Appendix SA4.1 Supplemental Discussion of Aggregation Bias

A4.1.1 Measures of Aggregation Bias

Total aggregation bias has been defined – for example, in Morimoto (1970) − as the difference between the vector of total outputs in the aggregated system and the vector obtained by aggregating the total outputs in the original unaggregated system. As in the last example, for some new vector of final demands, **f**, the total output vector in the unaggregated model is $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$. The total output vector in the aggregated model is $\mathbf{x}^* = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{f}^*$ and the total aggregation bias is defined as

$$\boldsymbol{\tau} = \mathbf{x}^* - \mathbf{S}\mathbf{x} \tag{A4.1.1}$$

That is, $\boldsymbol{\tau} = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{f}^* - \mathbf{S}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$, or $\boldsymbol{\tau} = \left[(\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{S} - \mathbf{S}(\mathbf{I} - \mathbf{A})^{-1}\right]\mathbf{f}$. Using the power series results,

$$\begin{aligned}\boldsymbol{\tau} &= \left[(\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2} + \ldots)\mathbf{S} - \mathbf{S}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots)\right]\mathbf{f} \\ &= \left[(\mathbf{A}^*\mathbf{S} - \mathbf{S}\mathbf{A}) + (\mathbf{A}^{*2}\mathbf{S} - \mathbf{S}\mathbf{A}^2) + \ldots\right]\mathbf{f}\end{aligned} \tag{A4.1.2}$$

The first term in this series has been defined as the “first-order” aggregation bias (Theil, 1957); that is,

$$\boldsymbol{\varphi} = (\mathbf{A}^*\mathbf{S} - \mathbf{S}\mathbf{A})\mathbf{f} \tag{A4.1.3}$$

A4.1.2 Aggregation Bias Theorems

We present two basic theorems regarding aggregation bias and, in particular, when it will vanish. One has to do with the nature of the **A** and $\mathbf{A}^*$ matrices, that is, with the structural characteristics of the economy; the other has to do with the nature of the final-demand vectors, **f** and $\mathbf{f}^*$, being studied. The first theorem is:

Theorem 4.1. The total aggregation bias vanishes (i.e., $\boldsymbol{\tau} = \mathbf{0}$) for any $\boldsymbol{\varphi}$ if and only if $\mathbf{A}^*\mathbf{S} = \mathbf{S}\mathbf{A}$. This follows from the expression for τ in (4.26) since, if $\mathbf{A}^*\mathbf{S} = \mathbf{S}\mathbf{A}$, then

$$\mathbf{A}^{*2}\mathbf{S} - \mathbf{S}\mathbf{A}^2 = \mathbf{A}^*\mathbf{A}^*\mathbf{S} - \mathbf{S}\mathbf{A}\mathbf{A} = \mathbf{A}^*(\mathbf{S}\mathbf{A}) - (\mathbf{A}^*\mathbf{S})\mathbf{A} = \mathbf{0}$$

and similarly, for higher-order terms in the series. This theorem suggests that if two (or more) sectors have identical interindustry structures (i.e., equal columns in the **A** matrix, as we found in the example), then aggregation of these sectors will result in zero total aggregation bias. For example, consider a three-sector economy in which sectors 1 and 3 have the same interindustry input structure:

$$\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \ \mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The corresponding transactions matrix is found by

$$\mathbf{Z}=\mathbf{A}\hat{\mathbf{x}}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix}=\begin{bmatrix} a_{11}x_1 & a_{12}x_2 & a_{13}x_3 \\ a_{21}x_1 & a_{22}x_2 & a_{23}x_3 \\ a_{31}x_1 & a_{32}x_2 & a_{33}x_3 \end{bmatrix}$$

The proper aggregation matrix for combining sectors 1 and 3 is $\mathbf{S}=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Hence, the aggregated transactions matrix and total outputs vector are

$$\mathbf{Z}^*=\mathbf{SZS}'=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_{11}x_1 & a_{12}x_2 & a_{13}x_3 \\ a_{21}x_1 & a_{22}x_2 & a_{23}x_3 \\ a_{31}x_1 & a_{32}x_2 & a_{33}x_3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

or

$$\mathbf{Z}^*=\begin{bmatrix} a_{11}x_1+a_{31}x_1+a_{11}x_3+a_{31}x_3 & a_{12}x_2+a_{32}x_2 \\ a_{21}x_1+a_{21}x_3 & a_{22}x_2 \end{bmatrix}$$

and $\mathbf{x}^*=\mathbf{Sx}=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} x_1+x_3 \\ x_2 \end{bmatrix}$. Hence, the aggregated technical coefficients matrix is found by

$$\mathbf{A}^*=\mathbf{Z}^*(\hat{\mathbf{x}}^*)^{-1}=\begin{bmatrix} \dfrac{(a_{11}+a_{31})(x_1+x_3)}{x_1+x_3} & \dfrac{(a_{12}+a_{32})x_2}{x_2} \\ \dfrac{a_{21}(x_1+x_3)}{x_1+x_3} & \dfrac{a_{22}x_2}{x_2} \end{bmatrix}=\begin{bmatrix} a_{11}+a_{31} & a_{12}+a_{32} \\ a_{21} & a_{22} \end{bmatrix}$$

Theorem 4.1 asserts that there will be no aggregation bias when two columns are identical, that is, when **A*S** = **SA**. For our general example this can be shown by

$$\mathbf{A}^*\mathbf{S}=\begin{bmatrix} a_{11}+a_{31} & a_{12}+a_{32} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}=\begin{bmatrix} a_{11}+a_{31} & a_{12}+a_{32} & a_{11}+a_{31} \\ a_{21} & a_{22} & a_{21} \end{bmatrix}$$

and

$$\mathbf{SA}=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} a_{11}+a_{31} & a_{12}+a_{32} & a_{11}+a_{31} \\ a_{21} & a_{22} & a_{21} \end{bmatrix},$$

which are the same.

