2 Chapter 2 Solution Set

Problems

2.1 Prove that the real and imaginary parts of the analytic signal are connected by Hilbert transforms. HINT: Use fact that $f^{(+)}(t)$ is analytic in l.h.p. and goes to zero as $|t| \to \infty$ in that half-plane.

Since $f^{(+)}(t)$ is analytic in the l.h.p. and goes to zero as $|t| \to \infty$ in that half-plane we can use Cauchy's integral theorem to find that

$$f^{(+)}(t_0) = -\frac{1}{2\pi i} \int_C dt \, \frac{f((+)(t)}{t - t_0}$$

where t_0 lies on the real axis and the contour C lies along the real axis from $-\infty$ to $+\infty$ and is deformed above the pole at $t = t_0$. We can break the contour C into the principle part integral from $-\infty$ to $t_0 - \epsilon$ and then from $t_0 + \epsilon$ to $+\infty$ and a semi-circle C_ϵ of radius ϵ in the u.h.p. centered at t_0. We then find that

$$f^{(+)}(t_0) = -\frac{1}{2\pi i} P \int_{-\infty}^{\infty} dt \, \frac{f((+)(t)}{t - t_0} - \frac{1}{2\pi i} \int_{C_\epsilon} dt \, \frac{f((+)(t)}{t - t_0}$$

The integral around C_ϵ is easily evaluated at found to be $f(t_0)/2$ which then leads to

$$f^{(+)}(t_0) = -\frac{1}{\pi i} P \int_{-\infty}^{\infty} dt \, \frac{f((+)(t)}{t - t_0}.$$

As a final step we set

$$f^{(+)}(t) = u(t) + iv(t)$$

to arrive at our final result

$$u(t) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} dt \, \frac{v(t)}{t - t_0}, \quad v(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dt \, \frac{u(t)}{t - t_0}.$$

2.2 Let

$$\epsilon_k(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, k^2(\omega) e^{-i\omega\tau},$$

where $k(\omega)$ is the wavenumber of a Causal medium.

1. Prove that $\epsilon_k(\tau)$ is causal; i.e., vanishes for negative τ.

   This follows immediately since the complex wavenumber $k(\omega)$ of a causal medium is analytic with no singularities throughout the u.h.p. and goes to $(\omega/c)^2$ as $|\omega| \to \infty$. If $\tau < 0$ we can then close the contour in the u.h.p. yielding zero.
2. Compute $\epsilon_k(\tau)$ as a generalized function for a non-dispersive medium where $k^2(\omega) = \omega^2/c^2$.

   We have

$$\epsilon_k(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\,(\frac{\omega}{c})^2 e^{-i\omega\tau} = -\frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\{\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega\tau}\} = -\frac{1}{c^2}\delta^{(2)}(\tau),$$

   where $\delta^{(2)}(\tau)$ is the second derivative of the delta function.
3. Prove that the real and imaginary parts of $k^2(\omega) - (\omega/c)^2$ are connected via Hilbert transforms.

   From the discussion given above it follows that $k^2(\omega) - (\omega/c)^2$ is analytic with no singularities in the u.h.p. and tends to zero as $|\omega| \to \infty$ in that half-plane. We can then follow the same treatment given in the solution of Problem 2.1 but where now the contour integral is performed in the upper-half of the complex ω plane rather than the lower-half of the complex t plane. This changes the signs in the Hilbert transform relationships and we obtain

$$\Re[k^2(\omega_0) - (\omega_0/c)^2] = \frac{1}{\pi}P\int_{-\infty}^{\infty} d\omega\,\frac{\Im[k^2(\omega) - (\omega/c)^2]}{\omega - \omega_0},$$
$$\Im[k^2(\omega_0) - (\omega_0/c)^2] = -\frac{1}{\pi}P\int_{-\infty}^{\infty} d\omega\,\frac{\Re[k^2(\omega) - (\omega/c)^2]}{\omega - \omega_0}.$$

2.3 Derive the equation satisfied by the time-dependent field $u_+(\mathbf{r}, t)$ radiated by the source $q(\mathbf{r}, t)$ if $k^2(\omega) \Leftrightarrow \epsilon_k(\tau)$. Show that this equation reduces to the usual wave equation in the special case when $k(\omega) = \omega/c$.

