X-Rays and Extreme Ultraviolet Radiation: Principles and Applications
Chapter 3. Wave Prop. & Refractive Index Homework Problems

(Consult the website: http://www.cxro.LBL.gov/optical_constants)

3.1
(a) Write an equation for the real part of the refractive index, δ, of silicon by assuming a semi-classical atomic model with two electrons associated with a K-edge at 1840 eV, eight electrons associated with an L-edge at 99.2 eV, and four electrons associated with a UV resonance at 6 eV. Consult Eqs. (2.80) and (3.13a).
(b) Graph your result for the real part of the refractive index in the above simplified model on a log scale extending from 30 eV to 30 keV. Use a linear ordinate that covers refractive values extending only from -2.0 to +2.0. For all resonances assume $\gamma/\omega_s = 10^{-2}$.

3.2
(a) Graph the δ value of the refractive index of silicon using tabulated values of f_1^0. Consult Appendix C or the website above. Use the same coordinates as in question 3.1 for ease of comparison.
(b) Why is it appropriate to use the f_1^0 approximation?
(c) Why is this graph different than that obtained in question 3.1?
(d) Consider the advantages and disadvantages of both approaches.

3.3
(a) Graph the δ and β values of the refractive index for carbon for photon energies extending from 30 eV to 30 keV. Present the results in a log-log format. Assume a density of 2.27 g/cm^3, as given in the Periodic Table of the Elements on the inside back cover of the book.
(b) Repeat the above for aluminum, ruthenium, and gold.

3.4
Confirm that equation (3.9) follows from equation (3.7). For the approximation to be valid, factors of the term $e^2n_a/\varepsilon_0 m\omega^2$ must be much less than unity. Demonstrate that this approximation is valid, away from the resonance, by considering various atoms, and frequencies corresponding to both EUV and x-ray radiation. Atomic densities of atoms in their natural form are given in the Periodic Chart of the Elements, on the inside back cover of the book. As an example, calculate the above factor for silver at photon energies of 80 eV, 1 keV and 10 keV.

3.5
(a) Discuss the physical significance of the phase velocity $v_\phi = \omega / k$, and the group velocity $v_g = \partial\omega / \partial k$, for propagation of a transverse electromagnetic wave in a material of single electron atoms, each having a single resonance at $\omega = \omega_s$. Assume $\gamma/\omega_s \ll 1$, and an atomic density n_a. Consult slides for Chapter 3 in the "Supporting Material" section at the book website, www.cambridge.org/xrayeuv .
(b) Develop expressions for f_1^0 and f_2^0 .
(c)) Write an expression for the refractive index n in terms of ω, ω_s, γ and ω_p, where $\omega_p^2 = e^2n_a/\varepsilon_0 m$.
(d) Neglecting γ for frequencies away from the resonance, write an expression for the dispersion relation between ω and k.
(e) Using the expression obtained in part (c), discuss v_ϕ for $\omega < \omega_s$ and $\omega > \omega_s$.
(f) Discuss v_g for $\omega < \omega_s$ and $\omega > \omega_s$ in this same approximation.
(g) Can v_g exceed c in the immediate vicinity of the resonance (even with $\gamma/\omega \ll 1$)? Consult J.D. Jackson, Ref. 2 of chapter 3, his p.325.

3.6

Confirm that for $\mathbf{H} = n\sqrt{\varepsilon_0 / \mu_0}\, \mathbf{k_0} \times \mathbf{E}$, where n is complex, the average intensity can be written as in equation (3.20). Consult if necessary J. Stratton, and M. Born and E. Wolf, Refs. 3 and 1, respectively, of Chapter 3.

3.7

(a) Determine the complex refractive index of both silicon and molybdenum (separately) for λ = 13.5 nm, in terms of δ and β. Assume densities of 2.33 g/cm^3 an and 10.2 g/cm^3, respectively. updated values from Appendix C or the website above.
(b) Calculate the absorption depth in each material (separately) at this wavelength.
(c) What is the phase shift, relative to vacuum, of the wave propagating through a 200 nm thickness of these two materials?
(d) Calculate the critical angle for total external reflection for each material.
(e) How is it that 13.5 nm radiation is not largely absorbed in a Mo/Si multilayer coating with d = 6.75 nm at the Bragg angle?

