Exercise Problems for Input-Output Analysis: Foundations and Extensions

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Chapter 2, Foundations: Exercise Problems

2.1 Dollar values of last year's interindustry transactions and total outputs for a two-sector economy (agriculture and manufacturing) are as shown below:

$$\mathbf{Z} = \begin{bmatrix} 500 & 350 \\ 320 & 360 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1,000 \\ 800 \end{bmatrix}$$

What are the two elements in the final-demand vector $\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$?

(a) Suppose that f_1 increases by \$50 and f_2 decreases by \$20. What new gross outputs would be necessary to satisfy the new final demands?

(b) Find an approximation to the answer by using the first five terms in the power series, $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + ... + \mathbf{A}^n$.

(c) Find the exact answer using the Leontief inverse.

2.2 Interindustry sales and total outputs in a small three-sector national economy for year t are given in the following table, where values are shown in thousands of dollars. (S_1, S_2 and S_3 represent the three sectors.)

	Interindustry Sales S_1	S_2	S_3	Total Output
S_1	350	0	0	1,000
S_2	50	250	150	500
S_3	200	150	550	1,000

(b) Find the technical coefficients matrix, **A**, and the Leontief inverse matrix, **L**, for this economy.

(c) Suppose that because of government tax policy changes, final demands for the outputs of sectors 1, 2 and 3 are projected for next year (year $t + 1$) to be 1,300, 100, and 200, respectively (also measured in thousands of dollars). Find the total outputs that would be necessary from the three sectors to meet this projected demand, assuming that there is no change in the technological structure of the economy (that is, assuming that the **A** matrix does not change from year t to year $t + 1$).

(d) Find the original (year t) final demands from the information in the table of data. Compare with the projected (year $t + 1$) final demands. Also, compare the original total outputs with the outputs found in part (b). What basic feature of the input-output model do these two comparisons illustrate?

2.3 Using the data of problem 2.1, above, suppose that the household (consumption) expenditures part of final demand is \$90 from sector 1 and \$50 from sector 2. Suppose, further, that payments from sectors 1 and 2 for household labor services were \$100 and \$60, respectively; that total household (labor) income in the economy was \$300; and that household purchases of labor services were \$40. Close the model with respect to households and find the impacts on sectors 1 and 2 of the new final demands in part (b) of problem 2.1, using the Leontief inverse for the new 3×3 coefficient matrix. Assume that these changes occur in the export part of final demand.

2.4 Consider an economy organized into three industries: lumber and wood products, paper and allied products, and machinery and transportation equipment. A consulting firm estimates that last year the lumber industry had an output valued at \$50 (assume all monetary values are in units of \$100,000), 5 percent of which it consumed itself; 70 percent was consumed by final demand; 20 percent by the paper and allied products industry; and 5 percent by the equipment industry. The equipment industry consumed 15 percent of its own products, out of a total of \$l00; 25 percent went to final demand; 30 percent to the lumber industry; 30 percent to the paper and allied products industry. Finally, the paper and allied products industry produced \$50, of which it consumed 10 percent; 80 percent went to final demand; and 5 percent went to the lumber industry, 5 percent to the equipment industry.

(a) Construct the input-output transactions matrix for this economy using these estimates from last year's data. Find the corresponding matrix of technical coefficients and show that the Hawkins-Simon conditions are satisfied.

(b) Find the Leontief inverse for this economy.

A recession in the economy this year is reflected in decreased final demands, reflected in the following table:

Industry	Percent Decrease in Final Demand
Lumber & Wood Products	25
Machinery & Transportation Equipment	10
Paper & Allied Products	5

(a) What would be the total production of all industries required to supply this year's decreased final demand?
(b) Compute the value-added and intermediate output vectors for the new transactions table.

2.5 Consider a simple two-sector economy containing industries A and B. Industry A requires \$2 million worth of its own product and \$6 million worth of Industry B's output in the process of supplying \$20 million worth of its own product to final consumers. Similarly, Industry B requires \$4 million worth of its own product and \$8 million worth of Industry A's output in the process of supplying \$20 million worth of its own product to final consumers.

(a) Construct the input-output transactions table describing economic activity in this economy.
(b) Find the corresponding matrix of technical coefficients and show that the Hawkins-Simon conditions are satisfied.
(c) If in the year following the one in which the data for this model was compiled there were no changes expected in the patterns of industry consumption, and if a final demand of \$15 million worth of good A and \$18 million worth of good B were presented to the economy, what would be the total production of all industries required to supply this final demand as well as the interindustry activity involved in supporting deliveries to this final demand?

