Extra Exercises and Assignments for

Wakker (2010) "Prospect Theory: for Risk and Ambiguity"

July, 2013

ASSIGNMENT 1.3.6 [b] [Measuring subjective probabilities without using sure prospects]. Consider the same case as in Exercise 1.3.4, with you wanting to measure the subjective probabilities of the street vendor. There is, however, one complication. You know that the street vendor maximizes expected value, but only when no constant prospect is involved. When choosing between a constant prospect and a nonconstant prospect, the street vendor may not choose the one with maximal expected value. You do not know what he will do then. Hence the method of Eqs. 1.3.1 and 1.3.2 cannot be used. Can you think of a way to measure subjective probabilities in this case? □

EXERCISE 1.6.14 [b] [Perfect hedges]. Prospect y is a *perfect hedge* for prospect x if x(s) + y(s) is constant. That is, every fluctuation in x is exactly neutralized by an opposite fluctuation in y, so that a riskless position results. Perfect hedges are often used in finance, where they constitute the optimal combinations of portfolios. They are also useful to simplify theoretical analyses (e.g. in no-arbitrage analyses of binomial trees in Hull 2005). This exercise gives a simple demonstration of their theoretical convenience. I suppress the three events. Assume the model of Theorem 1.6.1, and (4,1,5) ~ (2,5,1). What is the CE of these two prospects? □

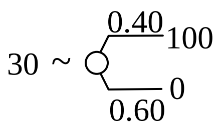
EXERCISE 1.6.15 [a] [Trade versus arbitrage]. Peter can easily produce a car and is willing to sell it for $8,000. Paul desparately needs a car and is willing to buy it for $14,000. Peter and Paul have no way to get in touch and trade with each other otherwise than through John. John buys the car at price $10,000 from Peter and sells it at price $12,000 to Paul. This takes John no effort or extra expenses, and he makes a sure $2,000 profit. Did he make a Dutch book/arbitrage? □

ASSIGNMENT 2.3.3[c] [Subjective probabilities different from objective ones if no richness]. Given an example with three states of nature, s_1,s_2, and s_3, where subjective expected value maximization holds with respect to subjective probabilities $Q(s_j) = q_j$, where also objective probabilities $P(s_j) = p_j$ are given, where decision under risk holds, but were $Q \neq P$. □

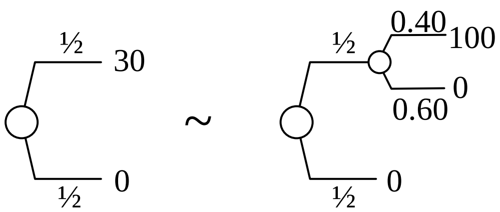
EXERCISE 2.6.7[a] [Numerical illustration of SG consistency]. Assume that U(0) = 0 and U(100) = 1.

a) Consider the following indifference. Calculate U(30).

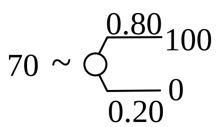
g



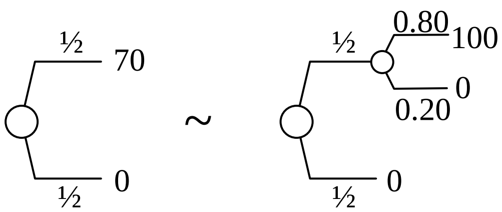
b) Consider the following indifference. Calculate U(30).



c) Consider the following indifference. Calculate U(70).



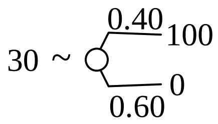
d) Consider the following indifference. Calculate U(70).



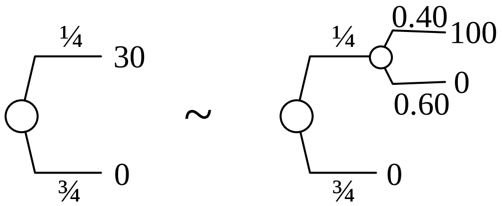
□

ASSIGNMENT 2.6.8[a] [Numerical illustration of SG consistency]. Assume that U(0) = 0 and U(100) = 1.

