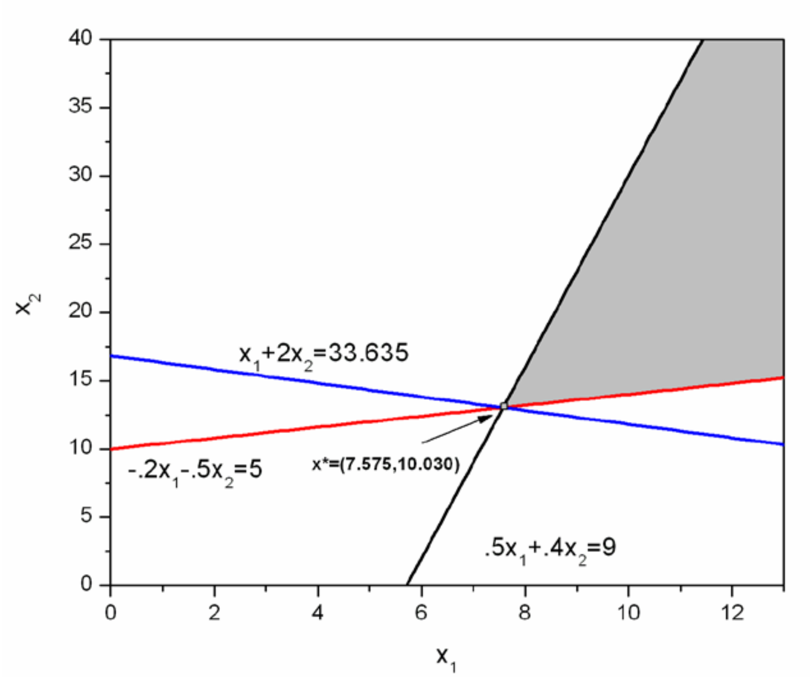
$\mathbf{f} = \mathbf{x} - \mathbf{A}\mathbf{x} = \begin{bmatrix} 3{,}653.3 \\ 79{,}571.8 \\ 118{,}102.9 \end{bmatrix}$.

PROBLEM 10.8

The linear programming (LP) formulation is $\begin{array}{l} Min\ \mathbf{v}'\mathbf{x} \\ (\mathbf{I}-\mathbf{A})\mathbf{x} \geq \mathbf{f} \\ \mathbf{x} \geq 0 \end{array}$ or $\begin{array}{l} Min\ .5x_1 + .4x_2 \\ .7x_1 - .1x_2 \geq 4 \\ -.2x_1 + .5x_2 \geq 5 \\ x_1, x_2 \geq 0 \end{array}$ where the graphical solution is



Note that it turns out that this LP problem has a "dual" formulation (discussed in Chapter 14) of maximizing the value of total final demand (or maximizing gross domestic product) subject to the technical coefficients and supply availability of value added factors, which may seem more intuitive. These dual LP problems have the same result such that maximized value of final demand equals the minimized cost of value added factors and that value is the gross domestic product, or the familiar equality of national product to national income. Applying the objective function to minimize pollution emissions, the solution is



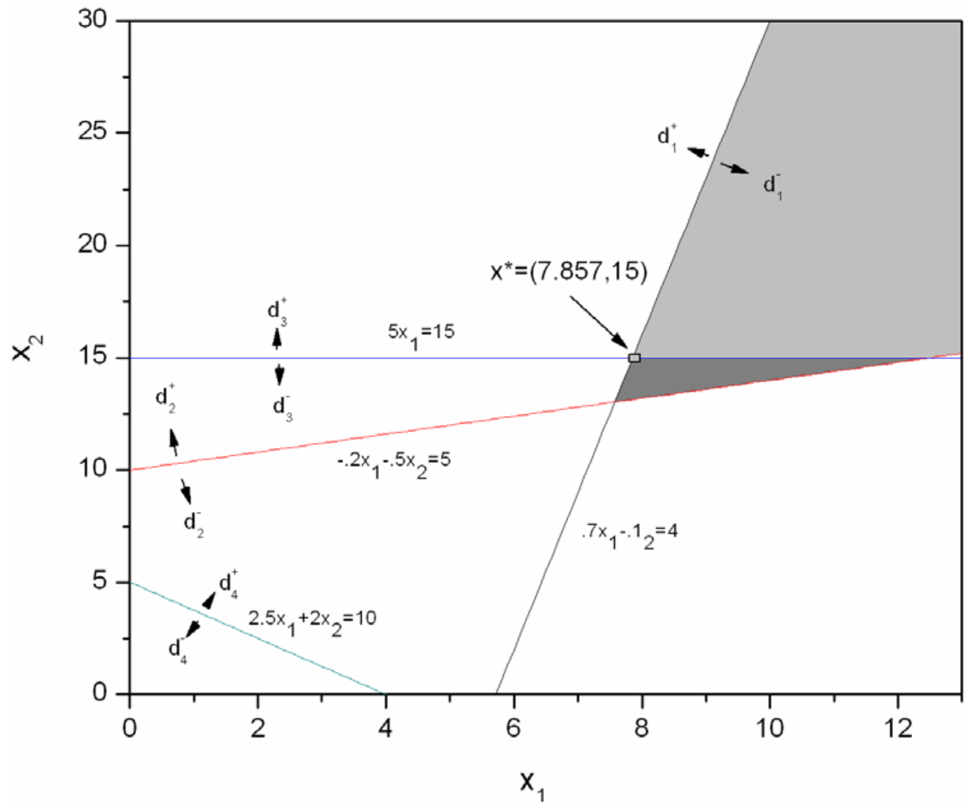
Note that the solution is the same as with the original objective function, which should not be surprising in this particular case since the minimum production levels defined in the constraints are already determined by the input-output relationships $(\mathbf{I}-\mathbf{A})\mathbf{x} \geq \mathbf{f}$.

PROBLEM 10.9

We can reformulate problem 10.8 as a goal programming problem:

$$
\begin{aligned}
Min\ \ & P_1(d_3^-) + P_2(d_1^- + d_2^-) + P_3(d_4^+) \\
& .7x_1 - .1x_2 + d_1^+ + d_1^- = 4 \\
& -.2x_1 - .5x_2 + d_2^+ + d_2^- = 5 \\
& .5x_2 + d_3^+ + d_3^- = 7.5 \\
& 2.5x_1 - 2x_2 + d_4^+ + d_4^- = 10
\end{aligned}
$$

