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Brief Overview of Production Theory for Analyzing Healthcare Performance

APPENDIX: CALCULUS AND DUAL SPACES

This section builds on Färe, Grosskopf, and Margaritis (2019).

As mentioned earlier, probably the most well-known calculus result in economics is Shephard's lemma Shephard's lemma (Shephard, 1953). The lemma states that when you take a derivative with respect to an input price in the cost function, you end up recovering that input's (dual) quantity; that is,

$$\frac{\partial C(y,w)}{\partial w_n} = x_n, n = 1,\ldots,N.$$

So Shephard's lemma links primal (quantity) and dual (price) spaces through calculus and what we now call duality theory. To see how this works, we need to formally introduce what we mean by primal and dual spaces. We think of quantities as belonging to a primal space and prices to a dual space. In the case of a single variable, the primal space is $\Re$ and the dual space is $\Re^*$. Mathematically $\Re = \Re^*$, but in economics we note that one can consume quantities but not prices; hence, we need to distinguish the two spaces. Formally, $\Re^*$ consists of all linear functionals

$$f : \Re \rightarrow \Re, \text{ where we let } q^0, q^1 \in \Re,$$

where

$$f(q^0 + q^1) = f(q^0) + f(q^1)$$

and

$$f(aq) = af(q), q \in \Re \text{ and } a \in \Re.$$

We say that a function

$$g : \Re \to \Re$$

is differentiable if there exists a linear function f such that

$$\lim_{h \to 0} \frac{g(q+h) - g(q) - f(h)}{h} = 0.$$

Because f is by definition linear, we have

$$\lim_{h \to 0} \frac{g(q+h) - g(q) - f(1)}{h} = 0.$$

Thus,

$$\lim_{h \to 0} \frac{g(q+h) - g(q)}{h} = f(1),$$

or

$$dg(q)/dq \ = \ f(1).$$

Thus, $dg(q)/dq$ belongs to the dual space $\Re^*$. Taking a derivative in primal space means we end up in dual space. Again, Shephard's lemma is an example of this result: The derivative in price space yields an element of the quantity space. The same applies to the distance functions only in reverse: There we take a derivative with respect to a quantity and obtain what may be called a shadow price.