2.6 Consider the following transactions table, $\mathbf{Z}$, and total outputs vector, $\mathbf{x}$, for two sectors, A and B:

$$\mathbf{Z} = \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$$

(a) Compute the value-added and final-demand vectors. Show that the Hawkins-Simon conditions are satisfied.
(b) Consider the r-order round-by-round approximation of $\mathbf{x} = \mathbf{Lf}$ to be:

$$\tilde{\mathbf{x}} = \sum_{i=0}^{r} \mathbf{A}^i \mathbf{f} \quad \text{(remember that } \mathbf{A}^0 = \mathbf{I}\text{)}$$

For what value of r do all the elements of $\tilde{\mathbf{x}}$ come within 0.2 of the actual values of $\mathbf{x}$?
(c) Assume that the cost of performing impact analysis on the computer using the round-by-round method is given by:

$$C_r = c_1 r + c_2 (r - 1.5)$$

where r is the order of the approximation (c_1 is the cost of an addition operation and c_2 is the cost of a multiplication operation). Also, assume that $c_1 = 0.5c_2$, that the cost of computing $(\mathbf{I}-\mathbf{A})^{-1}$ exactly is given by $C_e = 20c_2$ and the cost of using this inverse in impact analysis (multiplying it by a final-demand vector) is given by $C_f = c_2$. If we wish to compute the impacts (total outputs) of a particular (arbitrary) final-demand vector to within at least 0.2 of the actual values of $\mathbf{x} = \mathbf{L}\mathbf{f}_a$, where $\mathbf{f}_a$ is an arbitrary final-demand vector, should we use the round-by-round method or should we compute the exact inverse and then perform impact analysis? The idea is to find the least-cost method for computing the solution.

(d) Suppose we had five arbitrary final demand vectors whose impact we wanted to assess. How would you now answer part c?

(e) For what number of final-demand vectors does it not make any difference which method we use in answer to the question in part (c)?

2.7 Consider the following transactions and total output data for an eight-sector economy.

$$\mathbf{Z} = \begin{bmatrix} 8,565 & 8,069 & 8,843 & 3,045 & 1,124 & 276 & 230 & 3,464 \\ 1,505 & 6,996 & 6,895 & 3,530 & 3,383 & 365 & 219 & 2,946 \\ 98 & 39 & 5 & 429 & 5,694 & 7 & 376 & 327 \\ 999 & 1,048 & 120 & 9,143 & 4,460 & 228 & 210 & 2,226 \\ 4,373 & 4,488 & 8,325 & 2,729 & 2,9671 & 1,733 & 5,757 & 14,756 \\ 2,150 & 36 & 640 & 1,234 & 165 & 821 & 90 & 6,717 \\ 506 & 7 & 180 & 0 & 2,352 & 0 & 18,091 & 26,529 \\ 5,315 & 1,895 & 2,993 & 1,071 & 13,941 & 434 & 6,096 & 46,338 \end{bmatrix}$$

$$\mathbf{x}' = \begin{bmatrix} 37,610 & 45,108 & 46,323 & 41,059 & 209,403 & 11,200 & 55,992 & 161,079 \end{bmatrix}$$

a) Compute **A** and **L**.

b) If final demands in sectors 1 and 2 increase by 30 percent while that in sector 5 decreases by 20 percent (while all other final demands are unchanged), what new total outputs will be necessary from each of the eight sectors in this economy?

2.8 Consider the following two-sector input-output tables measure in millions of dollars:

	Manuf.	Services	Final Demand	Total Output
Manufacturing	10	40	50	100
Services	30	25	85	140
Value Added	40	65	135	
Total Output	100	140		240

If labor costs in the services sector increase, causing a 25 percent in value added inputs required per unit of services and labor costs in manufacturing decrease by 25 percent, what are the resulting changes in relative prices of manufactured goods and services?

2.9 For the 2003 U.S. direct requirements table given in Table 2.6, what would be the impact on relative prices if a national corporate income tax increased total value added of primary industries (agriculture and mining) by 10 percent, construction and manufacturing by 15 percent, and all other sectors by 20 percent?

2.10 Consider an input-output economy with three sectors: agriculture, services, and personal computers. The matrix of interindustry transactions and vector of total outputs are given, respectively, by $\mathbf{Z} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$ so that $\mathbf{f} = \mathbf{x} - \mathbf{Zi} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. Notice that this is a closed economy where all industry outputs become inputs. In other words, with the given $\mathbf{x}$, the vector of total value added is found by $\mathbf{v}' = \mathbf{x}' - \mathbf{i}'\mathbf{Z} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ and, of course, gross domestic product is $\mathbf{v}'\mathbf{i} = \mathbf{i}'\mathbf{f} = 0$. Does $\mathbf{L}$ exist for this economy? Suppose we determine that all of the inputs for the personal computers sector are imported and we seek to create a domestic transactions matrix by "opening" the economy to imports, i.e., transfer the value of all inputs to personal computers to final demand. What are the modified values of **Z**, **f** and **v**? What is the new value of gross domestic product? Does **L** exist for this modified representation of the economy? If so, compute it.