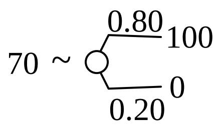
a) Consider the following indifference. Calculate U(30).



b) Consider the following indifference. Calculate U(30).



c) Consider the following indifference. Calculate U(70).



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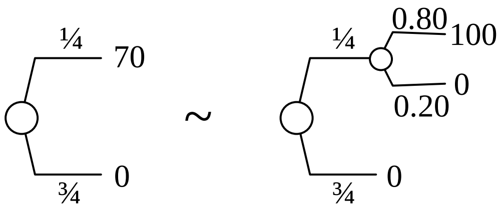

□

ASSIGNMENT 3.3.7[a] [Your first consultancy job]. This will be your first decision-analysis consultancy job. Choose one of your friends who has a bike insured, and preferably an open mind. Ask if she, in case of loss of bike and no insurance, could immediately buy a new one. If no (not enough money for it), she is not a good candidate for serving as your client, and have to search another one. If yes, can start the consultancy if she agrees to listen. Your task is to advise her to cancel the bike insurance, and to write a little report on the discussion you will have with her to that effect.

	bike lost	bike not lost
take no insurance	lose whole bike	lose nothing
take insurance	lose premium	lose premium

FIGURE

Start asking why she took the insurance. The likely answer will point to the advantage in the second ("bike lost") column in the figure, of losing only the premium and not the whole bike. Then you can point out the drawback, of losing the premium for nothing if no bike lost. Usually, conversations go in circles from here on, with one side always repeating the pro and the other side the con, without commensurance or convergence.

As you can explain to your client, to come to a sensible decision, a way must be found to compare the pro to the con, and to come to see which is more important.

What I can explain to you, but you cannot to your client, is that the big move of de Finetti's expected value and of expected utility is that these theories find a way to commensurate probabilities and outcomes, so that the pros and cons can be weighted against each other, leading to a sensible decision. Remember, never focus on one pro or one con in decisions, but always consider both sides and weigh them.

Another thing I have explained to you but you cannot to your client I guess, is a justification that works for one-shot decisions: For the moderate amounts as relevant here (because your client can easily buy a new bike if needed) additivity and absence of arbitrage are reasonable, and this implies that expected value is the right thing. This means not insuring.

Therefore, to consult your client, you cannot use the above arguments, and have to use another argument. Please use long-run arguments, based on something known as the law of large numbers (no problem if you don't know it). Tell your client that it is a well-known fact from statistics, for which she can take your word, that taking small insurances such as for bikes all life long, surely makes you pay more premiums in total than you get back from reimbursement when bike-loss. For example, not paying the premiums to the insurance company but putting them in a savings account and paying bikes lost from that account, makes you better off than taking insurance. This life-long perspective weighs the pro against the con and shows that the con of insuring weighs more than the pro.

Can be prepared for the following escape arguments:

1. Insurance just so as not to worry. Answer: Just don't worry without insurance. Just pay the losses whenever they come. Worrying is only something you do to yourself.

2. If insured then can be careless with bikes, making life easier. Answer: The insurance company reckons with this and takes care the premium is still high enough. This is, then, an extra reason for not taking insurance. Even if you are careless without the insurance, still worthwhile not to insure but to keep the premiums. Being more careful without insurance is simply an extra option available.

Extra arguments reinforcing no-insurance:
1. No administration costs.
2. The premium is even higher than appropriate for careless behavior, because it must also cover the expenses of the small part of clients who fraud.

Do point out to your client that the life-long reasoning only works if, when bike lost, immediately a new bike can be bought, so that the loss is only monetary and there are no big extra inconveniences generated. For big losses that cannot easily be covered, the above reasoning does not work, and insurance is a good thing that can be recommended.