Where the graphical solution, $\mathbf{x}^* = (7.857,15)$, is found by



PROBLEM 10.10

From Appendix B we can define $\mathbf{A}^{1997}$, $\mathbf{x}^{1997}$, $\mathbf{A}^{2005}$ and $\mathbf{x}^{2005}$. From these quantities we can compute $\mathbf{L}^{1997} = (\mathbf{I} - \mathbf{A}^{1997})^{-1}$ and $\mathbf{L}^{2005} = (\mathbf{I} - \mathbf{A}^{2005})^{-1}$ as well as $\mathbf{f}^{1997} = \mathbf{x}^{1997} - \mathbf{A}^{1997}\mathbf{x}^{1997}$ and $\mathbf{f}^{2005} = \mathbf{x}^{2005} - \mathbf{A}^{2005}\mathbf{x}^{2005}$. The corresponding pollution coefficients are $\mathbf{d}^{1997} = [2 \quad 3 \quad 4 \quad 7 \quad 10 \quad 5 \quad 4]'$ and $\mathbf{d}^{2005} = [2 \quad 3 \quad 3.4 \quad 6.3 \quad 10 \quad 5 \quad 4]'$ (reflecting emissions reduction technologies). Hence the total pollution impacts are $\Delta p = \mathbf{i}'\left[\mathbf{T}^{2005}\mathbf{f}^{2005} - \mathbf{T}^{1997}\mathbf{f}^{1997}\right] = 40{,}956{,}897$ where $\mathbf{T}^{2005} = \left[\mathbf{d}^{2005}\right]'\mathbf{L}^{2005}$ and $\mathbf{T}^{1997} = \left[\mathbf{d}^{1997}\right]'\mathbf{L}^{1997}$ or equivalently $\left[\mathbf{d}^{2005}\right]'\mathbf{x}^{2005} - \left[\mathbf{d}^{1997}\right]'\mathbf{x}^{1997}$. In this case the decrease in pollution coefficients was offset by growth in output levels so the net result was an increase in pollution impacts.

Solutions to Chapter 11 Problems

PROBLEM 11.1

In order to produce a balanced set of SAM accounts, X = 43. The SAM is

	Prod	Consumers	Capital	ROW	Total
Producers	0	552	65	**43**	660
Consumers	608	0	-25	17	600
Capital	0	40	0	0	40
Rest of World	52	8	0	0	60
	660	600	40	60	

The SAM reflecting allocation of 150 of consumer demand to final use and 150 of consumer inputs to valued added can be reflected by

	Prod	Intermed. Consumers	Final Consumers	Capital	ROW	Total
Producers	0	402	150	65	43	660
Intermed. Consumers	608	0	0	-25	17	600
Final Consumers	0	150	0	0	0	150
Capital	0	40	0	0	0	40
Rest of World	52	8	0	0	0	60
	660	600	150	40	60	

PROBLEM 11.2

		Production		Consumption		Cap.	ROW	Total	
		Manuf.	Services	Manuf.	Services				
Production	Manuf.	0	0	158	96	28	18	300	660
	Services	0	0	94	203	37	25	360	
Consumption	Manuf.	284	0	0	0	-12	13	285	600
	Services	5	319	0	0	-13	4	315	
Capital		0	0	20	20	0	0	40	
ROW		10	41	3	5	0	0	60	
Total		300	360	276	324	40	60		
		660		600					

PROBLEM 11.3

		Commodities		Industries		Total Final Demand			Total	
		Manuf.	Services	Manuf.	Services	PCE	Cap.	Exports		
Commodities	Manuf.	0	0	94	96	64	28	18	300	660
	Services	0	0	94	117	86	37	25	360	
Industrieis	Manuf.	284	0	0	0	0	-12	13	285	600
	Services	5	319	0	0	0	-13	4	315	
Value Added	Consumer	0	0	73	77	0	0	0	150	250
	Capital	0	0	20	20	0	0	0	40	
	Imports	10	41	4	5	0	0	0	60	
Total		300	360	285	315	150	40	60		
		660		600		250				

PROBLEM 11.4

(a) The matrix of total expenditure shares is $\bar{\mathbf{S}} = \begin{bmatrix} 0 & 0 & .331 & .305 & .427 & .7 & .3 \\ 0 & 0 & .331 & .371 & .573 & .925 & .417 \\ .95 & 0 & 0 & 0 & 0 & -.3 & .217 \\ .017 & .884 & 0 & 0 & 0 & -.325 & .067 \\ 0 & 0 & .257 & .244 & 0 & 0 & 0 \\ 0 & 0 & .07 & .063 & 0 & 0 & 0 \\ .033 & .116 & .011 & .016 & 0 & 0 & 0 \end{bmatrix}$

. (b) The SAM coefficient matrix is simply $\mathbf{S} = \begin{bmatrix} 0 & 0 & .331 & .305 \\ 0 & 0 & .331 & .371 \\ .95 & 0 & 0 & 0 \\ .017 & .884 & 0 & 0 \end{bmatrix}$, the upper left partition of $\bar{\mathbf{S}}$. (c) The matrix of "direct effect" multipliers is

$$\mathbf{M} = (\mathbf{I} - \mathbf{S})^{-1} = \begin{bmatrix} 1.812 & .726 & .840 & .822 \\ .865 & 1.835 & .894 & .945 \\ 1.721 & .690 & 1.798 & .781 \\ .794 & 1.634 & .804 & 1.849 \end{bmatrix}.$$

PROBLEM 11.5

First reorder the table so that the Surplus/Deficit and Rest of World sectors become sectors 5 and 6 in the table by $\mathbf{Z} = \mathbf{S}\bar{\mathbf{Z}}\mathbf{S}'$ where $\bar{\mathbf{Z}}$ is the original SAM and $\mathbf{S}$ is given by

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

. The direct, indirect, cross and total multipliers, respectively, are given

by

$$\mathbf{N}_1 = \begin{bmatrix} 5.7286 & 4.7756 & 4.7756 & 5.2105 & 0.0000 & 0.0000 \\ 7.2309 & 7.3029 & 7.3029 & 6.6610 & 0.0000 & 0.0000 \\ 0.5550 & 0.5605 & 1.5605 & 0.5772 & 0.0000 & 0.0000 \\ 7.4538 & 7.5279 & 7.5279 & 8.2133 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix},$$

$$\mathbf{N}_2 = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 26.9953 & 5.9390 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 34.7835 & 7.2176 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 3.2188 & 0.5400 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 42.5529 & 9.6888 \\ -0.2506 & -0.2511 & -0.2511 & -0.2472 & 0.0000 & 0.0000 \\ 1.5382 & 1.5302 & 1.5302 & 1.6014 & 0.0000 & 0.0000 \end{bmatrix},$$

$$\mathbf{N}_3 = \begin{bmatrix} 1.5617 & 1.5182 & 1.5182 & 1.9068 & 0.0000 & 0.0000 \\ 1.4871 & 1.4301 & 1.4301 & 1.9388 & 0.0000 & 0.0000 \\ -0.0168 & -0.0224 & -0.0224 & 0.0271 & 0.0000 & 0.0000 \\ 2.8569 & 2.7889 & 2.7889 & 3.3954 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -1.0000 & -0.2075 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 6.2000 & 1.3405 \end{bmatrix}$$

and

$$\mathbf{N}_T = \begin{bmatrix} 7.2903 & 6.2938 & 6.2938 & 7.1173 & 26.9953 & 5.9390 \\ 8.7180 & 8.7329 & 8.7329 & 8.5997 & 34.7835 & 7.2176 \\ 0.5382 & 0.5382 & 1.5382 & 0.6043 & 3.2188 & 0.5400 \\ 10.3106 & 10.3168 & 10.3168 & 11.6087 & 42.5529 & 9.6888 \\ -0.2506 & -0.2511 & -0.2511 & -0.2472 & 0.0000 & -0.2075 \\ 1.5382 & 1.5302 & 1.5302 & 1.6014 & 6.2000 & 2.3405 \end{bmatrix}.$$

PROBLEM 11.6

The resulting balanced SAM using RAS is:

	Prod.	Cons.	Capital	ROW	Totals
Producers|	0	560	40	60	660
Consumers	600	0	0	0	600
Capital	0	40	0	0	40
Rest of World	60	0	0	0	60
Totals	660	600	40	60	1360

PROBLEM 11.7

The resulting balanced SAM using RAS with the additional fixed cells is the following (which should look familiar—see problem 11.1).

