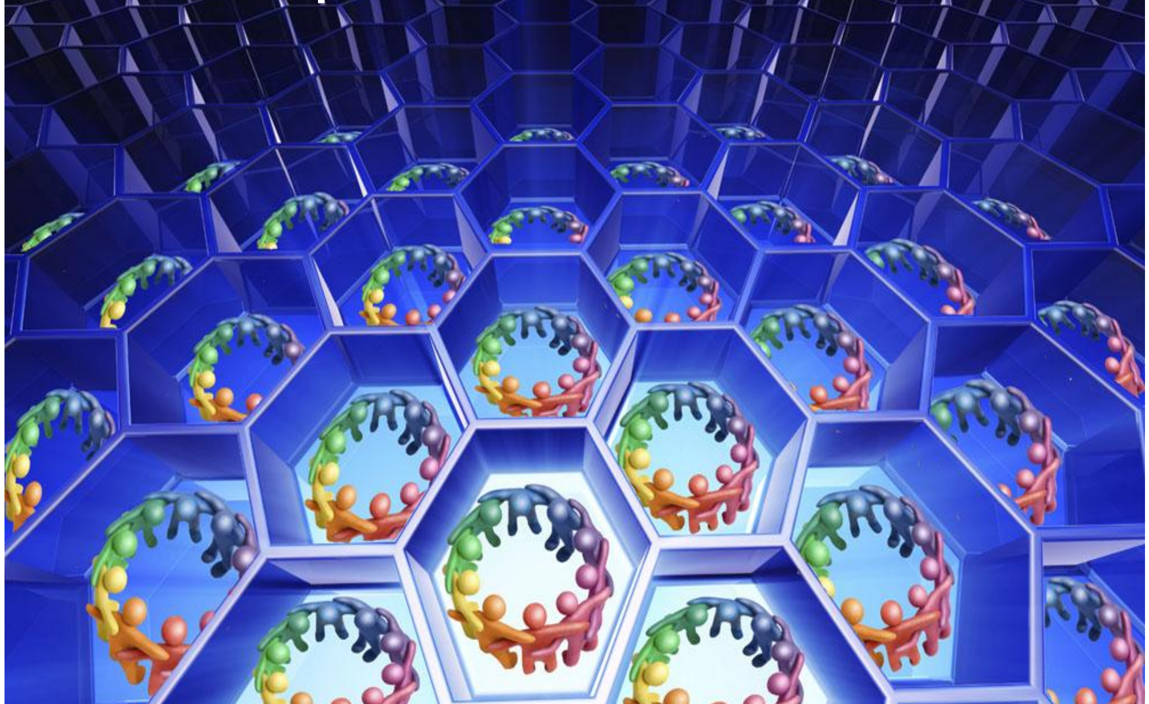
Cooperative Communications and Networking

Chapter 9

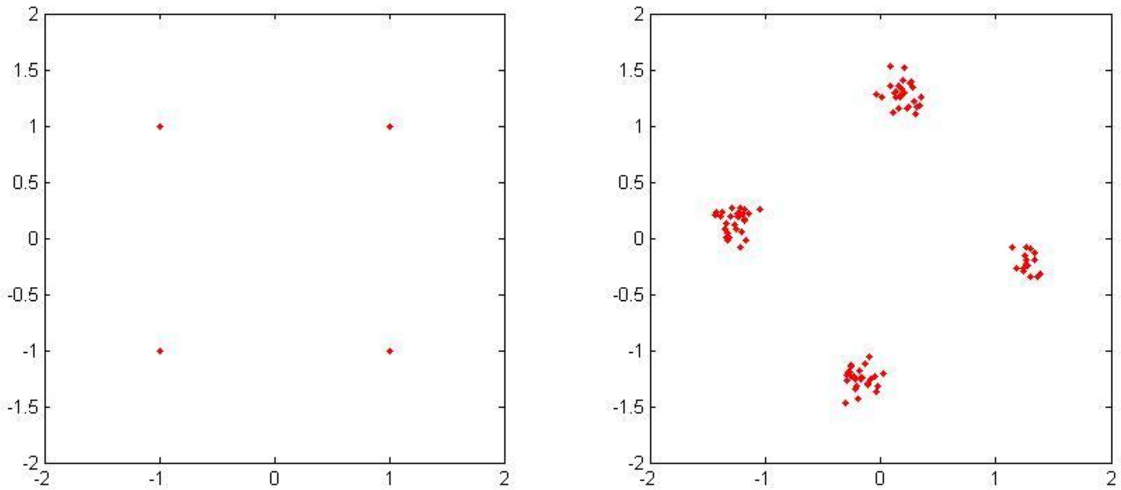
Differential Modulation for Cooperative Communications



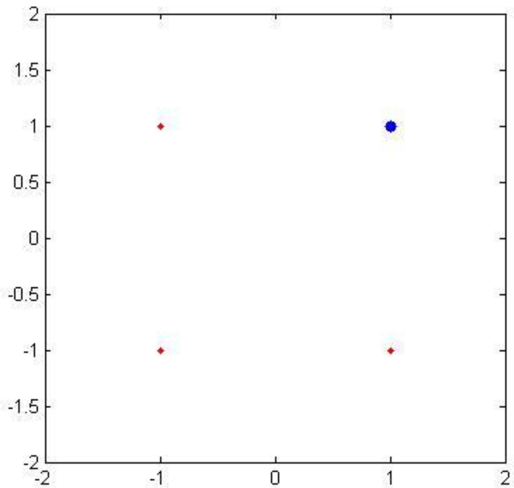
Motivation

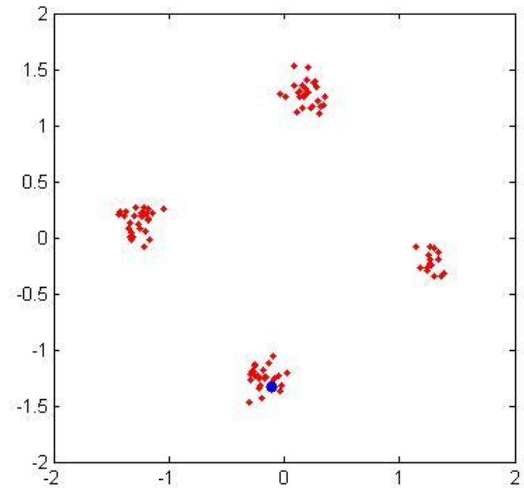
- The schemes we have seen so far in this seminar are all based on the assumption of perfect channel state information
- This requires channel estimation, but…
  - *Add complexity to the receiver*
  - *Add overhead (pilot/ training seq), waste resources*
  - *Not practical in some cases*
- Possible solutions
  - *Blind detection*
  - *Differential modulation*

Differential Modulation

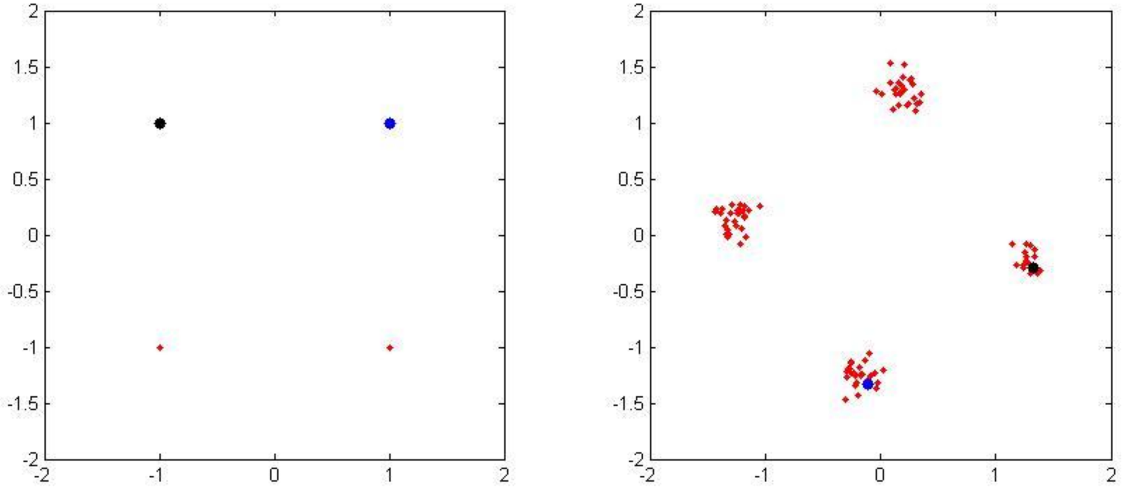


Differential Modulation

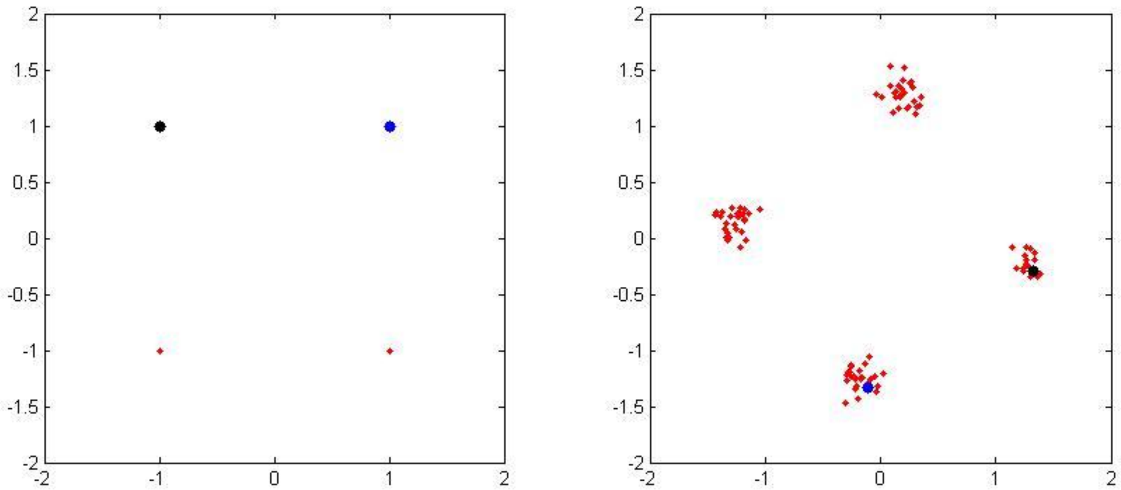




Differential Modulation



Differential Modulation



- Assumption: channels remains constant in a short period
- Differential modulation: information is conveyed in the difference between two consecutive symbols
- Non-coherent demodulation: without channel information

Model

- Information to transmit $v_m \in \Omega$

$$\Omega = \{e^{j2\pi m/M}, m = 0, 1, \ldots, M-1\} \text{ for DMPSK}$$

- Differential modulation $s^\tau = v_m s^{\tau-1}$
- Transmission under the channel $h^{\tau-1} = h^\tau$

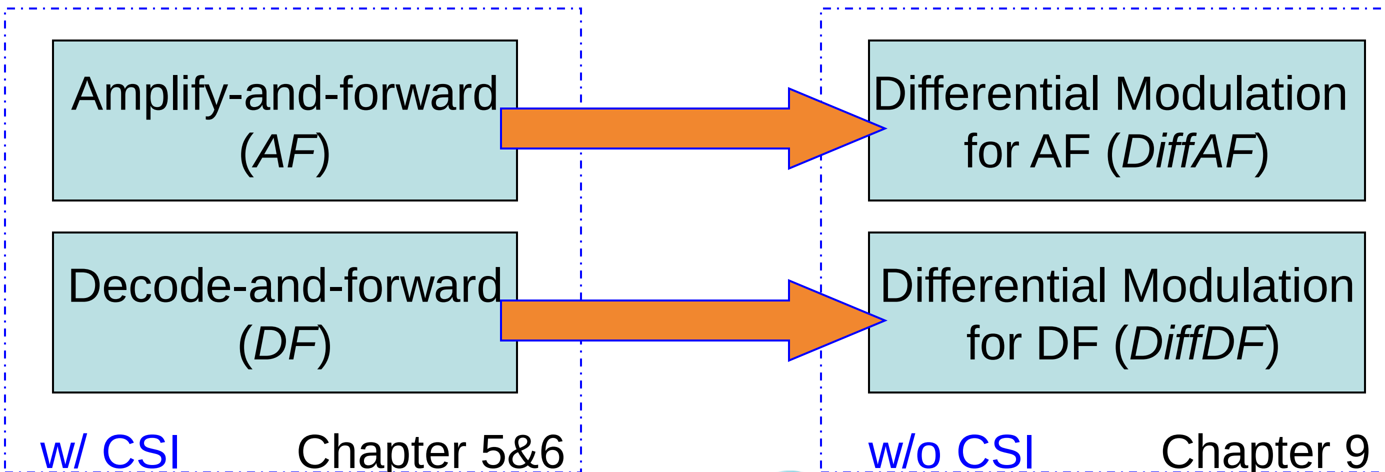
$$\begin{aligned} y^\tau &= \sqrt{\rho} h^\tau s^\tau + w^\tau \\ y^{\tau-1} &= \sqrt{\rho} h^{\tau-1} s^{\tau-1} + w^{\tau-1} \end{aligned} \Longrightarrow \begin{aligned} y^\tau &= y^{\tau-1} v_m - v_m w^{\tau-1} + w^\tau \\ &= y^{\tau-1} v_m + \underbrace{\sqrt{2}}_{\text{3dB loss!}} \tilde{w}^\tau, \end{aligned}$$

- Maximum-likelihood detection

$$\begin{aligned} \hat{m} &= \arg \min_{m \in 0,1,\ldots M-1} |y^\tau - v_m y^{\tau-1}|^2 \\ &= \arg \max_{m \in 0,1,\ldots,M-1} Re\{\underbrace{v_m^*}_{\text{match}} \underbrace{\left((y^{\tau-1})^* y^\tau\right)}_{\text{difference}}\} \end{aligned}$$

Research Methodology

- Relationship



- Similarity: what to explore?
  - *Model/scheme, BER, bounds, diversity order, optimal power allocation, from single relay to multiple, …*
- Difference: where comes the challenge?