The second theorem on aggregation bias is the following:

Theorem 4.2. If some sectors are not aggregated and the new final demands occur only in unaggregated sectors, the first-order aggregation bias will vanish.

For a general three-sector economy, the unaggregated and aggregated technical coefficients matrices, **A** and $\mathbf{A}^*$, respectively, are

$$\mathbf{A} = \begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}}{x_2} & \frac{z_{13}}{x_3} \\ \frac{z_{21}}{x_1} & \frac{z_{22}}{x_2} & \frac{z_{23}}{x_3} \\ \frac{z_{31}}{x_1} & \frac{z_{32}}{x_2} & \frac{z_{33}}{x_3} \end{bmatrix} \text{ and } \mathbf{A}^* = \begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}+z_{13}}{x_2+x_3} \\ \frac{z_{21}+z_{31}}{x_1} & \frac{z_{22}+z_{23}+z_{32}+z_{33}}{x_2+x_3} \end{bmatrix}$$

The unaggregated sector is sector 1 (in both the aggregated and unaggregated models). Consider final-demand vectors for which only the unaggregated elements are nonzero:

$\mathbf{f} = \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{f}^* = \mathbf{Sf} = \begin{bmatrix} f_1 \\ 0 \end{bmatrix}$. This theorem asserts that the first-order aggregation bias,

$\boldsymbol{\varphi} = \left(\mathbf{A}^*\mathbf{S} - \mathbf{SA}\right)\mathbf{f}$ is zero for final demands such as those given as **f** and **f*** above. For the example:

$$\mathbf{SA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}}{x_2} & \frac{z_{13}}{x_3} \\ \frac{z_{21}}{x_1} & \frac{z_{22}}{x_2} & \frac{z_{23}}{x_3} \\ \frac{z_{31}}{x_1} & \frac{z_{32}}{x_2} & \frac{z_{33}}{x_3} \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}}{x_2} & \frac{z_{13}}{x_3} \\ \frac{z_{21}+z_{31}}{x_1} & \frac{z_{22}+z_{32}}{x_2} & \frac{z_{23}+z_{33}}{x_3} \end{bmatrix}$$

and

$$\mathbf{A}^*\mathbf{S} = \begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}+z_{13}}{x_2+x_3} & \frac{z_{12}+z_{13}}{x_2+x_3} \\ \frac{z_{21}+z_{31}}{x_1} & \frac{z_{22}+z_{23}+z_{32}+z_{33}}{x_2+x_3} & \frac{z_{22}+z_{23}+z_{32}+z_{33}}{x_2+x_3} \end{bmatrix}.$$

Hence, the first-order bias, $\boldsymbol{\varphi}$, as defined earlier, is $\boldsymbol{\varphi} = \left(\mathbf{A}^*\mathbf{S} - \mathbf{SA}\right)\mathbf{f} =$

$$\begin{bmatrix} 0 & \left(\frac{z_{12}+z_{13}}{x_2+x_3} - \frac{z_{12}}{x_2}\right) & \left(\frac{z_{12}+z_{13}}{x_2+x_3} - \frac{z_{13}}{x_3}\right) \\ 0 & \left(\frac{z_{22}+z_{23}+z_{32}+z_{33}}{x_2+x_3} - \frac{z_{22}+z_{32}}{x_2}\right) & \left(\frac{z_{22}+z_{23}+z_{32}+z_{33}}{x_2+x_3} - \frac{z_{23}+z_{33}}{x_3}\right) \end{bmatrix} \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, if one is studying the effect of new final demand only for sector 1's output in an n-sector model, any and all combinations of sectors 2 through n into fewer sectors will generate no first-order aggregation bias. Although these theorems are stated in terms of sectoral aggregation, they also have implications for spatial aggregation in interregional models. In general, the conditions of Theorem 4.1 are almost certain not to be met as one combines regions in an interregional input–output model, but the conditions of Theorem 4.2 will be met in many cases. Additional general theorems on sectoral aggregation bias based on statistical properties are discussed in Gibbons, Wolsky and Tolley (1982).

A4.1.3 Spatial Aggregation Bias

Aggregation bias in interregional and multiregional input–output models, or spatial aggregation, is a very straightforward extension of sectoral aggregation. Most simply it can be thought of as aggregating regions each with the same level of sectoral detail to a reduced number of the regions considered, although including regions at different sectoral aggregations would, of course, also be possible.