Write a report of between 150 and 300 words about the results. Did you convince your client? If not, which counterargument kept your client from following your advise? Did your client only stick with the pro of insurance without trying to commensurate with the con? One of the escape arguments? Other escape arguments?
□

EXERCISE 3.3.8^b [Drawing and analyzing decision tree; taken from LaValle 1972, Example 1.5.1a]. You have contracted to deliver a special-purpose analog computer to the government at a price that will yield you a profit of $250,000—barring unforeseen failure of the computer to perform its function on a space vahicle. The only possible cause of failure would be defectiveness of a crucial electronic subsystem which you have subcontracted to another firm for political reasons despite the occasionally slipshod practices in the final-assembly operations of that company, which claims that the probability of defect in the subsystem is .001, a figure which you suspect was pulled from a hat and which you would revise to .10. If the computer malfunctions during its mission, your company will be subject to a $500,000 penalty and will also lose prestige in the industry. After careful consideration, you decide that your available courses of action are: (a) install as is, i.e. install the subsystem as is and take your chances with it; (b)rebuild, i.e. have your own people tear it down, inspect it, and carefully rebuild it, at a cost of $100,000, thus ensuring that the system will function properly.

a) Draw a decision tree with all probabilities and outcomes indicated.

b) Do not determine quantitative utilities to be used in an expected utility analysis, but use qualitative grounds to make a first decision.

c) Although at first it may seem that there are three unknowns for utility (U at the three outcomes that may occur), in reality there is only one unknown regarding utility. If this is a mystery to you, think about Exercise 2.6.4. Say which unknown it is, and determine its threshold value (making the two available prospects indifferent).

d) What would you do? □

EXERCISE 3.3.9^{b} [Analyzing oil dilling example]. Consider the oil drilling example in §5.10 of Winkler (1972), which is a simplified version of an actual decision analysis carried out by

Grayson, C. Jackson Jr. (1979), "*Decisions under Uncertainty: Drilling Decisions by Oil and Gas Operators*." Arno Press, New York; first version 1960, Harvard Business School.

The author suggests that the problem may be sensitive to changes in utility (p. 282, penultimate para). We will investigate this. Assume that utility is not as in Winkler (1972), but instead it is

$$U(\alpha) = (\alpha+150000)^{r},$$

with the power r specified below.[1] With this utility function, determine which of the five prospects in Winkler (1972) is most preferred under expected utility. Note that, if a prospect x has EU(x) = μ, then for certainty equivalent can be calculated as

$$CE(x) = U^{-1}(\mu) = \mu^{1/r} - 150{,}000.$$

For all five prospects, give their expected utility and their certainty equivalents, for the cases in parts (a)-(f). Then answer the summarizing questions in part (g).

I add a utility calculation and a CE calculation for each case, that can help you check that you programmed these calculations correctly.

(a) r = 1.210 (U(10000) = 3400223.08); EU(x) = 3400223.08 then CE(x) = 100000);

(b) r = 0.961; (U(10000) = 153964.41; EU(x) = 153964.41 then CE(x) = 100000);

(c) r = 0.936; (U(10000) = 112842.28; EU(x) = 112842.28 then CE(x) = 100000);

(d) r = 0.912; (U(10000) = 83737.75; EU(x) = 83737.75 then CE(x) = 100000);

(e) r = 0.500; (U(10000) = 500; EU(x) = 500 then CE(x) = 100000);

(f) r = 0.010; (U(10000) = 1.13; EU(x) = 1.13 then CE(x) = 100000);

(g) For which r are the most risky option, drill 100%, and the safest option, override 1/16, approximately equivalent? Do you recognize a pattern in how the r's determine which prospect is best? For which r do the CEs best fits the CE of Winkler? Do the results suggest to you that utility is a sensitive parameter in this problem? □

ASSIGNMENT 3.4.2[a] [More risk averse and risk premiums]. Consider Theorem 3.4.1. Show that person 2 is more risk averse than person 1 if and only if the risk premiums of person 2 (weakly) exceed those of person 1. □

ASSIGNMENT 3.4.3[b] [Decreasing risk aversion]. Show that decreasing (absolute) risk aversion holds if and only if [for all $\varepsilon \geq 0$: $\alpha+\varepsilon \sim x+\varepsilon \Rightarrow \alpha \succcurlyeq x$]. Some convenient

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[1] I denote the power by r and not by θ as in most of my book so as to avoid confusion with Winkler's notation, who uses θ for another purpose.

notation: Take any $\varepsilon \geq 0$, write $U_\varepsilon(\alpha) = U(\alpha + \varepsilon)$, and $\succcurlyeq_\varepsilon$ for the preference relation maximizing EU with utility function U_ε. □

ASSIGNMENT 4.6.3^c [Behavioral foundations, such as Theorem 4.6.4, which were derived under a richness assumption, applied to cases where the richness assumption is not satisfied, such as in a finite case]. This assignment is very difficult.