The frequency domain radiated field by the source $Q(\mathbf{r}, \omega)$ satisfies the Helmholtz equation

$$[\nabla^2 + k^2(\omega)]U_+(\mathbf{r}, \omega) = Q(\mathbf{r}, \omega).$$

Under a Fourier transformation the product of two functions transforms to the convolution of their transforms so that

$$k^2(\omega)U_+(\mathbf{r}, \omega) \to \int d\tau\, \epsilon_k(t - \tau)u_+(\mathbf{r}, \tau)$$

and we find that

$$\nabla^2 u_+(\mathbf{r}, t) + \int d\tau\, \epsilon_k(t - \tau)u_+(\mathbf{r}, \tau) = q(\mathbf{r}, t).$$

In the special case when $k(\omega) = (\omega/c)^2$ we showed in Problem 2.2 that

$\epsilon(\tau) = -\delta^{(2)}(\tau)/c^2$ so that

$$\int d\tau\, \epsilon_k(t-\tau)u_+(\mathbf{r},\tau) = -\frac{1}{c^2}\int d\tau\, \frac{\partial^2}{\partial\tau^2}\delta(t-\tau)u_+(\mathbf{r},\tau) = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u_+(\mathbf{r},t) \tag{2.1}$$

where we have made use of the fact that

$$\int d\tau\, \delta^{(n)}(\tau)f(\tau) = (-1)^n f^{(n)}(0).$$

On making use of Eq.(2.1) we then obtain

$$\nabla^2 u_+(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u_+(\mathbf{r},t) = q(\mathbf{r},t).$$

2.4 Prove that $k(-\omega) = -k^*(\omega)$ if ω is real valued.

We have that

$$k(\omega) = \frac{\omega}{c}n(\omega) \rightarrow k(-\omega) = -\frac{\omega}{c}n(-\omega) = -k^*(\omega)$$

since $n(-\omega) = n^*(\omega)$ for real ω.

2.5 Compute the incoming and outgoing wave Green function $G_-(\mathbf{R},\omega)$ and $G_+(\mathbf{R},\omega)$ from their Fourier integral representations using contour integration techniques.

These two Green functions admit the Fourier integral representation (cf. Eq.(2.15))

$$G_\pm(\mathbf{R},\omega) = \frac{1}{(2\pi)^3}\int d^3K\, \frac{e^{i\mathbf{K}\cdot\mathbf{R}}}{k^2 - K^2},$$

where the integration contour for G_+ lies above the pole at $K = -k$ and below the pole at $K = +k$ and for G_- lies below the pole at $K = -k$ and above the pole at $K = +k$. Since the transforms depend only on $K = |\mathbf{K}|$ it is convenient to transform the above integrals to spherical polar coordinates yielding

$$G_\pm(\mathbf{R};\omega) = \frac{-1}{(2\pi)^3}\int_{C_\pm} dK\, \frac{K^2}{K^2 - k^2}\int_{4\pi} d\Omega\, e^{iK\mathbf{s}\cdot\mathbf{R}},$$

where $\mathbf{s}$ is the unit radial vector in $\mathbf{K}$ space and $d\Omega$ the differential solid angle in $\mathbf{K}$ space. Here, the integration contour C_+ yields G_+ and extends from $K = 0$ to $+\infty$ below the pole at $K = k$ and C_- extends from $K = 0$ to $+\infty$ above the pole at $K = k$. By aligning the polar axis along the direction of $\mathbf{R}$ we have that $\mathbf{s}\cdot\mathbf{R} = R\cos\theta$ where θ is the polar angle in $\mathbf{K}$ space. The angular integral is then easily performed and we find that

$$G_\pm(\mathbf{R};\omega) = \frac{i}{(2\pi)^2 R}\int_{C_\pm} KdK\, \frac{e^{iKR}}{K^2 - k^2} - \frac{i}{(2\pi)^2 R}\int_{C_\pm} KdK\, \frac{e^{-iKR}}{K^2 - k^2}$$

