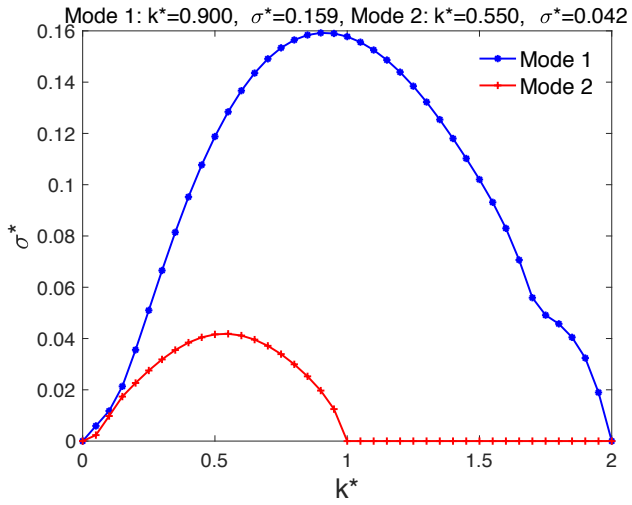

Figure 1: Dispersion relation for the Bickley jet, showing the sinuous and varicose modes.

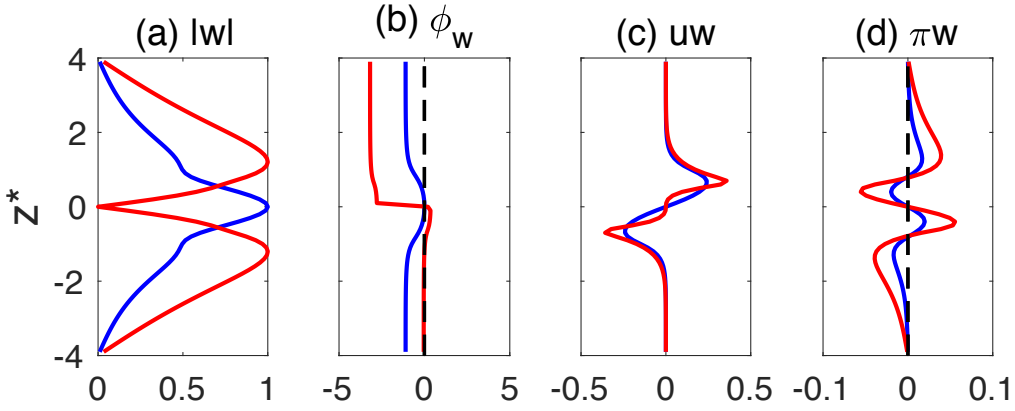

Figure 2: Eigenfunction, momentum flux and energy flux for the fastest growing modes of the Bickley jet (blue=sinuous, red=varicose).

11: The Bickley jet

The growth rate $\sigma^\star(k^\star)$ for the Bickley jet $U^\star = \mathrm{sech}^2(z^\star)$ is shown in figure 1. "Mode 1" and "Mode 2" will be identified as the sinuous and varicose modes, respectively. The sinuous mode grows faster and also has a shorter wavelength.

(a)

- Profiles of $\hat{w}$ (amplitude and phase), $\overline{u'w'}$ and $\overline{\pi'w'}$ are shown on figure 2.
- Profiles of the kinetic energy $K'(z)$, the shear production rate $SP(z)$ the flux convergence $FC(z)$ are shown on figure 3, plus the energy flux.
- The vertical structures of the sinuous and varicose modes are similar. Perturbation kinetic energy is created (converted from the mean flow, that is) in regions surrounding the inflection points on the upper and lower flanks of the jet. The momentum flux carries the positive momentum of the jet both upwards and downwards. Energy is fluxed upward and downward from each of those layers, accumulating both in the outer regions of the jet and near the jet tip ($z = 0$).
- Profiles of $2\sigma_r K'(z)$ and $SP(z) + FC(z)$ are combined on figure 4, and show that the kinetic energy budget balances to within a small tolerance.