	Prod.	Cons.	Capital	ROW	Totals
Producers|	0	550	65	45	660
Consumers	610	0	-25	15	600
Capital	0	40	0	0	40
Rest of World	50	10	0	0	60
Totals	660	600	40	60	1360

PROBLEM 11.8

The direct, indirect, cross, and total multipliers, respectively, are given by

$$\mathbf{M}_1 = \left[\begin{array}{ccccc|c} 1 & .897 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & \\ .602 & .54 & 1 & 0 & 0 & \mathbf{0} \\ .322 & .289 & 0 & 1 & 0 & \\ .306 & .274 & 0 & .95 & 1 & \\ \hline \multicolumn{5}{c|}{\mathbf{0}} & \mathbf{I} \end{array}\right],$$

$$\mathbf{M}_2 = \left[\begin{array}{cccccc|cccccc} & & & & & & .714 & .524 & .795 & .581 & 0 & 0 \\ & & & & & & .796 & .584 & .887 & .648 & 0 & 0 \\ \multicolumn{6}{c|}{\mathbf{I}} & .43 & .315 & .479 & .35 & 0 & 0 \\ & & & & & & .23 & .169 & .256 & .363 & 0 & 0 \\ & & & & & & .24 & .216 & .243 & .345 & 0 & 0 \\ \hline 0 & 0 & .847 & 0 & .588 & 0 & & & & & & \\ .078 & 0 & .153 & 0 & .078 & 0 & & & & & & \\ 0 & 0 & 0 & 0 & .334 & 0 & & & & & & \\ 0 & .10 & 0 & .05 & 0 & 0 & \multicolumn{6}{c}{\mathbf{I}} \\ 0 & .003 & 0 & 0 & 0 & 0 & & & & & & \\ -.002 & 0 & 0 & 0 & 0 & 0 & & & & & & \end{array}\right],$$

$$\mathbf{M}_3 = \left[\begin{array}{ccccc|cccccc}
1.206 & .331 & 3.296 & .167 & 3.455 & & & & & & \\
.23 & 1.369 & 3.675 & .186 & 3.851 & & & & & & \\
.124 & .199 & 2.984 & .1 & 2.079 & & & \mathbf{0} & & & \\
.071 & .132 & 1.136 & 1.066 & 1.191 & & & & & & \\
.078 & .135 & 1.2 & .068 & 2.246 & & & & & & \\
\hline
 & & & & & 3.469 & 1.931 & 2.669 & 2.488 & 0 & 0 \\
 & & & & & .677 & 1.526 & .734 & .67 & 0 & 0 \\
 & & \mathbf{0} & & & .413 & .333 & 1.441 & .452 & 0 & 0 \\
 & & & & & .439 & .339 & .477 & 1.436 & 0 & 0 \\
 & & & & & .013 & .01 & .014 & .012 & 1 & 0 \\
 & & & & & -.007 & -.005 & -.008 & -.007 & 0 & 1
\end{array}\right] \text{ and}$$

$$\mathbf{M} = \left[\begin{array}{ccccc|cccccc}
4.301 & 4.189 & 3.296 & 3.448 & 3.455 & 3.415 & 2.640 & 3.713 & 3.321 & 0 & 0 \\
3.68 & 4.67 & 3.675 & 3.844 & 3.851 & 3.807 & 2.943 & 4.14 & 3.703 & 0 & 0 \\
2.589 & 2.521 & 2.984 & 2.075 & 2.079 & 2.056 & 1.589 & 2.235 & 1.999 & 0 & 0 \\
1.463 & 1.444 & 1.136 & 2.197 & 1.191 & 1.177 & .91 & 1.28 & 1.322 & 0 & 0 \\
1.509 & 1.488 & 1.2 & 2.201 & 2.246 & 1.237 & .995 & 1.319 & 1.352 & 0 & 0 \\
\hline
3.080 & 3.011 & 3.233 & 3.052 & 3.082 & 3.469 & 1.931 & 2.669 & 2.488 & 0 & 0 \\
.849 & .828 & .807 & .758 & .762 & .677 & 1.526 & .734 & .67 & 0 & 0 \\
.504 & .497 & .401 & .736 & .751 & .413 & .333 & 1.441 & .452 & 0 & 0 \\
.44 & .538 & .423 & .494 & .444 & .439 & .339 & .477 & 1.436 & 0 & 0 \\
.012 & .016 & .012 & .013 & .013 & .013 & .01 & .014 & .012 & 1 & 0 \\
-.009 & -.009 & -.007 & -.007 & -.007 & -0.007 & -0.005 & -0.008 & -0.007 & 0 & 1
\end{array}\right]$$

PROBLEM 11.9

The direct multipliers in the additive form are the same as those in the multiplicative form, i.e., $\mathbf{M}_1 = \mathbf{N}_1$, which will always be the case as discussed in section 11.10.4.

PROBLEM 11.10

$$\mathbf{i}'\mathbf{M} = [3.245 \quad 3.053 \quad 3.38 \quad 3.647 \quad 3.581 \quad 2.949 \quad 2.769 \quad 2.588 \quad 2.645 \quad 1 \quad 1 \quad 1 \quad 3.302 \quad 2.691 \quad 4 \quad 3.16 \quad 1 \quad 1]$$

Solutions to Chapter 12 Problems

PROBLEM 12.1

(a) $\mathbf{G}=(\mathbf{I}-\mathbf{B})^{-1}=\begin{bmatrix}1.468 & .455 & .558 & .692\\ .376 & 1.393 & .384 & .418\\ .253 & .155 & 1.300 & .375\\ .336 & .268 & .399 & 1.466\end{bmatrix}$ for $\mathbf{B}=\hat{\mathbf{x}}^{-1}\mathbf{Z}=\begin{bmatrix}.168 & .194 & .213 & .283\\ .155 & .193 & .134 & .123\\ .105 & .031 & .126 & .165\\ .134 & .095 & .164 & .186\end{bmatrix}$. (b) Since $GDP=\mathbf{i}'\mathbf{f}=\mathbf{v}'\mathbf{i}$, $GDP=\mathbf{v}'\mathbf{i}=21{,}246$ for $(\mathbf{v}^{new})'=\begin{bmatrix}4{,}558 & 5{,}665 & 2{,}050 & 5{,}079\end{bmatrix}$. (c) For $\mathbf{x}^{new}=(\mathbf{I}-\mathbf{B}')^{-1}\mathbf{v}^{new}$, $\mathbf{x}^{new}=\begin{bmatrix}13{,}928.5 & 12{,}518.4 & 6{,}606.6 & 11{,}313.2\end{bmatrix}'$

