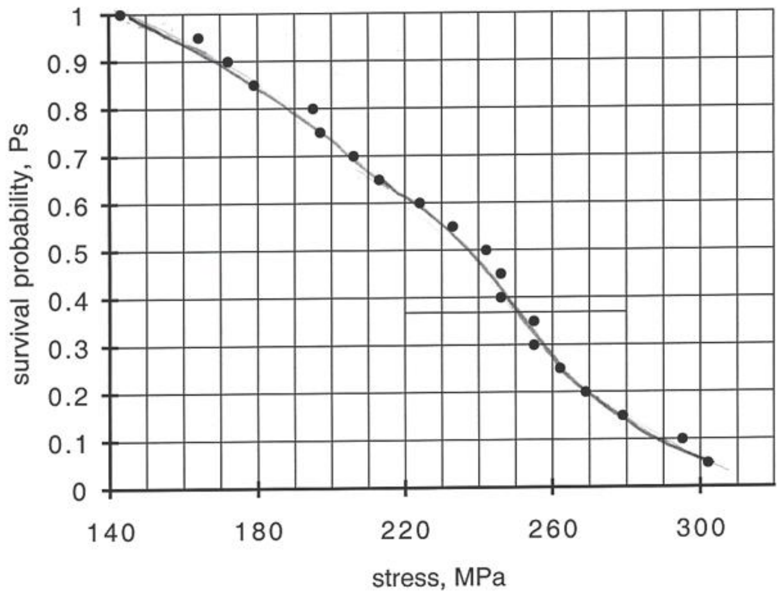
Chapter 19

1. Estimate the ratio of the elastic moduli of cubic AlSb and cubic ZnSe. Both have the sphalerite structure. The lattice parameters are 0.6136 and 0.5669 nm respectively.
solution: $E_1/E_2 = (z_1/z_2)^2/(r_1/r_2) = (9/4)/(.6136/.5669) = 2.43$.

2. Find the number of independent slip systems in rutile (TiO) for which slip occurs on the <110>{1»10} and <110>{001} systems.
solution: The three normal strains, e_1, e_2 and e_3 can be accommodated, but only two of these are independent. None of the shear strains, γ_{23}, γ_{31}, or γ_{12} can be accommodated. So there are two independent slip systems.

3. From a large number of tests on a certain material, it has been learned that 50% of them will break when loaded in tension at stress equal to or less than 520 MPa and that 30 % will break at stress equal to or less than 500 MPa. Assuming that the fracture statistics follow a Weibul distribution,
A. What are the values of σ_o and m in the equation $P_S = \exp[-(\sigma/\sigma_o)^m]$?
B. What is the maximum permissible stress if the probability of failure is to be kept less than 0.001% (i.e., one failure in 100,000)?
solution: A. $-\ln(P_S) = (\sigma/\sigma_o)^m$, $m = \ln\{\ln(P_{S1})/\ln(P_{S2})\}/\ln(\sigma_1/\sigma_2) =$
$\ln\{\ln(.5)/\ln(.7)\}/\ln(520/500) = 16.9$
$(\sigma/\sigma_o) = [-\ln(P_S)]^{1/m}$; $(500/\sigma_o) = [-\ln(.7)]^{1/16.9} = 0.941$, $\sigma_o = 500/.941 = 531\text{MPa}$.
B. $\ln P_S = -(\sigma/531)^{16.9}$; $P_S = .99999$, $10^{-5} = (\sigma/531)^{16.9}$; $\sigma = 531(10^{-5})^{1/16.9} = 269$ MPa.

4. Twenty ceramic specimens were tested to fracture. The measured fracture loads in N were: 248, 195, 246, 302, 255, 262, 164, 242, 197, 224, 255, 248, 213, 172, 179, 143, 206, 233, 246, 295.
a. Determine the Weibul modulus, m.
b. Find the load for which the probability of survival is 99%
solution: A. Plotting the survival and choosing two points on the curve,
$P_S = .9$ at 170 MPa and $P_s = .1$ at 290 MPa,
$m = \ln\{\ln(P_{S1})/\ln(P_{S2})\}/\ln(\sigma_1/\sigma_2) = \ln[\ln(.9)/\ln(.1)]/\ln(170/290) = 5.77$ or
$m = = \ln[\ln(.8)/\ln(.2)]/\ln(190/270) = 5.62$. take m = 5.7



B. From the plot, the stress at $P_S = 0.368$ is 250 MPa = σ_0.
For $P_S = 0.99$ □ $\exp[-(\sigma/\sigma_0)^m] = 0.99$, $(\sigma/\sigma_0) = [-\ln(0.99)]^{1/5.7} = .44$
$\sigma = (250)(0.44) = 111$ MPa

5. Equations 19.6 and 19.7 relate the surface stress, σ, in bending to the load and dimensions of the bending specimens, assuming elastic behavior. The corresponding strain on the surface depends also on the elastic properties of the material and the ratio of t/b. Express the strain, e, in terms of the load and dimensions for
A. $t/b \ll 1$
B. $t/b \gg 1$
<u>solution:</u> For $t/b \ll 1$, the stress state at the surface will be in plane stress (uniaxial tension) so $e = \sigma/E$.
For $t/b \gg 1$, the stress state at the surface will be in plane strain tension so $e = \sigma(1-\upsilon)/E$.

6. What percent reduction of the elastic modulus of alumina would be caused by 1% porosity? See Figure 19. 6.
<u>solution</u>: At low porosity, p, the plot is linear so the fractional modulus, E', can be approximated by $E' = 1-2p$. For $p = 0.01$, $E' = .98$ so there is a 2% decrease

7. Pottery is generally fired at a high temperature, then it is cooled and a glaze is applied. On reheating the glaze melts and spreads over the surface. On cooling again, residual stresses may develop in the glaze. To insure that these are compressive, what relation is necessary between the properties of the body of the ceramic and of the glaze?
<u>solution:</u> The coefficient of thermal expansion of the glaze, α, should be less than that of the body. (The body will contract more thermally so the glaze will have to be compressed elastically.)