Outline

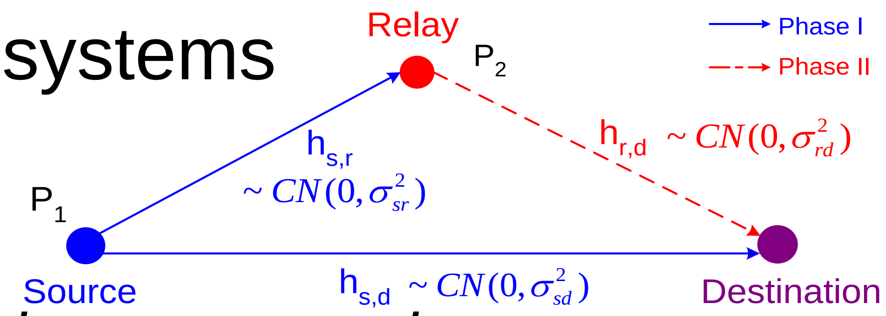
- Motivation
- DiffAF
- DiffDF
- Conclusions

Outline

- Motivation
- **DiffAF**
  - ***Protocol***
  - ***BER & bounds***
  - ***Optimal power allocation***
  - ***Simulation results***
  - ***Extend to the multi-relay case***
- DiffDF
- Conclusions

DiffAF protocol

- Single relay cooperative systems
  - *Two-phase transmission*
  - *Rayleigh fading channels*
  - *Differential modulation at the source node*



- Signal models
  - *Phase 1*

$$y^{\tau}_{s,d} = \sqrt{P_1} h^{\tau}_{s,d} x^{\tau} + w^{\tau}_{s,d},$$
$$y^{\tau}_{s,r} = \sqrt{P_1} h^{\tau}_{s,r} x^{\tau} + w^{\tau}_{s,r}.$$

$w^{\tau}_{s,d}$ and $w^{\tau}_{s,r}$ are $\mathcal{CN}(0, \mathcal{N}_0)$

  - *Phase 2*

$$y^{\tau}_{r,d} = \sqrt{\frac{P_2}{P_1 \boxed{\sigma^2_{s,r}} + \mathcal{N}o}} h^{\tau}_{r,d} y^{\tau}_{s,r} + w^{\tau}_{r,d}$$

$|h_{s,r}|^2$ (Coherent)

DiffAF protocol

- The receiver combines signals from two links

$$y = a_1 \left(y_{s,d}^{\tau-1}\right)^* y_{s,d}^{\tau} + a_2 \left(y_{r,d}^{\tau-1}\right)^* y_{r,d}^{\tau},$$

with weights $a_1 = \frac{1}{\mathcal{N}_0}, \quad a_2 = \frac{P_1\sigma_{s,r}^2 + \mathcal{N}_0}{\mathcal{N}_0(P_1\sigma_{s,r}^2 + P_2\sigma_{r,d}^2 + \mathcal{N}_0)}.$

- Decoder:

$$\hat{m} = \arg \max_{m\ =\ 0,1,\ldots,M-1} \mathrm{Re}\left\{v_m^* y\right\}.$$

- Challenge:
  - *The optimal weights are* $\hat{a}_1 = \frac{1}{\mathcal{N}_0}, \quad \hat{a}_2 = \frac{P_1\sigma_{s,r}^2 + \mathcal{N}_0}{\mathcal{N}_0(P_1\sigma_{s,r}^2 + P_2\boxed{|h_{r,d}|^2} + \mathcal{N}_0)}.$
    but NOT practical because channel is unknown
  - *Results with other weights are not available because* $\gamma \neq \gamma_1 + \gamma_2$

BER analysis

- Solutions:
  - *Use the optimal weights as a performance benchmark*
  - *See if the gap is significant through simulations*
- Conditional BER on channel realizations:

$$P_{b|\gamma} = \frac{1}{16\pi} \int_{-\pi}^{\pi} f(\theta) \exp\left[-\alpha(\theta)\gamma\right] d\theta$$

where

$$f(\theta) = \frac{b^2(1-\beta^2)\left[3+\cos(2\theta)-(\beta+1/\beta)\sin\theta\right]}{2\alpha(\theta)}$$

$$\alpha(\theta) = \frac{b^2(1+2\beta\sin\theta+\beta^2)}{2} \qquad \beta = a/b$$

$$\gamma = \gamma_1 + \gamma_2 = \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} + \frac{P_1 P_2 |h_{s,r}|^2 |h_{r,d}|^2}{\mathcal{N}_0 \left(P_1 \sigma_{s,r}^2 + P_2 |h_{r,d}|^2 + \mathcal{N}_0\right)}$$

For example,

DBPSK $a = 10^{-3}$ $b = \sqrt{2}$

DQPSK $a = \sqrt{2-\sqrt{2}}$ $b = \sqrt{2+\sqrt{2}}$

BER analysis

- Exact BER formulation
  - *average over all channel realizations*

$$\boxed{P_b = \frac{1}{16\pi}\int_{-\pi}^{\pi} f(\theta)\mathcal{M}_{\gamma_1}(\theta)\,\mathcal{M}_{\gamma_2}(\theta)\,d\theta}$$

  - *where*

$$\mathcal{M}_{\gamma_1}(\theta) = \frac{1}{1+k_{s,d}(\theta)}$$

$$\mathcal{M}_{\gamma_2}(\theta) = \frac{1}{1+k_{s,r}(\theta)}\left(1+\frac{k_{s,r}(\theta)}{1+k_{s,r}(\theta)}\frac{P_1\sigma_{s,r}^2+\mathcal{N}_o}{P_2}\frac{1}{\sigma_{r,d}^2}\int_0^\infty \frac{\exp\left(-u/\sigma_{r,d}^2\right)}{u+R(\theta)}du\right)$$

$$k_{s,d}(\theta) \triangleq \alpha(\theta)P_1\sigma_{s,d}^2/\mathcal{N}_0$$

$$k_{s,r}(\theta) \triangleq \alpha(\theta)P_1\sigma_{s,r}^2/\mathcal{N}_0$$

$$R(\theta) \triangleq \frac{P_1\sigma_{s,r}^2+\mathcal{N}_o}{P_2\left(1+k_{s,r}(\theta)\right)}$$

BER Bounds

- Double integration: difficult to calculate

$$P_b = \frac{1}{16\pi} \int_{-\pi}^{\pi} f(\theta) \mathcal{M}_{\gamma_1}(\theta) \mathcal{M}_{\gamma_2}(\theta)\, d\theta$$

$$\mathcal{M}_{\gamma_2}(\theta) = \frac{1}{1+k_{s,r}(\theta)} \left(1 + \frac{k_{s,r}(\theta)}{1+k_{s,r}(\theta)} \frac{P_1 \sigma_{s,r}^2 + \mathcal{N}_o}{P_2} \frac{1}{\sigma_{r,d}^2} \underline{\int_0^{\infty} \frac{\exp\left(-u/\sigma_{r,d}^2\right)}{u + R(\theta)} du} \right)$$

- Bound the integration by constants

$$R(\theta) \geq \frac{P_1 \sigma_{s,r}^2 + \mathcal{N}_o}{P_2} \left[1 - \frac{P_1 \sigma_{s,r}^2 b^2 (1+\beta)^2}{2\mathcal{N}_0} \right]^{-1} \triangleq R_{min}$$

$$R(\theta) \leq \frac{P_1 \sigma_{s,r}^2 + \mathcal{N}_o}{P_2} \left[1 + \frac{P_1 \sigma_{s,r}^2 b^2 (1-\beta)^2}{2\mathcal{N}_0} \right]^{-1} \triangleq R_{max}$$

$$Z_{min} \leq \int_0^{\infty} \frac{\exp\left(-u/\sigma_{r,d}^2\right)}{u + R(\theta)} du \leq Z_{max}$$

$$Z_{max} = \int_0^{\infty} \exp(-u/\sigma_{r,d}^2) \left[u + R_{min}\right]^{-1} du \quad Z_{min} = \int_0^{\infty} \exp(-u/\sigma_{r,d}^2) \left[u + R_{max}\right]^{-1} du$$

BER Lower Bound

- Replace the integration by its lower bound,

$$P_b \geq \frac{1}{16\pi} \int_{-\pi}^{\pi} f(\theta) \frac{1}{[1 + k_{s,d}(\theta)][1 + k_{s,r}(\theta)]} \left(1 + \frac{P_1 \sigma_{s,r}^2 + 1}{P_2 \sigma_{r,d}^2} \frac{k_{s,r}(\theta)}{1 + k_{s,r}(\theta)} Z_{min} \right) d\theta$$

- For high SNR, we ignore all 1's in the denominator to obtain an approximate bound

$$P_b \geq \frac{\left(P_1 \sigma_{s,r}^2 + 1\right) Z_{min} + P_2 \sigma_{r,d}^2}{P_1^2 P_2 \sigma_{s,d}^2 \sigma_{s,r}^2 \sigma_{r,d}^2} \mathcal{N}_0^2 C\left(\beta\right)$$

where

$$C\left(\beta\right) = \frac{1}{8\pi b^4} \int_{-\pi}^{\pi} \frac{(1 - \beta^2)[3 + \cos(2\theta) - (\beta + 1/\beta)\sin\theta]}{(1 + 2\beta \sin\theta + \beta^2)^3} d\theta$$

BER Upper Bound

- Similarly,

$$P_b \le \frac{1}{16\pi}\int_{-\pi}^{\pi} F(\theta)\frac{1}{[1+k_{s,d}(\theta)][1+k_{s,r}(\theta)]}\left(1+\frac{P_1\sigma_{s,r}^2+1}{P_2\sigma_{r,d}^2}\frac{k_{s,r}(\theta)}{1+k_{s,r}(\theta)}Z_{max}\right)d\theta,$$

$$P_b \le \frac{\left(P_1\sigma_{s,r}^2+1\right)Z_{max}+P_2\sigma_{r,d}^2}{P_1^2P_2\sigma_{s,d}^2\sigma_{s,r}^2\sigma_{r,d}^2}\mathcal{N}_0^2 C(\beta)$$ for high SNR

- Diversity order
  - *Let* $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$
  - *High SNR, P>>1*

$$P_b \le \frac{\alpha\sigma_{s,r}^2 Z_{max}+(1-\alpha)\sigma_{r,d}^2}{\alpha^2(1-\alpha)\sigma_{s,d}^2\sigma_{s,r}^2\sigma_{r,d}^2}\left(\frac{P}{\mathcal{N}_0}\right)^{-2}C(\beta)$$