PROBLEM 12.2

(a) $\mathbf{B}=\hat{\mathbf{x}}^{-1}\mathbf{Z}=\begin{bmatrix}.049 & .285 & .171\\ .195 & .077 & .176\\ .150 & .152 & .208\end{bmatrix}$ and, since $\mathbf{Z}^{new}=\mathbf{Z}+\Delta\mathbf{Z}=\begin{bmatrix}13 & 80 & 45\\ 63 & 21 & 48\\ 67 & 68 & 108\end{bmatrix}$ and $\mathbf{x}^{new}=\mathbf{f}+\mathbf{Z}^{new}\mathbf{i}=\begin{bmatrix}466.6\\ 537.4\\ 950.6\end{bmatrix}$, so $\mathbf{B}^{new}=(\hat{\mathbf{x}}^{new})^{-1}\mathbf{Z}^{new}=\begin{bmatrix}.028 & .171 & .096\\ .117 & .039 & .089\\ .070 & .072 & .114\end{bmatrix}$. The mean absolute percentage deviation [$MAPD=(1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n}[\left|b_{ij}-b_{ij}^{new}\right|/b_{ij}]\times 100$] between $\mathbf{B}$ and $\mathbf{B}^{new}$ is 46.3125. (b) For $\mathbf{G}=(\mathbf{I}-\mathbf{B})^{-1}=\begin{bmatrix}1.195 & .427 & .353\\ .307 & 1.235 & .341\\ .284 & .317 & 1.394\end{bmatrix}$ and $\mathbf{G}^{new}=(\mathbf{I}-\mathbf{B}^{new})^{-1}=\begin{bmatrix}1.063 & .2 & .136\\ .139 & 1.075 & .123\\ .096 & .103 & 1.149\end{bmatrix}$, the MAPD between $\mathbf{G}$ and $\mathbf{G}^{new}$ is 45.4684.

PROBLEM 12.3

For $\mathbf{B}=\hat{\mathbf{x}}^{-1}\mathbf{Z}=\begin{bmatrix}.154 & .208 & .332\\ .029 & .045 & .442\\ .224 & .003 & .127\end{bmatrix}$, $\mathbf{G}=(\mathbf{I}-\mathbf{B})^{-1}=\begin{bmatrix}1.37 & .3 & .673\\ .205 & 1.093 & .631\\ .352 & .08 & 1.32\end{bmatrix}$ so the relative price changes are $\boldsymbol{\pi}=(\hat{\mathbf{x}})^{-1}\mathbf{G}'\mathbf{v}^{new}=\begin{bmatrix}1.389 & 1.512 & 1.319\end{bmatrix}'$.

PROBLEM 12.4

First $\mathbf{v}_c^{new} = \hat{\mathbf{x}}^{-1}\mathbf{v}^{new} = \begin{bmatrix} .8 & .833 & .5 \end{bmatrix}'$ so $\tilde{\mathbf{p}} = (\mathbf{I} - \mathbf{A}')^{-1}\mathbf{v}_c^{new} = \mathbf{L}'\mathbf{v}_c^{new} = \begin{bmatrix} 1.389 & 1.512 & 1.319 \end{bmatrix}'$

where $\mathbf{L} = \begin{bmatrix} 1.37 & .626 & .561 \\ .098 & 1.093 & .252 \\ .422 & .201 & 1.32 \end{bmatrix}$ and $\boldsymbol{\pi} = \tilde{\mathbf{p}}$.

PROBLEM 12.5

$$\text{First, } \mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .090 & .134 & .111 & .324 \\ .183 & .103 & .017 & .227 \\ .090 & .137 & .172 & .265 \\ .057 & .009 & .139 & .114 \end{bmatrix}, \ \mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 1.207 & .227 & .264 & .579 \\ .280 & 1.182 & .138 & .447 \\ .214 & .241 & 1.332 & .539 \\ .114 & .065 & .228 & 1.256 \end{bmatrix},$$

$$\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z} = \begin{bmatrix} .090 & .149 & .127 & .201 \\ .165 & .103 & .018 & .128 \\ .079 & .133 & .172 & .144 \\ .091 & .017 & .256 & .114 \end{bmatrix} \text{ and } \mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1} = \begin{bmatrix} 1.207 & .251 & .303 & .360 \\ .253 & 1.182 & .142 & .251 \\ .187 & .233 & 1.332 & .293 \\ .183 & .116 & .419 & 1.256 \end{bmatrix}.$$

(a) The direct and total backward linkages are, $\mathbf{i}'\mathbf{A} = \begin{bmatrix} .420 & .384 & .440 & .930 \end{bmatrix}$ and $\mathbf{i}'\mathbf{L} = \begin{bmatrix} 1.815 & 1.716 & 1.962 & 2.820 \end{bmatrix}$. (b) The direct and total forward linkages are $\mathbf{Bi} = \begin{bmatrix} 0.568 & 0.414 & 0.528 & 0.479 \end{bmatrix}'$ and $\mathbf{Gi} = \begin{bmatrix} 2.121 & 1.828 & 2.046 & 1.974 \end{bmatrix}'$.

PROBLEM 12.6

Using the matrix notation $\overline{B}(d)^{rr} = (1/n)\mathbf{i}'\mathbf{A}^{rr}\mathbf{i}$, $\overline{B}(d)^{sr} = (1/n)\mathbf{i}'\mathbf{A}^{sr}\mathbf{i}$, $\overline{B}(t)^{rr} = (1/n)\mathbf{i}'\mathbf{L}^{rr}\mathbf{i}$ and $\overline{B}(t)^{sr} = (1/n)\mathbf{i}'\mathbf{L}^{sr}\mathbf{i}$. For the Japanese IRIO model, $n = 5$ and $B(d)^c = B(d)^{cc} + B(d)^{nc} + B(d)^{sc}$, $B(d)^n = B(d)^{nn} + B(d)^{cn} + B(d)^{sn}$, $B(d)^s = B(d)^{ss} + B(d)^{cs} + B(d)^{ns}$ are the direct backward linkages for the central, north and south regions, respectively. Analogous notation applies for the total backward linkages and the direct and total forward linkages. The results are provided in the following table:

	r = Central	r = North	r = South
$b(d)^r$	.865	.741	.939
$b(t)^r$	3.18	2.731	3.434
$f(d)^r$	.579	.453	.597
$f(t)^r$	2.615	2.483	2.595

PROBLEM 12.7

First retrieve **A** and **x** for the U.S. 2005 input-output table from Appendix B. (a) To hypothetically extract the agriculture sector (sector 1), set the first row and first column of **A** to zero [call the result $\mathbf{A}^{(1)}$] and the first element of **f** to zero [call the result $\mathbf{f}^{(1)}$]. Then compute $\mathbf{L}^{(1)} = (\mathbf{I} - \mathbf{A}^{(1)})^{-1}$ and subsequently $t_1 = \mathbf{i}'\mathbf{x} - \mathbf{i}'\mathbf{L}^{(1)}\mathbf{f}^{(1)} = 54{,}744{,}946$, which would be the reduction in total output of the economy if the agriculture sector were extracted. (b) If we define $p_i = 100 \times (\mathbf{i}'\mathbf{x} - \mathbf{i}'\mathbf{L}^{(i)}\mathbf{f}^{(i)}) / \mathbf{i}'\mathbf{x}$ as the percentage reduction in total output by extracting industry *j*, then we can compute all 7 p_i's as $\mathbf{p} = \begin{bmatrix} 2.4 & 2.6 & 11.5 & 29.8 & 22.0 & 54.8 & 18.8 \end{bmatrix}'$, which indicates that the services sector (sector 6) would yield the highest reduction in output from a hypothetical extraction with a 54.8 percent reduction in output.