8. Why is there so little concern about thermal shock in metals and polymers while there is much concern with ceramics? Why aren't the parameters $R_1 = \sigma_f(1-2\square)/(E\alpha)$ and $R_2 = \square_{Ic}/(E\alpha)$ useful in predicting fracture in metals?
<u>solution:</u> These parameters neglect plastic strain. They presume that at high stress all of the strain must be elastic or thermal.

9. Rapid heating and cooling can cause cracks to start at the surface of a ceramic material. Sometimes the cracks meet the surface at 90° and sometimes at 45° to the surface. By noting the orientation of a crack, how can one tell whether it initiated on the heating or the cooling portion of the thermal cycle?
<u>solution</u>: Cracks at 90° occur under tension. The surface is under tension during cooling when its greater thermal contraction must be accommodated by tension.
Cracks at 45° occur under compression. The surface is under compression during heating when its greater thermal expansion must be accommodated by compression. The surface may also be left under compression after it is cooled if the surface yielded in tension during cooling or the interior yielded in compression.

10. A glass retort is to be used under conditions that the temperature of the inside of the wall is 100°C and the outside is 0°C. What stress will develop on the outside? The properties of the glass are: $\alpha = 1.5\text{x}10^{-6}/°C$, $E = 70$ GPa, $\upsilon = 0.30$.
Hint: Assume a linear temperature gradient and for simplicity take the midpoint strain as zero.
<u>solution:</u> Let the base temperature and dimensions be at the center where
$T = 50°$. The surface is under biaxial tension so $e = (1/E)(\sigma \square \upsilon\sigma) + \alpha\Delta T = 0$,
$\sigma = -\alpha\Delta TE/(1-\upsilon) = -(1.5\text{x}10^{-6})(-50)(70\ \text{GPa})/(1-.3) = 7.5$ MPa

Chapter 20

1. A piece of PVC at 75°C was suddenly stretched by 1%. What would the stress? See Figure 20.6. If it were held in the stretched position, what would the stress be after a minute? After an hour? After a day?

solution: σ□= eE =.01E. immediately, σ = - .01(1.8)GPa = 15MPa
after I min, σ□= - .01(0.7)GPa = 7 MPa
after I hr, σ = - .01(0.3)GPa = 0.3 MPa
after I day, σ = - .01(0.03)GPa = 0.03 MPa.

2. Examine Figure 20.10, which relates the shear modulus of rubber to temperature. At 100°C, G increases with increasing % sulfur.
A. Find the slope of a G vs. %S plot at 100°C.
B. Since G = NkT, this slope must be related to N. Write an expression relating N to % S at 100°C.

solution: A. At 100°C, G = 5×10^{-3} for 30% S; G = 2×10^{-3} for 15% S; so the slope is about 2.5×10^{-3}/15%S = 1.7×10^{-4} GPa/%S
B. dG/dN = kT = $(13.8\times10^{-24}$ J/K)(373K) = 5.15×10^{-22} GPa(m^3)
dG/d%S = 1.7×10^{-4} GPa so dN/d%S = (dG/d%S/(dG/dN) =
1.7×10^{-4} GPa/5.15×10^{-22} GPa.m^3= 3.2×10^{17} (cross links/m^3)/%S.

3. Evaluate the coefficient of thermal expansion, α, for rubber under an extension of 100% at 20°C. A rubber band was stretched from 6 inches to 12 inches. While being held under a constant stress, it was heated from 20°C to 40°C. What change in length, ΔL, would the heating cause?

solution: $\lambda = 2.0$. From equation 20.7, $\alpha = -(1/T)(1-\lambda^{-2})/(1+2\lambda^{-3}) = -(1/293)(1-1/4)/(1+2/8)$ $=-3.2\times10^{-3}$. $\Delta L = L\alpha\Delta T = 12(-3.2\times10^{-3})(20) = -0.77$ in.

4. From Figure 20.17, estimate the % elongation associated with necking in polyethylene

solution: The width of the necked region appears to be about 1/4 of the width of the undeformed region. If the reduction at 90° is the same, the area after necking, A_n,=$(1/4)^2A_o$ = $(1/16)A_o$. If volume is conserved,$A_oL_o = A_nL_n$. $L_n/L_o = 16$.

5 A. Use the general statement of the flow rule, $d\varepsilon_{ij} = d\lambda(\partial f/\partial\sigma_{ij})$, with the yield criteria, equation 20.10 , to derive the expression predicting the relative volume change, $(dv/v)/d\varepsilon_1$, in tension as a function of C/T.
B. Using this expression and the data in Figure 20.23 to predict C/T for HIPS.

solution: A $d\varepsilon_1 = d\lambda(\partial f/\partial\sigma_1) = 2d\lambda(2\sigma_1 - \sigma_2 - \sigma_3 + C - T)$ In a 1-direction tension test,
$d\varepsilon_1 = 2d\lambda(2\sigma_1 + C - T) = = 2d\lambda(T + C)$
$dv/v = d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 6d\lambda(C-T)$
$(dv/v)/d\varepsilon_1 = 3(C-T)/(C+T) = 3(C/T-1)/(C/T+1)$
B. $(C/T)(dv/v)/d\varepsilon + (dv/v)/d\varepsilon = 3C/T - 3$
$(C/T) = [3 + (dv/v)/d\varepsilon_1]/[3 - (dv/v)/d\varepsilon_1]$

From Figure 20.10, $(dv/v)/d\varepsilon_1 = 5/6$, so $C/T = (3+5/6)/(3-5/6) = 1.77$

6. An isotropic pressure-dependent yield criterion for polymers of the form, $f = A(\sigma_1 + \sigma_2 + \sigma_3) + B[(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2]^{1/2} = 1$ has been proposed.