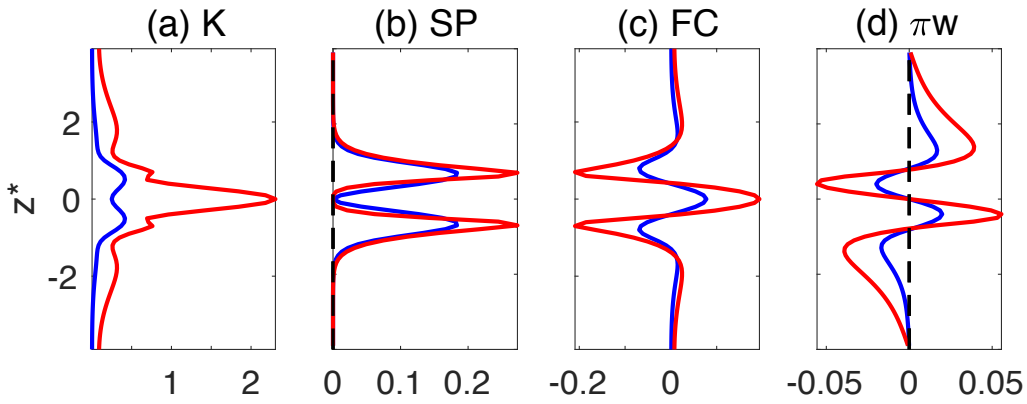

Figure 3: Terms in the energy budget (blue=sinuous, red=varicose).

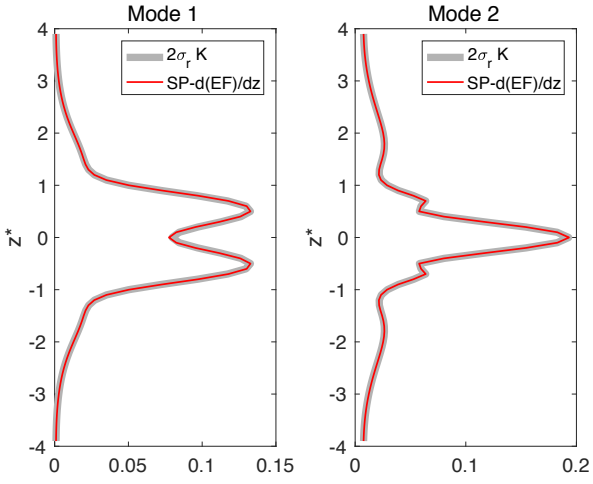

Figure 4: Balancing the energy budget.

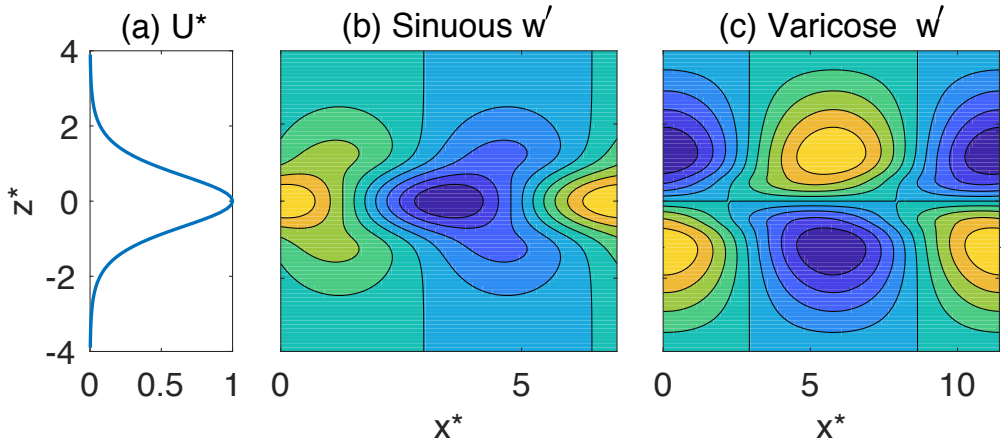

Figure 5: Mean velocity (a) . The vertical velocity perturbation for the fastest-growing sinuous (b) and varicose (c) modes.

- Figure 5 shows $w'(x,z)$ for sinuous (b) and varicose (c) modes, together with the vertical velocity profile (a) for reference. For the sinuous mode, the vertical velocity field indicates a smooth, periodic displacement of the jet core. For the varicose mode, the jet is periodically pinched and expanded, like a varicose vein. The tilt is consistent with positive shear production: the phase tilt leans against the shear on both the upper and lower flanks of the jet.

(**b**) The fastest growing mode of the sinuous instability has scaled wavelength $2\pi/0.9$. The scaled width of the jet is 2, so the ratio of the two is 3.5. Superimposed on figure 6 is a long arrow representing the wavelength and a short, perpendicular arrow whose length is the wavelength divided by 3.5. If the vortices are well represented by the sinuous mode, then the short arrow is the width of the jet. This is plausible, since the short arrow is approximately the width of the island (perpendicular to the wind direction).