PROBLEM 12.8

Initially, $\mathbf{A} = \begin{bmatrix} 0.0267 & 0.2560 & 0.4450 \\ 0.0933 & 0.1760 & 0.3850 \\ 0.1600 & 0.0960 & 0.1400 \end{bmatrix}$ and $\mathbf{L} = \begin{bmatrix} 1.2107 & 0.4738 & 0.8386 \\ 0.2557 & 1.3805 & 0.7503 \\ 0.2538 & 0.2423 & 1.4026 \end{bmatrix}$.

(a) If a_{13} is increased to 0.5785 ($\alpha = 30$) we find $\mathbf{L}^*_{(13)} = \begin{bmatrix} 1.2531 & 0.5144 & 1.0732 \\ 0.2647 & 1.3890 & 0.7999 \\ 0.2627 & 0.2507 & 1.4517 \end{bmatrix}$ and consequently $\mathbf{P}_{(13)} = 100\{[\mathbf{L}^*_{(13)} - \mathbf{L}] \oslash \mathbf{L}\} = \begin{bmatrix} 3.5069 & 8.5531 & 27.9806 \\ 3.5069 & 0.6201 & 6.6051 \\ 3.5069 & 3.5069 & 3.5069 \end{bmatrix}$. In this case, a_{13} is identified as inverse important because a 30 percent change in its value causes a greater than 5 percent change in three inverse elements—l_{12}, l_{13} and l_{23}. Notice that, as expected, the largest impact of a change in a_{13} is on the corresponding element in **L**, namely l_{13}. (b) With $\alpha = 20$ and $\beta = 10$, $\mathbf{L}^*_{(13)} = \begin{bmatrix} 1.2387 & 0.5005 & 0.9932 \\ 0.2616 & 1.3861 & 0.7830 \\ 0.2597 & 0.2478 & 1.4350 \end{bmatrix}$ and $\mathbf{P}_{(13)} = \begin{bmatrix} 2.3109 & 5.6362 & 18.4382 \\ 2.3109 & 0.4086 & 4.3525 \\ 2.3109 & 2.3109 & 2.3109 \end{bmatrix}$, so a_{13} would still be classified as inverse important, since there is (now only) one element, l_{13}, that is changed by more than $\beta = 10$ percent. (c) With $\alpha = 10$ and $\beta = 10$, $\mathbf{L}^*_{(13)} = \begin{bmatrix} 1.2245 & 0.4870 & 0.9150 \\ 0.2586 & 1.3832 & 0.7664 \\ 0.2567 & 0.2450 & 1.4186 \end{bmatrix}$ and $\mathbf{P}_{(13)} = \begin{bmatrix} 1.1423 & 2.7859 & 9.1138 \\ 1.1423 & 0.2020 & 2.1514 \\ 1.1423 & 1.1423 & 1.1423 \end{bmatrix}$. In this case, a_{13} would not be labeled inverse important; the largest percentage change in any element of the Leontief inverse is less than the $\beta = 10$ threshold.

PROBLEM 12.9

The data in Appendix B includes **A** and **x**. We can compute $\mathbf{Z} = \mathbf{A}\hat{\mathbf{x}}$ from which we can subsequently find $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z}$, $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ (shown below) and $\mathbf{v}' = \mathbf{x}' - \mathbf{i}'\mathbf{Z} = [123{,}475 \quad 233{,}770 \quad 648{,}605 \quad 1{,}487{,}054 \quad 2{,}137{,}239 \quad 6{,}324{,}050 \quad 1{,}501{,}645]$. As an example, a 10 percent reduction in construction primary inputs (sector 3) would reduce v_3 to 583,745. If we define the new **v** incorporating the reduced manufacturing labor input as $\overline{\mathbf{v}}$, then $\overline{\mathbf{x}}' = \overline{\mathbf{v}}'\mathbf{G} = [312{,}584 \quad 396{,}496 \quad 1{,}237{,}177 \quad 4{,}483{,}860 \quad 3{,}354{,}712 \quad 10{,}471{,}611 \quad 2{,}523{,}091]$, which represents a 0.33 percent reduction in total output $[100 \times (\mathbf{i}'\mathbf{x} - \mathbf{i}'\overline{\mathbf{x}}) / \mathbf{i}'\mathbf{x}]$. For comparison, a 10 percent reduction in the services sector (sector 6) generates a 5.08 percent reduction in total output.

$$\mathbf{B} = \begin{bmatrix} 0.2258 & 0.0000 & 0.0063 & 0.5513 & 0.0014 & 0.0585 & 0.0056 \\ 0.0021 & 0.1432 & 0.0245 & 0.7635 & 0.3105 & 0.0108 & 0.0447 \\ 0.0012 & 0.0001 & 0.0010 & 0.0061 & 0.0096 & 0.0573 & 0.0417 \\ 0.0136 & 0.0078 & 0.0752 & 0.3222 & 0.0409 & 0.1323 & 0.0569 \\ 0.0076 & 0.0050 & 0.0392 & 0.1328 & 0.0704 & 0.1044 & 0.0367 \\ 0.0025 & 0.0048 & 0.0152 & 0.0502 & 0.0564 & 0.2783 & 0.0489 \\ 0.0012 & 0.0015 & 0.0048 & 0.0390 & 0.0286 & 0.0780 & 0.0240 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1.3139 & 0.0129 & 0.1027 & 1.1313 & 0.0811 & 0.3446 & 0.0987 \\ 0.0365 & 1.1863 & 0.1691 & 1.5050 & 0.4944 & 0.4018 & 0.1883 \\ 0.0026 & 0.0010 & 1.0054 & 0.0257 & 0.0190 & 0.0930 & 0.0499 \\ 0.0301 & 0.0169 & 0.1284 & 1.5707 & 0.0997 & 0.3280 & 0.1182 \\ 0.0165 & 0.0102 & 0.0674 & 0.2631 & 1.1072 & 0.2229 & 0.0716 \\ 0.0085 & 0.0102 & 0.0379 & 0.1502 & 0.1005 & 1.4409 & 0.0868 \\ 0.0041 & 0.0036 & 0.0154 & 0.0863 & 0.0454 & 0.1363 & 1.0390 \end{bmatrix}$$