A. Evaluate A and B in terms of T and C. Here T is the yield strength in tension and C is the absolute magnitude of the compressive strength. Consider a 1-direction tension test in which $\sigma_1 = T$, $\sigma_2 = \sigma_3 = 0$ at yielding and a 1-direction compression test in which $\sigma_3 = C$, $\sigma_2 = \sigma_3 = 0$ at yielding.

solution: For a 1-direction tension test, $\sigma_1 = T$, $\sigma_2 = \sigma_3 = 0$, $AT + \sqrt{2}BT = 1$,

For a 1-direction compression test, $\sigma_1 = -C$, $\sigma_2 = \sigma_3 = 0$, $-AC + \sqrt{2}BC = 1$,

Substituting $A = 1/T - \sqrt{2}B$ into $-A = 1/C - \sqrt{2}B$,

$1/T - \sqrt{2}B = -1/C + \sqrt{2}B$, $B = (1/T + 1/C)/(2\sqrt{2})$; $A = 1/T - (1/T + 1/C)/2 = (1/T - 1/C)/2$.

7. Cut a strip from a commercial food wrap. Using your fingers pull it in tension. Then pull it in tension 90° to the original direction of stretching. Compare your observations with Figure 20.31. Explain why crazing is more likely when the loading is normal to the direction of prior extension.

solution: Crazing occurs easily when pulled at 90° to direction in which the molecules are aligned. This is what Figure 20.31 predicts. Prior stretching has aligned the molecules parallel to the extension direction. Perpendicular to this they are bonded only by van der Waals bonds.

8. Consider equation 20.13, which relates crazing to stress state.

A. Write an expression for the value of σ_1 to cause crazing in terms of the stress ratio, $\alpha = \sigma_2/\sigma_1$, and A and B. Assume plane stress, $\sigma_3 = 0$.

B. Compare the values of σ_1 for $\alpha = 0$ and $\alpha = 1$.

solution: A. Substituting $\alpha = \sigma_2/\sigma$, equation 20.13, $\sigma_1 - \upsilon\sigma_2 = A + B/(\sigma_1 + \sigma_2)$, becomes

$\sigma_1(1 - \upsilon\alpha) = A + B/[\sigma_1(1 + \alpha)]$.

$\sigma_1 = \{A \pm [A^2 + 4(1 - \upsilon\alpha)B/(1 + \alpha)]^{1/2}/[2(1 - \upsilon\alpha)]$

B. For $\alpha = 0$, $\sigma_1 = A \pm (A^2 + 4B)^{1/2}/2$

For $\alpha = 1$, $\sigma_1 = A \pm [A^2 + 8(1 - \upsilon)B]^{1/2}/[2(1 - \upsilon)]$

9. According to equations 20.10 and 20.11, yielding is possible under a state of pure hydrostatic tension, σ_H. Find that value of σ_H according to both equations if the tensile and compressive yield strengths are 80 and 100 MPa respectively.

solution: Equation 20.10, with $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ becomes $3\sigma = CT/(C-T)$ σ□$= (80)(100)/20x3 = 133$MPa.

Equation 20.11, with $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ becomes $\sigma = (1/3)\sqrt{(K_2/K_1)}$ but using example problem 20.5, $K_2/K_1 = 2T^2C^2/(C^2 - T^2) = 2(100^2)(80^2)/(100^2 - 80^2) = 35{,}556$ so $\sigma = \sqrt{(35{,}556)}/3 = 62.8$ MPa

10. The tensile and compressive yield stresses of polystyrene are 73 and 92 MPa respectively. Poison's ratio is 1/3. Crazing occurs at 47 MPa in uniaxial tension and 45 MPa in equal biaxial tension. Use equation 20.10 to predict the stress state, σ_1, σ_2, with $\sigma_3 = 0$ at which yielding and crazing occur simultaneously?

solution: For yielding , $2(\sigma_2^2 - \sigma_2\sigma_1 + \sigma_1^2) + 19(\sigma_1 + \sigma_2) = 2x92x73$

$\sigma_1^2(\alpha^2 - \alpha + 1) + 9.5\sigma_1(1+\alpha) = 6716$

$\sigma_1 = \{[6716-9.5(1+\alpha)]/(\alpha^2 - \alpha + 1)\}^{1/2}$ (1)

For crazing $\sigma_1 - \upsilon\sigma_2 = A + B/(\sigma_1 + \sigma_2)$.

For crazing in tension, 47 = A+ B/47,

For crazing in biaxial tension, (2/3)(45) = A + B/90 = 30, A = 30 – B/90

Solving simultaneously, 47 = 30 – B/90 + B/47;

B = 17/(-1/90 + 1/47) = 1672, A = 30 – 1672/90 = 11.4 so for crazing

$\sigma_1^2[1- (1/3)\alpha] - 11.4\sigma_1 - 1672/(1+\alpha) = 0$ (2)

Solving (1) and (2) numerically, $\alpha = 2.996$, σ_1= 9965MPa, σ_2= 29860 MPa

This solution isn't reasonable because these stresses are far above the yield strength.

12. The compressive strength of Kevlar is about 1/8 of its tensile strength. If it is bent to small radius of curvature it will kink as shown Figure 20.34. Estimate the smallest diameter rod on which Kevlar 49 fiber of 12 μm diameter fibers can be wound without kinking. The tensile strength is 2.8 GPa, Young's Modulus is 125 GPa, Poisson's ratio is 1/3 and the tensile strain to fracture is 2.3%.

solution: Assume there is no net tension. As it is bent elastically, the stress at the surface is $\sigma = Ee = Ed/2\rho$, where $d = 12x10^{-6}$ and ρ = the radius of curvature. Yielding will occur in compression when σ = 2.8GPa/8

$\rho = Ed/2\sigma$ = (125 GPa)($12x10^{-6}$)/[2(2.8/8GPa)] = 2.14 mm

Chapter 21

1. Calculate the volume fraction fiber in the several composites described below:

A. Maximum possible fiber fraction for unidirectionally aligned cylindrical fibers with negligible spacing between. [Assume a hexagonal array.]