To illustrate spatial aggregation of MRIO models, consider three regions, $k = 1, 2$ and 3 with transactions matrices for each two-sector region defined by $\mathbf{Z}^1 = \begin{bmatrix} 20 & 70 \\ 50 & 50 \end{bmatrix}$, $\mathbf{Z}^2 = \begin{bmatrix} 20 & 10 \\ 70 & 70 \end{bmatrix}$, and $\mathbf{Z}^3 = \begin{bmatrix} 90 & 40 \\ 50 & 80 \end{bmatrix}$, with corresponding total outputs vectors defined by $\mathbf{x}^1 = \begin{bmatrix} 340 \\ 350 \end{bmatrix}$, $\mathbf{x}^2 = \begin{bmatrix} 250 \\ 300 \end{bmatrix}$, and $\mathbf{x}^3 = \begin{bmatrix} 325 \\ 400 \end{bmatrix}$. Since $\mathbf{A}^k = \mathbf{Z}^k \hat{\mathbf{x}}^k$ defines the MRIO matix of regional technical coefficients, we can write:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^1 & 0 & 0 \\ 0 & \mathbf{A}^2 & 0 \\ 0 & 0 & \mathbf{A}^3 \end{bmatrix} = \left[\begin{array}{cc:cc:cc} .059 & .2 & 0 & 0 & 0 & 0 \\ .147 & .143 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & .08 & .033 & 0 & 0 \\ 0 & 0 & .28 & .233 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & .277 & .1 \\ 0 & 0 & 0 & 0 & .154 & .2 \end{array}\right].$$

Now, assume the trade flows among the three regions of the two industrial commodities, $i, j = 1$ and 2, are given by $\mathbf{Z}_1 = \begin{bmatrix} 9 & 16 & 10 \\ 5 & 6 & 8 \\ 4 & 10 & 18 \end{bmatrix}$ and $\mathbf{Z}_2 = \begin{bmatrix} 19 & 2 & 19 \\ 11 & 11 & 7 \\ 20 & 10 & 6 \end{bmatrix}$. The matrix of trade coefficients, defined in Chapter 3 by $c_i^{ab} = \dfrac{z_i^{ab}}{T_i^b}$; for z_i^{ab}, the flow of commodity i between regions a and b; and T_i^b, the total shipments of commodity i from all regions into region b (the column sums of $\mathbf{Z}_k$), is found as

$$\mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{11} & \hat{\mathbf{c}}^{12} & \hat{\mathbf{c}}^{13} \\ \hat{\mathbf{c}}^{21} & \hat{\mathbf{c}}^{22} & \hat{\mathbf{c}}^{23} \\ \hat{\mathbf{c}}^{31} & \hat{\mathbf{c}}^{32} & \hat{\mathbf{c}}^{33} \end{bmatrix} = \left[\begin{array}{cc:cc:cc} .5 & 0 & .5 & 0 & .278 & 0 \\ 0 & .38 & 0 & .087 & 0 & .594 \\ \hdashline .278 & 0 & .188 & 0 & .222 & 0 \\ 0 & .22 & 0 & .478 & 0 & .219 \\ \hdashline .222 & 0 & .313 & 0 & .5 & 0 \\ 0 & .4 & 0 & .435 & 0 & .188 \end{array}\right]$$

For aggregation from three to two regions, the aggregated commodity flows can be computed by constructing a spatial aggregation matrix, **R.** As an example, $\mathbf{R}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\end{bmatrix}$ will be used to aggregate sectors 2 and 3 in the three-region model to the second region of the two-region model—**R** is distinct from the sectoral aggregation matrix, **S**, defined earlier. The aggregated (2×2) interregional flow matrices, $\mathbf{Z}_i^*$, are found by $\mathbf{Z}_i^* = \mathbf{R}\mathbf{Z}_i\mathbf{R}'$ for industries i = 1 and 2, i.e., $\mathbf{Z}_1^* = \mathbf{R}\mathbf{Z}_1\mathbf{R}' = \begin{bmatrix}9 & 26\\9 & 42\end{bmatrix}$ and $\mathbf{Z}_2^* = \mathbf{R}\mathbf{Z}_2\mathbf{R}' = \begin{bmatrix}19 & 21\\31 & 34\end{bmatrix}$ and it follows that the matrix of trade coefficients for the aggegated two-sector model, $\mathbf{C}^*$, becomes

$$\mathbf{C}^* = \left[\begin{array}{cc:cc} .5 & 0 & .382 & 0\\ 0 & .38 & 0 & .382\\ \hdashline .5 & 0 & .618 & 0\\ 0 & .62 & 0 & .618\end{array}\right]$$

Similarly, we compute the aggregated regional transactions matrices as $\mathbf{Z}^{1*} = \mathbf{Z}^1 = \begin{bmatrix}20 & 70\\50 & 50\end{bmatrix}$ and $\mathbf{Z}^{2*} = \mathbf{Z}^2 + \mathbf{Z}^3 = \begin{bmatrix}110 & 50\\120 & 150\end{bmatrix}$ as well as the aggregated regional total outputs vectors as $\mathbf{x}^{1*} = \mathbf{x}^1 = \begin{bmatrix}340\\350\end{bmatrix}$ and $\mathbf{x}^{2*} = \mathbf{x}^2 + \mathbf{x}^3 = \begin{bmatrix}575\\700\end{bmatrix}$ so that $\mathbf{A}^{1*} = \mathbf{Z}^{1*}\hat{\mathbf{x}}^{1*}$ and $\mathbf{A}^{2*} = \mathbf{Z}^{2*}\hat{\mathbf{x}}^{2*}$ or $\mathbf{A}^* = \begin{bmatrix}\mathbf{A}^{1*} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}^{2*}\end{bmatrix} = \left[\begin{array}{cc:cc} .059 & .2 & 0 & 0\\ .147 & .143 & 0 & 0\\ \hdashline 0 & 0 & .191 & .071\\ 0 & 0 & .209 & .214\end{array}\right]$. The reader can verify that we could equivalently compute $\mathbf{A}^*$ by first creating $\mathbf{Z}^* = \begin{bmatrix}\mathbf{Z}^{1*} & \mathbf{0}\\ \mathbf{0} & \mathbf{Z}^{2*}\end{bmatrix} = \mathbf{SZS}'$ and $\mathbf{x}^* = \begin{bmatrix}\mathbf{x}^{1*}\\ \mathbf{x}^{2*}\end{bmatrix} = \mathbf{Sx}$ for $\mathbf{Z} = \begin{bmatrix}\mathbf{Z}^1 & 0 & 0\\ 0 & \mathbf{Z}^2 & 0\\ 0 & 0 & \mathbf{Z}^3\end{bmatrix}$; $\mathbf{x} = \begin{bmatrix}\mathbf{x}^1 & \mathbf{x}^2 & \mathbf{x}^3\end{bmatrix}'$ and $\mathbf{S} = \left[\begin{array}{cc:cc:cc} 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ \hdashline 0 & 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0 & 1\end{array}\right]$. $\mathbf{A}^*$ is then found as $\mathbf{A}^* = \mathbf{Z}^*\hat{\mathbf{x}}^*$.

