Exercises
Chapter 17

Exercise 17.1 Cooling of planetary lava flows

The radiative heat flux from a hot surface at temperature T_{surf} in an environment having temperature T_a is given, per unit surface area, by

$$q_{rad} = \varepsilon\sigma\left(T_{surf}^4 - T_a^4\right) \ ,$$

in which ε is the emissivity of the hot body (typically ~0.9 for a lava surface) and σ is the Stefan-Boltzmann constant (5.67 x 10^{-8} W m^{-2} K^{-4}).

The convective heat flux from a hot surface due to the presence of atmospheric gases is given per unit surface area by

$$q_{conv} = h_c\left(T_{surf} - T_a\right) \ ,$$

where h_c is the convective heat transfer coefficient, given by

$$h_c = \frac{\mathrm{Nu}k_a}{L} \ .$$

Here, k_a is the thermal conductivity of the mixture of atmospheric gases, and L is the length scale of the cooling body.

For *free* convection (i.e., in the absence of wind), the Nusselt number Nu is given by

$$\mathrm{Nu} = 0.14\,\mathrm{Ra}^{1/3} = 0.14(\mathrm{Gr}\,\mathrm{Pr})^{1/3} \ ,$$

where Ra, Gr, and Pr are, respectively, the Rayleigh, Grashof, and Prandtl numbers.

The Prandtl number (ratio of momentum diffusivity to thermal diffusivity) is given by

$$\mathrm{Pr} = \frac{\nu_a}{\kappa_a} \ ,$$

where ν_a is the atmospheric kinematic viscosity ($\nu_a = \eta_a/\rho_a$), and κ_a is the atmospheric thermal diffusivity ($\kappa_a = k_a/\rho_a c_a$), with η_a being the dynamic viscosity of the atmospheric gas, and c_a the specific heat capacity at constant pressure of the atmospheric gas.

The Grashof number (ratio of buoyancy force to viscous force)is given by

$$\text{Gr} = \frac{\alpha g \rho_a^2 L^3 \left(T_{surf} - T_a\right)}{\eta_a^2} \ ,$$

where α is the volume expansion coefficient of the atmospheric gas, and g is the acceleration due to gravity.

When a wind blows over the hot surface at a velocity w, *forced* convection conditions exist, and the Nusselt number is instead given by

$$\text{Nu} = \text{Pr}^{1/3}(0.036\,\text{Re}^{0.8} - 836) \ ,$$

in which Re is the Reynolds number of the atmospheric flow, given by

$$\text{Re} = \frac{wL\rho_a}{\eta_a} \ .$$

(a) Using the above information, write expressions for convective heat flux q_{conv} for free and forced convection, in terms of the atmospheric properties η_a, k_a, c_a, ρ_a. For free convection show that the heat flux per unit area is independent of the length scale of the flow.

(b) Using information given in Table 17.1 of Chapter 17, and the atmospheric properties data given in the table below, calculate the radiative heat flux q_{rad}, the convective fluxes q_{conv} (free and forced), and total heat flux q_{tot} from a lava flow surface at several temperatures ranging from ~1500 K to ~100 K above ambient temperature for lava flows on Earth, Mars, Venus, and the Moon. Use a length scale (typically assumed to be the average of flow width and length) L of 1000 m, and wind speeds of 1 and 10 m s^{-1}. Plot graphs of radiative and convective heat fluxes against temperature to help visualize the differences in cooling mechanisms between the four environments, and plot total heat flux against temperature for each body to consider the implications for the rate of cooling of flows on these four bodies.

Planet	**ρ_a (kg m^{-3})**	**η_a (Pa s)**	**k_a (W m^{-1} K^{-1})**	**c_a (J kg^{-1} K^{-1})**	**α (K^{-1})**
Earth	1.2	2.4×10^{-4}	0.025	1000	3.4×10^{-3}
Mars	1.5×10^{-2}	1.1×10^{-5}	0.016	840	4.7×10^{-3}
Venus	60	1.8×10^{-4}	0.057	1140	1.3×10^{-3}

Exercise 17.2 Volatile exsolution and magma fragmentation

Consider the ascent of basaltic magma through the lithospheres of Earth, Mars and Venus. A simple solubility law for magmatic water in basaltic magma is given by

$n_d = 6.8 \times 10^{-6} P^{0.7}$

where n_d is the mass fraction of water dissolved in the magma, and P is the pressure experienced by the magma at depth z. This pressure is equal to the sum of the lithostatic pressure ($=\rho\, g\, z$, where ρ is the density of the country rock) and the external atmospheric pressure, P_a. The mass fraction of exsolved water water n_{ex} is then equal to the total magmatic water content n_t minus the mass fraction remaining in solution n_d.

(a) For magmatic H_2O contents of 1, 0.3 and 0.03 wt.%, calculate the depths at which volatile exsolution commences on Earth, Mars, and Venus. Use information from Table 17.1 of Chapter 17 and assume a country rock density of 2700 kg m^{-3}. What is the issue on Venus?

(b) Assuming that magmas containing 1, 0.3 and 0.03 wt.% total H_2O undergo fragmentation at pressures of ~3.2, 0.84 and 0.06 MPa, respectively, calculate the corresponding fragmentation depths on Earth and Mars.

(c) How much magmatic water would a venusian magma need to contain for (i) volatile exsolution and (ii) an explosive eruption to occur?