Assume a finite state space S, say $S = \{s_1,s_2\}$. You, however, cannot observe the space of all prospects $\mathbb{R}^2$, as in Theorem 4.6.4. Instead you can only observe finitely many preferences. Say you only consider the outcome space {0,1,2}, i.e. $\{(x_1,x_2): 0 \leq x_j \leq 2$ and x_j is an integer for all j$\}$, leaving you with $3^2 = 9$ prospects. You have observed all preferences of a subject between these 9 prospects. Checking each preference condition of Theorem 4.6.4 for all possible cases, you find that:

(a) transitivity is not violated;

(b) completeness is not violated;

(c) monotonicity is not violated;

(d) tradeoff consistency is not violated.

Further, you find no violation of continuity because a finite data set can never reveal a violation of continuity. Can you conclude that expected utility holds, i.e. that the subject maximizes expected utility?

The same question, with the same answer, can be asked for every behavioral foundation with a richness condition in the literature. In the solution to this question (only provided to teachers) the same question will be answered for Savage's (1954) behavioral foundation, and references to general discussions in the literature will be given. □

ASSIGNMENT 4.7.2^a **{new in July 2013}** [Inconsistencies in probabilities lead to inconsistent utility tradeoffs]. Assume, with E denoting [cand_1 wins] and E^c denoting [cand_2 wins], that the decision maker is a quasi-SEU maximizer. He is consistent in utility, with always $U(\alpha) = \sqrt{\alpha}$, but he is inconsistent in probability as follows. He has $P(E) = ⅔$ if E has the worse outcome but $P(E) = ⅓$ if E has the better outcome. He evaluates $\alpha_E\beta$ by $\frac{2}{3}U(\alpha) + \frac{1}{3}U(\beta)$ if $\alpha \le \beta$, but by $\frac{1}{3}U(\alpha) + \frac{2}{3}U(\beta)$ if $\alpha \ge \beta$. He is a kind of pessimist, thinking the bad event is twice as likely as the good event.

We observe $\alpha^{j+1}{}_E0 \sim \alpha^j{}_E1$ for $a^0 = 16$, $\alpha^1 = 36$, $\alpha^2 = 64$, giving the sequence $\alpha^0,\alpha^1,\alpha^2$ = 16, 36, 64 of outcomes equally-spaced in utility units (U difference 2), and giving $64\ominus36 \sim^t 36\ominus16$. We next observe $\beta_E36 \sim 36_E100$ and $36_E36 \sim 16_E100$. Calculate what β is. Verify that the latter two indifferences imply $\beta\ominus36 \sim^t 36\ominus16$. Verify that $\beta \ne 64$. That is, the inconsistent treatment of probability has generated an inconsistency in our utility measurement, and a violation of tradeoff consistency. This violation shows to the researcher that SEU is violated, and that a different model will have to be invoked to analyze the preferences and measure utility. □

ASSIGNMENT 4.8.7^b [Additivity ==> sure-thing principle]. Assume weak ordering. Show that additivity, defined in Chapter 1, implies the sure-thing principle. Because we do not assume continuity or the existence of certainty equivalents, you cannot use theorems from the book, and have to find a direct derivation. □

ASSIGNMENT 4.9.2^c **{new in July 2013}** [Additivity is needed in Theorem 4.9.4] . Consider Eq. 4.9.2´:

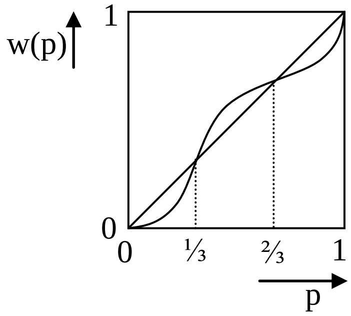
For each event E, a matching probability q exists. (4.9.2´)