B. Maximum possible fiber fraction for unidirectionally aligned cylindrical fibers of 100 μm diameter coated with 10 μm thick coating. [Assume a hexagonal array.]

C. Maximum possible fiber fraction for alternating layers of unidirectionally aligned fibers as shown Figure 21.20C.

D. Maximum possible fiber fraction alternating layers of unidirectionally aligned fibers of 100 μm diameter coated with 10 μm thick coating as shown in Figure 21.20D.

C

D

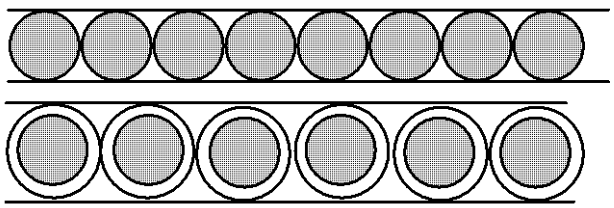


Figure 21.20. A ply of unidirectionally aligned fibers (top) and a ply of unidirectionally aligned coated fibers (bottom).

solution: A. For a hexagonal array, the area of an inscribed circle is πr^2. The area of the hexagon is $(6/\sqrt{3})r^2$ so the max fraction $= \pi/(6/\sqrt{3}) = 0.907$.

B. The area of the hexagon is $(6/\sqrt{3})(60)^2$ so the max fraction $= \pi(50)^2/(6/\sqrt{3})(60)^2 = 0.630$.

C. max fraction $= \pi/4 = 0.785$.

D. max fraction $= (\pi/4)\,(50)^2/(60)^2 = 0.545$.

2. Calculate the elastic modulus of a composite of 40 volume % continuous aligned boron fibers in an aluminum matrix (E = 70 GPa)

A. Parallel to the boron fibers.

B. Perpendicular to the boron fibers.

solution: A. $E = 0.4E_B + 0.6E_{Al}$. Substituting from Table III, E_B = 390 GPa. $E = .4(390 \text{ GPa}) + .6(70) = 198$ GPa

B. $1/E = .4/(390 \text{ GPa}) + .6/(70) = .00960$: E = 104 GPa.

3. In all useful fiber-reinforced composites, the elastic moduli of the fibers are higher than those of the matrix. Explain why this is so.

solution: The fibers can carry the higher stress only if they have the higher moduli.

4. Consider the matrix of elastic constants for a composite consisting of an elastically soft matrix reinforced by a 90° cross ply of stiff fibers. The general form of the matrix of elastic constants is:

$$\begin{array}{cccccc}
s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\
s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\
s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & s_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s_{44} & 0
\end{array}$$

0 0 0 0 0 s_{66}

Young's modulus for the composite when loaded parallel to one of the sets of fibers is 100 GPa so $s_{11} = 10\times10^{-12}Pa^{-1}$. Of the values listed below, which is most likely for s_{12}? for s_{66}?

a. $10\times10^{-10}Pa^{-1}$; b. $30\times10^{-12}Pa^{-1}$; c. $10\times10^{-12}Pa^{-1}$; d. $3\times10^{-12}Pa^{-1}$; e. $100\times10^{-12}Pa^{-1}$;
f. $-10\times10^{-10}Pa^{-1}$; g. $-30\times10^{-12}Pa^{-1}$; h. $-10\times10^{-12}Pa^{-1}$; i. $-3\times10^{-12}Pa^{-1}$; j. $-100\times10^{-12}Pa^{-1}$.

solution: The most likely value of s_{12} is (i) $-3\times10^{-12}Pa^{-1}$ (This corresponds to $\upsilon = 1/3$); The most likely value of s_{16} is (a) $10\times10^{-10}Pa^{-1}$(This corresponds to the lowest value of $G = 1/s_{16}$.)

5. What would be the critical length, L*, for maximum load in a 10 μm diameter fiber with a fracture strength of 2 GPa embedded in a matrix such that the shear strength of the matrix-fiber interface is 100 MPa.

solution: Setting x = x* and using equation 21.14 with $\sigma_x =$ 2GPa, $\tau =$ 2GPa, D = 10 μm, x* = (1/4)(10 μm)(2GPa)/(100MPa) = 50 μm. L* = 2x* = 100μm.

6. Estimate the greatest value of the elastic modulus that can be obtained by long randomly oriented fibers of E-glass embedded in an epoxy resin if the volume fraction is 40%. Assume the modulus of the epoxy is 5 GPa.

solution: Using equation 21.8, $E = (3/8)E_{parallel} + (5/8)E_{perpendicular}$, with
$E_{parallel}$ = 0.4(70GPa) + 0.6(5GPa) = 31 GPa and
$E_{parallel}$ = 1/{.4/(70GPa) + .6/(5GPa)] = 7.95.
E = (3/8)(31) + (5/8)(7.95) = 16.6 GPa

7. Carbide cutting tools are composites of very hard tungsten carbide particles in a cobalt matrix. The elastic moduli of tungsten carbide and cobalt are 102×10^6 and 30×10^6 psi respectively. It was experimentally found that the elastic modulus of a composite containing 52 volume % carbide was 60×10^6 psi. What value of the exponent, n, in equation 21.26 would this measurement suggest? (A trial and error solution is necessary to solve this. Note that n = 0 is a trivial solution)

solution: A trial and error solution of $60^n = 0.52(102)^n + 0.48(30)^n$, gives n = 0.307

8. A steel wire (1.0 mm diameter) is coated with aluminum, 0.20 mm thick.
A. Will the steel or the aluminum yield first as tension is applied to the wire?
B. What tensile load can the wire withstand without yielding
C. What is the composite elastic modulus?
D. Calculate the composite thermal expansion coefficient.

data:

	Young's modulus	yield strength	Poisson's ratio	linear coef. of thermal
aluminum	70	65	.3	24×10^{-6}
steel	210	280	.3	12×10^{-6}

solution: A. yielding occurs when the strain reaches YS/E. For Al this is 1.08×10^{-3} and for steel this is 1.33×10^{-3}. The aluminum will yield first.