Optimal power allocation

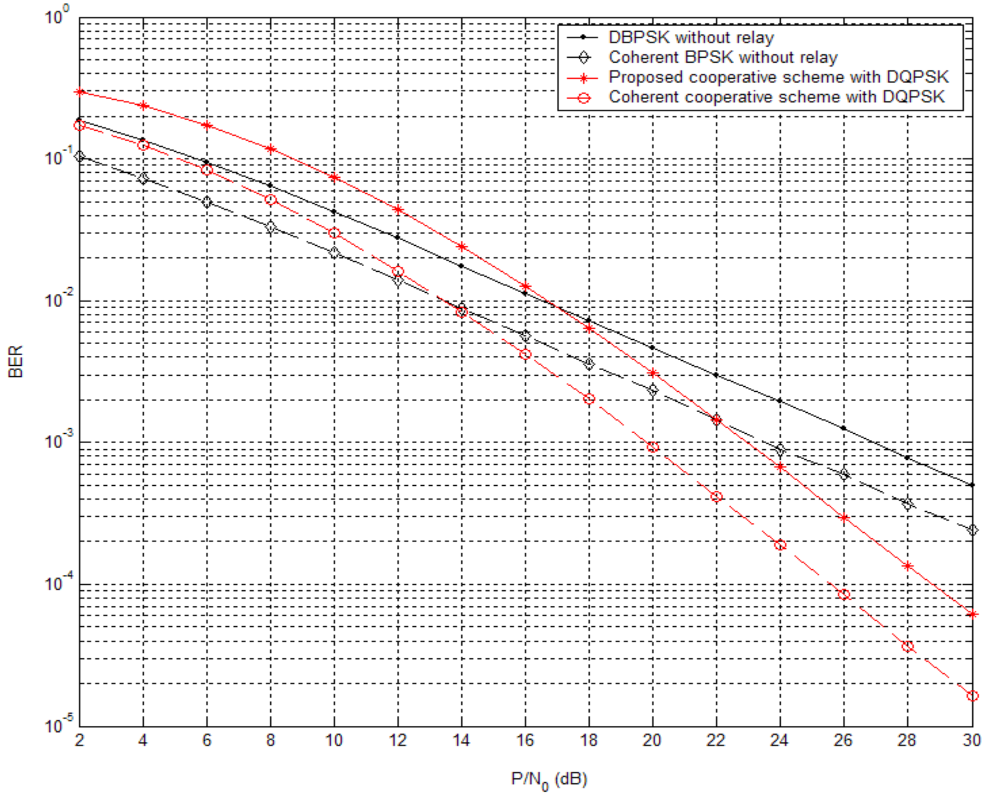
- Numerical: exhaustive search through simulation
- Analytical: derive the optimal values
  - *Based on the approximated lower bound (very tight, shown by simulation results later)*
  - *Optimization problem*

$$\boxed{\begin{aligned}&\arg\min_{P1,P2} \frac{\left(P_1\sigma_{s,r}^2+1\right)Z_{min}+P_2\sigma_{r,d}^2}{P_1^2P_2\sigma_{s,d}^2\sigma_{s,r}^2\sigma_{r,d}^2}\mathcal{N}_0^2C\left(\beta\right)\\ &\text{s. t. } P_1+P_2\le P, P_1\ge 0, P_2\ge 0\end{aligned}}$$

  - *Solvable by the Lagrangian method*

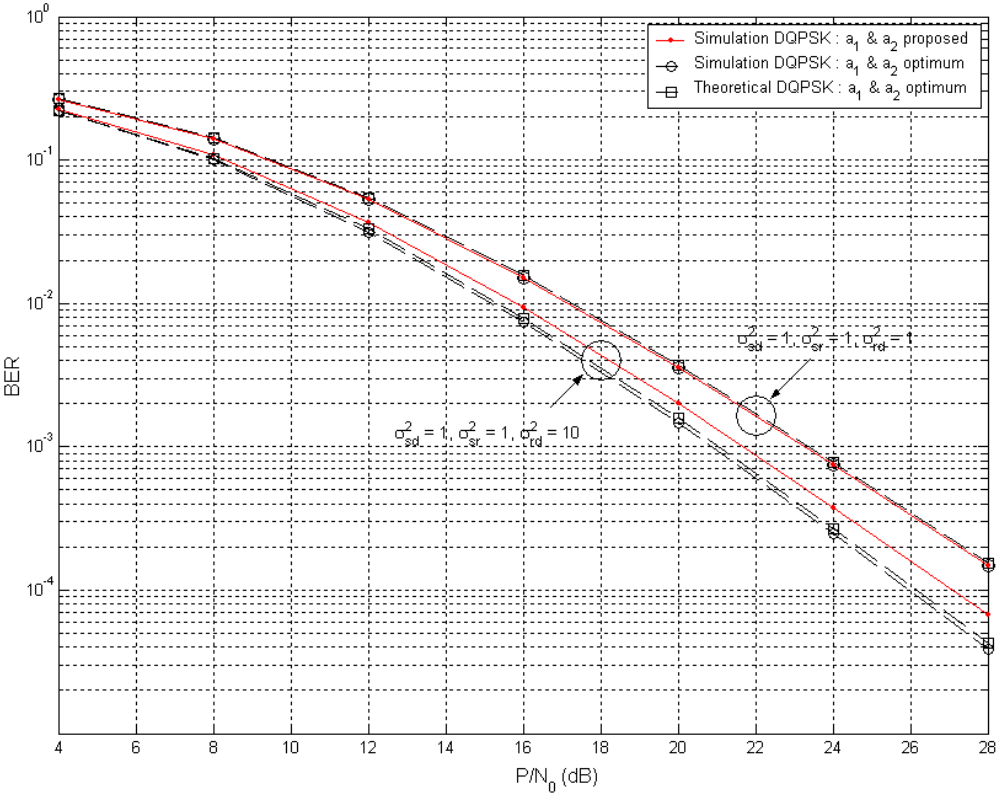
Simulation (1)

- Cooperative vs. Non-cooperative, Coherent vs. Differential modulation



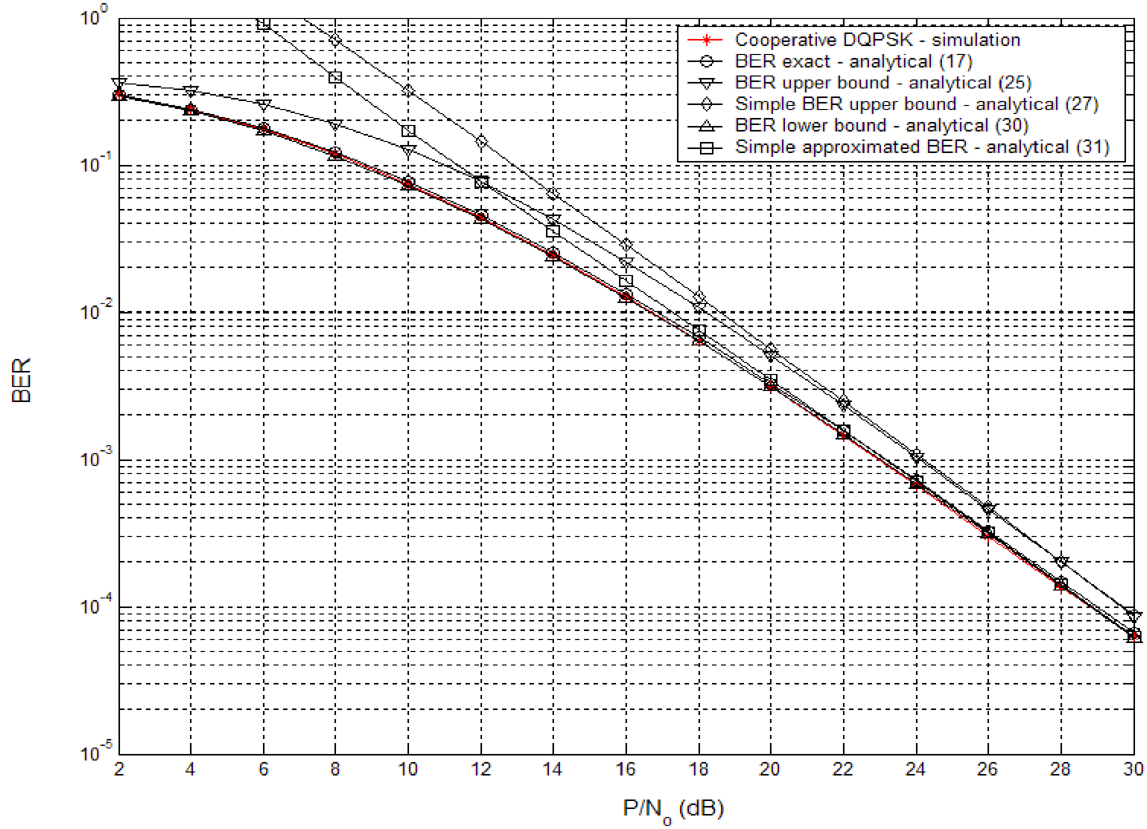
Simulation (2)

- Optimal weights vs. Practical weights



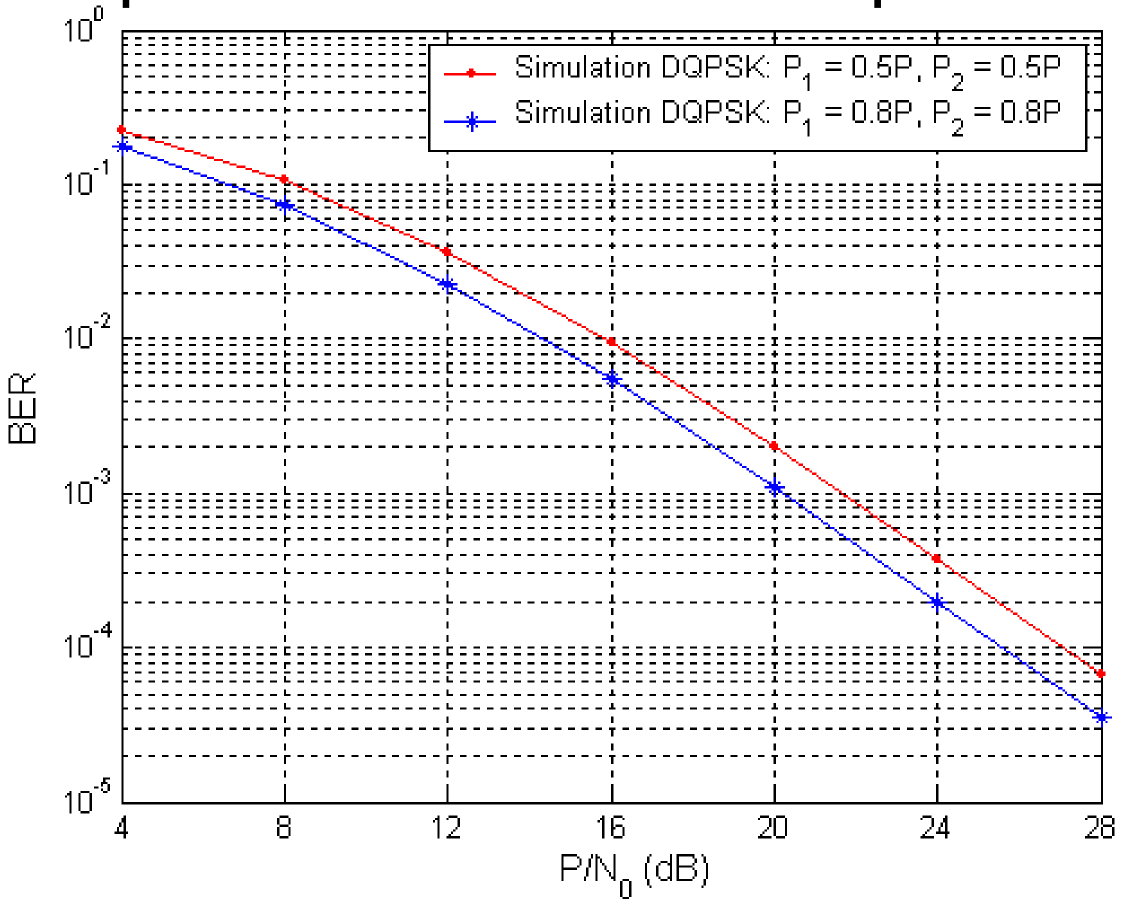
Simulation (3)