This formulation will be useful next in computing measures of bias.

Finally, in computing the total aggregation bias, we presume a projected unaggregated final demand vector of $\tilde{\mathbf{f}}' = \left[\begin{array}{cc:cc:cc}100 & 100 & 100 & 100 & 100 & 100\end{array}\right]$. The corresponding aggregated vector of final demands would then be $\tilde{\mathbf{f}}'^* = \mathbf{S}\tilde{\mathbf{f}}' = \begin{bmatrix}100 & 100 & 200 & 200\end{bmatrix}$ and, for these final demands, we compute and $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{x}}^*$ as the following:

$$\tilde{\mathbf{x}} = \left(\mathbf{I} - \mathbf{C}\,\mathbf{A}\right)^{-1}\mathbf{C}\tilde{\mathbf{f}} = \left[\begin{array}{cc:cc:cc}142.9 & 155.4 & 126.5 & 150.8 & 140.9 & 155.3\end{array}\right]'$$

$$\tilde{\mathbf{x}}^* = \left(\mathbf{I} - \mathbf{C}^*\mathbf{A}^*\right)^{-1}\mathbf{C}^*\tilde{\mathbf{f}}^* = \left[\begin{array}{cc:cc}147.8 & 161.5 & 263.3 & 299.8\end{array}\right]'$$

It follows that total aggregation bias, as defined earlier, is computed as the sum of absolute differences of $\mathbf{S}\tilde{\mathbf{x}}$ and $\tilde{\mathbf{x}}^*$ or, expressed as a percentage of the value of total outputs of the unaggregated economy, is

$$100\left(\frac{\left|\mathbf{S}\tilde{\mathbf{x}}-\tilde{\mathbf{x}}^{*}\right|\mathbf{i}}{\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}}\right)=100\left(\frac{21.4}{871.8}\right)=2.46\%.$$

As a general matter it appears that spatial aggregation in both IRIO and MRIO models produce only modest aggregation bias. Hence, for questions pertaining to one or more specific regions, it appears that an MRIO (or IRIO) model in which those regions are distinct, while the rest of the economy is spatially aggregated into the "remaining" region, can often prove to be entirely adequate. The subjects of spatial aggregation applied to IRIO and MRIO models are discussed in more detail in Miller and Blair (1981) and Blair and Miller (1983), respectively. Examples of spatial aggregation for the three-region Japanese interregional and the US multiregional input–output models are included for the interested reader below.

In recent years with considerably expanded computational capacity increasingly available, very large MRIO models, including global models, have been widely applied (discussed in more detail in Chapters 3 and 13). Lenzen (2019) and others have considered methods for optimizing aggregation of such very large models and concluded that while clustering sectors with similar characteristics was frequently identified as the method associated with the lowest general error level, especially where considerable aggregation is necessary, for large MRIO systems such methods remain computationally challenging. Lenzen found Structural Path Analysis (SPA), discussed in Chapter 8, to be the most intuitively appealing and provides "a straightforward approach to realise groupings that account for the specific purpose of a specific study (Lenzen, 2019, p. 19). Other researchers over the years have explored a wide range of approaches for minimizing error in aggregation of input-output tables, such as Fisher (1969), and Neudecker (1970), Blin and Cohen (1977), Roy, Batton and Lesse (1982), Cabrer, Contreras and Miravete (1991), Oksanen and Williams (1992), Olsen (1993 and 2001), Andrew, Peters and Lennox (2009) and, as noted above, Lenzen (2019).

A4.1.4 Examples of Spatial Aggregation in IRIO and MRIO Models

We consider two examples of spatial aggregation for two multiple region input–output models: (1) a three-region interregional (IRIO) model for Japan and (2) the US multi-regional (MRIO) model and using the basic measures of aggregation bias introduced in Section 4.8.2.

A4.1.4.1 Spatial Aggregation of IRIO Models

Spatial aggregation of IRIO models is in many respects identical to sectoral aggregation. As an example for the IRIO case, we consider a highly aggregated, three-region, five-sector version of the Japanese IRIO model defined in Table A4.1.1. In the following we consider the case of aggregating this model to two regions, the first being region 1 (Central), unaggregated, of the three-region model. The second aggregated model region is to be composed by combining regions 2 (North) and 3 (South) of the three-region model. Hence, using the notation of Chapter 3 for IRIO transactions and denoting the regions of the aggregated model by a (Central) and b (North plus South), the new transactions matrix is found by (for $i, j = 1, 2, \ldots, 5$ in all cases) $z_{ij}^{aa}=z_{ij}^{11}, z_{ij}^{ab}=z_{ij}^{12}+z_{ij}^{13}, z_{ij}^{ba}=z_{ij}^{21}+z_{ij}^{31}, z_{ij}^{bb}=z_{ij}^{22}+z_{ij}^{23}+z_{ij}^{32}+z_{ij}^{33}$. Similarly, total outputs are $\mathbf{x}_i^a=\mathbf{x}_i^1$ and $\mathbf{x}_i^b=\mathbf{x}_i^2+\mathbf{x}_i^3$.