Chapter 3, Regional Input-Output Analysis: Exercise Problems

3.1 The data in problem 2.2 described a small national economy. Consider a region within that national economy that contains firms producing in each of the three sectors. Suppose that the technological structure of production of firms within the region is estimated to be the same as that reflected in the national data, but that there is need to import into the region (from producers elsewhere in the country) some of the inputs used in production in each of the regional sectors. In particular, the percentages of required inputs from sectors 1, 2, and 3 that come from within the region are 60, 90, and 75, respectively. If new final demands for the outputs of the regional producers are projected to be 1300, 100, and 200, what total outputs of the three regional sectors will be needed in order to meet this demand?

3.2 The following data represent sales (in dollars) between and among two sectors in regions r and s.

	r		s	
r	40	50	30	45
	60	10	70	45
s	50	60	50	80
	70	70	50	50

In addition, sales to final demand purchasers were $\mathbf{f}^r = \begin{bmatrix} 200 \\ 200 \end{bmatrix}$ and $\mathbf{f}^s = \begin{bmatrix} 300 \\ 400 \end{bmatrix}$. These data are sufficient to create a two-region interregional input-output model connecting regions r and s. If, because of a stimulated economy, household demand increased by \$280 for the output of sector 1 in region r and by \$360 for the output of sector 2 in region r, what are the new necessary gross outputs from each of the sectors in each of the two regions to satisfy this new final demand? That is, find $\Delta\mathbf{x} = \begin{bmatrix} \Delta\mathbf{x}^r \\ \Delta\mathbf{x}^s \end{bmatrix}$ associated with $\Delta\mathbf{f}$.

3.3 Suppose that you have assembled the following information on the dollar values of purchases of each of two goods in each of two regions and on the shipments of each of the two goods between regions:

Purchases in Region r		Purchases in Region s	
$z_{11}^r = 40$	$z_{12}^r = 50$	$z_{11}^s = 30$	$z_{12}^s = 45$
$z_{21}^r = 60$	$z_{22}^r = 10$	$z_{21}^s = 70$	$z_{22}^s = 45$
Shipments of Good 1		**Shipments of Good 2**	
$z_1^{rr} = 50$	$z_1^{rs} = 60$	$z_2^{rr} = 50$	$z_2^{rs} = 80$
$z_1^{sr} = 70$	$z_1^{ss} = 70$	$z_2^{sr} = 50$	$z_2^{ss} = 50$

These data are sufficient to generate the necessary matrices for a two-region multiregional input- output model connecting regions r and s. There will be six necessary matrices— $\mathbf{A}^r$, $\mathbf{A}^s$, $\hat{\mathbf{c}}^{rr}$, $\hat{\mathbf{c}}^{rs}$, $\hat{\mathbf{c}}^{sr}$ and $\hat{\mathbf{c}}^{ss}$. All of these will be 2×2 matrices. If the projected demands for the coming period are $\mathbf{f}^r = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$ and $\mathbf{f}^s = \begin{bmatrix} 40 \\ 60 \end{bmatrix}$, find the gross outputs for each sector in each region necessary to satisfy this new final demand; that is, find $\mathbf{x}^r$ and $\mathbf{x}^s$.

3.4 A federal government agency for a three-region country has collected the following data on input purchases for two sectors, (1) manufacturing and (2) agriculture, for last year, in dollars. These flows are not specific with respect to region of origin; that is, they are of the $z_{ij}^{\bullet s}$ sort. Denote the three regions by *A*, *B* and *C*.

	Region *A*		Region *B*		Region *C*	
	1	2	1	2	1	2
1	200	100	700	400	100	0
2	100	100	100	200	50	0

Also, gross outputs for each of the two sectors in each of the three regions are known. They are:

$$\mathbf{x}^A = \begin{bmatrix} 600 \\ 300 \end{bmatrix}, \ \mathbf{x}^B = \begin{bmatrix} 1,200 \\ 700 \end{bmatrix} \text{ and } \mathbf{x}^C = \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

The agency hires you to advise them on potential uses for this information.

(a) Your first thought is to produce a regional technical coefficients table for each region. Is it possible to construct such tables? If so, do it; if not, why not?
(b) You also consider putting the data together to generate a national technical coefficients table. Is this possible? If so, do it; if not, why not?
(c) Why is it not possible to construct from the given data a three-region multiregional input-output model?
(d) If the federal government is considering spending \$5,000 on manufactured goods and \$4,500 on agricultural products next year, what would you estimate as the national gross outputs necessary to satisfy this government demand?
(e) Compare the national gross outputs for sectors 1 and 2 found in d, above, with the original gross outputs, given in the data set from last year. What feature of the input-output model does this comparison illustrate?

