Chapter 17
Answers to Exercises

Exercise 17.1

(a) For *free convection*, substituting the expressions for Gr and Pr, and recognizing that $\nu_a/\kappa_a = \eta_a c_a/k_a$, gives the Nusselt number in the form

$$\mathrm{Nu} = 0.14L\left(\frac{\alpha c_a g \rho_a^2 (T_{surf} - T_a)}{k_a \eta_a}\right)^{1/3}.$$

Multiplying this expression by $k_a(T_{surf}-T_a)/L$ yields the expression for q_{conv} (free), in which the length scale cancels:

$$q_{conv} = 0.14\left(\frac{\alpha c_a g k_a^2 \rho_a^2 (T_{surf} - T_a)^4}{\eta_a}\right)^{1/3}.$$

For *forced convection*, substituting for Pr and Re in the expression for Nusselt number yields

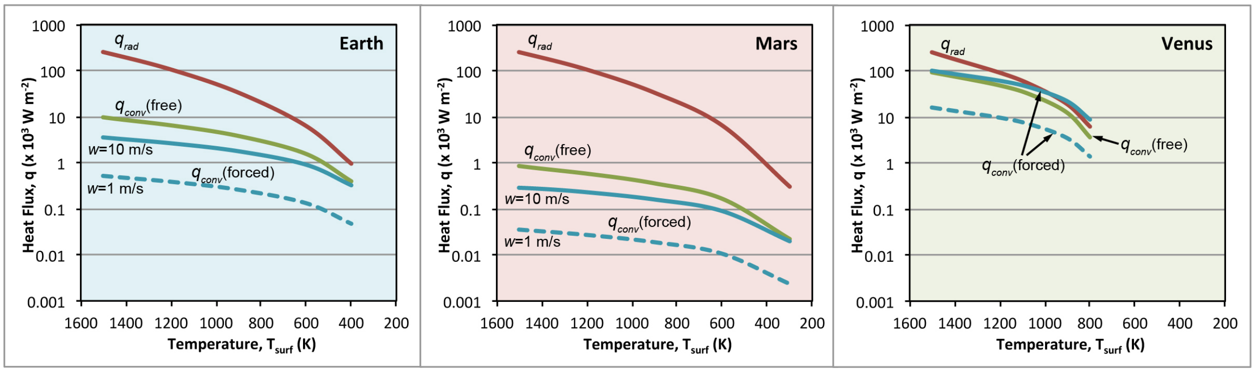
$$\mathrm{Nu} = \left(\frac{\eta_a c_a}{k_a}\right)^{1/3}\left[0.036\left(\frac{wL\rho_a}{\eta_a}\right)^{4/5} - 836\right],$$

which gives the following for the forced convective heat flux

$$q_{conv} = \frac{1}{L}\left(\eta_a c_a k_a^2\right)^{1/3}\left(T_{surf} - T_a\right)\left[0.036\left(\frac{wL\rho_a}{\eta_a}\right)^{4/5} - 836\right].$$

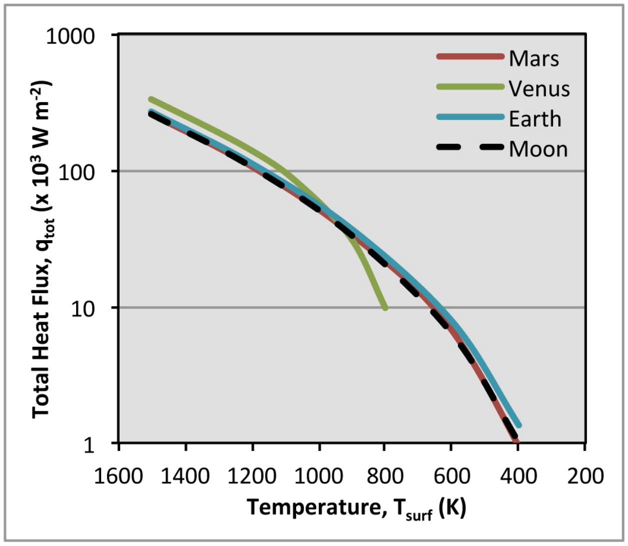
Note that this expression depends only weakly on L.