Table A4.1.1 Input Coefficients for the Five-Sector, Three-Region Interregional Input–Output Table for Japan (1965)

	Central					North					South					Total Output*
	1	3	3	4	5	1	3	3	4	5	1	3	3	4	5	
Central																
1 Agriculture	0.053	0.000	0.009	0.011	0.009	0.001	0.000	0.007	0.000	0.001	0.001	0.000	0.001	0.000	0.000	1,307
2 Mining	0.000	0.001	0.001	0.001	0.002	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	123
3 Const. & Manuf.	0.428	0.723	0.250	0.240	0.180	0.012	0.004	0.052	0.001	0.013	0.017	0.005	0.044	0.000	0.014	16,400
4 Transportation	0.000	0.001	0.010	0.090	0.012	0.000	0.000	0.002	0.015	0.001	0.000	0.000	0.001	0.007	0.001	1,342
5 Other	0.012	0.029	0.042	0.117	0.125	0.000	0.001	0.015	0.001	0.010	0.000	0.000	0.007	0.001	0.014	8,591
North																
1 Agriculture	0.004	0.000	0.000	0.000	0.000	0.089	0.001	0.017	0.039	0.021	0.002	0.000	0.000	0.000	0.000	1,308
2 Mining	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.002	0.007	0.011	0.000	0.000	0.000	0.000	0.000	201
3 Const. & Manuf.	0.068	0.041	0.020	0.000	0.002	0.362	0.521	0.160	0.233	0.129	0.034	0.028	0.012	0.000	0.001	4,167
4 Transportation	0.000	0.002	0.000	0.014	0.000	0.000	0.008	0.010	0.025	0.011	0.000	0.000	0.000	0.023	0.000	394
5 Other	0.003	0.034	0.001	0.000	0.001	0.010	0.033	0.027	0.095	0.103	0.002	0.008	0.000	0.000	0.001	2,759
South																
1 Agriculture	0.002	0.000	0.002	0.000	0.000	0.002	0.000	0.006	0.000	0.000	0.072	0.000	0.011	0.016	0.010	2,131
2 Mining	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.004	0.001	0.002	0.004	267
3 Const. & Manuf.	0.036	0.021	0.082	0.000	0.013	0.012	0.012	0.056	0.000	0.007	0.473	0.719	0.303	0.264	0.196	22,053
4 Transportation	0.000	0.000	0.001	0.024	0.000	0.000	0.000	0.001	0.022	0.000	0.000	0.003	0.009	0.068	0.012	1,546
5 Other	0.001	0.005	0.006	0.000	0.003	0.000	0.001	0.009	0.000	0.003	0.012	0.050	0.037	0.112	0.110	9,968

*Total Output is measure in billions of Japanese Yen.

Note, as noted earlier, that we can easily accomplish this spatial aggregation by constructing an aggregation matrix, **S**, as we did in the case of sectoral aggregation:

$$\mathbf{S}=\left[\begin{array}{ccccc|ccccc|ccccc}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
\hline
0&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&1
\end{array}\right]$$

We can use **S** to create $\mathbf{x}^*=\mathbf{Sx},\mathbf{Z}^*=\mathbf{SZS}'$ where $\mathbf{x}^*$ is the 10 × 1 aggregated vector of final demands (the unaggregated vector, **x**, is 15 × 1); $\mathbf{Z}^*$ is the aggregated 10 × 10 interindustry transactions matrix (the unaggregated transactions matrix, **Z**, is 15 × 15).

We can subsequently compute the new aggregated total outputs vector as $\mathbf{x}^*=\mathbf{Sx}=\left[1307\quad 123\quad 16400\quad 1342\quad 8591 \mid 3440\quad 468\quad 26220\quad 1940\quad 12727\right]'$. The new aggregated matrix of IRIO input coefficients is

$$\mathbf{A}^*=\mathbf{Z}\left(\hat{\mathbf{x}}^*\right)^{-1}=\left[\begin{array}{ccccc|ccccc}
.053&0&.009&.011&.009&.001&0&.002&0&0\\
0&.001&.001&.001&.002&0&0&0&0&0\\
.428&.723&.25&.24&.18&.015&.005&.045&0&.014\\
0&.001&.01&.09&.012&0&0&.001&.009&.001\\
.012&.029&.042&.117&.125&0&0&.008&.001&.013\\
\hline
.006&0&.002&0&0&.08&0&.013&.021&.012\\
0&0&0&0&0&.001&.004&.001&.003&.006\\
.104&.062&.102&0&.015&.456&.655&.299&.258&.184\\
0&.002&.001&.038&0&0&.005&.009&.082&.012\\
.004&.039&.007&0&.004&.012&.048&.037&.109&.110
\end{array}\right]$$