3.8

Calculate the critical angle for total external reflection from nickel (ρ = 8.91 g/cm^3) and from gold (ρ = 19.3 g/cm^3), for wavelengths of 13.5 nm (EUV), 2.50 nm (SXR), and 0.100 nm (x- ray).

3.9

Calculate the normal incidence reflectivity at a vacuum-copper (Z = 29) interface for photon energies of 80 eV (EUV), 500 eV (SXR), and 8 keV (x-ray).

3.10

Calculate the glancing incidence reflectivity of ruthenium at an angle of incidence of 30 mrad (1.7˚) for photon energies of 100 eV, 1 keV, and 10 keV.

3.11 (a) What is the normal incidence reflectivity for 13.5 nm radiation at a single vacuum-molybdenum (Z = 42) surface? (b) What is it for a single silicon surface at normal incidence and this wavelength? (c) Make an educated guess as to how many Mo/Si interfaces would be required in a multilayer mirror to achieve a reflectance of approximately 40−60%. Neglect absorption, approximate silicon by a vacuum (n = 1) spacer, assume the interface spacings permit positive interference of the reflections from all interfaces, and recall that the phase change upon reflection varies in sign depending on the sequence of high and low refractive index materials. (d) What is the required bi-layer periodicity (d-spacing) required for positive interference? Note that this problem will be considered computationally, including absorption, in Chapter 10, problem 10.5.

3.12
(a) Calculate reflectivity versus photon energy for iridium ($Z = 77$) at 0.75° glancing incidence (45 arc min). Use scattering factors from the CXRO website.
(b) How might a mirror of iridium be used in an x-ray telescope?

3.13
In designing a filter to transmit K_α characteristic emission from atoms of a single element (C, O, or Cu, etc.), while blocking other wavelengths, what is generally a good material to start with? Draw an atomic energy diagram showing the K-absorption edge and K_α to explain your answer.

3.14
(a) For the case of *p*-polarized radiation in which the electric field is parallel to the plane of incidence, what is the expression for the reflected electric field in terms of E_0, n, and ϕ as given in equation (3.54).
(b) From this derive the angle of minimum reflectance, known as Brewster's angle or the polarizing angle.
(c) Give a physical description as to why one would expect little or no reflection of *p*-polarized radiation at this angle.
(d) Why is there no Brewster's effect for *s*-polarized radiation?

3.15
(a) Calculate Brewster's angle for visible light incident from air on a glass surface assuming $n = 1.5$.
(b) How can this be used to minimize cavity losses and control the polarization of visible or near IR laser? Consult the texts by Silfvast and by Siegman, Refs. 9 and 7, respectively, in Chapter 9, for further discussion of Brewster's angle optics in lasers.

3.16
(a) Calculate Brewster's angle for 6.8 nm wavelength radiation incident from vacuum on a carbon mirror.
(b) How should the surface be oriented to suppress horizontally polarized radiation while reflecting vertically polarized radiation?

3.17
(a) What is Brewster's angle for 14.0 nm wavelength radiation incident from vacuum on a silicon surface?
(b) What is Brewster's angle for nickel at a photon energy of 730 eV?
(c) What is Brewster's angle for nickel at a photon energy of 10 keV?

3.18 Equation (3.90) describes the current density induced by an incident electric field in a collection of identical atoms of number density n_a. The contributions of all bound electrons are accounted for in $f^0(\omega)$. The superscript zero indicates that this formulation is appropriate for both forward scattering and long wavelength. Beginning with the general definition of current density

$$\mathbf{J} = qn\mathbf{v}$$

use equation (2.65) to show that in these regions of validity ($f \to f^0$), the product $n\mathbf{v}$ is replaced by

$$n\,\mathbf{v} \to n_a \sum_{s=1}^{Z} \mathbf{v}_s = n_a f\ \ (\omega)\,\mathbf{v}_\mathrm{e}$$

where $\mathbf{v}_e$ is the response of a single free electron to an incident electric field $\mathbf{E}_i$. Note that in the text $\mathbf{v}_e$ is replaced by $\mathbf{v}$ for simplicity in equation (3.90). Equations (3.90) and (3.92) then follow directly.