B. The composite yields (becomes non-linear) when the strain is 1.08×10^{-3}. At this p0oint, the load = $[\pi(.001)^2/4](210\times10^9)(1.08\times10^{-3}) + [\pi(.0014)^2/4 - (\pi(.001)^2/4)](70\times10^9)(1.08\times10^{-3})$ = 235 N.

C. Area fraction steel = $.5^2/.7^2 = 0.51$; $A_S/A_{Al} = 0.5^2/(0.7^2 - 0.5^2) = 1.04$

E = .51(210) + .49(70) = 141 GPa

D. $e = (1/E_S)\sigma_S + \alpha_S\Delta T = (1/E_{Al})\sigma_{Al} + \alpha_{Al}\Delta T$

A force balance gives $\sigma_S A_S = \sigma_{Al}A_{Al}$, so $\sigma_{Al} = -\sigma_S(A_S/A_{Al}) = -1.04\sigma_S$

Substituting $(1/E_S)\sigma_S + \alpha_S\Delta T = -(1/E_{Al})\sigma_S(A_S/A_{Al}) + \alpha_{Al}\Delta T$

$\sigma_S[(1/E_S) + (1/E_{Al})(A_S/A_{Al})] = (\alpha_{Al} - \alpha_S)\Delta T$

$\sigma_S = (\alpha_{Al} - \alpha_S)\Delta T/[(1/E_S) + (1/E_{Al})(A_S/A_{Al})] =$

$(12\times10^{-6})\Delta T/[1/270\times10^9 + 1/(1.04\times70\times10^9) = 6.88\times10^5\Delta T$

$e = (1/E_S)\sigma_S + \alpha_S\Delta T = (1/270\times10^9)(6.88\times10^5\Delta T) + \alpha_S\Delta T$

$= 14.6\times10^{-6}\Delta T$; $\alpha_{composite} = 14.6\times10^{-6}/°C$

9. Consider a carbon-reinforced epoxy composite containing 45 volume % unidirectionally aligned carbon fibers. A. Calculate the composite modulus. B. Calculate the composite tensile strength. Assume both the epoxy and carbon are elastic to fracture.

data:

	Young's modulus	tensile strength
epoxy	3 GPa	55 MPa
carbon	250 GPa	2.5 GPa

solution: A. E = .45(250) + 0.55(3) = 114 GPa

B The fibers will break when the strain is 2.5/250 = 0.01

The epoxy will break when the strain is 0.055/3 = 0.08

The maximum strength will be reached when the strain is 0.01 so

TS = 114(0.01) = 1.14 GPa

Chapter 22

1. A small special alloy shop received an order for slabs 4 inches wide and 1/2 inch thick of an experimental superalloy. The shop cast ingots 4inx4inx12in and hot rolled them in a 12 in. diameter mill, making reductions of about 5% per pass. On the fifth pass, the first slab split longitudinally parallel to the rolling plane. The project engineer, the shop foreman, and a consultant met to discuss the problem. The consultant proposed applying forward and back tension during rolling, the project engineer suggested reducing the reduction per pass and the shop foreman favors higher reductions per pass. With whom would you agree? Explain your reasoning.

solution: Δ should be decreased. At the start , $\Delta = (4)/\sqrt{[(6)(.05x.4)]} = 4.1$

Increasing the reduction per pass will increase Δ.

2. A high-strength steel bar must be cold reduced from a diameter of 1.00 in. to 0.65 in. A number of schedules have been proposed. Which of the schedules below would you choose to avoid drawing failure and minimize the likelihood of centerline bursts? Explain. Assume η = 0.50.

A. A single reduction in a die having a die angle of 8°.

B. Two passes (1.00 to 0.81 in and 0.81 to 0.65 in. using dies with angles of α = 8°.

C. Three passes (1.00 to 0.87 in and 0.87 to 0.75 in, and 0.75 to 0.65 in. using dies with angles of 8°.

D., E., and F. Same schedules as A, B, and C, except using dies with α = 15°.

solution: Drawing failure will occur if $\varepsilon = \eta = 0.5$. This corresponds to a diameter reduction of $d/d_o = \exp(-0.5/2) = 0.78$. The overall strain is $2\ln(1/0.65) = 0.86$. This will require at least two passes. To minimize □, the reduction should be large and the die angle small. Therefore choose 2 passes with lower die angle, i.e. schedule B.

3. Your company is planning to produce niobium wire and you have been asked to decide how many passes would be required to reduce the wire from 0.125 to 0.010 inches in diameter. In laboratory experiments with dies having the same angle as will be used in the operation it was found that the efficiency increased with reduction, $\eta = 0.65 + \Delta\varepsilon/3$, where $\Delta\varepsilon$ is the strain in the pass. Assume that in practice the efficiency will be only 75% of that found in the laboratory experiments. To insure no failures, stress on the drawn section of wire must never exceed 80% of its strength. Neglect work hardening.

solution: To find the minimum number of dies, design using the largest reduction permissible, i.e. $\sigma_d = 0.6\sigma$. Assume no work hardening, so that $\sigma_d = (1/\eta)\sigma\Delta\varepsilon$. The maximum value of σ_d is $(0.75)(0.80)\sigma = 0.60\sigma$. Substituting, $0.60\sigma = (1/\eta)\sigma\Delta\varepsilon$ or $\Delta\varepsilon = 0.6\eta$. Now substituting $\eta = 0.65 + \Delta\varepsilon/3$, $\Delta\varepsilon = 0.6(0.65 + \Delta\varepsilon/3)$; $0.8\Delta\varepsilon = 0.39$, Δ□$= 0.39/.8 = 0.487$. (This is the maximum strain per pass.)

The total strain in reducing the wire from 0.125 to 0.010 inches is e = $\ln\{(\pi/4)(0.125)^2/[(\pi/4)(0.1010)^2] = 2\ln(.125/.010) = 5.05$.

The number of passes required is 5.05/0.487 = 10.4. 11 passes will be required.