The corresponding Leontief inverse is

$$(\mathbf{I}-\mathbf{A}^*)^{-1} = \left[\begin{array}{rrrrr|rrrrr}
1.063 & .012 & .015 & .019 & .014 & .004 & .004 & .005 & .002 & .002 \\
.001 & 1.002 & .001 & .002 & .003 & 0 & .001 & .001 & 0 & 0 \\
.639 & 1.016 & 1.380 & .413 & .299 & .075 & .081 & .101 & .041 & .050 \\
.008 & .013 & .016 & 1.107 & .019 & .002 & .002 & .003 & .012 & .002 \\
.050 & .088 & .071 & .170 & 1.161 & .012 & .016 & .021 & .011 & .023 \\
\hline
.013 & .008 & .007 & .004 & .002 & 1.099 & .018 & .023 & .033 & .021 \\
.001 & .001 & .001 & 0 & 0 & .003 & 1.007 & .003 & .005 & .007 \\
.267 & .267 & .217 & .092 & .076 & .754 & 1.050 & 1.480 & .477 & .335 \\
.005 & .009 & .006 & .049 & .003 & .009 & .018 & .017 & 1.098 & .018 \\
.021 & .064 & .020 & .015 & .010 & .049 & .105 & .065 & .155 & 1.140
\end{array}\right]$$

Let us now compute the aggregation bias introduced by grouping regions 2 and 3. Consider the following vector of final demands for the unaggregated (three-region, five-sector) model of $\tilde{\mathbf{f}} = [100 \quad 100 \quad \cdots \quad 100]'$. The corresponding aggregated (two-region, five-sector) version is

$$\tilde{\mathbf{f}}^* = [100 \quad 100 \quad 100 \quad 100 \quad 100 \mid 200 \quad 200 \quad 200 \quad 200 \quad 200]'$$

We can compute $\tilde{\mathbf{x}}^* = (\mathbf{I}-\mathbf{A}^*)^{-1}\tilde{\mathbf{f}}^*$ and $\mathbf{x}^* = (\tilde{\mathbf{I}}-\mathbf{A}^*)^{-1}\tilde{\mathbf{f}}$ where $\mathbf{A}$ is the original unaggregated technical coefficients matrix. In order to compare $\tilde{\mathbf{x}}^*$ and $\tilde{\mathbf{x}}$, we must aggregate $\tilde{\mathbf{x}}$, which can be accomplished with the sectoral aggregation matrix, **S**, given earlier, that is, $\mathbf{S}\tilde{\mathbf{x}}$. Table A4.1.2 gives the vectors $\mathbf{S}\tilde{\mathbf{x}}$, $\tilde{\mathbf{x}}^*$ and the differences between the corresponding elements. The sum of absolute differences between $\mathbf{S}\tilde{\mathbf{x}}$ and $\mathbf{x}^*$ for the unaggregated region *a* (Central) as a percentage of the total outputs in that region, $\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}$, is $100\left(\frac{|\mathbf{S}\tilde{\mathbf{x}}-\tilde{\mathbf{x}}^*|\mathbf{i}}{\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}}\right) = 100\left(\frac{3.768}{954.792}\right) = 0.395\%$ and the corresponding value for region *b* (North and South) is $100\left(\frac{73.319}{1851.735}\right) = 3.959\%$. This indicates, not surprisingly, that more error is introduced into the prediction of outputs in the aggregated region than in the unaggregated region. The overall error (for both regions) is $100\left(\frac{77.087}{2806.527}\right) = 2.747\%$.

Table A4.1.2 Spatial Aggregation of IRIO Models: Results for Japanese IRIO Table

		Aggregated Gross Outputs from the Three-Region Model $\mathbf{S\tilde{x}}$	Outputs from the Aggregated Two-Region Model $\tilde{\mathbf{x}}^*$	Aggregation Error $\mathbf{S\tilde{x}} - \tilde{\mathbf{x}}^*$	Aggregation Error as a Percent of Gross Outputs of the Three-Region Model $100\left(\frac{\lvert \mathbf{S\tilde{x}} - \tilde{\mathbf{x}}^* \rvert}{\mathbf{S\tilde{x}i}}\right)$
Region *a*	Sector				
	1	116.801	115.749	1.052	.901
	2	101.649	101.394	.255	.251
	3	443.529	444.330	−.801	−.181
	4	121.260	120.363	.896	.739
	5	171.553	170.789	.764	.446
Region *a* Total Absolute)		954.792	952.625	3.768	
Region *b*	Sector				
	1	246.876	242.116	4.769	1.928
	2	206.519	205.343	1.176	.570
	3	853.242	911.145	−57.904	−6.786
	4	235.381	238.800	−3.418	−1.452
	5	309.717	315.778	−6.061	−1.957
Region *b* Total (Absolute)		1851.735	1913.182	73.319	
Total (Absolute)		2806.527	2865.807	77.087	

Notice from the table that the aggregation bias is quite small in all three calculations, that is, region *a*, region *b*, and overall, particularly in the unaggregated region. Miller and Blair (1981) show that spatial aggregation of IRIO models generally seems to introduce only modest bias. This suggests, for example, that if one is interested in the impacts in one region in an interconnected interregional system of a change in final demands for some of the sectors in that region (e.g., effects on the California economy of new federal spending in California, which is one of the interconnected 48 continental states), then a "two-region" model of California and the rest of the United States may be sufficient.

A4.1.4.2 Spatial Aggregation of MRIO Models

Consider a highly aggregated (three-region, five-sector) MRIO input–output model of the United States given in Table A4.1.3. We consider the case of aggregating regions 2 (Central) and 3 (West) of the basic three-region model, leaving region 1 (East) unaggregated. We designate the regions in the aggregated model by superscripts a (East) and b (Central plus West) so that the new intraregional flow matrices are found by (for i, j = 1,2, … 5 in all cases) the following: $z_{ij}^a = z_{ij}^1, z_{ij}^b = z_{ij}^2 + z_{ij}^3$. Similarly, total regional outputs are $x_i^a = x_i^1, x_i^b = x_i^2 + x_i^3$. Hence, the input coefficients for the aggregated model are found by $a_{ij}^a = \frac{z_{ij}^a}{x_j^a}, a_{ij}^b = \frac{z_{ij}^b}{x_j^b}$.