That is, it weakens Eq. 4.9.2 by removing the additivity part. Now consider the variation of Theorem 4.9.4 with additivity in Statement (ii) replaced by Eq. 4.9.2´. Does this variation of Theorem 4.9.4´ hold? In words: Can additivity be derived from Eq. 4.9.3 and, hence, be removed from Eq. 4.9.2? Show by proof or counterexample. □

ASSIGNMENT 4.12.2^b [Violating sure-thing principle]. Consider Figure 2.4.1. Show that Figs. 2.4.1g and h can be used to test the sure-thing principle. A difficulty of this exercise is, of course, that the sure-thing principle has been defined for event-contingent prospects, whereas Figs. 2.4.1g and h concern probability-contingent prospects. □

Assume weak ordering. Show that additivity, defined in Chapter 1, implies the sure-thing principle. Because we do not assume continuity or the existence of certainty equivalents, you cannot use theorems from the book, and have to find a direct derivation. □

ASSIGNMENT 6.3.2^a [Reading weighting function]. Assume DUR and RDU with linear utility and the following probability weighting function



Give a two-outcome prospect with strict risk aversion (expected value strictly more preferred than the prospect) and a two-outcome prospect with strict risk seeking (expected value strictly less preferred than the prospect). □

ASSIGNMENT 6.3.3^a. For this assignment, do not use a pocket calculator or computer, but calculate by hand. Assume RDU with $U(\alpha) = \alpha^2$ and $w(p) = p^2$. What is the certainty equivalent of $4_{0.5}\sqrt{3}$? Is it consistent with risk aversion or with risk seeking? □

ASSIGNMENT 6.4.5^c [Overweighting disappointing outcomes].

Routledge, Bryan R. & Stanley E. Zin (2010) "Generalized Disappointment Aversion and Asset Prices," *The Journal of Finance* 64, 1303–1332

proposed a variation of Gul's (1991) disappointment aversion model where, for a prospect x, all outcomes below δCE are overweighted by a factor θ (Gul 1991 is the special case of $\delta = 1$). They motivate their choice by having the overweighting depend on the prospect considered (unlike loss aversion) and by being on the spirit of value at risk. Exercise 6.4.4 showed that value at risk is a special case of rank dependence. An analog of Routledge & Zin's idea, capturing the same intuitions, can be developed for rank dependence. Imagine that you want to maximize expected utility with one exception: all outcomes below the 0.25 quantile are qualified as disappointing and should be weighted "twice as much" as the others. Think what this "twice as much" can mean exactly, and indicate how this can be obtained using RDU with an appropriate weighting function. □

ASSIGNMENT 6.4.6^b [Worst-case analysis as an extreme case of the certainty effect]. Assume a decision maker who goes by worst-case analysis: a prospect is evaluated by its worst outcome; more precisely, by the worst outcome that has positive probability. For example, $0_{1/1000}1000 \sim 0_{8/9}10 \sim 0$. Model this by RDU with linear utility, where w may be nondecreasing and need not be increasing. Indicate what w is. □

ASSIGNMENT 7.4.3^c [Assumptions in Theorem 7.4.1]. Assume RDU with $U(\alpha) = \alpha$ and w constant 0 on [0,0.5], and w constant 1 on (0.5,1]. That is, prospects are evaluated by their median outcome. Then w is not convex. Show that $\succcurlyeq$ is quasiconvex.

Given that, according to Theorem 7.4.1, quasiconvexity of $\succcurlyeq$ implies convexity of w, at least one of the assumptions in Theorem 7.4.1 must be violated. Which? □

ASSIGNMENT 7.7.2^c [Likelihood insensitivity is not a local property]. p. 225, penultimate para, claims that insensitivity is not a local property. To see this point, assume that the insensitivity region $[b_{rb}, w_{rb}] = [0.05, 0.95]$ has been chosen. Assume that Eq. 7.7.5 holds for all small p, meaning here for all $0 \le p \le 0.01$. Show that Eq. 7.7.5 need not hold for larger p. □

ASSIGNMENT 8.1.1^{a} [The endowment effect as a riskless version of loss aversion]. The choices considered in this assignment are a small variation of an experiment described by

Kahneman, Daniel, Jack L. Knetsch, & Richard H. Thaler (1991) "The Endowment Effect, Loss Aversion, and Status Quo Bias: Anomalies," *Journal of Economic Perspectives* 5 no. 1, 193–206.