If we now make the transformation $K \rightarrow -K$ in the second integral we find that the above reduces to

$$G_\pm(\mathbf{R};\omega) = \frac{i}{(2\pi)^2 R}\int_{\tilde{C}_\pm} KdK\, \frac{e^{iKR}}{K^2 - k^2}$$

where now $\tilde{C}_+ = C_+ - C_+(-K)$ extends from $-\infty$ to $+\infty$ and lies above the pole at $K = -k$ and below the pole at $K = +k$ and $\tilde{C}_- = C_- - C_-(-K)$ extends from $-\infty$ to $+\infty$ and lies below the pole at $K = -k$ and above the pole at $K = +k$

Since $R > 0$ the integration contours can be closed in the u.h.p. and can thus be readily performed with the help of Cauchy's residue theorem. In the case of G_+ the u.h.p. pole is located at $K = +k$ and for G_- is at $K = -k$ and we find that

$$G_\pm(\mathbf{R};\omega) = -\frac{1}{4\pi}\frac{e^{\pm ikR}}{R}.$$

2.6 Using the Fourier integral representations of the frequency domain outgoing and incoming wave Green functions $G_\pm(\mathbf{R},\omega)$ and Cauchy's integral theorem show that the free field propagator $G_f(\mathbf{R},\omega) = G_+(\mathbf{R},\omega) - G_-(\mathbf{R},\omega)$ satisfies the homogeneous Helmholtz equation.

Using the results from the previous problem we have that

$$G_f(\mathbf{R},\omega) = G_+(\mathbf{R},\omega) - G_-(\mathbf{R},\omega)$$
$$= \frac{i}{(2\pi)^2 R}\int_{\tilde{C}_+} KdK\,\frac{e^{iKR}}{K^2 - k^2} - \frac{i}{(2\pi)^2 R}\int_{\tilde{C}_-} KdK\,\frac{e^{iKR}}{K^2 - k^2}$$
$$= \frac{i}{(2\pi)^2 R}\int_{C} KdK\,\frac{e^{iKR}}{K^2 - k^2}$$

where $\tilde{C}_+$ extends from $-\infty$ to $+\infty$ and lies above the pole at $K = -k$ and below the pole at $K = +k$ and $\tilde{C}_-$ extends from $-\infty$ to $+\infty$ and lies below the pole at $K = -k$ and above the pole at $K = +k$ and where $C = \tilde{C}_+ - \tilde{C}_-$ is the sum of the closed contour surrounding the pole at $K = -k$ in the counter clockwise direction and the closed contour surrounding the pole at $K = +k$ in the clockwise direction. We then find that

$$[\nabla^2 + k^2]G_\pm = -\frac{i}{(2\pi)^2 R}\int_C KdK\, e^{iKR} = 0$$

since no poles lie within the contour C.

2.7 Use the second Helmholtz identity in the time-domain Eq.(1.36b) to derive Eq.(2.31b). Can Eq.(1.36b) be derived from Eq.(2.31b) for dispersive media? What does this say about the validity of this identity in the time domain for dispersive media?

The frequency domain version Eq.(2.31b) of Eq.(1.36b) follows immediately from the well-known relationship between a convolution of two functions and the inverse Fourier transform of the product of the transforms of the two functions:

$$\int_{-\infty}^{\infty} dt'\, f(t-t')g(t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \tilde{F}(\omega)\tilde{G}(\omega)e^{-i\omega t}.$$

By the same argument the time-domain version Eq.(1.36b) of Eq.(2.31b) must also hold for dispersive media which means that the second Helmholtz identity in the time-domain Eq.(1.36b) holds for both dispersive as well as non-dispersive media.

2.8 Show that if $\mathbf{r} \in \tau$ that the back propagated field $\Phi(\mathbf{r},\omega)$ defined in Eq.(2.33a) can be expressed in terms of the free field propagator in the form

$$\Phi(\mathbf{r},\omega) = -\int_{\partial\tau} dS'\,[U_+(\mathbf{r}',\omega)\frac{\partial}{\partial n'}G_f(\mathbf{r}-\mathbf{r}',\omega) - G_f(\mathbf{r}-\mathbf{r}',\omega)\frac{\partial}{\partial n'}U_+(\mathbf{r}',\omega)].$$

This follows immediately from the second Helmholtz identity Eq.(2.31b).