4. One stand of a hot -rolling mill is being designed. It will reduce 60 in. wide sheet from 0.150 to 0.120 in. thickness at an exit speed of 20 feet per second. Assume that the flow stress of the steel at the temperature and strain rate in the rolling mill is 1500 psi. If the deformation

efficiency is 82% and the efficiency of transferring energy from the motor to the mill is 85%, what horsepower motor should be used?

solution: The rate of doing work = (vol/time)(work/vol)

(vol/time) = (0.12 in)(20 ft/s(12 in/ft) = 1728 in^3/s

(work/vol) = (1/0.82)(1500ln(.15/.12) = 4.085 in-lbs/in^3

rate of doing work on metal = (1728)(4.085)in-lbs/s = 5883 ft-lbs/s =

(5883)[1.818×10^{-3} hp/(ft-lbs/s)] = 107 hp

accounting for the transfer efficiency, the motor horsepower should be 107/.85 = 126 hp

5. A typical aluminum beverage can is 2.6 in. in diameter and 4.8 in. high. The thickness of the bottom is 0.010 in. and the wall thickness is 0.004. The cans are produced from circular blanks 0.010 in thick by drawing, redrawing and ironing to a height of 5.25 in before trimming.

A. Calculate the diameter of the initial circular blank.

B. Calculate the total effective strain at the top of the cup from rolling, drawing, redrawing and ironing.

solution: A. $(\pi D_o{}^2/4)t_o = (\pi D_1{}^2/4)t_0 + (\pi D_1 h)t_1$

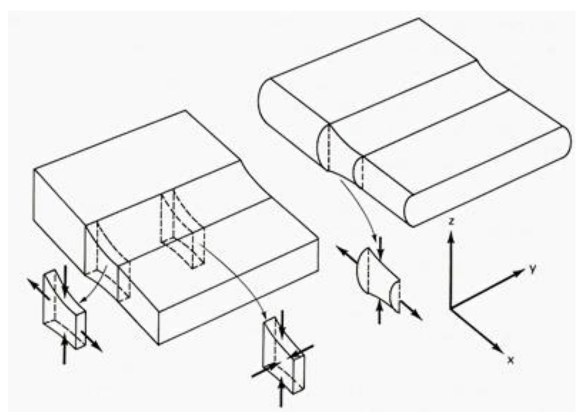
$D_o{}^2 t_o = D_1{}^2 t_0 + 4D_1 h t_1$

$D_o = \sqrt{}(D_1{}^2 + 4D_1 h t_1/t_o) = \sqrt{}[(2.6)^2 + 4(2.6)(5.25)(.4)] = 5.34$ in,

B. »$\varepsilon = \ln[\pi(5.34)(0.01)]/[\pi(2.6)(0.004)] = 1.64$

6. Figure 22.12 shows that much reductions in rolling can be achieved before edge cracking occurs if the edges are maintained square instead of being allowed become rounded. Figure 22.25 below shows the edge elements. Explain in terms of the stress state at the edge why the higher strains are possible with square edges.

Figure 22.25. The difference between the stress states at the edges of square-edge and round-edge strips during rolling. From W. F. Hosford and R. M. Caddell, *Metal Forming: Mechanics and Met* *2nd Ed.* Prentice Hall, 1993, p. 141.



solution: The extreme elements with curved edges are not under as much compressive stress as those with square edges. Therefore their elongation requires more tension in the rolling-direction .

7. When aluminum alloy 6061-T6 is cold drawn through a series of dies with a 25% reduction per pass, a loss of density is noted as shown in Figure 22.26. Explain why the density loss increases with higher angle dies.

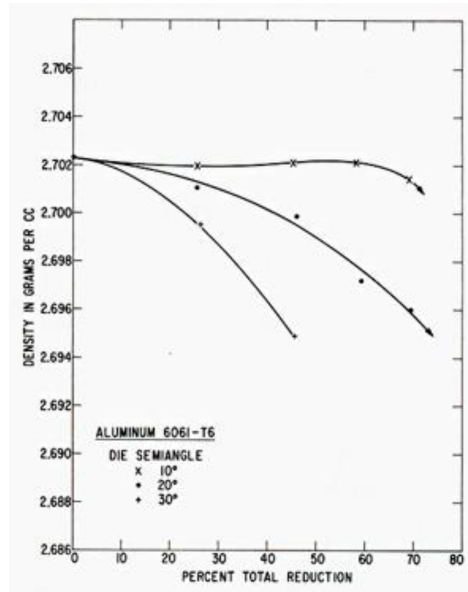


Figure 22.26. Density changes in aluminum alloy 6161-T6 during drawing. From H. C. Rogers, *General Electric Co. Report No. 69-C-260,* 1969.

solution: For the same reduction, Δ increases with die angle and hydrostatic tension at the centerline increases with Δ .

8. Assuming that in drawing of cups, the thickness of the cup bottom and wall is the same as that of the original sheet, find an expression for the ratio of the cup height to diameter, h/d_1, in terms of the ratio of blank diameter to cup diameter, d_o/d_1. Evaluate h/d_1 for d_o/d_1 = 1.5, 1.75, 2.0, and 2.25 and plot h/d_1 vs. d_o/d_1.