The situation considered here is somewhat similar to Figure 8.1.1. In Fig. a, subjects can buy a mug for \$5 or not. In Fig. b, subjects can choose between receiving a mug or receiving \$5. In Fig. c, subjects are first given a mug, and next can sell it for \$5 or keep it.

FIGURE.

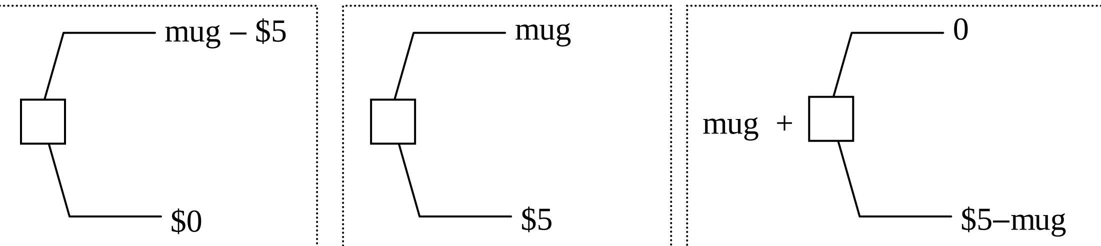


FIG. a [Buy]. Buy mug for \$5 or not.

FIG. b [Choice]. Choose between mug and \$5.

FIG. c [Sell]. Prior endowment with mug, followed by offer to sell mug for \$5 or not.

a) If the subjects are all very rational, then will the choices observed in Fig. b be closer to those in Fig. a or in Fig. c?

b) The experiment found that the choices in Fig. b were closer to those in Fig. a than in Fig. c. In Figs. a and b similarly large majorities chose the lower branch, but in Fig. c it was different and the majority chose the upper branch. Here is an open question: Can you speculate on an explanation why Fig. c gave a different result than Fig. b?

c) Here is another open question: Can you speculate on an explanation why Fig. a gave a similar result as Fig. b? (This question is difficult.) □

EXERCISE 9.2.1^{a} [Calculating PT]. Consider (0.1:9, 0.3:1, 0.5:–1, 0.1:–4) and (0.5:3, 0.5:–2). Assume PT with $w^{+}(p) = p^{2}$, $w^{-}(p) = \sqrt{p}$, $U(\alpha) = u(\alpha) = \alpha^{0.6}$ if $\alpha \geq 0$, and $U(\alpha)$

$= 2.25 \times u(\alpha) = -2.25(-\alpha)^{0.8}$ if $\alpha < 0$. Calculate the PT value of both prospects, and determine which is preferred. □

EXERCISE 9.3.12[a] [Using PT as descriptive theory and EU as normative theory; see Bleichrodt, Pinto, & Wakker (2001)]. Reconsider the analysis of the medical example in §3.1. Now assume, however, that the probability p in Figure 3.1.2 that gives indifference is not 0.90 as it was in §3.1, but is 0.97.

a) Use the result of Exercise 3.1.1 to immediately conclude what would now be the preferred treatment under the "EU-allthrough" analysis of §3.1.
b) Now consider an alternative analysis. Assume that the patient does not behave according to EU in Figure 3.1.2, but, instead, according to PT. Immediate death is taken as reference point with utility 0, and U(normal voice) = 1. w^+ is as in Eq. 9.3.3 with c = 0.61. What is U(artificial speech)?
c) Assume that PT is accepted as best descriptive theory so that the U value derived from part b is accepted. Assume that EU is taken as best normative theory, to be used to determine the best solution in Figure 3.1.1. Assume that in the latter analysis, the U value derived in part (b) is used as the proper utility value. What decision is recommended now? □

EXERCISE 9.5.3[b] [Loss aversion versus basic utility]. Imagine a person has just enough money to pay for all needs and to continue living as is, which living is fine as is. For each nontrivial loss of money, something dear must be given up and life style and habits have to worsen considerably. Thus in the current position a loss is felt 2.25 more intensely than a corresponding gain. Is this person loss averse? Is this attitude irrational? □

ASSIGNMENT 9.5.4[b] [Data fitting for prospect theory, showing importance of normalizing utility to have derivative 1 at zero]. We consider again the eight indifferences from Tversky & Kahneman (1992) of Exercise 3.6.5. We assume PT, but now allow the risk attitudes for gains and losses to be completely independent.