- BER: exact BER and all the bounds



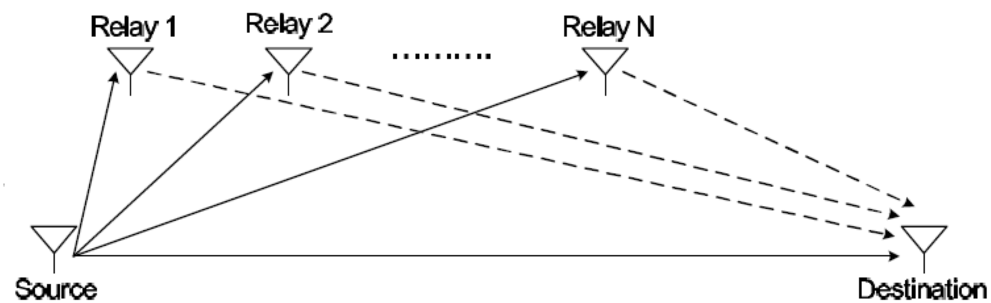
Simulation (4)

- Optimal power allocation vs. Equal allocation



Multi-relay DiffAF

$$y^{\tau}_{r_i,d} = \frac{\sqrt{P_i}}{\sqrt{P_s\sigma^2_{s,r_i} + \mathcal{N}_0}} h^{\tau}_{r_i,d} y^{\tau}_{s,r_i} + \eta^{\tau}_{r_i,d}$$



$$y^{AF} = w^{AF}_s \left(y^{\tau-1}_{s,d}\right)^* y^{\tau}_{s,d} + \sum_{i=1}^{N} w^{AF}_i \left(y^{\tau-1}_{r_i,d}\right)^* y^{\tau}_{r_i,d}$$

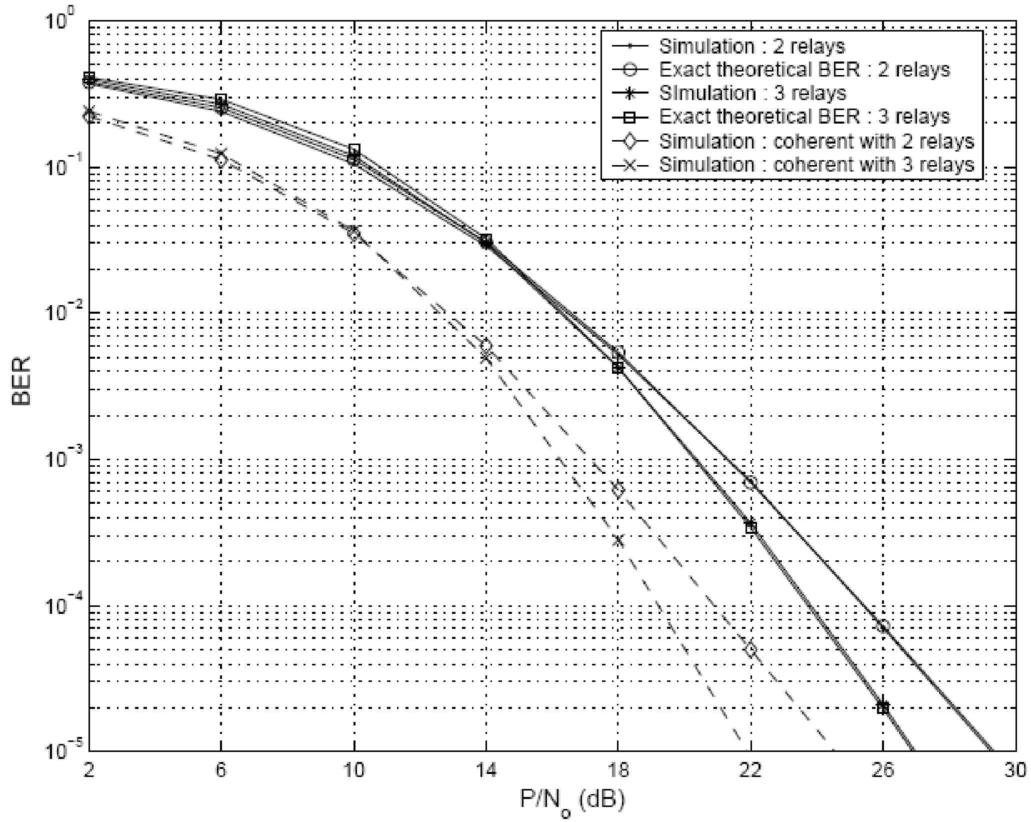
$$\hat{w}^{AF}_s = \frac{1}{\mathcal{N}_0}, \quad \hat{w}^{AF}_i = \frac{P_s\sigma^2_{s,r_i} + \mathcal{N}_0}{\mathcal{N}_0(P_s\sigma^2_{s,r_i} + P_i|h^{\tau}_{r_i,d}|^2 + \mathcal{N}_0)} \qquad \gamma^{AF} = \gamma^{AF}_s + \sum_{i=1}^{N} \gamma^{AF}_i$$

$$P^{AF}_b = \frac{1}{2^{2(N+1)}\pi} \int_{-\pi}^{\pi} f(\theta, \beta, N+1) \mathcal{M}_{\gamma^{AF}_s}(\theta) \prod_{i=1}^{N} \mathcal{M}_{\gamma^{AF}_i}(\theta)\, d\theta$$

$$P^{AF}_b \approx \frac{C(\beta, N+1)\mathcal{N}_0^{N+1}}{P_s\sigma^2_{s,d}} \prod_{i=1}^{N} \frac{P_i\sigma^2_{r_i,d} + (P_s\sigma^2_{s,r_i} + \mathcal{N}_0) Z_{i,min}}{P_s P_i \sigma^2_{s,r_i}\sigma^2_{r_i,d}}$$

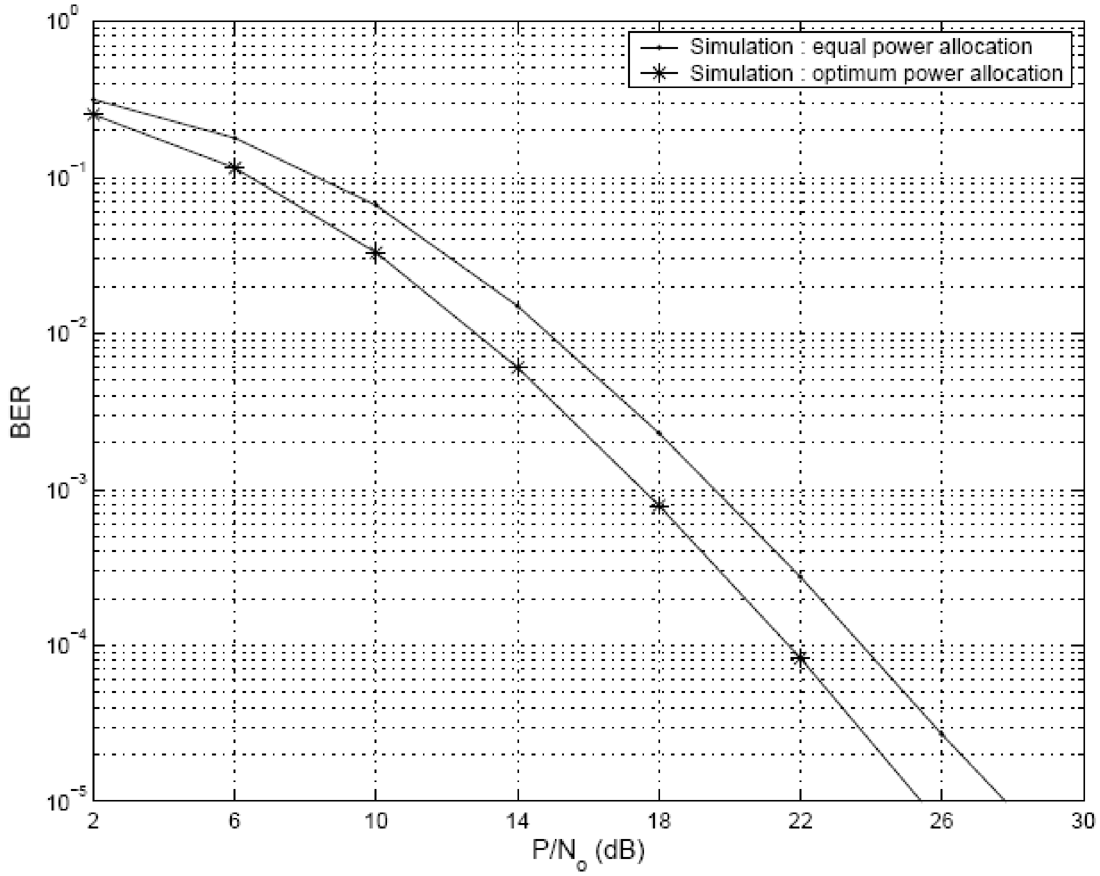
Simulation (1)

- Coherent vs. Differential modulation, 2-relay and 3-relay case



Simulation (2)

- Optimal power allocation vs. Equal allocation

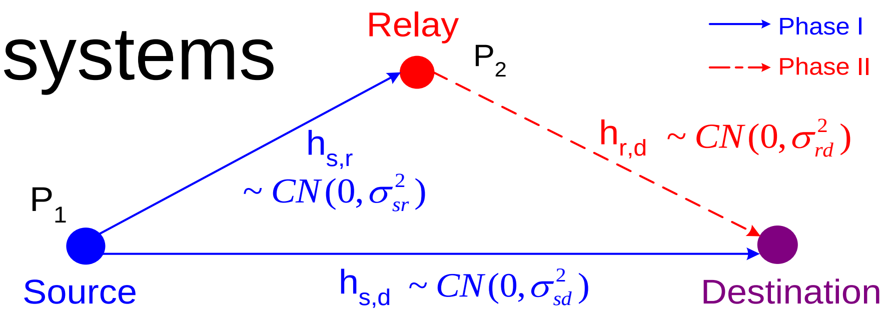


Outline

- Motivation
- DiffAF
- DiffDF
  - *Protocol*
  - *BER & bounds*
  - *Optimal threshold & power allocation*
  - *Simulation results*
  - *Extend to the multi-relay case*
- Conclusions

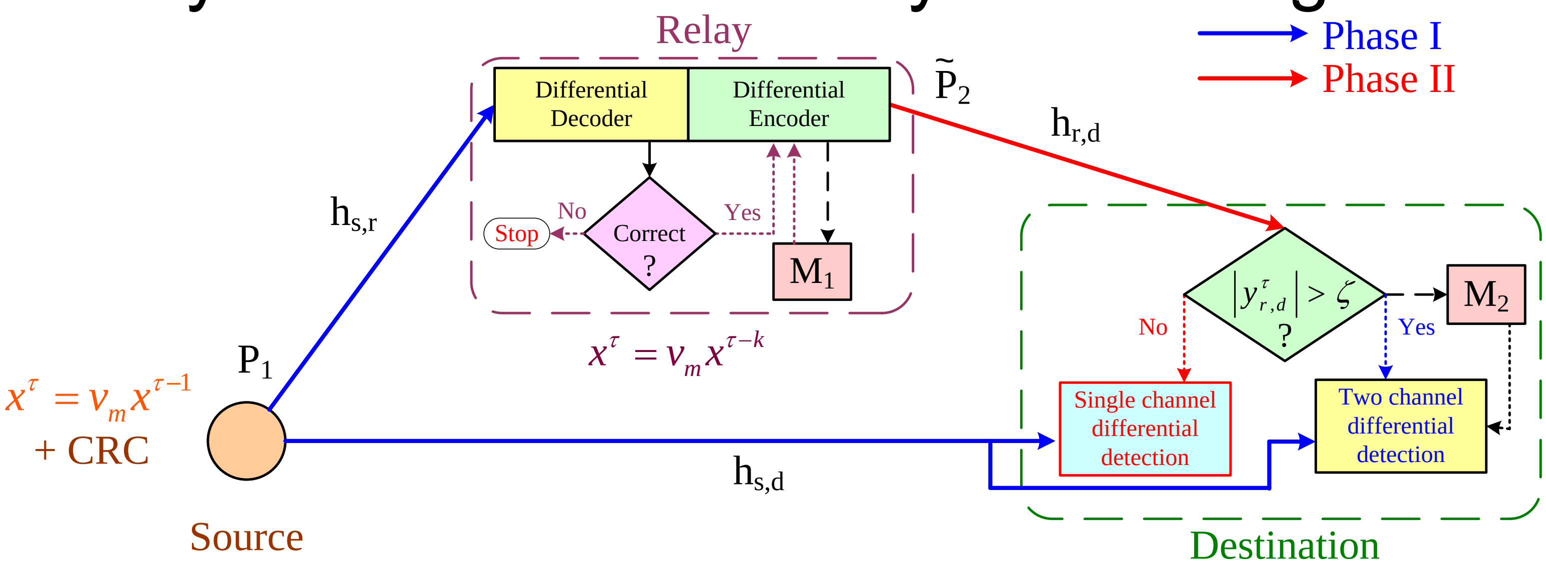
DiffDF protocol

- Single relay cooperative systems
  - *Two-phase transmission*
  - *Rayleigh fading channels*
  - *Differential modulation at both the source node and the relay node*
  - *The relay node only transmits when information is correctly decoded*
- Challenge
  - *How can the destination node keep "sync" with the relay node?*



