**Exercises**

**Chapter 17**

**Exercise 17.1 Cooling of planetary lava flows**

The radiative heat flux from a hot surface at temperature *Tsurf* in an environment having temperature *Ta* is given, per unit surface area, by

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in which ** is the emissivity of the hot body (typically ~0.9 for a lava surface) and ** is the Stefan-Boltzmann constant (5.67 x 10-8 W m-2 K-4).

The convective heat flux from a hot surface due to the presence of atmospheric gases is given per unit surface area by

 ,

where *hc* is the convective heat transfer coefficient, given by

****.

Here, *ka* is the thermal conductivity of the mixture of atmospheric gases, and *L* is the length scale of the cooling body.

For *free* convection (i.e., in the absence of wind), the Nusselt number Nu is given by

 ,

where Ra, Gr, and Pr are, respectively, the Rayleigh, Grashof, and Prandtl numbers.

The Prandtl number (ratio of momentum diffusivity to thermal diffusivity) is given by

 ,

where *a* is the atmospheric kinematic viscosity (*a* = *a*/*a*), and *a* is the atmospheric thermal diffusivity (*a* = *ka*/*aca*), with *a* being the dynamic viscosity of the atmospheric gas, and *a* the specific heat capacity at constant pressure of the atmospheric gas.

The Grashof number (ratio of buoyancy force to viscous force)is given by

 ,

where ** is the volume expansion coefficient of the atmospheric gas, and *g* is the acceleration due to gravity.

When a wind blows over the hot surface at a velocity *w*, *forced* convection conditions exist, and the Nusselt number is instead given by

 ,

in which Re is the Reynolds number of the atmospheric flow, given by

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1. Using the above information, write expressions for convective heat flux *qconv* for free and forced convection, in terms of the atmospheric properties *a*, *ka*, *ca*, *a*. For free convection show that the heat flux per unit area is independent of the length scale of the flow.
2. Using information given in Table 17.1 of Chapter 17, and the atmospheric properties data given in the table below, calculate the radiative heat flux *qrad*, the convective fluxes *qconv* (free and forced), and total heat flux *qtot* from a lava flow surface at several temperatures ranging from ~1500 K to ~100 K above ambient temperature for lava flows on Earth, Mars, Venus, and the Moon. Use a length scale (typically assumed to be the average of flow width and length) *L* of 1000 m, and wind speeds of 1 and 10 m s-1. Plot graphs of radiative and convective heat fluxes against temperature to help visualize the differences in cooling mechanisms between the four environments, and plot total heat flux against temperature for each body to consider the implications for the rate of cooling of flows on these four bodies.

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| --- | --- | --- | --- | --- | --- |
| **Planet** | ***a* (kg m-3)** | ***a* (Pa s)** | ***ka* (W m-1 K-1)** | ***ca* (J kg-1 K-1)** | **** (K-1)** |
| Earth | 1.2 | 2.4 x 10-4 | 0.025 | 1000 | 3.4 x 10-3 |
| Mars | 1.5 x 10-2 | 1.1 x 10-5 | 0.016 | 840 | 4.7 x 10-3 |
| Venus | 60 | 1.8 x 10-4 | 0.057 | 1140 | 1.3 x 10-3 |

**Exercise 17.2 Volatile exsolution and magma fragmentation**

Consider the ascent of basaltic magma through the lithospheres of Earth, Mars and Venus. A simple solubility law for magmatic water in basaltic magma is given by

*nd* = 6.8 x 10-6 *P*0.7

where *nd* is the mass fraction of water dissolved in the magma, and *P* is the pressure experienced by the magma at depth *z*. This pressure is equal to the sum of the lithostatic pressure (=* g z*, where **is the density of the country rock) and the external atmospheric pressure, *Pa*. The mass fraction of exsolved water water *nex* is then equal to the total magmatic water content *nt* minus the mass fraction remaining in solution *nd*.

1. For magmatic H2O contents of 1, 0.3 and 0.03 wt.%, calculate the depths at which volatile exsolution commences on Earth, Mars, and Venus. Use information from Table 17.1 of Chapter 17 and assume a country rock density of 2700 kg m-3. What is the issue on Venus?
2. Assuming that magmas containing 1, 0.3 and 0.03 wt.% total H2O undergo fragmentation at pressures of ~3.2, 0.84 and 0.06 MPa, respectively, calculate the corresponding fragmentation depths on Earth and Mars.
3. How much magmatic water would a venusian magma need to contain for (i) volatile exsolution and (ii) an explosive eruption to occur?