3-5 Consider the following two-region interregional input-output transactions table:

		North			South			
		Agric.(1)	Mining (2)	Constr. & Manuf.(3)	Agric.(1)	Mining (2)	Const. & Manuf. (3)	Total Output
North	Agriculture (1)	277,757	3,654	1,710,816	8,293	26	179,483	3,633,382
	Mining (2)	319	2,412	598,591	15	112	30,921	743,965
	Construction & Manufacturing (3)	342,956	39,593	6,762,703	45,770	3,499	1,550,298	10,931,024
South	Agriculture (1)	7,085	39	98,386	255,023	3,821	1,669,107	3,697,202
	Mining (2)	177	92	15,966	365	3,766	669,710	766,751
	Construction & Manufacturing (3)	71,798	7,957	2,017,905	316,256	36,789	8,386,751	14,449,941

(a) Find the final-demand vectors and the technical coefficients matrices for each region.
(b) Assume that the rising price of imported oil (upon which the economy is 99 percent dependent) has forced the construction and manufacturing industry (sector 3) to reduce total output by 10 percent in the South and 5 percent in the North. What are the corresponding amounts of output available for final demand? (Assume interindustry relationships remain the same, that is, the technical coefficients matrix is unchanged.)
(c) Assume that tough import quotas imposed in Western Europe and the U.S. on this country's goods have reduced the final demand for output from the country's construction and manufacturing industries by 15 percent in the North. What is the impact on the output vector for the North region? Use a full two-region interregional model.
(d) Answer the question in part c, above, ignoring interregional linkages, that is, using the Leontief inverse for the North region only. What do you conclude about the importance of interregional linkages in this aggregated version of this economy?

3.6 Consider the MRIO transactions table for China (2000) given below. Suppose all the inputs to the North region from the South region were replaced with corresponding industry production from the Rest of China region. How would you reflect such a situation in the MRIO model? What would be the impact on total outputs of all regions and sectors for a final demand of ¥100,000 on export demand for manufactured goods produced in the North?

China 2000		North			South			Rest of China		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North	Natural Resources	1,724	6,312	406	188	1,206	86	14	49	4
	Manuf. & Const.	2,381	18,458	2,987	301	3,331	460	39	234	57
	Services	709	3,883	1,811	64	432	138	5	23	5
South	Natural Resources	149	656	42	3,564	8,828	806	103	178	15
	Manuf. & Const.	463	3,834	571	3,757	34,931	5,186	202	1,140	268
	Services	49	297	99	1,099	6,613	2,969	31	163	62
ROC	Natural Resources	9	51	3	33	254	18	1,581	3,154	293
	Manuf. & Const.	32	272	41	123	1,062	170	1,225	6,704	1,733
	Services	4	25	7	25	168	47	425	2,145	1,000
Total Output		16,651	49,563	15,011	27,866	81,253	23,667	11,661	21,107	8,910

3.7 A three-region, five-sector version of the U.S. multiregional input-output economy is shown below (and in Table A4.1-3 of Appendix S4.1 to the next chapter). Suppose that a new government military project is initiated in the western United States which stimulates new final demand in that region of (in millions of dollars) $\Delta\mathbf{f}^{W} = \begin{bmatrix} 0 & 0 & 100 & 50 & 25 \end{bmatrix}'$. What is the impact on total production of all sectors in all three regions of the United States economy stimulated by this final demand in the West?

Five-Sector, Three-Region Multiregional Input–Output Tables for the United States (1963)

Regional Transactions (millions of dollars)

	Agric	Mining	Const. & Manuf.	Services	Transport & Utilities
East					
Agriculture	2,013	0	7,863	44	0
Mining	35	335	3,432	44	843
Const. & Manuf.	2,029	400	78,164	11,561	2,333
Services	1,289	294	19,699	26,574	2,301
Transport.& Util.	225	384	7,232	4,026	3,534
Central					
Agriculture	10,303	0	13,218	97	0
Mining	82	472	8,686	15	1,271
Const. & Manuf.	4,422	1,132	93,816	10,155	2,401
Services	4,952	2,378	21,974	22,358	2,473
Transport.& Util.	667	406	9,296	3,468	4,513
West					
Agriculture	2,915	0	3,452	65	0
Mining	4	292	2,503	0	353
Const. & Manuf.	1,214	466	27,681	4,925	1,015
Services	1,307	721	8,336	10,809	991
Transport.& Util.	338	160	2,936	1,659	1,576

Commodity Trade Flows and Total Outputs (millions of dollars)

	East	West	Central
Agriculture			
East	6,007	2,124	208
West	3,845	28,885	2,521
Central	403	2,922	7,028
Mining			
East	2,904	415	53
West	1,108	10,942	271
Central	71	772	3,996
Const. & Manuf.			
East	158,679	42,150	8,368
West	44,589	201,025	11,778
Central	4,702	6,726	61,385
Services			
East	146,336	16,116	2,955
West	9,328	121,079	3,185
Central	1,939	3,643	58,663
Transp. & Util.			
East	21,434	4,974	263
West	4,396	23,811	1,948
Central	1,009	1,334	9,635
Total Output			
Agriculture	10,259	33,939	9,753
Mining	4,084	12,129	4,319
Const. & Manuf.	207,948	249,840	81,512
Services	157,468	140,850	64,803
Transport.& Util.	26,847	30,130	11,841

3.8 Consider the three-region, five-sector version of an interregional input-output economy of Japan for 1965 given in Table A4.1-1 of Appendix S4.1. Suppose the same final demand vector given in problem 3.7 is placed on goods and services produced in Japan's South region. What is the impact on total production of all sectors in all three regions of Japan of this final demand in the South?