Table A4.1.3 Five-Sector, Three-Region Multiregional Input–Output Tables for the United States (1963)

Regional Transactions (millions of dollars)

	Agric	Mining	Const. & Manuf.	Services	Transport & Utilities
East					
Agriculture	2,013	0	7,863	44	0
Mining	35	335	3,432	44	843
Const. & Manuf.	2,029	400	78,164	11,561	2,333
Services	1,289	294	19,699	26,574	2,301
Transport.& Util.	225	384	7,232	4,026	3,534
Central					
Agriculture	10,303	0	13,218	97	0
Mining	82	472	8,686	15	1,271
Const. & Manuf.	4,422	1,132	93,816	10,155	2,401
Services	4,952	2,378	21,974	22,358	2,473
Transport.& Util.	667	406	9,296	3,468	4,513
West					
Agriculture	2,915	0	3,452	65	0
Mining	4	292	2,503	0	353
Const. & Manuf.	1,214	466	27,681	4,925	1,015
Services	1,307	721	8,336	10,809	991
Transport.& Util.	338	160	2,936	1,659	1,576

Commodity Trade Flows and Total Outputs (millions of dollars)

	East	West	Central
Agriculture			
East	6,007	2,124	208
West	3,845	28,885	2,521
Central	403	2,922	7,028
Mining			
East	2,904	415	53
West	1,108	10,942	271
Central	71	772	3,996
Const. & Manuf.			
East	158,679	42,150	8,368
West	44,589	201,025	11,778
Central	4,702	6,726	61,385
Services			
East	146,336	16,116	2,955
West	9,328	121,079	3,185
Central	1,939	3,643	58,663
Transp. & Util.			
East	21,434	4,974	263
West	4,396	23,811	1,948
Central	1,009	1,334	9,635
Total Output			
Agriculture	10,259	33,939	9,753
Mining	4,084	12,129	4,319
Const. & Manuf.	207,948	249,840	81,512
Services	157,468	140,850	64,803
Transport.& Util.	26,847	30,130	11,841

The resulting block diagonal aggregated technical coefficients matrix, which we denote $\mathbf{A}^*$, is given by

$$\mathbf{A}^* = \left[\begin{array}{ccccc:ccccc} .082 & .003 & .012 & .005 & .61 & 0 & 0 & 0 & 0 & 0 \\ 0 & .196 & .043 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .156 & .211 & .302 & .076 & .110 & 0 & 0 & 0 & 0 & 0 \\ .096 & .133 & .131 & .220 & .101 & 0 & 0 & 0 & 0 & 0 \\ .012 & .001 & .061 & .002 & .234 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & .046 & .002 & .030 & .007 & .075 \\ 0 & 0 & 0 & 0 & 0 & 0 & .302 & .057 & .001 & 0 \\ 0 & 0 & 0 & 0 & 0 & .103 & .143 & .281 & .075 & .115 \\ 0 & 0 & 0 & 0 & 0 & .207 & .151 & .127 & .216 & .101 \\ 0 & 0 & 0 & 0 & 0 & .010 & .001 & .075 & .002 & .230 \end{array}\right]$$

The *interregional* commodity flow matrices for the original unaggregated model are $z_i = z_i^{rs}$ for r, s = 1, 2, 3 regions and i = 1, 2, ... 5 sectors, a total of five 3 × 3 matrices. Aggregation from three to two regions for the commodity flows can be accomplished by constructing a spatial aggregation matrix **R**, as in the case of sectoral aggregation; for this example $\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.
We define **R** to be distinct from the sectoral aggregation this matrix, **S**, defined earlier. The aggregated (2 × 2) interregional flow matrices, $\mathbf{Z}_i^*$, are found by $\mathbf{Z}_i^* = \mathbf{R}\mathbf{Z}_i\mathbf{R}'$ for i = 1,2, ... 5 industries.

We can then construct the aggregated trade coefficients $c_i^{ab} = \dfrac{z_i^{ab}}{T_i^b}$. The matrix of trade coefficients for the aggregated MRIO model, **C***, is

$$\mathbf{C}^* = \begin{bmatrix} \hat{\mathbf{c}}^{aa} & \hat{\mathbf{c}}^{ab} \\ \hat{\mathbf{c}}^{ba} & \hat{\mathbf{c}}^{bb} \end{bmatrix} = \left[\begin{array}{ccccc:ccccc} .621 & 0 & 0 & 0 & 0 & .047 & 0 & 0 & 0 & 0 \\ 0 & .586 & 0 & 0 & 0 & 0 & .053 & 0 & 0 & 0 \\ 0 & 0 & .738 & 0 & 0 & 0 & 0 & .144 & 0 & 0 \\ 0 & 0 & 0 & .824 & 0 & 0 & 0 & 0 & .121 & 0 \\ 0 & 0 & 0 & 0 & .721 & 0 & 0 & 0 & 0 & .157 \\ \hdashline .379 & 0 & 0 & 0 & 0 & .953 & 0 & 0 & 0 & 0 \\ 0 & .414 & 0 & 0 & 0 & 0 & .947 & 0 & 0 & 0 \\ 0 & 0 & .262 & 0 & 0 & 0 & 0 & .856 & 0 & 0 \\ 0 & 0 & 0 & .176 & 0 & 0 & 0 & 0 & .879 & 0 \\ 0 & 0 & 0 & 0 & .279 & 0 & 0 & 0 & 0 & .843 \end{array}\right]$$