PROBLEM 12.10

b(d)	1	2	3	4	5	6	7	Ave.*
1919	0.556	0.748	0.729	0.722	0.57	0.546	0.524	0.628
1929	0.57	0.653	0.59	0.706	0.53	0.638	0.444	0.59
1939	0.624	0.724	0.517	0.807	0.639	0.449	0.626	0.627
1947	0.553	0.306	0.589	0.643	0.319	0.411	0.717	0.505
1958	0.581	0.433	0.582	0.641	0.325	0.472	0.782	0.545
1963	0.605	0.463	0.566	0.633	0.322	0.408	0.713	0.53
1967	0.618	0.46	0.559	0.629	0.324	0.411	0.703	0.529
1972	0.617	0.379	0.542	0.615	0.299	0.337	0.062	0.407
1977	0.622	0.357	0.578	0.64	0.353	0.333	0.072	0.422
1982	0.631	0.337	0.531	0.652	0.408	0.343	0.097	0.428
1987	0.63	0.361	0.53	0.592	0.37	0.384	0.093	0.423
1992	0.6	0.504	0.54	0.615	0.385	0.348	0.073	0.438
1997	0.631	0.569	0.551	0.669	0.421	0.353	0.007	0.457
2002	0.649	0.501	0.501	0.668	0.384	0.368	0.381	0.493
2003	0.586	0.465	0.494	0.648	0.332	0.391	0.394	0.473
2005	0.605	0.411	0.502	0.668	0.363	0.396	0.406	0.479

b(t)	1	2	3	4	5	6	7	Ave.*
1919	2.366	2.922	3.01	2.879	2.547	2.354	2.47	2.65
1929	2.359	2.5	2.475	2.701	2.292	2.415	2.141	2.412
1939	2.915	3.245	2.702	3.525	2.988	2.401	2.978	2.965
1947	2.164	1.582	2.287	2.439	1.642	1.861	2.464	2.063
1958	2.281	1.919	2.309	2.482	1.667	2.032	2.682	2.196
1963	2.345	1.937	2.246	2.446	1.633	1.836	2.479	2.132
1967	2.362	1.935	2.221	2.421	1.63	1.838	2.436	2.121
1972	2.298	1.68	2.09	2.262	1.5	1.606	1.109	1.792
1977	2.338	1.672	2.21	2.351	1.627	1.622	1.137	1.851
1982	2.387	1.611	2.109	2.36	1.736	1.639	1.188	1.862
1987	2.319	1.642	2.038	2.183	1.645	1.692	1.174	1.813
1992	2.227	1.923	2.073	2.252	1.672	1.612	1.137	1.842
1997	2.377	2.08	2.133	2.431	1.766	1.636	1.013	1.919
2002	2.384	1.929	2.018	2.408	1.695	1.647	1.71	1.97
2003	2.208	1.866	1.985	2.316	1.593	1.7	1.734	1.915
2005	2.3	1.77	2.033	2.386	1.662	1.723	1.771	1.949

f(d)	1	2	3	4	5	6	7	Ave.*
1919	0.821	0.806	0.671	0.531	0.679	0.307	0.631	0.635
1929	0.743	0.835	0.624	0.554	0.691	0.412	0.52	0.625
1939	0.717	0.871	0.732	0.571	0.959	0.18	0.879	0.701
1947	0.886	0.928	0.249	0.528	0.375	0.38	0.381	0.675
1958	0.835	0.95	0.18	0.569	0.403	0.397	0.756	0.727
1963	0.845	0.953	0.174	0.56	0.393	0.397	0.599	0.703
1967	0.854	0.942	0.171	0.549	0.387	0.43	0.655	0.713
1972	0.851	0.063	0.163	0.557	0.39	0.379	0.124	0.504
1977	0.789	0.376	0.218	0.573	0.416	0.358	0.153	0.555
1982	0.792	0.146	0.197	0.569	0.446	0.376	0.152	0.525
1987	0.859	0.125	0.201	0.559	0.414	0.394	0.176	0.532
1992	0.836	0.199	0.235	0.563	0.405	0.373	0.139	0.535
1997	0.848	0.184	0.095	0.586	0.389	0.421	0.088	0.516
2002	0.848	0.304	0.148	0.629	0.386	0.45	0.166	0.562
2003	0.835	0.32	0.126	0.634	0.382	0.447	0.176	0.56
2005	0.849	0.299	0.117	0.649	0.396	0.456	0.177	0.563

f(t)	1	2	3	4	5	6	7	Ave.*
1919	3.357	3.081	2.725	2.36	2.902	1.8	2.571	2.685
1929	2.941	3.038	2.444	2.316	2.75	1.947	2.269	2.529
1939	3.153	4.006	3.379	2.789	3.981	1.579	3.702	3.227
1947	3.22	2.981	1.514	2.105	1.805	1.74	3.694	2.437
1958	3.159	3.139	1.407	2.242	1.886	1.843	4.92	2.657
1963	3.172	3.1	1.383	2.207	1.841	1.821	4.444	2.567
1967	3.154	3.028	1.383	2.172	1.828	1.882	4.539	2.569
1972	3.062	3.161	1.29	2.088	1.729	1.702	1.233	2.038
1977	2.851	3.983	1.412	2.166	1.809	1.676	1.301	2.171
1982	2.872	3.408	1.368	2.134	1.868	1.718	1.296	2.095
1987	3.041	3.282	1.353	2.075	1.776	1.708	1.328	2.08
1992	2.898	3.603	1.408	2.094	1.765	1.664	1.254	2.098
1997	3.052	3.569	1.175	2.186	1.756	1.787	1.173	2.1
2002	3.106	3.726	1.266	2.253	1.732	1.822	1.304	2.173
2003	2.995	3.912	1.208	2.238	1.72	1.808	1.321	2.172
2005	3.085	3.981	1.197	2.292	1.759	1.835	1.33	2.211

*The last column in each table includes the simple averages of the first seven columns.

Solutions to Chapter 13 Problems

PROBLEM 13.1

First $\mathbf{A}^0 = \mathbf{Z}^0(\hat{\mathbf{x}}^0)^{-1} = \begin{bmatrix} .097 & .239 & .333 \\ .077 & .072 & .250 \\ .065 & .287 & .333 \end{bmatrix}$ and $\mathbf{A}^1 = \mathbf{Z}^1(\hat{\mathbf{x}}^1)^{-1} = \begin{bmatrix} .500 & .438 & .300 \\ .320 & .450 & .167 \\ .100 & .075 & .133 \end{bmatrix}$ so

$\mathbf{L}^0 = (\mathbf{I} - \mathbf{A}^0)^{-1} = \begin{bmatrix} 1.199 & .562 & .455 \\ .111 & 1.221 & .308 \\ .398 & 1.125 & 1.910 \end{bmatrix}$ and $\mathbf{L}^1 = (\mathbf{I} - \mathbf{A}^1)^{-1} = \begin{bmatrix} 1.214 & .566 & .82 \\ .15 & 1.289 & .558 \\ .182 & 1.61 & 1.82 \end{bmatrix}$. Then

$\Delta\mathbf{f} = \mathbf{f}^1 - \mathbf{f}^0 = \begin{bmatrix} 15 \\ 10 \\ -10 \end{bmatrix}$ and $\Delta\mathbf{L} = \mathbf{L}^1 - \mathbf{L}^0 = \begin{bmatrix} .015 & .004 & .365 \\ .039 & .068 & .251 \\ -.216 & -.515 & -.09 \end{bmatrix}$. By using

$\Delta\mathbf{x} = (1/2)\underbrace{(\Delta\mathbf{L})(\mathbf{f}^0 + \mathbf{f}^1)}_{\text{Technology change}} + (1/2)\underbrace{(\mathbf{L}^0 + \mathbf{L}^1)(\Delta\mathbf{f})}_{\text{Final demand change}}$, the basic structural decomposition relationship,

we write $\Delta\mathbf{x} = \begin{bmatrix} 35 \\ 27.5 \\ -50 \end{bmatrix}$, $(\Delta\mathbf{L})(\mathbf{f}^0 + \mathbf{f}^1) = \begin{bmatrix} 35.3 \\ 34.6 \\ -88.8 \end{bmatrix}$ and $(\mathbf{L}^0 + \mathbf{L}^1)(\Delta\mathbf{f}) = \begin{bmatrix} 34.7 \\ 20.4 \\ -11.2 \end{bmatrix}$.