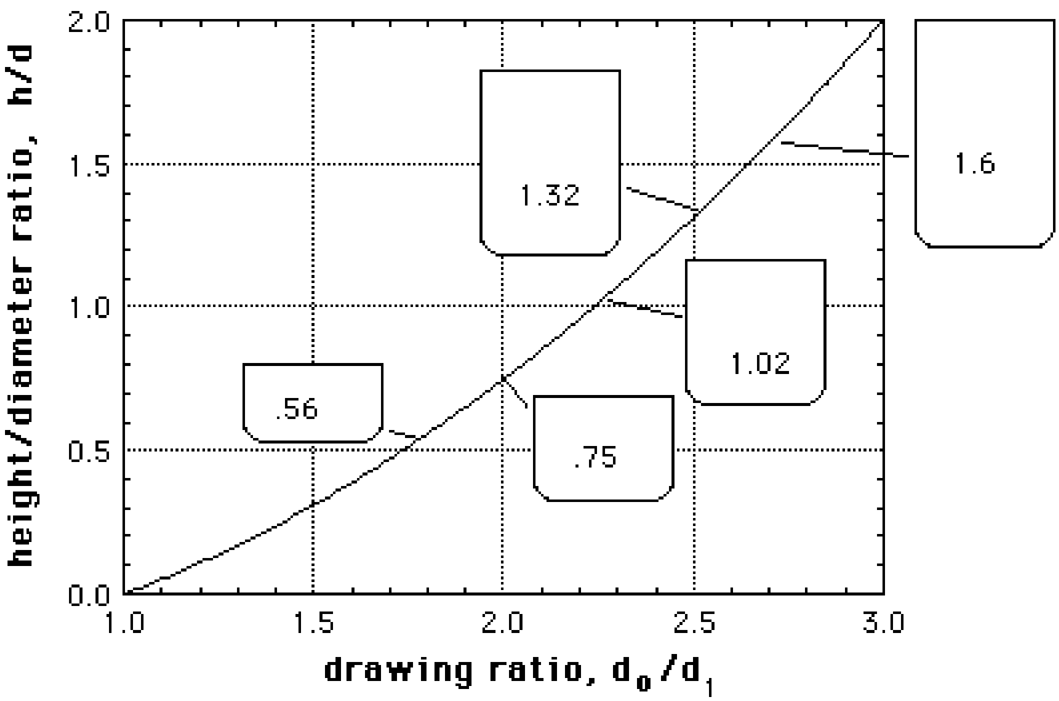
solution:

With constant thickness the surface areas of the blank and final cup are equal. $\pi r_1^2 + 2\pi r_1 h = \pi r_o^2$; $2r_1h = r_o^2 - r_1^2$;

$h/r_1 = [(r_o/r_1)^2-1]/2 = [(d_o/d_1)^2-1]/2$. $h/d_1 = (1/2)h/r_1 = [(d_o/d_1)^2-1]/4$.

for d_o/d_1 = 1.8, h/r_1 = 0.56; for d_o/d_1 = 2, h/r_1 = 0.75;

for d_o/d_1 = 2.25, h/r_1 =1.016; for d_o/d_1 = 2.5, h/r_1 = 1.312.



9. In drawing of cups with conical wall, the elements between the punch and the die must deform in such a way that their circumference shrinks. Otherwise they will buckle or wrinkle. The tendency to wrinkle can be decreased by applying a greater blankholder force as shown in Figure 22.27. This increases the radial tension between the punch and die. How would the R-value of the material affect how much blankholder force is necessary to prevent wrinkling?

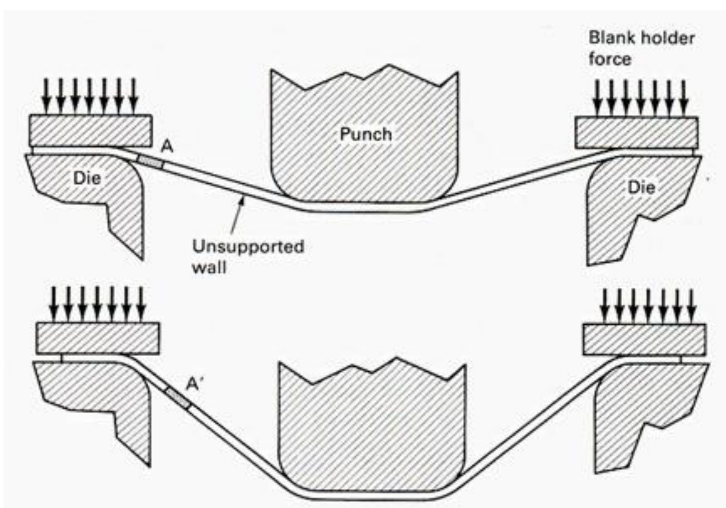


Figure 22.27. Drawing of a conical cup. As element A is drawn into the die cavity, its circumference must shrink. This requires enough tensile stretching in the radial direction. From W. F. Hosford and R. M. Caddell, *ibid.*

solution: There will be more circumferential contraction under radial tension than with a low R-value. Therefore less radial elongation is required to prevent wrinkling. This in turn means less blankholder pressure is needed to prevent wrinkling.

10. Figure 22.28 is a forming limit diagram for a low-carbon steel. This curve represents the combinations of strains that would lead to failure under plane stress ($\sigma_3 = 0$) loading.

A. Show the straining path inside a Marciniak defect under biaxial tension that would lead to necking at point N.

B. Plot carefully on the diagram the strain path that corresponds to uniaxial tension ($\sigma_3 = 0$).

C. Describe how this path would be changed for a material with a value of R>1..

solution: A. The strain path turns upward and is vertical at failure.($d\varepsilon_2/d\varepsilon_1 = 0$ for local necking)

C. The strain path has a slope $\varepsilon_1/\varepsilon_2 = -2$

D. For R >1, ε_2 would be more negative for the same value of ε_1 so the slope, $\varepsilon_1/\varepsilon_2$ = <-2 or tilted counter-clockwise.