3.9 Consider the year 2000 IRIO model for China, Japan, the United States and an aggregation of other Asian nations including Indonesia, Malaysia, the Philippines, Singapore and Thailand provided in the table below. Assume that annual final demand growth in China is 8 percent, growth in the U.S. and Japan is 4 percent, and that of other Asian nations is 3 percent. Compute the percentage growth in total output corresponding to the growth in final demand.

2000		United States			Japan			China			Rest of Asia		
		Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
U.S.	Nat. Res.	75,382	296,016	17,829	351	4,764	473	174	403	17	103	2,740	83
	Manuf. & Const.	68,424	#######	960,671	160	21,902	3,775	587	8,863	1,710	383	45,066	4,391
	Services	95,115	#######	3,094,357	118	6,695	807	160	1,466	296	197	7,393	953
Japan	Nat. Res.	7	52	53	8,721	78,936	11,206	13	66	2	14	180	27
	Manuf. & Const.	859	41,484	11,337	28,088	#######	484,802	764	20,145	2,809	462	72,258	4,108
	Services	97	4,390	1,424	24,901	662,488	#######	107	2,763	335	270	7,816	1,189
China	Nat. Res.	72	343	147	50	2,316	229	49,496	183,509	15,138	102	2,430	99
	Manuf. & Const.	331	15,657	6,442	93	10,199	1,989	89,384	892,227	181,932	157	15,093	1,237
	Services	38	2,218	1,099	17	1,780	280	25,391	210,469	136,961	23	2,078	132
ROA	Nat. Res.	322	1,068	203	64	11,906	266	64	1,475	14	12,153	92,647	6,402
	Manuf. & Const.	503	56,287	18,129	278	35,418	3,562	1,141	41,496	4,685	23,022	566,274	144,417
	Services	152	4,578	1,921	41	3,982	447	138	3,669	422	15,163	213,470	239,053
TOTAL OUTPUT		468,403	#######	########	140,622	#######	#######	408,153	#######	702,248	173,080	#######	#######

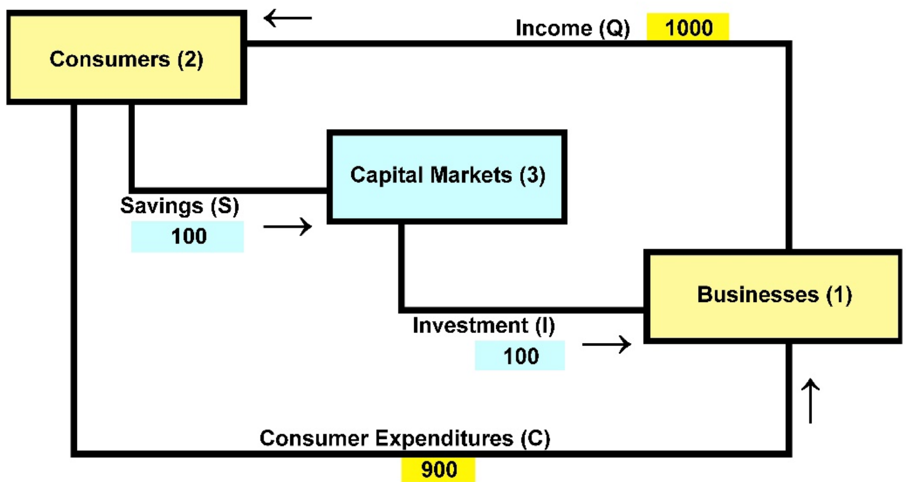
3.10 Assume that you have a very limited computer that can directly determine the inverse of matrices no larger than 5×5 (in practice this might be more like 500×500). Given this limited computer, explain how you could go about determining $\mathbf{L}$ for

$$\mathbf{A} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0.1 & 0.3 & 0.1 \\ 0.3 & 0 & 0 & 0.1 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.1 & 0.2 \end{bmatrix}.$$

(a) Compute the Leontief inverse in this manner.
(b) What implications does such a procedure have for the computation of very large matrices (e.g., $n > 10{,}000$)?

Chapter 4, Basic Data: Exercise Problems

4.1 Consider a macroeconomy show below where transactions are measured in millions of dollars. Create the corresponding set of "T" accounts for production, income and capital transactions. Write the account balance equations.



