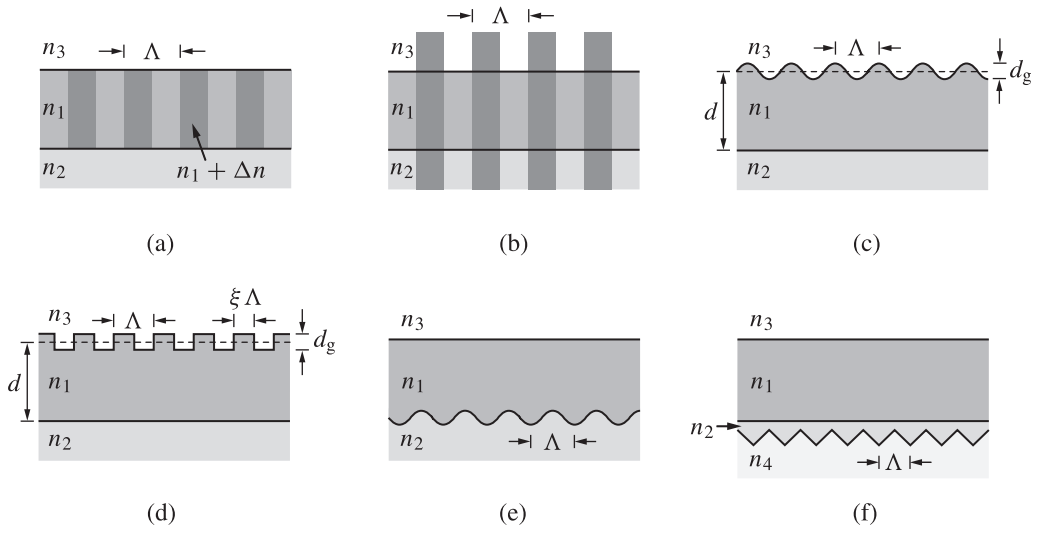

Figure 4.3 Structures of planar grating waveguide couplers with (a) and (b) periodic index modulation, (c), (d), (e), and (f) periodic structural corrugation.

EXAMPLE 4.2

Find the qth-order coupling coefficient $\kappa_{\nu\mu}(q)$ for a sinusoidal grating that has a period of Λ, as shown in Fig. 4.3(c), such that $\kappa_{\nu\mu}(z) = a\cos Kz$, where $K = 2\pi/\Lambda$. Find it for a square-function grating that has a period of Λ and a duty factor of ξ, as shown in Fig. 4.3(d), such that $\kappa_{\nu\mu}(z) = a$ for $0 < z < \xi\Lambda$ and $\kappa_{\nu\mu}(z) = -a$ for $\xi\Lambda < z < \Lambda$ within each period. In each case, which orders are useful for mode coupling?

Solution:

For the sinusoidal grating, we find by using (4.37) that

$$\begin{aligned}\kappa_{\nu\mu}(q) &= \frac{1}{\Lambda}\int_0^{\Lambda}\kappa_{\nu\mu}(z)\exp\left(-\mathrm{i}qKz\right)\mathrm{d}z\\ &= \frac{1}{\Lambda}\int_0^{\Lambda}a\cos Kz\exp\left(-\mathrm{i}qKz\right)\mathrm{d}z\\ &= \frac{a}{\Lambda}\int_0^{\Lambda}\frac{\exp\left(\mathrm{i}Kz-\mathrm{i}qKz\right)+\exp\left(-\mathrm{i}Kz-\mathrm{i}qKz\right)}{2}\mathrm{d}z.\\ &= \frac{a}{2}\left(\delta_{q,1}+\delta_{q,-1}\right),\end{aligned}$$

where $\delta_{q,1}$ and $\delta_{q,-1}$ are the Kronecker delta functions. Therefore, only the order $q = 1$ and the order $q = -1$ are useful for mode coupling because only these two orders have a nonzero coupling coefficient of $\kappa_{\nu\mu}(1) = \kappa_{\nu\mu}(-1) = a/2$.