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Project 11: Bickley jet

clear
close all

fs=18;
ms=16;
lw=2;


% dispersion relation

% wave vectors
ks=[0:.05:2.0];l=0;

% vertical grid
ztop=4;
```

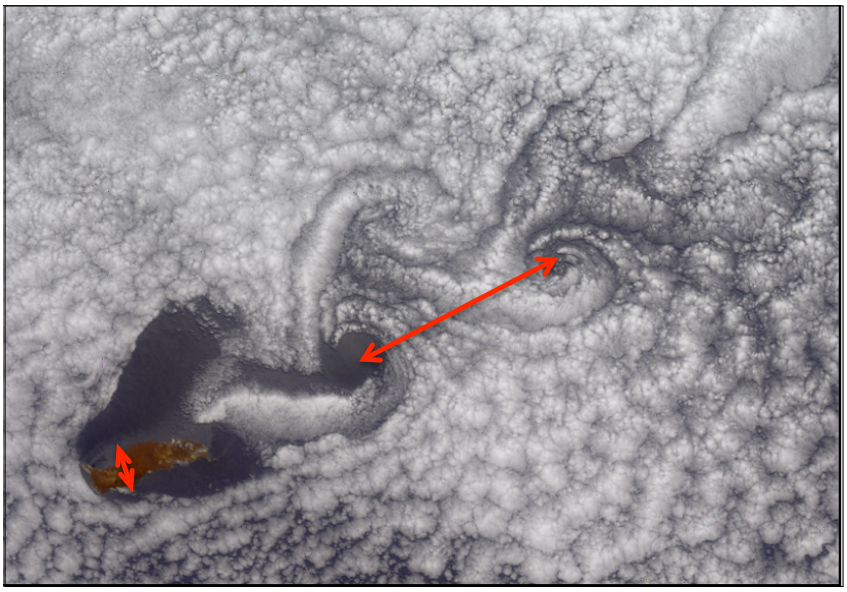

Figure 6: Cloud patterns over Guadalupe Island, near the Baja peninsula (NASA). The long arrow approximates the wavelength. The short arrow is the wavelength divided by 3.5.

```
zbot=-ztop;
dz=0.1;
z=[zbot+dz:dz:ztop-dz]';

% boundary conditions (1=impermeable; 2=asymptotic)
iBC=[1 1];

% background velocity profile
U=sech(z).^2;

% loop over k
for i=1:length(ks)
    k=ks(i);
    [sig1(i)]=Ray(z,U,k,l,iBC,1);
    [sig2(i)]=Ray(z,U,k,l,iBC,2);
end

s1FGM=max(real(sig1));
k1FGM=ks(real(sig1)==s1FGM);
s2FGM=max(real(sig2));
k2FGM=ks(real(sig2)==s2FGM);
titl=sprintf('Mode 1: k*=%.3f, \\sigma*=%.3f, Mode 2: k*=%.3f, \\sigma*=%.3f',...
    k1FGM,s1FGM,k2FGM,s2FGM)


figure
plot(ks,real(sig1),'b-*');hold on
```

```
hold on
plot(ks,real(sig2),'r-+');hold on
h=legend('Mode 1','Mode 2','location','northeast');
set(h,'fontsize',fs)
legend boxoff
set(gca,'fontsize',fs-2)
xlabel('k*','fontsize',fs+2)
ylabel('\sigma*','fontsize',fs+4)
title(titl,'fontsize',fs-2,'fontweight','normal')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.4)

% %% Vertical velocity fields
% % Sinuous mode
% k=k1FGM;l=0;
% iBC=[1 1];
% [sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,l,iBC,1);
%
% x=[0:.01:1]*2*pi/k;
% [X, Z]=meshgrid(x,z);
% ii=complex(0.,1.);
% ePhi=exp(ii*k*X);
% clear w
% for i=1:length(z)
%     w(i,:)=real(w1(i)*ePhi(i,:));
% end
%
% figure
% subplot(2,5,1)
% plot(U,z)
% ylim([-ztop ztop])
% ylabel('z*')
% title('(a) U*','fontweight','normal')
% set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)
%
%
% subplot(2,5,2:3)
% contourf(X,Z,w);
% xlabel('x*')
% set(gca,'yticklabel','')
% set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)
% title('(b) Sinuous w^\prime','fontweight','normal')
%
% % Varicose mode
% k=k2FGM;l=0;
% iBC=[1 1];
% [sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,l,iBC,2);
%
% x=[0:.01:1]*2*pi/k;
```

```
% [X, Z]=meshgrid(x,z);
% ii=complex(0.,1.);
% ePhi=exp(ii*k*X);
% clear w
% for i=1:length(z)
%     w(i,:)=real(w1(i)*ePhi(i,:));
% end
%
% subplot(2,5,4:5)
% contourf(X,Z,w);
% xlabel('x*','fontsize',fs+2)
% set(gca,'yticklabel','')
% set(gca,'fontsize',fs-2)
% title('(c) Varicose  w^\prime','fontweight','normal')
% set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)


%% Eigfns, fluxes
figure
clear sig1 w1 uw SP pw PKE

for i=1:2;
    if i==1
        k=k1FGM;
        imode=1;
        col='b';
    elseif i==2;
        k=k2FGM;
        imode=2;
        col='r';
    end