Threshold-based DiffDF

- Try to "track" the state by introducing a threshold



  - *M_1 stores the most recent transmitted signal at the relay*
  - *M_2 stores the most recent received signal that passes the threshold*
  - *Difficulty in analysis: possible mismatch of M_1 and M_2*

Signal Models

- Transmit:
  - *Phase I*

$$y_{s,d}^{\tau} = \sqrt{P_1} h_{s,d}^{\tau} x^{\tau} + w_{s,d}^{\tau} \qquad y_{s,r}^{\tau} = \sqrt{P_1} h_{s,r}^{\tau} x^{\tau} + w_{s,r}^{\tau}.$$

  - *Phase II*

$$\tilde{x}^{\tau} = v_m \tilde{x}^{\tau - k}$$

$$y_{r,d}^{\tau} = \begin{cases} \sqrt{P_2} h_{r,d}^{\tau} \tilde{x}^{\tau} + w_{r,d}^{\tau} & : \text{if relay correctly decodes } (\tilde{P}_2^{\tau} = P_2); \\ w_{r,d}^{\tau} & : \text{if relay incorrectly decodes } (\tilde{P}_2^{\tau} = 0), \end{cases}$$

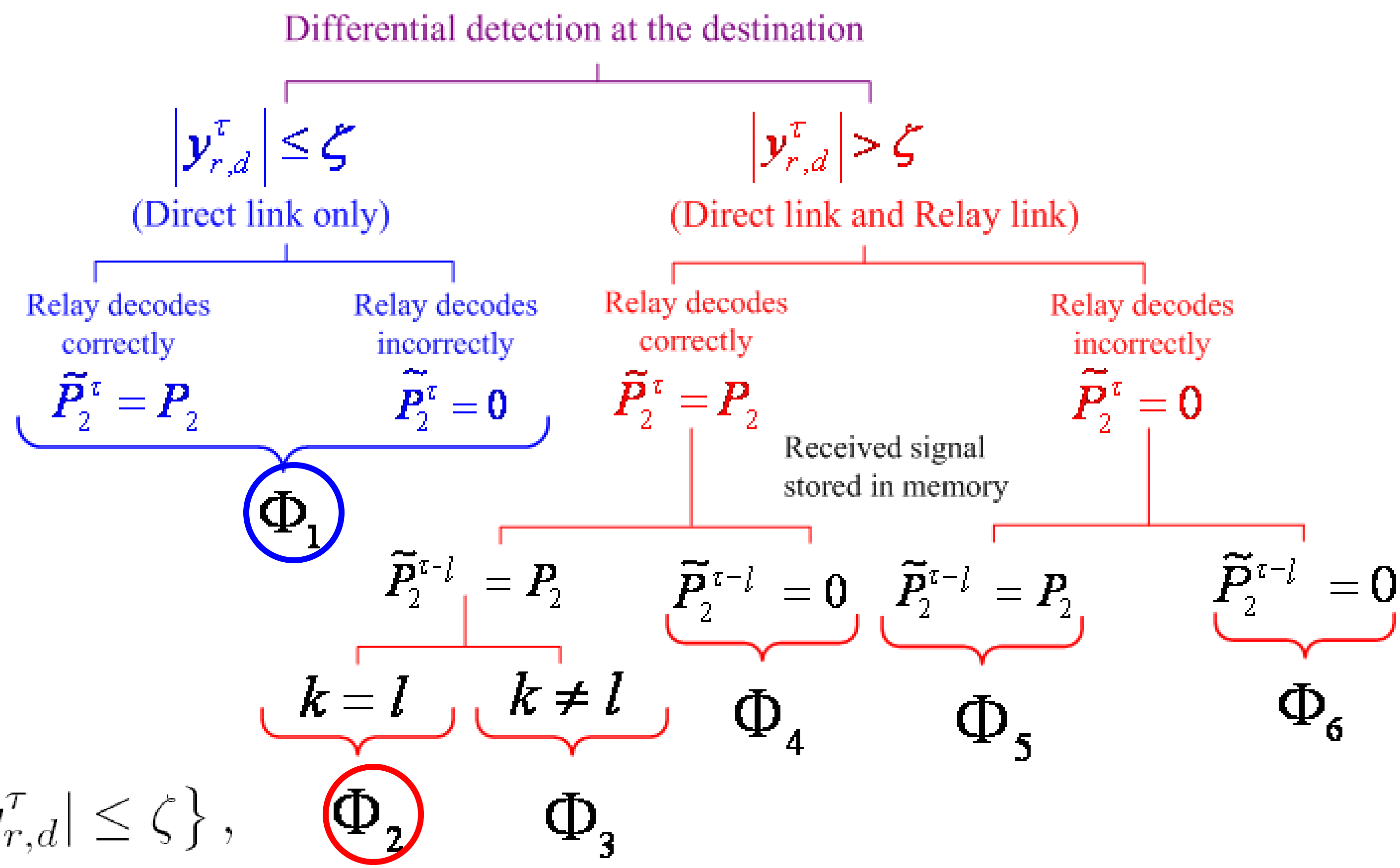
- Receive:
  - *Combine*

$$y = \begin{cases} a_1 (y_{s,d}^{\tau-1})^* y_{s,d}^{\tau} + a_2 (y_{r,d}^{\tau-l})^* y_{r,d}^{\tau} & \text{if } |y_{r,d}^{\tau}| > \zeta \\ (y_{s,d}^{\tau-1})^* y_{s,d}^{\tau} & \text{if } |y_{r,d}^{\tau}| \leq \zeta \end{cases}$$

  - *Demodulation*

$$\hat{m} = \arg \max_{m=0,1,\ldots,M-1} \mathrm{Re}\left\{ v_m^* y \right\}$$

Classification of scenarios



$$\Phi_1 \triangleq \{|y_{r,d}^{\tau}| \le \zeta\},$$

$$\Phi_2 \triangleq \{|y_{r,d}^{\tau}| > \zeta, \tilde{P}_2^{\tau} = P_2, \tilde{P}_2^{\tau-l} = P_2, l = k\}$$

BER Analysis

- Conditional BER on channel *h* and state Φ_i

$$P_{BER|\Phi_i}^{h}$$

- Average over six states to obtain conditional BER on channel realization *h*

$$P_{BER}^{h} = \sum_{i=1}^{6} P_{BER|\Phi_i}^{h} P_r^h(\Phi_i)$$

- Average over all channel realizations to get the overall BER formulation

$$\boxed{P_{BER} = \sum_{i=1}^{6} \mathrm{E}\left[P_{BER|\Phi_i}^{h} P_r^h(\Phi_i)\right] = \sum_{i=1}^{6} P_{BER}^{(i)}}$$

- What are needed? $P_{BER|\Phi_i}^{h}$ $P_r^h(\Phi_i)$

Probability of occurrence

- Probability of incorrect decoding at the relay

$$P_r^h\left(\tilde{P}_2^\tau=0\right)=\Psi(\gamma_{s,r}^\tau)\triangleq\frac{1}{\pi}\int_0^{(M-1)\pi/M}\exp\left[-g(\phi)\gamma_{s,r}^\tau\right]d\phi,$$

with $\gamma_{s,r}^\tau=P_1|h_{s,r}^\tau|^2/\mathcal{N}_0$ $\quad g(\phi)=\frac{\sin^2(\pi/M)}{1+\cos(\pi/M)\cos(\phi)}$

- Probability of state $\Phi_1 \triangleq \left\{|y_{r,d}^\tau|\leq\zeta\right\},$

$$P_r^h(\Phi_1)=P_r^h\left(|y_{r,d}^\tau|\leq\zeta \mid \tilde{P}_2^\tau=0\right)\Psi(\gamma_{s,r}^\tau)+P_r^h\left(|y_{r,d}^\tau|\leq\zeta \mid \tilde{P}_2^\tau=P_2\right)\left[1-\Psi(\gamma_{s,r}^\tau)\right]$$

$$=\left(1-\exp(-\zeta^2/\mathcal{N}_0)\right)\Psi(\gamma_{s,r}^\tau)+\left(1-\mathcal{M}\left(P_2|h_{r,d}^\tau|^2,\zeta\right)\right)\left[1-\Psi(\gamma_{s,r}^\tau)\right]$$

where $\mathcal{M}\left(P_2|h_{r,d}^\tau|^2,\zeta\right)\triangleq Q_1\left(\sqrt{\frac{P_2|h_{r,d}^\tau|^2}{\mathcal{N}_0/2}},\frac{\zeta}{\sqrt{\mathcal{N}_0/2}}\right)$, $\quad Q_1(\alpha,\beta)=\int_\beta^\infty\lambda\exp\left[-\left(\frac{\lambda^2+\alpha^2}{2}\right)\right]I_0(\alpha\lambda)d\lambda$

Probability of occurrence

- Probability of $\Phi_2 \triangleq \{|y_{r,d}^{\tau}| > \zeta, \tilde{P}_2^{\tau} = P_2, \tilde{P}_2^{\tau-l} = P_2, l = k\}$

$$P_r^h(\Phi_2) = P_r^h\left(|y_{r,d}^{\tau}| > \zeta, \tilde{P}_2^{\tau} = P_2, \tilde{P}_2^{\tau-l} = P_2, l = k \middle| |y_{r,d}^{\tau-l}| > \zeta\right)$$
$$= P_r^h\left(|y_{r,d}^{\tau}| > \zeta, \tilde{P}_2^{\tau} = P_2\right) P_r^h\left(\tilde{P}_2^{\tau-l} = P_2, l = k \;\middle|\; |y_{r,d}^{\tau-l}| > \zeta\right)$$
$$= P_r^h\left(|y_{r,d}^{\tau}| > \zeta \;\middle|\; \tilde{P}_2^{\tau} = P_2\right) P_r^h\left(\tilde{P}_2^{\tau} = P_2\right) \times$$
$$\sum_{k \geq 1} P_r^h\left(\tilde{P}_2^{\tau-k} = P_2 \middle| |y_{r,d}^{\tau-k}| > \zeta\right) P_r^h\left(|y_{r,d}^{\tau-k}| > \zeta\right) \prod_{i=1}^{k-1} P_r^h\left(|y_{r,d}^{\tau-i}| \leq \zeta\right) P_r^h\left(\tilde{P}_2^{\tau-i} = 0 \middle| |y_{r,d}^{\tau-i}| \leq \zeta\right)$$
$$\approx \frac{\mathcal{M}^2\left(P_2|h_{r,d}^{\tau}|^2, \zeta\right)\left(1 - \Psi(\gamma_{s,r}^{\tau})\right)^2}{(1 - e^{-\zeta^2/\mathcal{N}_0})\Psi(\gamma_{s,r}^{\tau})}$$

Assume channels stay almost constant for several time slots

- Probabilities of other states can be approached in the same way

Conditional BER

$$P^h_{BER}|_{\Phi_1} = \Omega_1(\gamma_1) \triangleq \frac{1}{4\pi}\int_{-\pi}^{\pi} f_1(\theta)\exp\left[-\alpha(\theta)\gamma_1\right]d\theta$$

$$\gamma_1 = \frac{P_1|h^\tau_{s,d}|^2}{\mathcal{N}_0}, \quad f_1(\theta) = \frac{1-\beta^2}{1+2\beta\sin\theta+\beta^2}, \quad \alpha(\theta) = \frac{b^2}{2\log_2 M}(1+2\beta\sin\theta+\beta^2) \quad \beta = a/b$$

$$P^h_{BER}|_{\Phi_2} = \Omega_2(\gamma_2) \triangleq \frac{1}{16\pi}\int_{-\pi}^{\pi} f_2(\theta)\exp\left[-\alpha(\theta)\gamma_2\right]d\theta,$$

$$\gamma_2 = \frac{P_1|h^\tau_{s,d}|^2}{\mathcal{N}_0} + \frac{P_2|h^\tau_{r,d}|^2}{\mathcal{N}_0}, \qquad f_2(\theta) = \frac{b^2(1-\beta^2)[3+\cos(2\theta)-(\beta+\frac{1}{\beta})\sin(\theta)]}{2\alpha(\theta)},$$

In other states, conditional BER can be obtained by treating relay signals as noise.