2.9 Verify that the contribution from the integral over the surface Σ_∞ in the derivation of Eqs.(2.43) vanishes.

We wish to show that

$$I = \int_{\Sigma_\infty} dS'\,[U_+(\mathbf{r}',\omega)\frac{\partial}{\partial n'}G_+(\mathbf{r}-\mathbf{r}',\omega) - G_+(\mathbf{r}-\mathbf{r}',\omega)\frac{\partial}{\partial n'}U_+(\mathbf{r}',\omega)] = 0.$$

To establish this result we use the asymptotic form for the outgoing wave Green function and its normal derivative

$$G_+(\mathbf{r}-\mathbf{r}',\omega) \sim -\frac{1}{4\pi}e^{-ik\hat{\mathbf{r}}'\cdot\mathbf{r}}\frac{e^{ikr'}}{r'}, \qquad \frac{\partial G_+(\mathbf{r}-\mathbf{r}',\omega)}{\partial r'} \sim -\frac{ik}{4\pi}e^{-ik\hat{\mathbf{r}}'\cdot\mathbf{r}}\frac{e^{ikr'}}{r'}$$

as $r' \to \infty$ and which then yields

$$G_+(\mathbf{r}-\mathbf{r}',\omega)\frac{\partial}{\partial n'}U_+(\mathbf{r}',\omega) - U_+(\mathbf{r}',\omega)\frac{\partial}{\partial n'}G_+(\mathbf{r}-\mathbf{r}',\omega)$$
$$\sim -\frac{1}{4\pi}e^{-ik\hat{\mathbf{r}}'\cdot\mathbf{r}}\frac{e^{ikr'}}{r'}\{\frac{\partial}{\partial n'}U_+(\mathbf{r}',\omega) - ikU_+(\mathbf{r}',\omega)\}.$$

The bracketed term in the above equation will go to zero faster than $1/r'$ according to the SRC. It then follows on setting $dS' = r'^2 d\Omega$ that the integral over Σ_∞ will vanish and $I = 0$ as required.

2.10 State and prove the frequency domain version of the source decomposition theorem 1.3.

The theorem in the frequency domain reads:

Theorem 2.1 (Frequency Domain Source Decomposition Theorem) *Let $Q(\mathbf{r},\omega)$ be a square integrable source compactly supported within τ_0. Then this source can be uniquely decomposed into an NR component $Q_{nr}(\mathbf{r},\omega)$ and a minimum norm component $\hat{Q}(\mathbf{r},\omega)$ such that*

$$\int_{\tau_0} d^3r\, Q_{nr}(\mathbf{r},\omega)\hat{Q}(\mathbf{r},\omega) = 0,$$
$$[\nabla_r^2 + k^2]\hat{Q}(\mathbf{r},\omega) = 0,$$
$$Q_{nr}(\mathbf{r},\omega) = [\nabla_r^2 + k^2]\Pi(\mathbf{r},\omega),$$

where $\Pi(\mathbf{r},\omega)$ is a square integrable function supported in τ_0 that has continuous first partial derivatives.

The first statement of the theorem follows immediately from the corresponding statement of Theorem 1.3 on the application of Parseval's Theorem while the second two statements follow from Fourier transformation of the corresponding two statements of Theorem 1.3.

2.11 Show that by using a coordinate system translation along the z axis that the RS solutions given in Eqs.(2.47) yield the solutions in Eqs.(2.48) which are valid for a boundary value plane $z = z_0$ with z_0 being arbitrary.

Although the solutions to the RS Dirichlet and Neumann boundary value problems given in Eqs.(2.47) assume that the boundary plane is the plane $z = 0$, the general case of a boundary plane located at $z = z_0$ is easily obtained via a coordinate system translation along the z axis. In particular, if we note that the arguments on the Green functions in Eqs.(2.47) depend only on the difference vector $\mathbf{r} - \boldsymbol{\rho}'$ we can translate the origin of the coordinate system so that the data plane $z = 0$ gets translated to $z = z_0$ so that Eqs.(2.47) transform to Eqs.(2.48).

2.12 Solve the one-dimensional inverse RS problem in the r.h.s. $z > z^+$ in terms of Dirichlet data on the line $z = z_0 > z^+$. Compare this solution with the exact field radiated by the source into this half-space.