PROBLEM 13.2

First we determine $\mathbf{x}_0 = \mathbf{f}_0 + \mathbf{Z}\mathbf{i} = \begin{bmatrix} 236 \\ 281 \\ 408 \end{bmatrix}$ and $\mathbf{A} = \mathbf{Z}(\hat{\mathbf{x}}_0)^{-1} = \begin{bmatrix} .059 & .270 & .113 \\ .229 & .078 & .012 \\ .288 & .253 & .230 \end{bmatrix}$. For sector 1 (manufactured goods), the level of final demand is exogenously specified and for sectors 2 and 3 (oil and electricity), levels of total output are specified for each sector, so we partition **A** (as in section 13.2.4) as $\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right] = \left[\begin{array}{c|cc} .059 & .27 & .113 \\ \hline .229 & .078 & .012 \\ .288 & .253 & .23 \end{array}\right]$, and with a vector of exogenously specified values $\left[\begin{array}{c} \bar{\mathbf{f}} \\ \hline \bar{\mathbf{x}} \end{array}\right]$ and the vector of endogenously determined values designated by $\left[\begin{array}{c} \mathbf{x} \\ \hline \mathbf{f} \end{array}\right]$ we write $\mathbf{M}\begin{bmatrix}\mathbf{x}\\ \hline \mathbf{f}\end{bmatrix}=\mathbf{N}\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}$ where $\mathbf{M}=\left[\begin{array}{c|c}(\mathbf{I}-\mathbf{A}_{11}) & \mathbf{0}\\ \hline -\mathbf{A}_{21} & -\mathbf{I}\end{array}\right]=\left[\begin{array}{c|cc}.941 & 0 & 0\\ \hline -.229 & -1 & 0\\ -.288 & 0 & -1\end{array}\right]$ and

$\mathbf{N}=\left[\begin{array}{c|c}\mathbf{I} & \mathbf{A}_{12}\\ \hline \mathbf{0} & -(\mathbf{I}-\mathbf{A}_{22})\end{array}\right]=\left[\begin{array}{c|cc}1 & .270 & .113\\ \hline 0 & -.922 & .012\\ 0 & .253 & -.770\end{array}\right]$. It follows that $\mathbf{M}^{-1}=\left[\begin{array}{c|cc}1.063 & 0 & 0\\ \hline -.243 & -1 & 0\\ -.306 & 0 & -1\end{array}\right]$.

(a) For $\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}130\\ \hline 200\\ 175\end{bmatrix}$ we find $\begin{bmatrix}\mathbf{x}\\ \hline \mathbf{f}\end{bmatrix}=\mathbf{M}^{-1}\mathbf{N}\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}216.7\\ \hline 132.6\\ 21.7\end{bmatrix}$. That is, total output of manufactured goods will be 216.7, and final demands presented to the economy for oil and electricity are 132.6 and 21.7, respectively. (b) For $\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}150\\ \hline 200\\ 175\end{bmatrix}$ we find $\begin{bmatrix}\mathbf{x}\\ \hline \mathbf{f}\end{bmatrix}=\mathbf{M}^{-1}\mathbf{N}\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}237.9\\ \hline 127.8\\ 15.6\end{bmatrix}$.

PROBLEM 13.3

(a) $\mathbf{A}=\begin{bmatrix}.500 & .438\\ .320 & .450\end{bmatrix}$, $\mathbf{L}=\begin{bmatrix}4.074 & 3.241\\ 2.370 & 3.704\end{bmatrix}$, $\Delta\mathbf{f}=\begin{bmatrix}0.9\\ 63.0\end{bmatrix}$ and, hence, $\Delta\mathbf{x}=\mathbf{L}\Delta\mathbf{f}=\begin{bmatrix}207.8\\ 235.47\end{bmatrix}$.

(b) We now have $\mathbf{A}=\begin{bmatrix}.500 & .438 & .001\\ .320 & .450 & .070\\ .020 & .050 & .150\end{bmatrix}$ and for $\Delta\mathbf{f}=\begin{bmatrix}0.9\\ 63.0\\ 135.0\end{bmatrix}$, $\Delta\mathbf{x}=\mathbf{L}\Delta\mathbf{f}=\begin{bmatrix}249.7\\ 282.9\\ 181.3\end{bmatrix}$ where

$$\mathbf{L}=\begin{bmatrix}4.130 & 3.310 & .277\\ 2.433 & 3.782 & .314\\ .240 & .300 & 1.201\end{bmatrix}.$$

PROBLEM 13.4

Reorder sectors, as in section 13.2.1, so that those with exogenously specified final demands are first (here this is only sector 3) and those with exogenously specified total outputs are second (here this is sectors 1, 2, and 4). We use the same general notation of problem 13-2, i.e., with $\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}$ defining the vector of the exogenously determined values, $\mathbf{M}\begin{bmatrix}\mathbf{x}\\ \hline \mathbf{f}\end{bmatrix}=\mathbf{N}\begin{bmatrix}\bar{\mathbf{f}}\\ \hline \bar{\mathbf{x}}\end{bmatrix}$ where

$\mathbf{M}=\left[\begin{array}{c|c}(\mathbf{I}-\mathbf{A}_{11}) & \mathbf{0}\\ \hline -\mathbf{A}_{21} & -\mathbf{I}\end{array}\right]=\mathbf{M}=\left[\begin{array}{c|ccc}.832 & 0 & 0 & 0\\ \hline -.194 & -1 & 0 & 0\\ -.105 & 0 & -1 & 0\\ -.178 & 0 & 0 & -1\end{array}\right]$ and

$$\mathbf{N} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{A}_{12} \\ \hline \mathbf{0} & -(\mathbf{I}-\mathbf{A}_{22}) \end{array}\right] = \left[\begin{array}{c|ccc} 1 & .155 & .213 & .212 \\ \hline 0 & -.807 & .168 & .115 \\ 0 & .025 & -.874 & .124 \\ 0 & .101 & .219 & -.814 \end{array}\right]$$. It follows that $$\mathbf{M}^{-1} = \left[\begin{array}{c|ccc} 1.202 & 0 & 0 & 0 \\ \hline -.233 & -1 & 0 & 0 \\ -.126 & 0 & -1 & 0 \\ -.214 & 0 & 0 & -1 \end{array}\right]$$

and $$\left[\begin{array}{c} \mathbf{x} \\ \hline \mathbf{f} \end{array}\right] = \mathbf{M}^{-1}\mathbf{N}\left[\begin{array}{c} \bar{\mathbf{f}} \\ \hline \bar{\mathbf{x}} \end{array}\right] = \left[\begin{array}{c} 6055 \\ \hline 969 \\ 3572.9 \\ 1347.4 \end{array}\right]$$ for $$\left[\begin{array}{c} \bar{\mathbf{f}} \\ \hline \bar{\mathbf{x}} \end{array}\right] = \left[\begin{array}{c} 2050 \\ \hline 4558 \\ 5665 \\ 5079 \end{array}\right]$$. That is, total output of sector 3 will be 6055, and amounts production from sectors 1, 2, and 4 available for final demand are 969, 3572.9, and 1347.4, respectively. The GDP is computed as the sum of all final demands, i.e., both exogenously specified and endogenously determined: 7939.3.