The corresponding matrix of MRIO multipliers is

$$
\left(\mathbf{I}-\mathbf{C}^{*}\mathbf{A}^{*}\right)^{-1}\mathbf{C}^{*}=\left[\begin{array}{ccccc|ccccc}
.658 & .004 & .012 & .005 & .039 & .053 & .002 & .006 & .006 & .015 \\
.004 & .680 & .032 & .003 & .004 & .002 & .088 & .014 & .002 & .003 \\
.124 & .180 & 1.007 & .084 & .129 & .045 & .078 & .271 & .037 & .071 \\
.103 & .142 & .161 & 1.031 & .127 & .055 & .068 & .077 & .193 & .065 \\
.017 & .015 & .063 & .008 & .895 & .008 & .010 & .036 & .005 & .243 \\
\hline
.425 & .013 & .028 & .007 & .061 & 1.008 & .051 & .048 & .012 & .095 \\
.013 & .678 & .066 & .008 & .014 & .013 & 1.358 & .1 & .01 & .017 \\
.118 & .202 & .493 & .064 & .128 & .153 & .264 & 1.213 & .109 & .189 \\
.138 & .176 & .131 & .281 & .111 & .237 & .274 & .218 & 1.115 & .18 \\
.021 & .022 & .066 & .009 & .433 & .025 & .025 & .105 & .013 & 1.083
\end{array}\right]
$$

We now compute the aggregation bias introduced by this spatial consolidation. Consider the following 15-element vector of hypothesized final demands for the unaggregated (three-region, five-sector) model $\tilde{\mathbf{f}}=\begin{bmatrix}100 & 100 & \ldots & 100\end{bmatrix}'$. The corresponding aggregated (two-region, five-sector) version is $\tilde{\mathbf{f}}^{*}=\left[\begin{array}{ccccc|ccccc}100 & 100 & 100 & 100 & 100 & 200 & 200 & 200 & 200 & 200\end{array}\right]'$.

We can compute $\tilde{\mathbf{x}}^{*}=(\mathbf{I}-\mathbf{C}^{*}\mathbf{A}^{*})^{-1}\mathbf{C}^{*}\tilde{\mathbf{f}}^{*}$ and $\tilde{\mathbf{x}}=(\mathbf{I}-\mathbf{C}\mathbf{A})^{-1}\mathbf{C}\tilde{\mathbf{f}}$ where **A** and **C** are from the original unaggregated model. In order to compare $\tilde{\mathbf{x}}^{*}$ and $\tilde{\mathbf{x}}$, we must aggregate $\tilde{\mathbf{x}}$, which, as noted earlier, we can accomplish by using a sectoral aggregation matrix, **S**, to compute:

$$
\mathbf{S}\tilde{\mathbf{x}}=\left[\begin{array}{ccccc|ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
117 \\ 127 \\ 254 \\ 240 \\ 155 \\ \hline 139 \\ 192 \\ 293 \\ 277 \\ 175 \\ \hline 126 \\ 145 \\ 191 \\ 219 \\ 135
\end{array}\right]
$$

Table A4.1.4 gives the vectors $\tilde{\mathbf{x}}^{*}$, $\mathbf{S}\,\tilde{\mathbf{x}}$, and the differences between corresponding elements. The sum of absolute differences between $\mathbf{S}\,\tilde{\mathbf{x}}$ and $\tilde{\mathbf{x}}^{*}$ for the unaggregated region *a* (East) as a percentage of the total outputs in that region, that is, $\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}$, is $100\left(\dfrac{\left|\mathbf{S}\tilde{\mathbf{x}}-\tilde{\mathbf{x}}^{*}\right|\mathbf{i}}{\mathbf{S}\tilde{\mathbf{x}}\mathbf{i}}\right)=100\left(\dfrac{8.352}{954.679}\right)=0.883\%$ and the corresponding value for region *b* (Central plus West) is $100\left(\dfrac{41.625}{1854.456}\right)=2.245\%$. This indicates, as in the IRIO case, that more error is introduced into the prediction of outputs in the aggregated region than in the unaggregated region. The overall error (for both regions) is $100\left(\dfrac{49.977}{2800.135}\right)=1.785\%$.

Table A4.1.4 Spatial Aggregation of MRIO Models: Results for US MRIO Model

		Aggregated Gross Outputs from the Three-Region Model $\mathbf{S\tilde{x}}$	Outputs from Aggregated Two-Region Model $\tilde{\mathbf{x}}^*$	Aggregation Error $\mathbf{S\tilde{x}}-\tilde{\mathbf{x}}^*$	Aggregation Error as a Percent of Gross Outputs of the Three-Region Model $100\left(\dfrac{\lvert \mathbf{S\tilde{x}}-\tilde{\mathbf{x}}^*\rvert}{\mathbf{S\tilde{x}}}\right)$
Region *a*	Sector				
	1	131.718	135.265	.547	.405
	2	109.863	110.036	.173	.157
	3	352.078	358.354	6.276	1.751
	4	133.305	134.171	.866	.645
	5	215.715	216.205	.490	.226
Region a Total (Absolute)		954.679	954.031	8.352	
Region *b*	Sector				
	1	311.061	318.149	7.088	2.228
	2	229.036	229.359	.324	.141
	3	658.678	633.958	−24.720	−3.899
	4	262.909	257.744	−5.164	−2.004
	5	392.772	388.443	−4.329	−1.115
Region *b* Total (Absolute)		1854.456	1827.653	41.625	
Total (Absolute)		2800.135	2781.684	49.977	

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