We assume that $w^+(p)$ is as in Eq. 7.2.1 with parameter c, and $w^-(p)$ is also as in Eq. 7.2.1 but has parameter c´ instead of c. For gains, U is exponential with parameter θ, and for losses U is exponential with parameter θ´. In each part, find the five parameters that minimize the distance measure of §A.2 to the data, for $0.3 \leq c = i/100 \leq 1.1$, $0.3 \leq c' = i'/100 \leq 1.1$, $0 < \lambda = k/100 < 3$, $-0.0010 \leq \theta \leq 0.0050$, and $-0.0080 \leq \theta' \leq 0.0010$ that best fit the data, and give the distance.

a) Take U exactly as in Eq. 3.5.4 and U´ also but only with θ´ instead of θ.

b) Take U exactly as in Eq. 3.5.5 and U´ also (so that they have derivatives 1 at 0) but only with θ´ instead of θ. □

ASSIGNMENT 10.1.1[b] [Modeling exercise for Ellsberg 2-urn]. Consider Example 10.1.1. Show that the majority preferences violate the sure-thing principle. If you have no clue how to proceed, then here is a hint.[2] [This is a useful assignment because it requires setting up a model.] □

EXERCISE 10.5.7[a] [Violation of the sure-thing principle]. Reconsider the majority choices in Figs. 2.4.1g and h. As explained in the elaboration of the extra Exercise 4.12.2, these choices entail a violation of the sure-thing principle, so that EU cannot hold. Explain directly that the rank-sure-thing principle for uncertainty is not violated, by verifying that the rank of outcome events is not constant, with event E_1 having probability 0.01, E_2 having probability 0.89, and E_3 having probability 0.10. □

ASSIGNMENT 10.8.1 [Duality convexity-concavity]. Consider a weighting function W, and its dual $Z(E) = 1 - W(E^c)$. Show that W is convex if and only if Z is concave. □

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[2] To apply the sure-thing principle, you have to define a state space, or at least events. For this purpose, remember that the book uses the term outcome events. Those events should surely be there. So ask yourself what the outcome events are. Then, for two events, their intersection is also an event. Thus add those intersections.

ASSIGNMENT 10.12.5^c [The Shapley value]. This exercise concerns weighting functions W and complete rankings of state spaces, and no decision theory otherwise. Assume a finite state space $S = \{s_1,\ldots,s_n\}$. The *Shapley value* $\sigma(s)$ of a state s is its average decision weight, where the average is taken over all of the n! complete rankings of the state space. For example, if $S = \{s_1,s_2,s_3\}$, $w(p) = p^2$, and $W(E) = w(\|E\|/3)$, then the Shapley value of s_1 can be calculated as follows, where we first list the decision weight for each of the six complete rankings by ordering them from best to worst: s_1, s_2, s_3: $\pi(s_1) = \pi(s_1{}^b) = 1/9$; s_1, s_3, s_2: $\pi(s_1) = \pi(s_1{}^b) = 1/9$; s_2, s_1, s_3: $\pi(s_1) = \pi(s_1{}^{\{s_2\}}) = (2/3)^2 - (1/3)^2 = 3/9$; s_2, s_3, s_1: $\pi(s_1) = \pi(s_1{}^w) = 5/9$; s_3, s_1, s_2: $\pi(s_1) = \pi(s_1{}^{\{s_3\}}) = 3/9$; s_3, s_2, s_1: $\pi(s_1) = \pi(s_1{}^w) = 5/9$. $\sigma(s_1) = (1/9+1/9+3/9+5/9+3/9+5/9)/6 = 1/3$. The *Shapley value* $\sigma(E)$ of an event E is the sum of the individual Shapley values. $\sigma(S)=1$ (S's decision weight for each complete ranking is 1), so that σ is a probability measure.