4.2 For the macroeconomy from problem 4.1, add a capital consumption allowance to account for depreciation of capital investments of 10 percent of total investment (I). Also add a "rest of world" account to accommodate purchases of imports of $75 million, sales of exports of $50 million, and savings made available to capital markets from overseas lenders of $25 million (resulting in a new total amount of capital available for businesses of $125 million). Construct the modified set of "T" accounts and the corresponding balance equations.

4.3 The national economic balance sheet for an economy is given by the following

Debits					Economic Transaction	Credits				
Prod.	Cons.	Capital Accum.	Govt	Rest of World		Prod.	Cons.	Capital Accum.	Govt	Rest of World
	475				Consumption Goods (C)	475				
		54			Capital Goods (I)	54				
				46	Exports (X)	46				
46					Imports (M)					46
554					Income (Q)		554			
		-29			Depreciation (D)		-29			
	30				Savings (S)			30		
			25		Govt. Expenditures (G)	25				
	20				Taxes (T)				20	
		5			Govt Deficit Spending (B)				5	
600	525	30	25	46	Totals	600	525	30	25	46

(a) Write the complete set of macro balance equations for this economy.

(b) Construct the matrix representation of the consolidated national accounts

4.4 Consider the following 4 sector input-output transactions table for the year 2015 along with industry prices for 2015 and 2020.

	Industry Transactions				Total	Price	Price
	1	2	3	4	Output	Year 2000	Year 2005
1	24	86	56	64	398	2	5
2	32	15	78	78	314	3	6
3	104	49	62	94	469	5	9
4	14	16	63	78	454	7	12

Compute the matrices of interindustry transactions and technical coefficients as well as the vector of total outputs deflated to year 2015 value terms.

4.5 Consider the transactions data given in problem 2.8. One way of assessing the effects of aggregation is as follows. Using a final-demand vector of all 1's, determine the effect on total outputs throughout the entire economy (i.e., summed over all the sectors) of the following set of increasingly aggregated models. (Remember to aggregate the final-demand vector appropriately each time you aggregate the sectors.)

- Case 1 (8×8) No sectoral aggregation
- Case 2 (7×7) Combine sector 6 with sector 2
- Case 3 (6×6) Also combine sector 5 with sector 1
- Case 4 (5×5) Also combine sector 8 with sector 3
- Case 5 (4×4) Also combine sector 7 with previously combined 6 and 2
- Case 6 (3×3) Also combine sector 4 with previously combined 5 and I

4.6 Consider the seven-sector input-output table of technical coefficients for the U.S. economy (1972) given in Appendix SD1 (located on the supplemental resources website). Given a vector of final demands of

$$\Delta \mathbf{f} = \begin{bmatrix} 100 & 100 & 100 & 100 & 100 & 100 & 100 \end{bmatrix}'$$

compute the first order and total aggregation bias associated with combining agriculture with mining, construction with manufacturing, and transportation-utilities with services and other sectors to yield a new three-sector model.

4.7 Consider the following national accounting equations:

(1) $Q + M = C + I + X + G$

(2) $C + S + T = Q + D$

(3) $L + I + D + B = S$

(4) $X = M + L$

(5) $G = T + B$

where $Q =$ total consumer income payments; $M =$ purchases of imports; $C =$ total sales of consumption goods; $S =$ total consumer savings; $T =$ total taxes paid to government;

$I =$ total purchases of capital goods; $D =$ total capital consumption allowance (depreciation); $L =$ net lending from overseas; $B =$ total government deficit spending; $X =$ total sales of exports; $G =$ total government purchases and the following are known: $Q = 500$, $M = 75$, $S = 60$, $T = 20$, $D = 10$, $L = 20$, and $B = 10$. Write the consolidated table of national accounts represented in matrix form.

4.8 Consider the following table of national accounts.

	Prod.	Cons.	Cap.	ROW	Govt.	Total
Prod.		410	80	55	30	575
Cons.	500		-10			490
Cap.		60				60
ROW	75		-20			55
Govt.		20	10			30
Total	575	490	60	55	30	

Suppose the following tables become available providing the interindustry supply and use detail for this economy.

Use of commodities by industries:

		Industry			Total Intermed. Output
		Nat. Res.	Manuf.	Serv.	
Commodity	Agriculture	20	12	18	50
	Mining	5	30	12	47
	Manufacturing	10	13	11	34
	Services	12	17	40	69

Final uses of commodity production:

	Households	Government	Investment	Exports
Agriculture	30	6	16	5
Mining	60	9	16	17
Manufacturing	50	3	40	22
Services	70	12	8	11
Totals	210	30	80	55

Supply of commodities by industries:

		Commodity				Total Industry Output
		Agric.	Mining	Manuf.	Services	
Industry	Natural Resources	99			10	109
	Manufacturing	8	143	137	10	298
	Services		6	12	150	168
	Total Commodity Output	107	149	149	170	575

Construct a consolidated set of supply and use accounts including the sector detail for interindustry transactions.