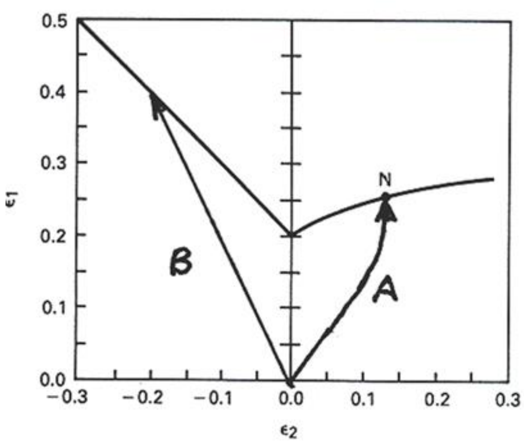
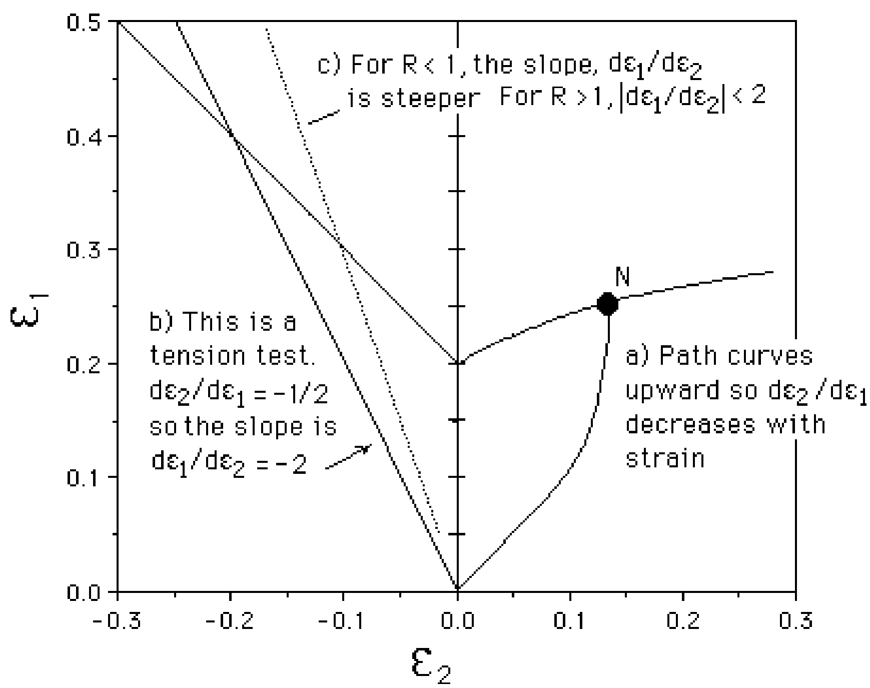


Figure 22.28. Forming limit diagram for a low-carbon steel. From W. F. Hosford and R. M. Caddell, *ibid.*

15-7



11. Consider drawing a copper wire from 0.125 to 0.100 in. diameter. Assume that $\sigma = (55\text{MPa})\varepsilon^{0.36}$ in a die for which $\alpha = 6°$.

A. Calculate the drawing strain.

B. Calculate the reduction of area.
C. Use Figure 22.4 to determine η and calculate the drawing stress.
D. Calculate the yield strength of the drawn wire.

solution: A. $\varepsilon = 2\ln(.125/.100) = 0.446$

B. $RA = 1 - (D/D_o)^2 = 1 - 0.8^2 = 0.36$ or 36%

C. $\eta = 0.76$.

D. $\sigma_d = (1/\eta)\int\sigma d\varepsilon = (1/0.76)(55\text{MPa})0.446^{1.36}/1.36 = 17.7$ MPa

Appendix I

Problem

1. Write the correct direction indices, [ ], and planar indices, ( ), for the directions and planes Figure AI.7 sketched below.

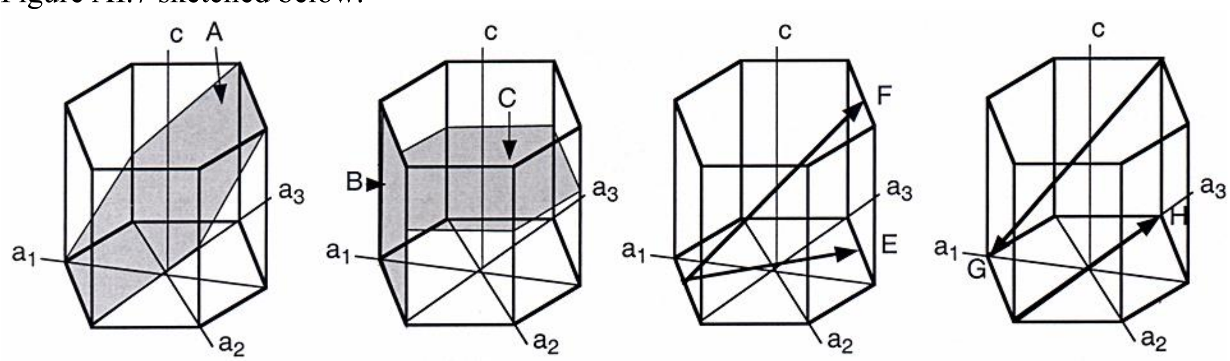


Figure AI.7. Several planes and directions for problem 1.

A. (10»12), B. (10»10), C (0001)

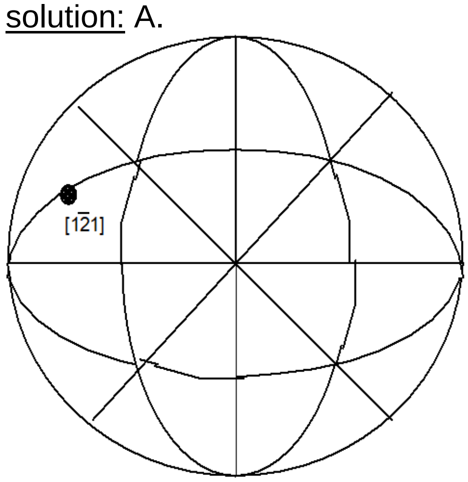
E. [»1010], F. [»1011], G. [10»1»1], H. [»1»120]

Appendix II

1. A Sketch a standard cubic projection with [100] at the center and [001] at the North Pole. Locate the [1Ÿ21] direction on this projection.

B If an fcc crystal were stressed in tension with tensile axis parallel to the [1Ÿ21] direction, on which of the <110>{11Ÿ1} slip system (or systems) would the shear stress be the highest?

solution: A.



B. [1»10](»1»11) and [0»11](1»1»1)