**Solutions to Numerical Exercises**

**Chapters 1-4**

**Exercise 1.16**

Rewriting Eq. (1.1) we have



The area *A* is given as 4 cm2, which in SI units becomes 4 × 10-4 m2. The stress is given as 1 MPa, which is 1 × 106 Pa (N m-2). Using these values, the above equation yields the axial force acting on the specimen as



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**Exercise 1.17**

Rewriting Eq. (1.3) we have



The strain ε is given as 0.4%, which is equal to 0.004. The original length *L* is 0.2 m. From the equation above we then obtain the shortening Δ*L* as



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**Exercise 1.18**

The measured strain is 0.12% which is equal to 0.0012. The compressive stress is 60 MPa, which in standard SI units is 6 × 107 Pa. From Eq. (1.6) we obtain Young's modulus *E* as



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**Exercise 2.9**

From Eq. (2.11) the vertical stress is



This result can be compared with the statement on page 53 that the vertical stress increases by about 25 MPa for every kilometre in the uppermost part of the crust.

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**Exercise 2.10**

We rewrite Eq. (2.11) to solve for the depth *z* and then substitute 1 × 108 Pa (100 MPa) for the vertical stress σv and 2700 kg m-3 for the crustal density ρr. Thus, we have



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**Exercise 2.11**

The vertical stress is the maximum principal stress and is given as 76.5 MPa (Exercises 2.9), whereas the minimum principal stress is horizontal and given as 50 MPa. The angle α is 90 − 70 = 20°. Thus, from Eq. (2.7) the normal stress on the normal fault at 3 km depth is



Similarly, from Eq. (2.8) the shear stress on the normal fault is (see Fig. 2.6 for the Mohr's circle construction)



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**Exercise 2.12**

The vertical stress is the minimum principal stress and equal to the overburden pressure (geostatic pressure). To calculate the vertical stress from Eq. (2.11) we need to know the rock density, which is not given. For a rift zone composed primarily of basaltic lava flows, as in Fig. 1.6 (referred to in this exercise), the density of the uppermost 100 m is likely to be between 2500 and 2600 kg m-3. This refers to the in-situ density; the densities of small laboratory specimens of basalt are generally higher (Appendix E.1). Here we take the density as 2500 kg m-3. Then, from Eq. (2.11) the vertical stress at 100 m depth is



We know that the maximum stress is horizontal and given as 20 MPa. The angle α is 90 − 75 = 15°. Thus, from Eq. (2.7) the normal stress on the normal fault at 100 m depth is



Similarly, from Eq. (2.8) the shear stress on the normal fault is



The Mohr's circle construction follows directly from Fig. 2.6.

Notice: in Exercises 2.11 and 2.12, Eqs. (2.5) and (2.6) could also be used, in which case the angle θ, rather than α, would be used to define the attitude of the fault plane. The relationship between these angles is shown in Fig. 2.19 (and in Example 2.7). If Eqs. (2.5) and (2.6) are used, then the Mohr's construction follows the procedure in Example 2.4.

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**Exercise 3.7**

Elongation is given by Eq. (3.11) as



Here ε = 0.002 and the extension or change in length of the rod is Δ*L* = 0.001 m (0.1 cm). We then rewrite Eq. (3.11) so as to solve for the original length of the rod, *Lu*, thus



Thus, the original length of the rod was 0.5 m.

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**Exercise 3.8**

We rewrite Eq. (1.6) and obtain Young's modulus thus



This is a low laboratory Young's modulus for rocks, but some sedimentary rocks and tuffs have similar values (Appendix D.1). However, the in-situ Young's modulus of many young (Holocene) lava flows is similar to this value (cf. page 108 in the book).

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**Exercise 3.9**

We rewrite Eq. (3.12) to obtain the deformed length *Ld* as



From Eq. (3.12) we get the elongation as



From Eq. (3.13), the quadratic elongation is



From Eq. (3.19) the natural strain of the fossil is



(The result in Example 3.2 should be divided by 2, yielding the correct result: -0.22)

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**Exercise 3.10**

The stretch of the volcanic rift zone is the difference between its undeformed width, 20000 m (*Lu*), the deformed width is 2000 + 0.5 = 20000.5 m (*Ld*). From Eq. (3.12) the stretch is



From Eq. (3.11) the elongation is



Strain rate is calculated from the elongation over time in seconds. The number of seconds in a year is about , so that the number of seconds in 20 years is about . Since the elongation (extension) is 2.5 × 10-5, we get the strain rate from Eq. (3.20) as



This is a typical geological strain rate and corresponds to about 2.5 centimetres (0.025 metres) per year. This is a common spreading rate - and 'spreading rate' is the other term used for a strain rate across a rift zone - for a slow-spreading ridge.