We solved the one-dimensional RS problem in the r.h.s. for Dirichlet data $U_0(\omega)$ specified at $z = 0$ in Example 2.7 where the solution was found to be

$$U(z, \omega) = U_0(\omega)e^{ikz}.$$

In the more general case where the source lies in the l.h.s. $z < z^+$ and Dirichlet data on the boundary $z = z_+$ the solution in the half-space $z > z_+$ is easily verified to be given by

$$U(z, \omega) = U_{z_+}(\omega)e^{ik(z-z_+)}.$$

We then find that

$$U(z_0, \omega) = U_{z_+}(\omega)e^{ik(z_0-z_+)} \rightarrow U_{z_+}(\omega) = U(z_0, \omega)e^{-ik(z_0-z_+)}$$

and the (exact) solution of the inverse RS problem in the half-space $z > z^+$ is then

$$U(z, \omega) = U(z_0, \omega)e^{-ik(z_0-z_+)}e^{ik(z-z_+)} = U(z_0, \omega)e^{ik(z-z_0)}.$$

2.13 Compute a one-dimensional non-radiating source distributed on the two points $z = z^{\pm}$.

The one-dimensional Green function was found in Example 2.2 to be given by

$$G_+(z - z', \omega) = -\frac{i}{2k}e^{ik|z-z'|}$$

which yields the solution to the one-dimensional radiation problem

$$U_+(z, \omega) = -\frac{i}{2k}\int_{z^-}^{z^+} dz'\, Q(z', \omega)e^{ik|z-z'|},$$

where the interval $[z^-, z^+]$ is the support of the source. The radiation pattern is immediately found from the above solution to be

$$f(s,\omega) = -\frac{i}{2k}\int_{z^-}^{z^+} dz'\, Q(z',\omega)e^{-iksz'} = -\frac{i}{2k}\tilde{Q}(sk,\omega)e^{-iksz'},$$

where $s = +1$ in the r.h.s. and $s = -1$ in the l.h.s. The condition for an NR source then reduces to

$$\tilde{Q}(sk,\omega) = 0, \quad s = \pm 1.$$

For a one-dimensional source distributed on the two points $z = z^\pm$ we have that

$$\tilde{Q}(sk,\omega) = \int_{z^-}^{z^+} dz'\, [Q^-\delta(z-z^-)+Q^+\delta(z-z^+)]e^{-iksz'} = Q^-e^{-iksz^-}+Q^+e^{-iksz^+}$$

where $Q^\pm$ are two functions of ω. We then find that the NR condition becomes

$$Q^-e^{-iksz^-} + Q^+e^{-iksz^+} = 0, \quad s = \pm 1.$$

We can express the above condition in the matrix form

$$\begin{bmatrix} e^{ikz^-} & e^{ikz^+} \\ e^{-ikz^-} & e^{-ikz^+} \end{bmatrix} \begin{bmatrix} Q^- \\ Q^+ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

which will possess a solution if and only if

$$\sin k(z^- - z^+) = 0 \rightarrow z^- - z^+ = m\frac{\lambda}{2},$$

where m is an arbitrary integer.

2.14 Use the general procedure employed in Section 2.8 to compute the field radiated by a source located in the l.h.s. in the presence of a Dirichlet plane (a plane over which the field vanishes) at $z = 0$. Express your answer in terms of a Dirichlet Green function.

We begin, as usual, with the equations satisfied by the Green function and radiated field

$$[\nabla^2_{r'} + k^2]G(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}'),$$
$$[\nabla^2_{r'} + k^2]U(\mathbf{r}',\omega) = Q(\mathbf{r}';\omega).$$

As boundary conditions we require that the radiated field satisfy the SRC throughout the left-half-space $z < 0$ and homogeneous Dirichlet conditions over the plane $z = 0$. If we also require that the Green function (as of now un-specified) also satisfy the SRC in the l.h.s. then standard Green function techniques yield the equation

$$U(\mathbf{r},\omega) = \int_{\tau_0} d^3r'\, G(\mathbf{r},\mathbf{r}',\omega)Q(\mathbf{r}';\omega)+\int_{\partial\tau} dS'\, [U(\mathbf{r}',\omega)\frac{\partial}{\partial z'}G(\mathbf{r},\mathbf{r}',\omega)-G(\mathbf{r},\mathbf{r}',\omega)\frac{\partial}{\partial z'}U(\mathbf{r}',\omega)]$$

where it is assumed that $\mathbf{r}$ is contained in the l.h.s. (i.e., that $z < 0$) and where $\partial\tau$ is the boundary plane $z' = 0$.