Average BER

$$\boxed{P_{BER} = P_{BER}^{(1)} + P_{BER}^{(2)} + P_{BER}^{(3)} + 2P_{BER}^{(4)} + P_{BER}^{(6)},}$$

$$P_{BER}^{(1)} = F_1\big(1+\alpha(\theta)P_1\sigma_{s,d}^2/\mathcal{N}_0\big)\Big[\big(1-e^{\frac{-\zeta^2}{\mathcal{N}_0}}\big)G\big(1+g(\theta)P_1\sigma_{s,r}^2/\mathcal{N}_0\big) + \Big(1-\int_0^\infty \frac{\mathcal{M}(P_2q,\zeta)}{\sigma_{r,d}^2}e^{\frac{-q}{\sigma_{r,d}^2}}dq\Big)\Big(1-G\Big(1+\frac{g(\theta)P_1\sigma_{s,r}^2}{\mathcal{N}_0}\Big)\Big)\Big]$$

$$P_{BER}^{(2)} \approx \frac{1}{\sigma_{r,d}^2}\int_0^\infty s_2(q)\mathcal{M}^2(P_2q,\zeta)e^{-\frac{q}{\sigma_{r,d}^2}}dq \cdot \frac{1}{\sigma_{s,r}^2}\int_0^\infty \frac{\big(1-\Psi(P_1u/\mathcal{N}_0)\big)^2}{1-(1-e^{-\zeta^2/\mathcal{N}_0})\Psi(P_1u/\mathcal{N}_0)}e^{-\frac{u}{\sigma_{s,r}^2}}du,$$

in which $s_2(q) = \frac{1}{16\pi}\int_{-\pi}^{\pi} f_2(\theta)\big(1+\frac{\alpha(\theta)P_1\sigma_{s,d}^2}{\mathcal{N}_0}\big)^{-1}e^{(-\alpha(\theta)\frac{P_2q}{\mathcal{N}_0})}d\theta.$

$$P_{BER}^{(3)} \approx \frac{1}{\sigma_{s,r}^2}\int_0^\infty\Big[\frac{1}{16\pi}\int_{-\pi}^{\pi} f_2(\theta)s_3(u,\theta)d\theta\Big]e^{-u/\sigma_{s,r}^2}du,$$

in which

$$s_3(u,\theta) = \frac{1}{\sigma_{s,d}^2\sigma_{r,d}^2}\int_0^\infty\int_0^\infty \exp\Big(-\frac{\alpha(\theta)P_1z}{\mathcal{N}_0+\frac{(P_2q+\mathcal{N}_0)^2}{P_1z}} - \frac{z}{\sigma_{s,d}^2} - \frac{q}{\sigma_{r,d}^2}\Big) \times \mathcal{M}^2(P_2q,\zeta)\big(1-\Psi(P_1u/\mathcal{N}_0)\big)^2 \times\Big(\frac{1}{\Gamma(P_1u,P_2q)} - \frac{1}{1-(1-e^{-\zeta^2/\mathcal{N}_0})\Psi(P_1u/\mathcal{N}_0)}\Big)\,dq\,dz.$$

$$P_{BER}^{(4)} = P_{BER}^{(5)} \approx \frac{1}{\sigma_{s,r}^2}\int_0^\infty\Big[\frac{1}{16\pi}\int_{-\pi}^{\pi} f_2(\theta)s_4(u,\theta)d\theta\Big]e^{-u/\sigma_{s,r}^2}du,$$

where

$$s_4(u,\theta) = \frac{1}{\sigma_{s,d}^2\sigma_{r,d}^2}\int_0^\infty\int_0^\infty \exp\Big(-\frac{\alpha(\theta)P_1z}{\mathcal{N}_0+\frac{P_2q+\mathcal{N}_0}{P_1z/\mathcal{N}_0}} - \frac{\zeta^2}{\mathcal{N}_0} - \frac{z}{\sigma_{s,d}^2} - \frac{q}{\sigma_{r,d}^2}\Big) \times \mathcal{M}(P_2q,\zeta)\frac{\Psi(P_1u/\mathcal{N}_0)\big(1-\Psi(P_1u/\mathcal{N}_0)\big)}{\Gamma(P_1u,P_2q)}\,dq\,dz.$$

$$P_{BER}^{(6)} \approx \frac{e^{-2\zeta^2/\mathcal{N}_0}}{\sigma_{s,r}^2\sigma_{r,d}^2}\int_0^\infty\int_0^\infty \frac{\Psi^2(P_1u/\mathcal{N}_0)}{\Gamma(P_1u,P_2q)}e^{-\frac{u}{\sigma_{s,r}^2}-\frac{q}{\sigma_{r,d}^2}}\,dq\,du \times \frac{1}{16\pi\sigma_{s,d}^2}\int_{-\pi}^{\pi} f_2(\theta)\int_0^\infty e^{\Big(-\frac{\alpha(\theta)P_1z}{\mathcal{N}_0+\mathcal{N}_0^2/P_1z}-\frac{z}{\sigma_{s,d}^2}\Big)}dz\,d\theta.$$

BER bounds

- States $\Phi_3, \Phi_4, \Phi_5, \Phi_6$ are not so important; by carefully choose the threshold, their probability can be made very small.
- Bounding $P_{BER}^h|_{\Phi_3}, P_{BER}^h|_{\Phi_4}, P_{BER}^h|_{\Phi_5}, \text{and } P_{BER}^h|_{\Phi_6}$

$$0 \leq P_{BER}^h|_{\Phi_i} \leq 1/2$$

$$P_{BER}^h|_{\Phi_3} \geq P_{BER}^h|_{\Phi_2}$$

- BER Bounds

$$P_{BER} \leq P_{BER}^{(1)} + P_{BER}^{(2)} + \frac{1}{2}\{P_r^h(\Phi_3) + 2P_r^h(\Phi_4) + P_r^h(\Phi_6)\}$$

$$P_{BER} \geq P_{BER}^{(1)} + \mathrm{E}[P_{BER}^h|_{\Phi_2} P_r^h(\Phi_2 \cup \Phi_3)]$$

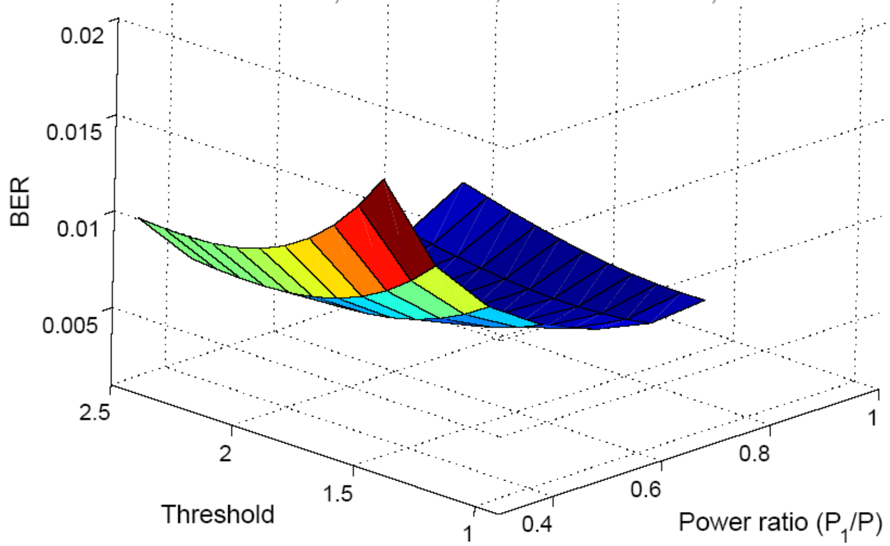
Optimal threshold and power

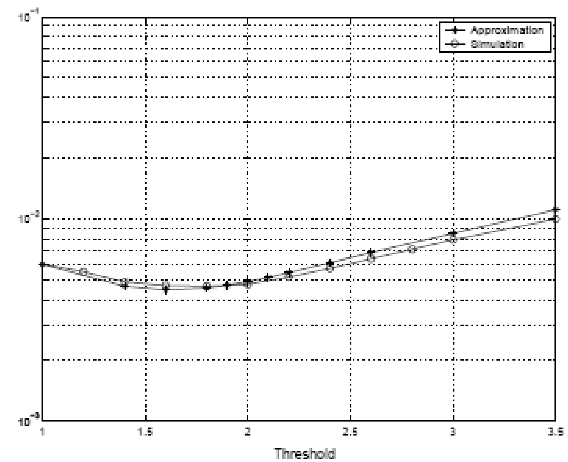
- Exhaustive search numerically

$$(\hat{\zeta}, \hat{r}) = \arg\min_{\zeta, r} \quad P_{BER}(\zeta, r),$$

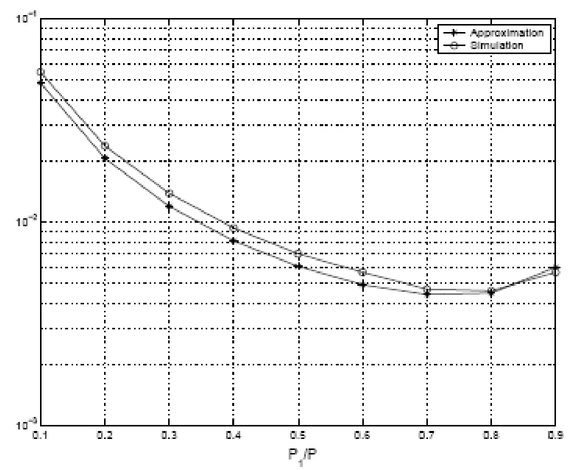
$$P_1 = \hat{r}P \text{ and } P_2 = (1 - \hat{r})P$$

$$\text{DQPSK: } \sigma^2_{s,d} = \sigma^2_{s,r} = 1,\ \sigma^2_{r,d} = 10$$





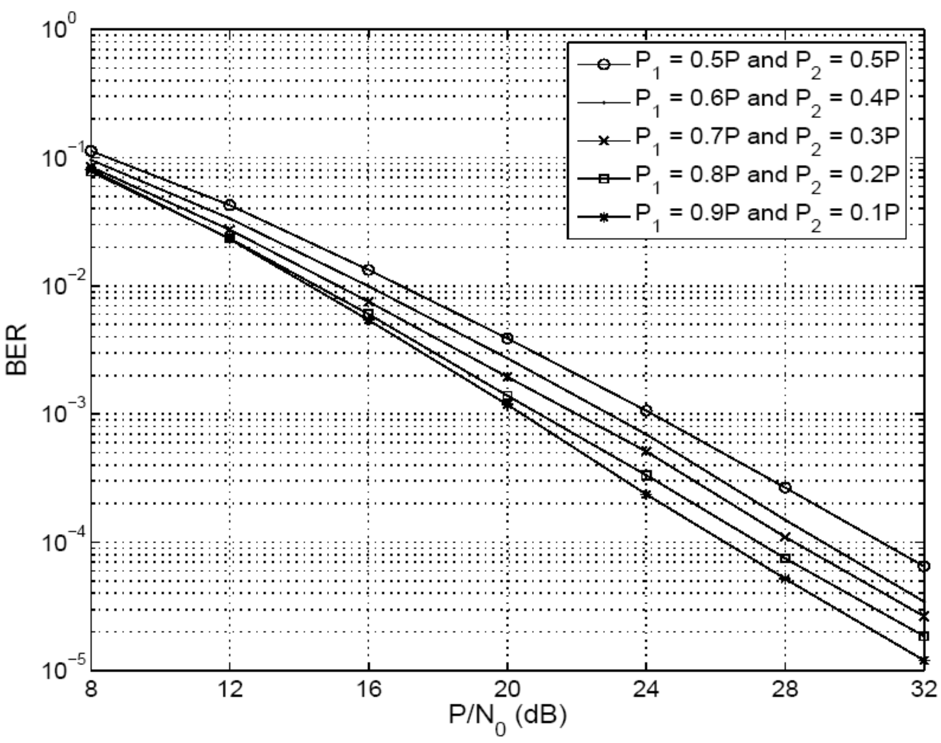
(e) Vary ζ, $P_1 = 0.8P$ and $P_2 = 0.2P$.