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**Exercise 4.15**

(a) The average density is given, so that we rewrite Eq. (4.45) so as to find the depth *z* for this state of stress, namely



Thus, the rock body with this state of stress would be at a depth of just over 400 m, or similar to the depth of exposure of the the normal fault in Fig. 2.4.

(b) The principal strains in terms of principal stresses and Young's modulus and Poisson's ratio follow from Eqs. (4.5-4.7). For the maximum principal compressive strain we have

 = 

For the intermediate principal strain we have

 

For the minimum principal strain we have

 

This shows that the minimum principal strain is negative, that is, tensile. Tensile strain (and stress) at the depth of about 400 m km is close to a typical depth of absolute tension during rifting events (cf. Chapters 7 and 8).

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**Exercise 4.16**

We first change the values given into SI values. Thus: the original diameter of the rod is 2 10-2 m, its original length 0.1 m, the increase in its diameter during loading 3 ×10-6m, and the decrease in its length 6 ×10-5m.

To find Young's modulus, we first need to know the elongation which, from Eq. (3.4), is obtained as



From Eq. (4.1) we then get Young's modulus as



For Poisson's ratio we need, in addition to the elongation (here shortening), the transverse strain which is obtained from Eqs. (3.3) or (3.5) as



We have to use minus in front of the increase in diameter, because strictly extension is negative whereas compression or, here, shortening is positive. Poisson's ratio is the negative of the lateral extension over the axial shortening, or



Here the shortening is here parallel with the y-axis and the extension parallel with the x-axis, in contrast to that in Example 4.2.

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**Exercise 4.17**

There are several ways to find Young's modulus *E* and Poisson's ratio ν when the shear modulus *G* (15.7 GPa) and the bulk modulus *K* (29 GPa) are known using the various relations among the elastic constants in Appendix D.2. Here we do as follows. We obtain Young's modulus using the relation



Similarly, we obtain Poisson's ratio from the relation



These are both very reasonable laboratory values for many solid rocks (Appendix D.1).

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**Exercise 4.18**

The theoretical horizontal stress, in relation to the vertical stress, when the horizontal strains are assumed zero can be obtained from Eqs. (4.51) or (4.53). Using Eq. (4.53), and remembering that *m* = 1/ν and thus 1/0.25 = 4 in this case, we have

 

At a depth of 2 km, and with a crustal density of 2500 kg m-3, the vertical stress is, from Eq. (4.45) obtained as



The horizontal stress is 1/3 of the vertical stress, or about 16.4 MPa, and the stress difference is thus . This stress difference is many times larger than the in-situ tensile strength, so that long before this difference could be reached (through reduction in the horizontal stress from its initial lithostatic value equal to the vertical stress) a dyke or an inclined sheet would be injected and bring the state of stress again close to lithostatic at the boundary of a fluid magma chamber.

Notice that in this exercise, Young's modulus, although given, is not needed. Normally, however, when Poisson's ratio is known and given, Young's modulus is also known and thus given here for completeness.

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**Exercise 4.19**

From Eq. (4.69) we obtain the strain energy density, that is, strain energy per unit volume, as



This is the strain energy per unit volume. The volume of the specimen is its area times its height. The area is *A* = *πR2*, where *R* = 0.01 m (1 cm). Thus, *A * 0.0314 m2. The height of the specimen is 0.1 m (10 cm), so that its volume is



It follows that the total strain energy stored in the the specimen is

*U = 50* × 0.003= 0.15 *J*

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**Exercise 4.20**

We have the following information: diameter of the bar is 4 cm = 4 10-2 m (and its radius, *R*, is thus 2 10-2 m), length of the bar is 2 m, and the applied tensile force is 80 kN. Since stress is force per unit area, we first find the cross-sectional area thus



The axial stress on the bar, from Eq. (1.1), is thus



The corresponding strain or elongation, from Eq. (4.1), is



The strain (elongation) has a negative sign since it is tensile, that is, the bar has increased its length. The strain energy per unit volume of bar (strain energy density) is, from Eq. (4.69), obtained as



The volume of the bar is its area times its height, that is



It follows that the total strain energy stored in the bar is

*U =* 2.5× 104 × 2.52 × 10-3 = 63 *J*

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