We now impose the physical requirement that the field satisfy homogeneous Dirichlet conditions on the plane $z'=0$. This annihilates the first term in the surface integral on the r.h.s. of the above equation. We can annihilate the second term in this integral if we require that the Green function also satisfy homogeneous Dirichlet conditions over this plane; i.e., if we select G to be the Dirichlet Green function relative to this plane. Under this choice of Green function we then find that the above equation reduces to

$$U(\mathbf{r},\omega)=\int_{\tau_0} d^3r'\, G_D(\mathbf{r},\mathbf{r}',\omega)Q(\mathbf{r}';\omega), \quad z<0, \tag{2.2}$$

which is the radiated field throughout the half-space $z<0$ that vanishes on the boundary $z=0$; i.e., is the solution to the radiation problem in the presence of a perfectly conducting infinite plane located at $z=0$. A completely parallel development can be employed for the case where the field must satisfy homogeneous Neumann conditions over an infinite plane.

2.15 Interpret the radiated field found in the previous problem in terms of a mirror image source relative to the plane $z=0$.

By substituting for the Dirichlet Green function we find that Eq.(2.2) can be written in the form

$$U(\mathbf{r},\omega)=\overbrace{\int_{\tau_0} d^3r'\, G_+(\mathbf{r}-\mathbf{r}',\omega)Q(\mathbf{r}';\omega)}^{U_+(\mathbf{r},\omega)}-\overbrace{\int_{\tau_0} d^3r'\, G_+(\mathbf{r}-\tilde{\mathbf{r}'},\omega)Q(\mathbf{r}';\omega)}^{U^{(s)}(\mathbf{r},\omega)},$$

where $\tilde{\mathbf{r}'}$ is the mirror image of the source point $\mathbf{r}'$ about the conducting plane. The first term in the above equation is the normal radiated field U_+ in an infinite homogeneous medium having no physical boundaries. The second term can be interpreted as the reflected (scattered) field produced by the perfecting conducting infinite plane at $z=0$. By noting that

$$|\mathbf{r}-\tilde{\mathbf{r}'}|=\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}$$

we can make the change of integration variable $z'\to -z'$ in the scattered field term in the above equation to obtain

$$U^{(s)}(\mathbf{r},\omega)=-\int_{\tau_0} d^3r'\, G_+(\mathbf{r}-\mathbf{r}',\omega)Q(\tilde{\mathbf{r}'};\omega)=-\int_{\tilde{\tau_0}} d^3r'\, G_+(\mathbf{r}-\mathbf{r}',\omega)\tilde{Q}(\mathbf{r}';\omega)$$

where $\tilde{\tau_0}$ is the mirror image of the source region τ_0 about the plane $z=0$ and $\tilde{Q}(\mathbf{r}',\omega)=Q(x',y',-z')$ is the mirror image of the source itself. Under this interpretation we see that the scattered field component of the total field is the negative of the field that would be radiated into an infinite homogeneous medium by the mirror image source about the conducting plane $z=0$.

2.16 Determine a cloaking surface source over a closed surface $\partial\tau_0$ directly in the frequency domain using NR surface sources constructed from the incident wavefield and its normal derivative over the cloaking surface.

This is done in complete analogy to the time-domain construction employed in Chapter 1. In this case we base the construction on the Helmholtz identities in the frequency domain in Section 2.5 so that the net field radiated into an interior region τ_0 from a source located outside of τ_0 and the surface source components satisfying the frequency domain version of Eqs.(1.73) will be null.

2.17 Prove that the cloaking surface source found in the previous problem with the incident wavefield replaced by the total wavefield (incident plus that generated from the cloaked object) radiates an identical wavefield into the interior τ_0 as that of the original surface source.

This proof follows identical lines as that employed in Problem 1.26 of Chapter 1.

2.18 Give an argument why the surface source found in the previous problem also cloaks the region τ_0; i.e., both cancels the incident wavefield within τ_0 and is NR outside of τ_0.

See solution of Problem 1.26 of Chapter 1.