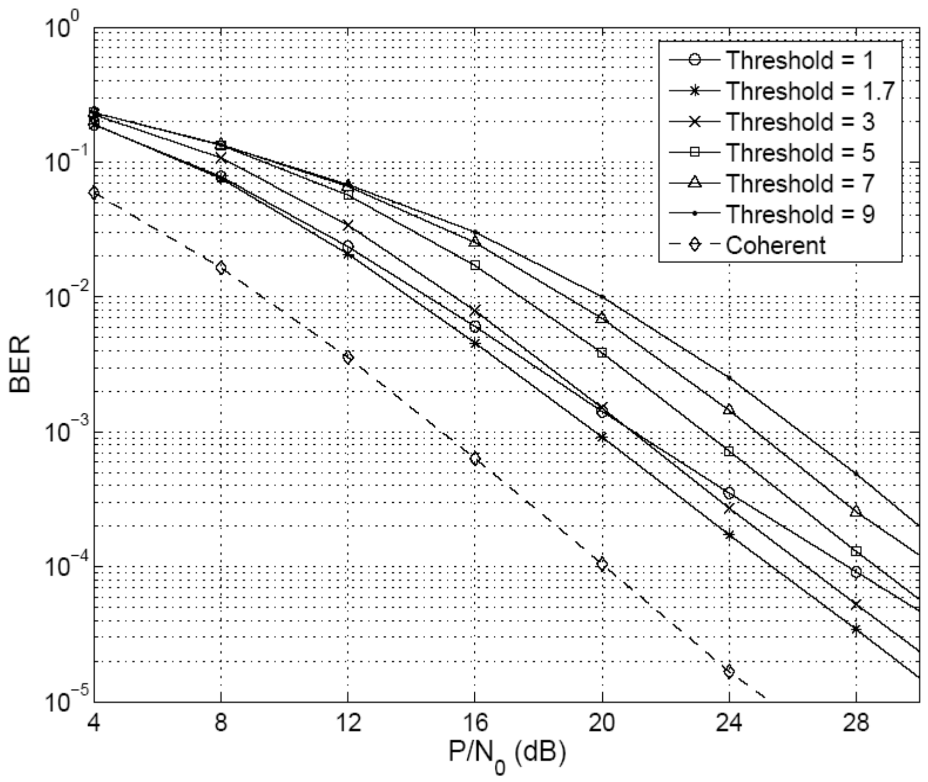


(f) Vary r, $\zeta = 1.7$.

Simulation (1)

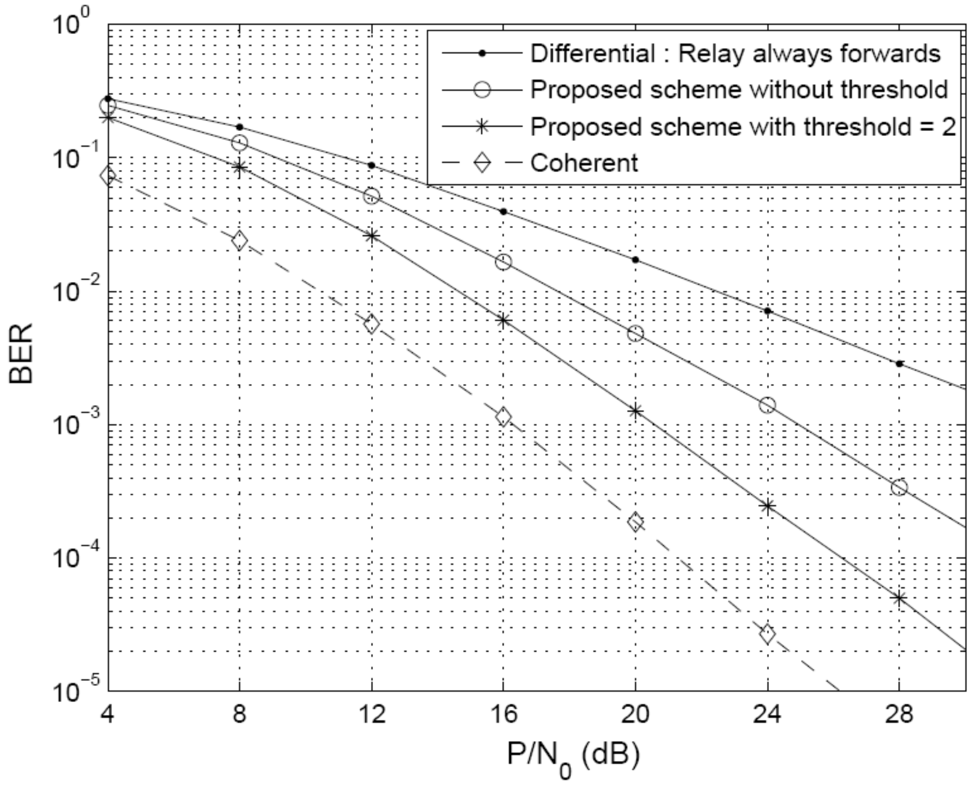
- Different thresholds or different power allocation





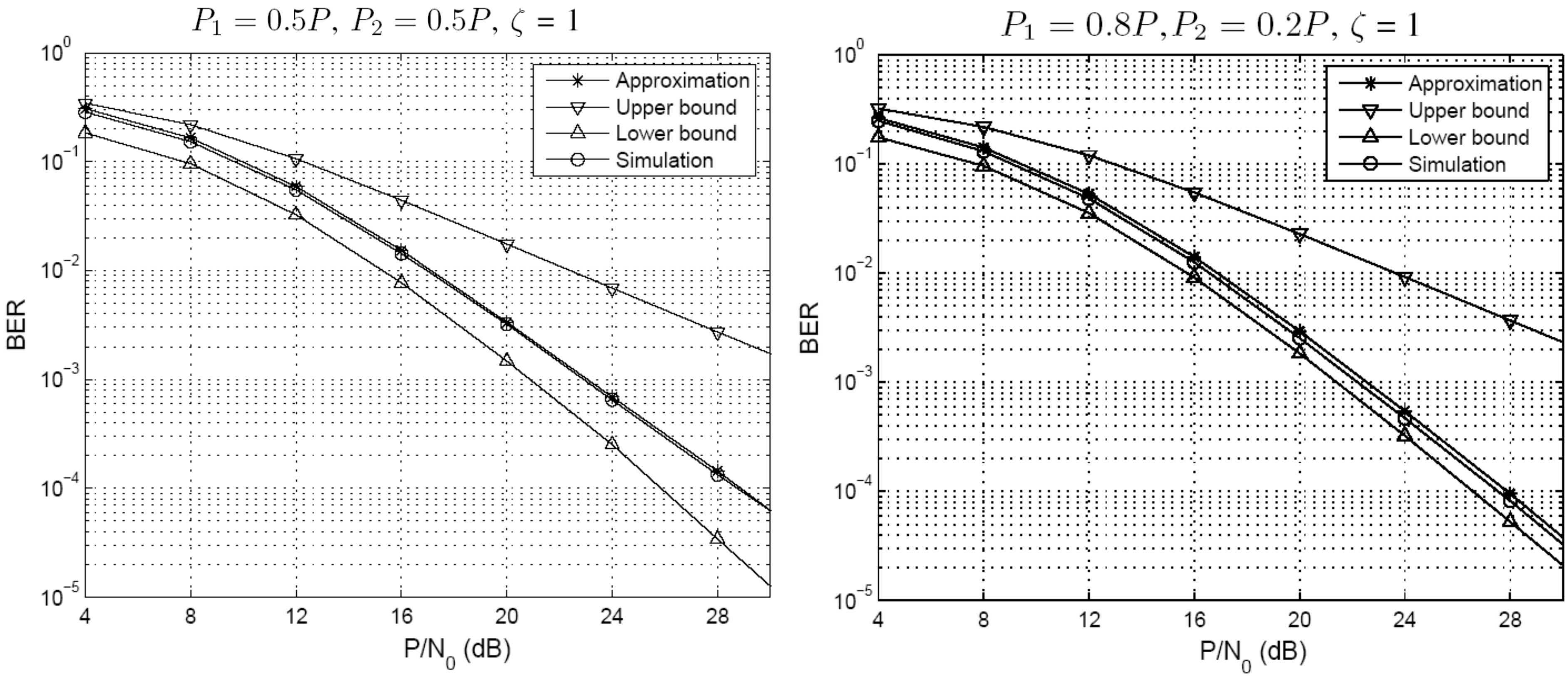
Simulation (2)

- Different cooperation schemes



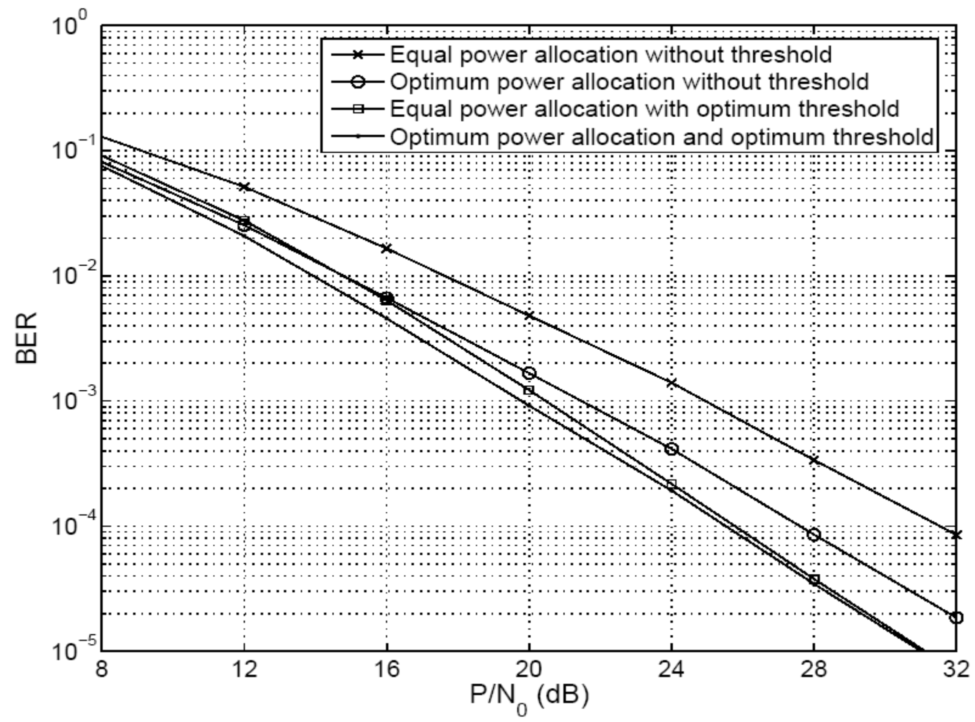
Simulation (3)

- BER: theoretical curve, simulation curve, and it bounds

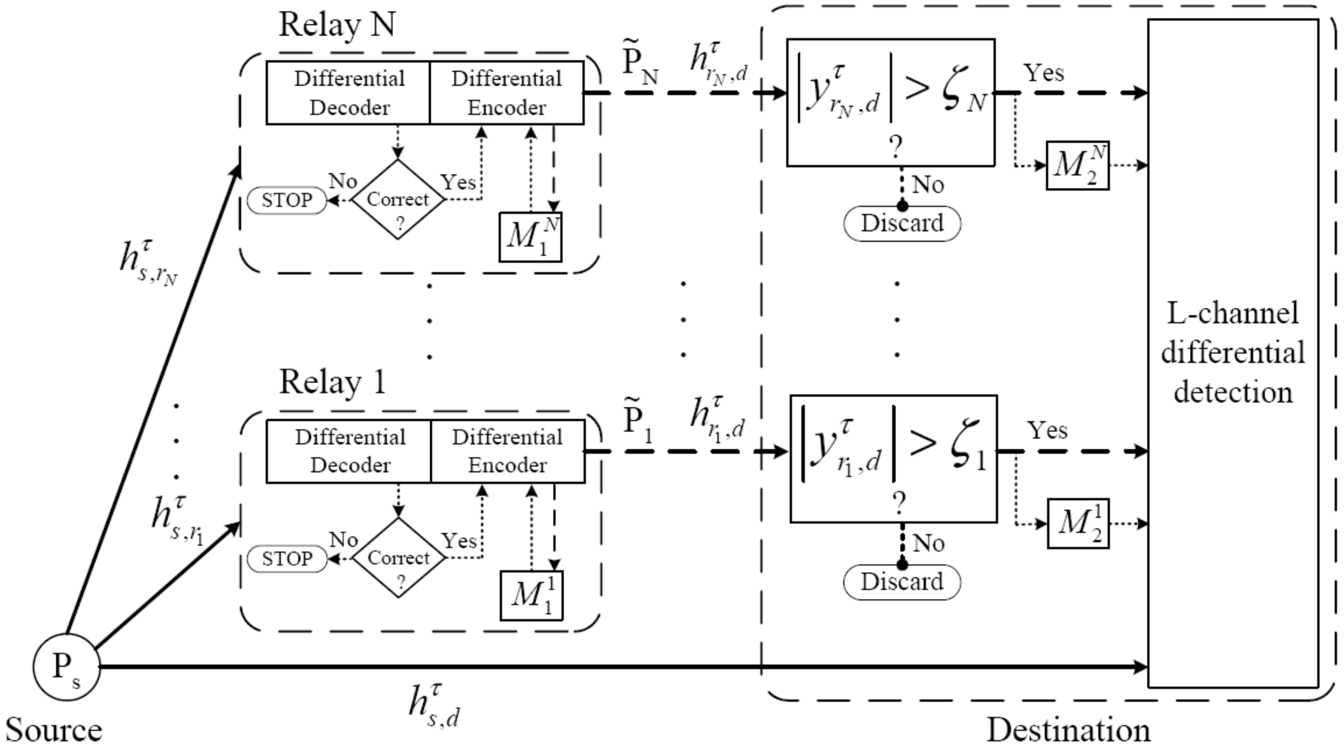


Simulation (4)

