$$\frac{2\pi n_1}{\lambda} > \beta > \frac{2\pi n_3}{\lambda} \quad \Rightarrow \quad 9.68 \times 10^6\ \mathrm{m}^{-1} > \beta > 9.43 \times 10^6\ \mathrm{m}^{-1}.$$

And the wavelength of a guided mode that has a propagation constant of $\beta = 1.5 \times 10^7\ \mathrm{m}^{-1}$ falls in the range:

$$\frac{2\pi n_1}{\beta} > \lambda > \frac{2\pi n_3}{\beta} \quad \Rightarrow \quad 645.1\ \mathrm{nm} > \lambda > 632.8\ \mathrm{nm}.$$

If the structure is immersed in CS_2 so that $n_3 = 1.63$, then $k_3 > k_1 > k_2$ because $n_3 > n_1 > n_2$. In this situation, the structure does not have any guided mode because the core has a lower refractive index than the cover. Only cover radiation modes and substrate–cover radiation modes can be found for this structure.

Guided TE Modes

For a TE mode, it is only necessary to find $\mathcal{E}_y$; then the other two nonvanishing field components $\mathcal{H}_x$ and $\mathcal{H}_z$ can be found by using (3.83) and (3.84), respectively. The boundary conditions require that $\mathcal{E}_y$, $\mathcal{H}_x$, and $\mathcal{H}_z$ be continuous at the interfaces at $x = \pm d/2$ between layers of different refractive indices. From (3.83) and (3.84), it can be seen that these boundary conditions are equivalent to requiring $\mathcal{E}_y$ and $\partial\mathcal{E}_y/\partial x$ be continuous at these interfaces.

For a guided mode, we know that the transverse field patterns in the core, substrate, and cover regions are respectively characterized by the transverse field parameters h_1, γ_2, and γ_3, given in (3.131). A guided TE mode field distribution that satisfies the boundary conditions for the continuity of $\mathcal{E}_y$ at $x = \pm d/2$ has the form:

$$\hat{\mathcal{E}}_y = C_{\mathrm{TE}} \begin{cases} \cos(h_1 d/2 - \psi) \exp[\gamma_3(d/2 - x)], & x > d/2, \\ \cos(h_1 x - \psi), & -d/2 < x < d/2, \\ \cos(h_1 d/2 + \psi) \exp[\gamma_2(d/2 + x)], & x < -d/2. \end{cases} \tag{3.134}$$

Application of the other two boundary conditions for the continuity of $\partial\mathcal{E}_y/\partial x$ at $x = \pm d/2$ yields two eigenvalue equations:

$$\tan h_1 d = \frac{h_1(\gamma_2 + \gamma_3)}{h_1^2 - \gamma_2\gamma_3} \tag{3.135}$$

and

$$\tan 2\psi = \frac{h_1(\gamma_2 - \gamma_3)}{h_1^2 + \gamma_2\gamma_3}. \tag{3.136}$$

A guided TE mode can be normalized using the orthonormality relation in (3.20) for

$$C_{\mathrm{TE}} = \sqrt{\frac{\omega\mu_0}{\beta d_{\mathrm{E}}}}, \tag{3.137}$$

where

$$d_{\mathrm{E}} = d + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} \tag{3.138}$$

is the *effective waveguide thickness* for a guided TE mode.