4.9 We define an input-output economy with $\mathbf{Z} = \begin{bmatrix} 500 & 0 & 0 \\ 50 & 300 & 150 \\ 200 & 150 & 550 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1,000 \\ 750 \\ 1,000 \end{bmatrix}$. Suppose this is a "U.S. style" input-output table in which interindustry transactions include competitive imports but the sum of all imports across all industries of a particular product is included as a negative component of final demand.

(a) If the vector of total value of competitive imports is found to be $\mathbf{m} = \begin{bmatrix} 150 \\ 105 \\ 210 \end{bmatrix}$, using the assumption of import similarity, compute the domestic transactions matrix where competitive imports are removed from interindustry transactions. Compute the corresponding **A** and **L**.

(b) If we subsequently learn that $\mathbf{M} = \begin{bmatrix} 100 & 0 & 0 \\ 25 & 50 & 30 \\ 25 & 50 & 100 \end{bmatrix}$, compute the domestic transactions matrix and the corresponding **A** and **L**.

(c) Now compute the mean absolute deviation (the average of the absolute value differences) between the total requirements matrices computed in (a) and (b).

4.10 Consider the three-region, three-sector 2000 Chinese interregional model specified in problem 3.6. Aggregate regions 1 and 2 and leave region 3 unaggregated to yield a two-region model. Calculate the aggregation bias measured as a percent of gross outputs with a reference vector of final demands given by $\tilde{\mathbf{f}} = \begin{bmatrix} 100 & 100 & \cdots & 100 \end{bmatrix}'$ for the unaggregated model.

4.11 Consider a six-sector input-output table for Nepal for the year 2013, defined by the matrix of interindustry transactions, **Z**, and vector of total outputs, **x**, in the following:

Interindustry Transactions	Agric.	Mining	Manuf.	Const.	Utilities	Services	Total Output
Agriculture	774	0	1,149	45	0	719	9,766
Mining	0	0	87	0	119	1	252
Manufacturing	1,037	22	2,029	166	1,654	1,743	13,015
Construction	47	5	109	47	79	376	834
Utilities	11	1	13	6	2	394	3,963
Services	780	22	781	201	377	3,443	20,446

We presume this table of transactions includes both domestic transactions and competitive imports, but we know only the total value of all imports of each commodity, defined by $\mathbf{m} = \begin{bmatrix} 68 & 48 & 3,227 & 65 & 1 & 457 \end{bmatrix}'$. As a first case, using an assumption of *import similarity*, compute the estimated matrix of domestic transactions and the corresponding matrices of domestic direct requirements and of total requirements.

As a second case, suppose we are provided with more detailed estimates of imports—a matrix of the competitive imports associated with each individual interindustry transaction rather than just the total level of imports of each commodity across the entire economy, defined by

$$\mathbf{M} = \begin{bmatrix} 18 & 0 & 30 & 1 & 0 & 18 \\ 0 & 0 & 20 & 0 & 28 & 0 \\ 406 & 16 & 793 & 124 & 672 & 1,216 \\ 5 & 0 & 12 & 5 & 10 & 33 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 19 & 2 & 54 & 10 & 22 & 350 \end{bmatrix}$$

. An element of $\mathbf{M}$, m_{ij} , defines the value of the intermediate inputs of sector i to sector j that is filled by imports. Using this new information, compute the matrix of domestic transactions and the corresponding matrices of domestic direct requirements and of total requirements. Compute the mean absolute deviation between the estimates of these two cases of domestic direct requirements and of total requirements.

Chapter 5, Commodity by Industry: Exercise Problems

5.1 In a system of commodity-by-industry accounts, suppose we have defined three commodities and two industries. The make matrix, **V**, and the use matrix, **U**, are given below.

$$\mathbf{U}=\begin{bmatrix}3 & 5\\ 2 & 7\\ 2 & 3\end{bmatrix},\quad \mathbf{V}=\begin{bmatrix}15 & 5 & 10\\ 5 & 25 & 0\end{bmatrix}$$

(a) Compute the vector of commodity final demands, the vector of industry value added inputs, the vector of total commodity outputs, and the vector of total industry outputs.
(b) Assuming an "industry-based" technology, compute the industry-by-commodity total requirements matrix.

5.2 Consider the following system of commodity and industry accounts for a region:

		Commodities		Industries		Final	Total
		1	2	1	2	Demand	Output
Commodities	1			1	2	7	10
	2			3	4	3	10
Industries	1	10	2				12
	2	0	8				8
Value Added				8	2	10	
Total Inputs		10	10	12	8		

(a) Compute the commodity-by-industry matrix of direct requirements.
(b) Compute the industry-by-commodity total requirements matrices under both assumptions of industry-based and commodity-based technology.
(c) If a new naval facility is being constructed in the region, represented by commodity final demands $\Delta\mathbf{e}=\begin{bmatrix}6 & 5\end{bmatrix}'$, what would be the total production of each industry in the region required to support this facility? Do this for both technology assumptions.