a) Show that not every weighting function is a strictly increasing transform of its Shapley value.
b) Give a proof or counterexample to the following claim: "If there exists a probability measure P on S such that W is a strictly increasing transform of P, then W is also a strictly increasing tranform of σ, so that σ could be taken instead of P." □

EXERCISE 11.3.3^b [Ambiguity if no ambiguity aversion]. Assume two sources $\mathcal{R}$ and $\mathcal{A}$. Assume that the two sources are rich in the sense that for each $A \in \mathcal{A}$ there exists an event $R \in \mathcal{R}$ such that

$$A \sim R \tag{*}$$

and, conversely, for each $R \in \mathcal{R}$ there exists $A \in \mathcal{A}$ such that (*) is satisfied. Assume that the events in $\mathcal{K}$ are probabilized in the sense of Structural Assumption 10.7.1, i.e. an objective probability is given for them and RDU for risk holds for them. Assume further, for all events $A \in \mathcal{A}$ and $R \in \mathcal{R}$,

$$A \sim R \Leftrightarrow A^c \sim R^c. \tag{**}$$

That is, there is source preference of $\mathcal{A}$ over $\mathcal{R}$ and also of $\mathcal{R}$ over $\mathcal{A}$. This can be called source indifference.

(a) What is the index of optimism? Do not try to find or prove the answer mathematically, but just gamble on what you guess is the plausible answer.
(b) Can there be any kind of ambiguity attitude with respect to $\mathcal{A}$, or are the events of $\mathcal{A}$ treated as unambiguous risky event in every respect and is there no more manifestation of ambiguity?

□

ASSIGNMENT 11.1.1^{b} **{new in July 2013}** [Source preference]. Assume that W satisfies solvability. Show that source preference holds for $\mathcal{A}$ over $\mathcal{B}$ if and only if:

$$\text{For all } A \in \mathcal{A} \text{ and } B \in \mathcal{B}\text{: } W(A) = W(B) \;\Rightarrow\; W(A^c) \geq W(B^c). \qquad (*)$$

□

ASSIGNMENT 11.4.2^{c} [Empty CORE]. Assume that W is concave and not additive. Show that its CORE is empty. □

EXERCISE A3.1^{c} [Degenerate optimal fits]. This exercise can only be done by students who know the definition of expected utility for decision under risk in Ch. 2 (Definition 2.5.3). It further illustrates the problems that can arise with data fitting if the distance measure taken can exhibit strong curvature, so that degenerate solutions can result. Further explanation is in the discussion added at the solution.

Assume a data set consisting of the following three indifferences:

$$0.09_{0.50}0.25 \sim 0.16,\ 0.64_{0.50}0 \sim 0.16,\ 0.16_{0.50}0 \sim 0.03.$$

We want to optimally fit the data using expected utility with power utility α^{θ}. The first two indifferences can be fit perfectly well with with square-root utility ($\theta = 0.5$), but in the third indifference the certainty equivalent 0.03 is one cent less than what square-root utility would predict, suggesting slightly more risk aversion. We, therefore, expect that a power slightly below 0.5 will optimally fit the data.

(a) Use the distance measure proposed in Appendix A.2, and used throughout this book. Find the power $\theta = j/1000 > 0$ (for an integer $0 < j < 1000$) such that EU with power utility $U(\alpha) = \alpha^\theta$ best fits the data. Give the distance. Predict the CE of $0.36_{0.50}0$, and the preference between $0.36_{0.50}0$ and 0.10.

(b) Now do not use the distance measure proposed in this appendix. Instead, use a distance in utility units. That is, use the distance measure of Example A.4. Find the power $\theta = j/1000 > 0$ (for an integer $0 < j < 1000$) such that EU with power utility $U(\alpha) = \alpha^\theta$ best fits the data. Give the distance. Predict the CE of $0.36_{0.50}0$, and the preference between $0.36_{0.50}0$ and 0.10.

(c) Use the same distance measure, in utility units, as in part (b). What is the distance of $\theta = 6.8$? Is it bigger or smaller than the optimal distance found in part (b)? For $\theta = 6.8$, predict the CE of $0.36_{0.50}0$, and the preference between $0.36_{0.50}0$ and 0.10. □