Answers to Exercises
Chapter 10

Exercise 10.1

(a) At the front of the surge, consider points A and B outside and inside the PDC, respectively. In A, the pressure is the sum of the dynamic pressure and of the static pressure of the atmosphere, so that

$$P_A = \tfrac{1}{2}\rho_0 u_c^{\,2} + \rho_0 gH . \qquad (10.1)$$

In B, the pressure is the sum of the static pressure of the surge and of the overlying atmosphere, so that

$$P_B = \rho_c g h_c + \rho_0 g(H - h_c). \qquad (10.2)$$

Assuming $P_A = P_B$, then

$$\frac{u_c}{\left[g h_c(\rho_c - \rho_0)/\rho_0\right]^{1/2}} = \frac{u_c}{(g' h_c)^{1/2}}, \qquad (10.3)$$

which is also the Froude number at the front and Fr = √2.

(b) Assuming Fr = √2, then

$$\rho_c = \frac{u_c \rho_0}{2 g h_c} + \rho_0, \qquad (10.4)$$

and $\rho_c \sim 1.4$ kg m^{-3}.

(c) The analysis is valid if $g' \leq g$, so that $(\rho_c\text{-}\rho_0)/\rho_0 \leq 1$, and $\rho_c \leq 2\rho_0$.

Exercise 10.2

Considering the Savage number (Sa, Table 10.1) and assuming that the shear rate $\gamma = u_f/h_f$, then

$$u_f = \left(\frac{\mathrm{Sa}(\rho_p - \rho_0) g h_f^{\,3}}{\rho_p d^2}\right)^{1/2}. \qquad (10.5)$$

With Sa = 0.1, then u_f= 0.49 m s^{-1} and the flows will be in frictional regime below that velocity, and in the collisional regime at greater velocities.

Exercise 10.3

From mass conservation $h_c = Q/\pi r^2$, and substituting this into the front condition, we find

$$u_c = \frac{dr}{dt} = \mathrm{Fr}\left(\frac{g'Q}{\pi r^2}\right)^{1/2}. \qquad (10.6)$$

Re-arranging this equation to separate all terms in r and integrating this equation subject to the boundary condition $r = 0$ at $t = 0$,

$$r = \left(\frac{2\mathrm{Fr}}{\pi^{1/2}}\right)^{1/2} (g'Q)^{1/4} t^{1/2}. \qquad (10.7)$$

The position of the front depends on $Q^{1/4}$.