    [sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,0,iBC,imode); % recalculate with budget terms
    FC=-ddz(z)*pw;

    subplot(2,4,1)
    plot(abs(w1),z,col,'linewidth',lw);hold on
    if i==1; title('(a) |w|','fontsize',fs,'fontweight','normal');end
    ylabel('z*','fontsize',fs)
    set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.3)

    subplot(2,4,2)
    plot(phase(w1),z,col,'linewidth',lw);hold on
    if i==1; title('(b) \phi_w','fontsize',fs,'fontweight','normal');end
    set(gca,'yticklabel','')
    plot([0 0],ylim,'k--')
    set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.3)
```

```
    subplot(2,4,3)
    plot(uw,z,col,'linewidth',lw);hold on
    if i==1; title('(c) uw','fontsize',fs,'fontweight','normal'); end
    set(gca,'yticklabel','')
    set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.3)

    subplot(2,4,4)
    plot(pw,z,col,'linewidth',lw);hold on
    if i==1; title('(d) \piw','fontsize',fs,'fontweight','normal');end
    set(gca,'yticklabel','','fontsize',fs-2)
    plot([0 0],ylim,'k--')

end

%% Energy budget terms

clear sig1 w1 uw SP pw PKE

for i=1:2;
    if i==1
        k=k1FGM;
        imode=1;
        col='b';
    elseif i==2;
        k=k2FGM;
        imode=2;
        col='r';
    end

    figure(100)
    [sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,0,iBC,imode); % recalculate with budget terms
    FC=-ddz(z)*pw;

    subplot(2,4,1)
    plot(PKE,z,col,'linewidth',lw);hold on
     title('(a) K','fontweight','normal')
    ylabel('z*','fontsize',fs)
    set(gca,'fontsize',fs-2)
    axis tight

    subplot(2,4,2)
    plot(SP,z,col,'linewidth',lw);hold on
     title('(b) SP','fontweight','normal')
    set(gca,'yticklabel','')
    plot([0 0],ylim,'k--')
    axis tight

    set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.4)
```

```
    subplot(2,4,3)
    plot(FC,z,col,'linewidth',lw);hold on
     title('(c) FC','fontweight','normal')
    set(gca,'yticklabel','')
    set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.4)
    axis tight

    subplot(2,4,4)
    plot(pw,z,col,'linewidth',lw);hold on
     title('(d) \piw','fontweight','normal')
    set(gca,'yticklabel','','fontsize',fs-2)
    plot([0 0],ylim,'k--')
    axis tight


    % Part D) check PKE budget
    figure(101)
    subplot(1,2,i)
    l(1)=plot(2*real(sig1)*PKE,z,'k','linewidth',lw*3);
    set(l(1),'color',.7*[1 1 1])
    hold on
    l(2)=plot(SP+FC,z,'r','linewidth',lw*.7)
    h=legend(l,'2\sigma_r K','SP-d(EF)/dz')
    set(h,'fontsize',fs-4)
    ylabel('z*','fontsize',fs)
    set(gca,'fontsize',fs-2)
    if i==1;
        title('Mode 1','fontweight','normal')
    elseif i==2;
        title('Mode 2','fontweight','normal')
    end

end


%% Vertical velocity fields
% Sinuous mode
k=k1FGM;l=0;
iBC=[1 1];
[sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,l,iBC,1);

x=[0:.01:1]*2*pi/k;
[X, Z]=meshgrid(x,z);
ii=complex(0.,1.);
ePhi=exp(ii*k*X);
clear w
for i=1:length(z)
```

```
    w(i,:)=real(w1(i)*ePhi(i,:));
end

figure
subplot(2,5,1)
plot(U,z)
ylim([-ztop ztop])
ylabel('z*')
title('(a) U*','fontweight','normal')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)


subplot(2,5,2:3)
contourf(X,Z,w);
xlabel('x*')
set(gca,'yticklabel','')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)
title('(b) Sinuous w^\prime','fontweight','normal')

% Varicose mode
k=k2FGM;l=0;
iBC=[1 1];
[sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,l,iBC,2);

x=[0:.01:1]*2*pi/k;
[X, Z]=meshgrid(x,z);
ii=complex(0.,1.);
ePhi=exp(ii*k*X);
clear w
for i=1:length(z)
    w(i,:)=real(w1(i)*ePhi(i,:));
end

subplot(2,5,4:5)
contourf(X,Z,w);
xlabel('x*','fontsize',fs+2)
set(gca,'yticklabel','')
set(gca,'fontsize',fs-2)
title('(c) Varicose  w^\prime','fontweight','normal')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.0,'titlefontsizemultiplier',1.0)
```