3.19

Using Snell's law, equation (3.38), derive the correction to Bragg's law, Eq. (3.95):

$$m\lambda = 2d\sin\theta\left(1 - \frac{4\bar{\delta}d^2}{m^2\lambda^2}\right)$$

where λ and θ are external to the multilayer, and δ is a density profile averaged value for a bi-layer pair. Start with Bragg's law within the periodic structure as given by equation (3.95b),

$$m\lambda' = 2d\sin\theta'$$

where λ' and θ' are within the multilayer (or crystal). Use Snell's law to replace λ' and θ', in the approximation $\beta/\bar{\delta}$ « 1. For further discussion consult Compton and Allison, p. 674 therein, given as Ref. 6 in Chapter 3; Richtmyer, Kennard, and Lauritsen, p. 392 therein, given as Ref. 28 in Chapter 1 of this book; or Leighten, p. 456 therein, given as Ref. 27 in Chapter 1 of this book.

3.20

(a) Describe some general considerations relevant to the choice of materials for a multilayer interference coating for use at a given photon energy.

(b) Comment on the choice of angle of incidence, required *d*-spacing, choice of the low-*Z* "spacer" material, choice of the high-*Z* "scattering" material, optimum choice of Γ, required number of bi-layer pairs (N), and specification on substrate roughness. Be sure to consider materials compatibility, interface roughness, and chemical stability at the exposed surface.

3.22

(a) Plot the reflectivity of a Mo/Si coating for wavelengths from 11 to 15 nm, at normal incidence ($\phi = 0$, $\theta = \pi/2$), assuming d = 6.7 nm, $\Gamma = 0.4$, and N = 40. Show results for a perfectly smooth, infinitely sharp interface ($\sigma = 0$), and for interfaces characterized by $\sigma = 0.1$ nm rms and 0.5 nm rms.

(b) What does this indicate regarding surface finish specifications for high mirror reflectivity?

(c) What throughput would be expected for a nine-mirror optical system with $\sigma = 0.5$ nm rms? Feel free to use the multilayer sub-routine at the website http://www.cxro.lbl.gov/optical_constants.

3.22

3.23

(a) Repeat problem 3.22 for a Mo/Be multilayer coating with d = 5.7 nm, $\Gamma = 0.40$, N = 40, and $\sigma = 0.1$ nm rms, for wavelengths from 11 to 15 nm.
(b) Compare the peak reflectivity and relative spectral bandpass to that of Mo/Si at $\sigma = 0.1$ nm rms.
(c) How does this change if one employs 70 bi-layer pairs of Mo/Be?
(d) What are the implications for an EUV optical stepper (lithographic optical reduction print camera) with seven multilayer coated surfaces?
(e) What role might the emission spectrum of the source play in the apparent advantage of Mo/Be?
(f) What disadvantages might there be?

Note: There are additional multilayer homework problems in Chapters 10 and 11.

3.24

You are interested in probing the magnetic properties of iron containing materials using polarized radiation near the iron L-edge.
(a) What photon energy range would be appropriate? You plan to use Brewster's angle optics to polarize the incident radiation and to measure Faraday rotation of the polarization vector as it traverses the material while exposed to an applied magnetic field.
(b) Describe the parameters of a W/C multilayer mirror to achieve the desired polarization, including the *d*-spacing, Γ, N, and σ.
(c) How can a polarimeter be constructed? Refer to the paper of J.B. Kortright *et al* in Phys. Rev. B 51, 10240 (1995).

3.25

An x-ray fluorescence microprobe uses multilayer coated Kirkpatrick-Baez (KB) optics to focus hard x-rays to a 1 μm diameter spot size to study elemental compositions of samples for materials science, environmental sciences, and parts-per-billion (ppb) impurity analyses in which fluorescent emissions are detected from elements simultaneously. This is done with thick samples, in air, and in the presence of higher *Z* materials if necessary.

(a) Plot the reflectivity of a single W/C multilayer mirror from zero to 12 keV photon energy, for d = 4.0 nm, $\Gamma = 0.40$, N = 50 layer pairs, and a glancing incidence angle of 1.2°. Assume $\sigma = 0.1$ nm rms.
(b) What is the relative spectral bandpass?
(c) How does this compare to 1/N?
(d) For what elements could multi-keV fluorescence emission from K-shells be observed?
(e) From L-shells of which elements?

3.22