PROBLEM 13.5

For **A** and **B** provided, $\mathbf{G} = (\mathbf{I}-\mathbf{A}+\mathbf{B}) = \begin{bmatrix} .69 & -.103 \\ -.205 & .48 \end{bmatrix}$ and $\mathbf{G}^{-1} = \begin{bmatrix} 1.548 & .332 \\ .661 & 2.225 \end{bmatrix}$. The "dynamic multipliers" are $\mathbf{R} = \mathbf{G}^{-1}\mathbf{B} = \begin{bmatrix} .017 & .011 \\ .018 & .046 \end{bmatrix}$, $\mathbf{R}^2\mathbf{G}^{-1} = \begin{bmatrix} .001 & .002 \\ .003 & .006 \end{bmatrix}$ and $\mathbf{R}^3\mathbf{G}^{-1} = \begin{bmatrix} .00006 & .00009 \\ .00018 & .00029 \end{bmatrix}$. Then we can construct the difference equations in matrix terms as

$$\mathbf{D} = \begin{bmatrix} \mathbf{G} & -\mathbf{B} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} & -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G} & -\mathbf{B} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1} & \mathbf{R}^2\mathbf{G}^{-1} & \mathbf{R}^3\mathbf{G}^{-1} \\ \mathbf{0} & \mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1} & \mathbf{R}^2\mathbf{G}^{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}^{-1} \end{bmatrix}$$ so that $$\left[\begin{array}{c} \mathbf{x}^0 \\ \hline \mathbf{x}^1 \\ \hline \mathbf{x}^2 \\ \hline \mathbf{x}^3 \end{array}\right] = \mathbf{D}\left[\begin{array}{c} \mathbf{f}^0 \\ \hline \mathbf{f}^1 \\ \hline \mathbf{f}^2 \\ \hline \mathbf{f}^3 \end{array}\right]$$ and

$\mathbf{x}^0 = \begin{bmatrix} 197.7 \\ 315 \end{bmatrix}$, $\mathbf{x}^1 = \begin{bmatrix} 257.7 \\ 468.3 \end{bmatrix}$, $\mathbf{x}^2 = \begin{bmatrix} 302.8 \\ 521.2 \end{bmatrix}$ and $\mathbf{x}^3 = \begin{bmatrix} 352.8 \\ 567.3 \end{bmatrix}$.

PROBLEM 13.6

Here $\mathbf{B}^{-1} = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$. Solving the characteristic equation $|\mathbf{Q}-\mathbf{I}| = 0$, we find $\lambda_{\max} = c = 5$.

PROBLEM 13.7

(a) Here $\mathbf{Q}=\mathbf{B}^{-1}(\mathbf{I}-\mathbf{A}+\mathbf{B})=\begin{bmatrix}10 & -2\\ -3 & 7\end{bmatrix}$ and $\lambda_{\max}=11.37$. (b) With $\mathbf{B}=\begin{bmatrix}.1 & 0\\ .1 & .1\end{bmatrix}$ we find $\mathbf{Q}=\begin{bmatrix}10 & -2\\ -12 & 9\end{bmatrix}$ and $\lambda_{\max}=14.42$. The turnpike growth, $\lambda_{\max}$, rate has increased, indicating expanded economic growth.

PROBLEM 13.8

For **A** and **B** provided, $\mathbf{G}=(\mathbf{I}-\mathbf{A}+\mathbf{B})=\begin{bmatrix}.78 & -.102\\ -.303 & .49\end{bmatrix}$ and $\mathbf{G}^{-1}=\begin{bmatrix}1.395 & .29\\ .863 & 2.220\end{bmatrix}$. The "dynamic multipliers" are $\mathbf{R}=\mathbf{G}^{-1}\mathbf{B}=\begin{bmatrix}.029 & .006\\ .024 & .024\end{bmatrix}$, $\mathbf{R}^2\mathbf{G}^{-1}=\begin{bmatrix}.002 & .001\\ .002 & .002\end{bmatrix}$ and $\mathbf{R}^3\mathbf{G}^{-1}=\begin{bmatrix}.00006 & .00004\\ .00010 & .00007\end{bmatrix}$ such that $\Delta\mathbf{x}^{-3}=\mathbf{R}^3\mathbf{G}^{-1}\Delta\mathbf{f}^0$, $\Delta\mathbf{x}^{-2}=\mathbf{R}^2\mathbf{G}^{-1}\Delta\mathbf{f}^0$ and $\Delta\mathbf{x}^{-1}=\mathbf{R}\mathbf{G}^{-1}\Delta\mathbf{f}^0$ or, in matrix terms, $\begin{bmatrix}\mathbf{x}^{-3}\\ \mathbf{x}^{-2}\\ \mathbf{x}^{-1}\\ \mathbf{x}^{0}\end{bmatrix}=\begin{bmatrix}\mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1} & \mathbf{R}^2\mathbf{G}^{-1} & \mathbf{R}^3\mathbf{G}^{-1}\\ \mathbf{0} & \mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1} & \mathbf{R}^2\mathbf{G}^{-1}\\ \mathbf{0} & \mathbf{0} & \mathbf{G}^{-1} & \mathbf{R}\mathbf{G}^{-1}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{f}^{-3}\\ \mathbf{f}^{-2}\\ \mathbf{f}^{-1}\\ \mathbf{f}^{0}\end{bmatrix}$.

PROBLEM 13.9

Using the same procedure outlined in the solution to problem 13.1, the following table summarizes the results:

Sector	$\Delta\mathbf{x}$	Technology	Final Demand
1	186,559	-215,862	588,981
2	154,131	54,745	253,517
3	801,570	-147,102	1,750,242
4	3,089,223	-418,484	6,596,931
5	2,434,476	-7,500	4,876,451
6	8,426,367	2,448,634	14,404,100
7	1,985,075	218,564	3,751,585

PROBLEM 13.10

Using the same procedures outlined in the solutions to problems 13.2 and 13.4, the following table summarizes the results:

	Endogenous		Exogenous	
	f	x	f	x
1 Agriculture		333,767	51,968	
2 Mining		414,773	-130,561	
3 Construction		1,426,583	1,265,103	
4 Manufactuirng		4,748,959	1,653,196	
5 Trade Transport & Utilities	2,144,061			3,557,300
6 Services	6,034,580			11,106,299
7 Other	2,203,480			2,677,904