- Different power allocations and decision thresholds



Multi-relay extension



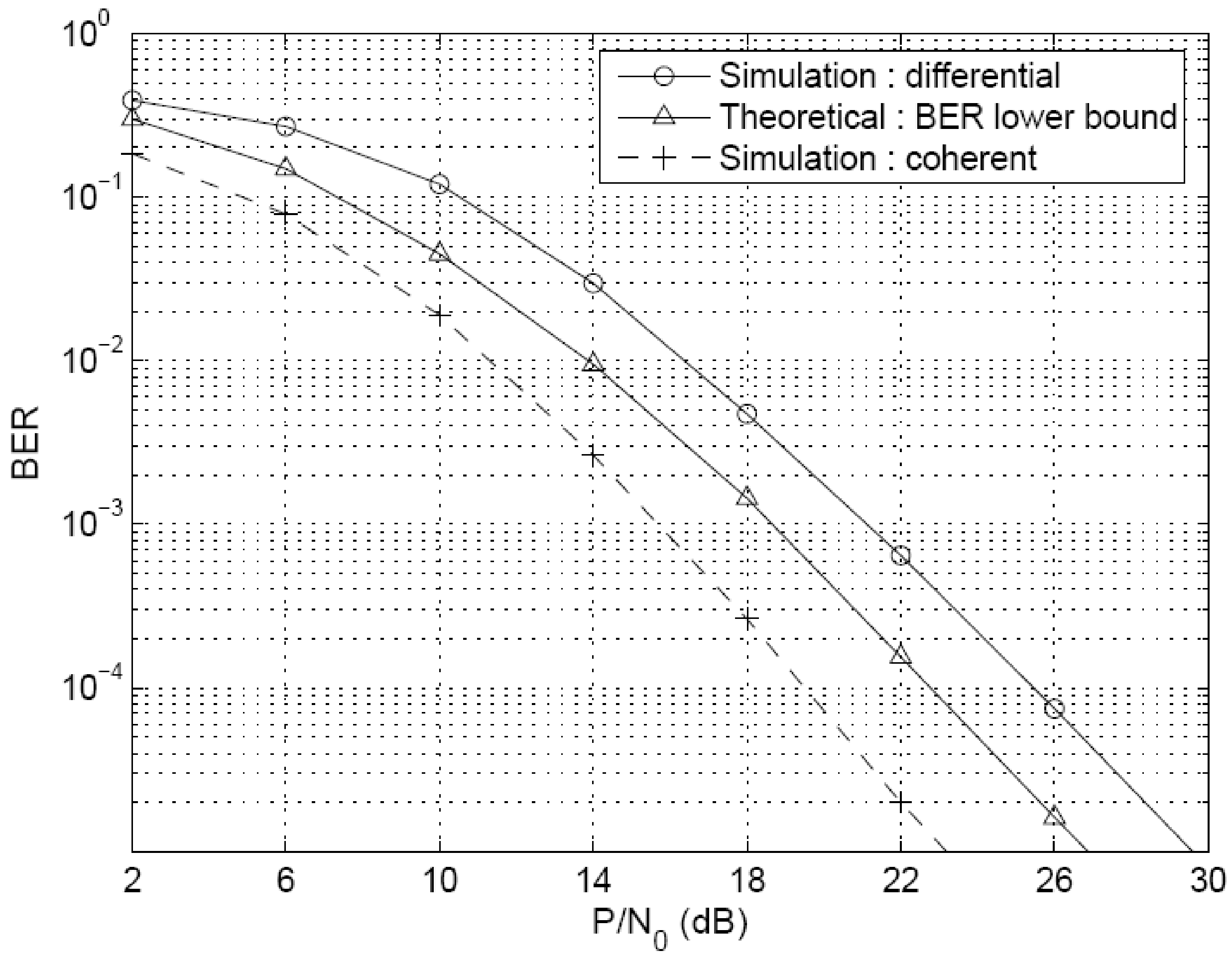
$$y^{DF} = w_s^{DF}(y_{s,d}^{\tau-1})^* y_{s,d}^\tau + \sum_{i=1}^{N} w_i^{DF} I_{\zeta_i}[|y_{r_i,d}^\tau|](y_{r_i,d}^{\tau-l_i})^* y_{r_i,d}^\tau$$

Multi-relay extension

- For each relay, there are six states with the same probability as the single-relay case. The same approach can be applied to derive BER.
- However, there are 6^N states in total! (relays are independent with each other)
- Compact form: two subsets of states
  - *Important states: each relay is either in state 1 or state 2. 2^N such states in total.*
  - *Less important states: all the other states*
- Search optimal parameters numerically

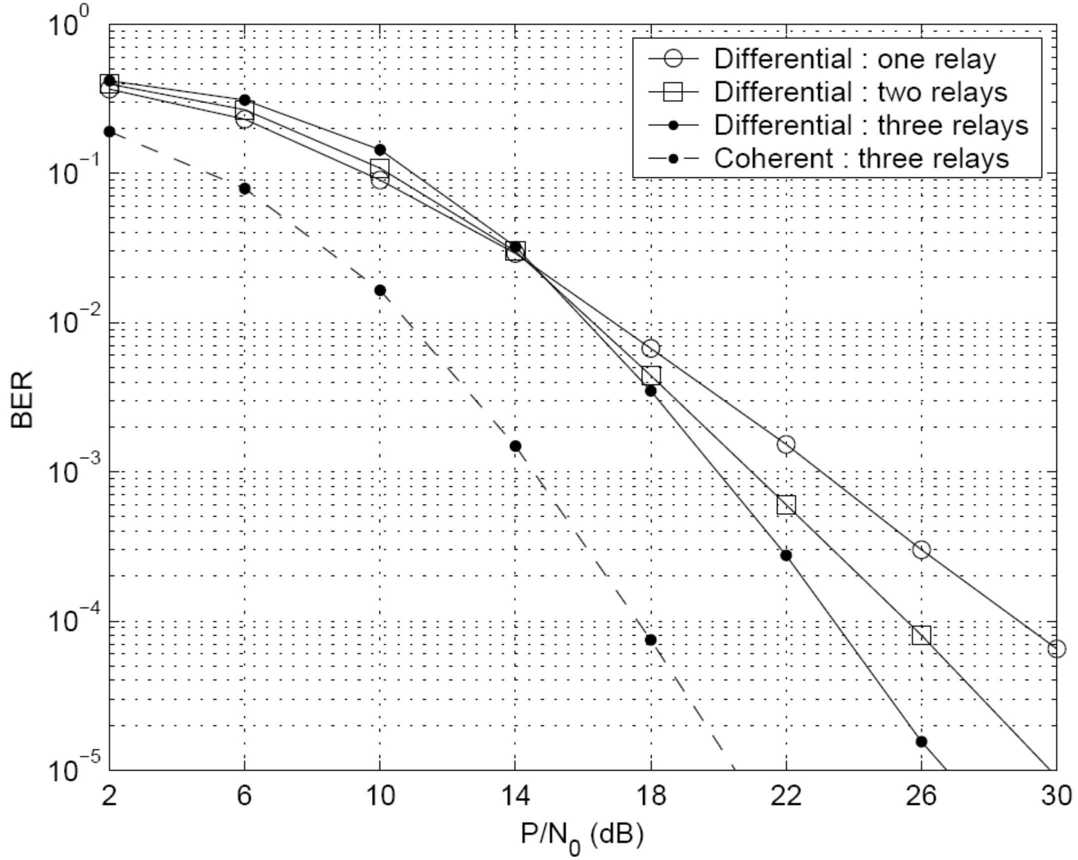
Simulation (1)

- BER



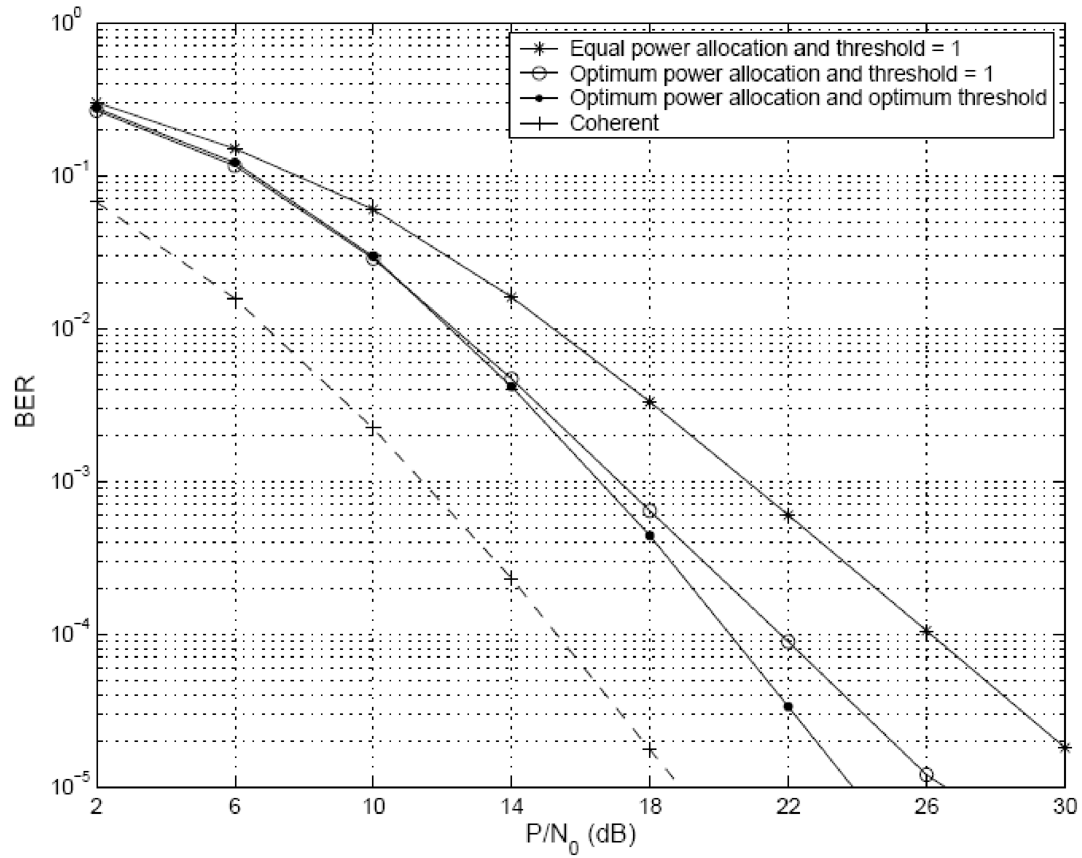
Simulation (2)

- Different number of relays



Simulation (3)

- Power allocation and threshold



Conclusions

- With differential modulation, information is conveyed in the difference between two consecutive symbols. Non-coherent detection can be used without the knowledge of channel states information.
- Two protocols have been proposed, i.e., DiffAF and DiffDF, which combine differential modulation with AF and DF cooperative communications.
- BER and its bounds have been derived to evaluate the proposed algorithms. The optimal parameters can be found either numerically or analytically. Simulation results are also presented to illustrate the performance.