(b) Calculations for radiative, convective and total heat fluxes are given in OS_Ch17_HeatFluxes.xls. Figure OS17.1 shows a comparison of the heat fluxes for terrestrial, martian and venusian atmospheric conditions. As can be seen, the radiative flux dominates the total heat flux on Earth and Mars, especially at high temperatures, because of the T_{surf}^4 dependence. On Venus, the high atmospheric density means that convective heat fluxes are typically on the same order as the radiative heat flux. Also shown is the influence of wind speed on forced convective heat flux.



[**Fig. OS17.1**](#) Comparison of radiative and convective heat fluxes on Earth, Mars, and Venus.

Figure [OS17.2](#) shows a comparison of total heat fluxes (where $q_{tot} = q_{rad} + q_{conv}$(free)) for the Earth, Mars, Venus, and the Moon. It can be seen, that because of the dominance of radiative cooling at elevated temperatures, the total heat flux does not differ significantly among Earth, Mars and the Moon. Only at low temperatures is the total heat flux for the terrestrial case greater than for Mars or the Moon; this is because of the greater atmospheric density on Earth. This suggests that lava flow surfaces would not cool and solidify significantly faster on Earth vs. Mars or the Moon, though they may take a little longer to reach their final temperature. Instead, other factors have to be considered (e.g., effusion rate) to explain flows on Mars and the Moon that are significantly longer than typical flows on the Earth. For venusian conditions, however, the dense atmosphere makes a noticeable difference to the total heat flux from a high-temperature lava surface. This suggests that flow surfaces might initially cool more rapidly to produce an insulating surface crust. However, as the flow surface continues to cool, the total heat flux declines below that of the Earth (or Mars or the Moon), because the high ambient temperature on Venus acts to diminish the temperature differentials driving both convective ($T_{surf} - T_a$) and radiative ($T_{surf}^4 - T_a^4$) cooling. Inhibited cooling may thus act to promote continued flow of the lava, and produce systematically longer flows on Venus than on Earth for the same eruption conditions.



[**Fig. OS17.2**](#)**.** Comparison of total heat fluxes for Earth, Mars, Venus, and the Moon.

Exercise 17.2

(a) Below the volatile exsolution depth, the n_d is equal to n_t. When exsolution takes place, n_d becomes just less that n_t, i.e., when $6.8 \times 10^{-6} P^{0.7} < n_t$. The total pressure experienced by the magma is $P = \rho gz + P_a$, so that we have

$$6.8 \times 10^{-6} \left(\rho gz + P_a\right)^{0.7} < n_t .$$

Rearranging for z, we find that exsolution takes place when the depth becomes less than

$$z = \frac{(1.47 \times 10^5 n_t)^{1/0.7} - P_a}{\rho g} .$$

This leads to the following results for Earth, Mars and Venus:

Table OS17.1. Depths at which magmatic water begins to exsolve.

Total H_2O content, n_t(wt.%)	**Earth**	**Mars**	**Venus**
1.0	907	2394	631
0.3	159	429	–
0.03	2.3	16	–

From these calculations, we understand that water will not exsolve in magmas under Venusian conditions if the magmatic water content is as low as 0.3 wt%.

(b) The corresponding fragmentation depths on Earth and Mars are given in Table OS17.2. From this we see that a magma containing 0.03 wt.% water will not produce an explosive eruption on Earth.

Table OS17.2. Depths at which H_2O-driven fragmentation takes place.

Total H_2O content, n_t (wt.%)	**Earth**	**Mars**
1.0	117	318
0.3	28	83
0.03	–	6

(c) The minimum water content required for exsolution to occur is found by understanding that the exsolution depth z must be greater than 0, so we have

$$z = \frac{(1.47 \times 10^5 n_t)^{1/0.7} - P_a}{\rho g} > 0 .$$

Rearranging this expression leads to $n_t > 6.8 \times 10^{-6} P_a^{0.7}$, which for Venusian atmospheric pressure implies n_t must exceed 0.5 wt.%.

On Venus, the fragmentation pressure must just exceed the atmospheric pressure for fragmentation to take place at $z = 0$ (which admittedly would be an extremely weak explosive eruption). Extrapolating the relationship between fragmentation pressure and total water content used in part (b) of this question leads to an estimate of ~2.8 wt.% for the minimum water content required for an explosive eruption to take place on Venus. Note that all calculations have been performed for atmospheric pressure values typical of the mean planetary radius. If eruptions were to take place at the lower pressures corresponding to higher altitudes on the volcanoes, lower water contents would be required to ensure exsolution and fragmentation (e.g., 0.33 and 1.5 wt.%, respectively, at 10 km above the mean planetary radius) on Venus.