5.3 Consider again the system of accounts given in problem 5.1. Suppose we can split **V** into two components, $\mathbf{V}_1=\begin{bmatrix}5 & 5 & 5\\ 5 & 5 & 0\end{bmatrix}$ and $\mathbf{V}_2=\begin{bmatrix}10 & 0 & 5\\ 0 & 20 & 0\end{bmatrix}$ such that $\mathbf{V}=\mathbf{V}_1+\mathbf{V}_2$. Which of the two "mixed technology" assumptions that were covered in sections 5.7.1 and 5.7.2 can we invoke in computing the industry-by-commodity total requirements matrix for this system of accounts? Compute the matrix. Why can we not invoke the other assumption? Can we invoke either the commodity-based or industry-based technology assumptions?

5.4 Use both mixed technology assumptions in deriving industry-by-commodity total requirements matrices for the system of accounts given in problem 5.2.

5.5 In a system of commodity-by-industry accounts, suppose we have defined four commodities and three industries. The make matrix, **V**, and the use matrix, **U**, are given below.

$$\mathbf{U} = \begin{bmatrix} 20 & 12 & 18 \\ 5 & 30 & 12 \\ 10 & 13 & 11 \\ 12 & 17 & 40 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 99 & 0 & 0 & 10 \\ 8 & 143 & 137 & 10 \\ 0 & 6 & 12 & 150 \end{bmatrix}$$

(a) Is it possible to compute the total commodity-by-industry total requirements matrix using the assumption of industry-based technology? If not, why not. If so, calculate that matrix.
(b) Using the assumption of an industry-based technology, calculate the industry-by-commodity requirements matrix for commodity driven demand.

(c) Aggregate the first two commodities to one in the make and use matrices. Assume that you can decompose the resulting aggregated **V** into $\mathbf{V}_1$ and $\mathbf{V}_2$, where $\mathbf{V}_1 = \begin{bmatrix} 99 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 30 \end{bmatrix}$. Assume a commodity-based technology for $\mathbf{V}_1$ and an industry-based technology for $\mathbf{V}_2$. Calculate the four total requirements matrices (i.e., commodity-by-commodity, industry-by-commodity, commodity-by-industry and industry-by-industry) to be used with commodity-driven demand calculations.

5.6 The numerical results in section 5.7.3 illustrate that column sums of both the **R** and **T** matrices are one.
(a) Prove that $\mathbf{i}'\mathbf{C}_1 = \mathbf{i}'$ and $\mathbf{i}'\mathbf{R} = \mathbf{i}'$.
(b) Prove that $\mathbf{i}'\mathbf{D}_1 = \mathbf{i}'$ and $\mathbf{i}'\mathbf{T} = \mathbf{i}'$.

5.7 For the make and use matrices specified in problem 5.5, assume that the three industries are: Agriculture, Oil Production, and Manufacturing. The four commodities are Agricultural Products, Crude Oil, Natural Gas, and Manufactured Products. We can interpret this as meaning in this case that natural gas is considered a secondary product of the oil industry. For a final demand of 100 of manufactured products what levels of oil industry output are generated and how much natural gas production is generated to satisfy this final demand?

5.8 Consider the following make and use matrices:

$$\mathbf{U} = \begin{bmatrix} 20 & 15 & 18 \\ 5 & 30 & 12 \\ 10 & 16 & 11 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 30 & 0 & 0 \\ 10 & 50 & 35 \\ 0 & 25 & 150 \end{bmatrix}$$

Compute the corresponding commodity-by-commodity transactions table using the assumption of commodity-based technology. Notice that there are negative elements. Use the iterative procedure developed in Appendix 5.2 to generate a revised commodity-by-commodity transactions table that includes no negative entries.

5.9 Consider the use and make matrices for the U.S. input-output tables for 2003 shown below and used to construct an industry-based technology commodity-by-industry **A** and **L**. Also below is a table providing the detail of the components of total final demand. Note that the total final demand entry for mining is negative due to a negative trade balance, i.e., the value of net exports (exports minus imports) is negative and is sufficiently large to offset other components of final demand to render total final demand negative. Suppose that the value for total imports of manufactured goods is projected to increase by $1 trillion from its 2003 value with, for simplicity, all other elements of total final demand remaining identical to those for 2003. What is the impact on gross national product and on total output of all sectors of the economy?

US Use Table for 2003	1	2	3	4	5	6	7
1 Agriculture	61,946	1	1,270	147,559	231	18,453	2,093
2 Mining	441	33,299	6,927	174,235	89,246	1,058	11,507
3 Construction	942	47	1,278	8,128	10,047	65,053	48,460
4 Manufacturing	47,511	22,931	265,115	1,249,629	132,673	516,730	226,689
5 Trade, Transport & Utils	24,325	13,211	100,510	382,630	190,185	297,537	123,523
6 Services	25,765	42,276	147,876	509,084	490,982	2,587,543	442,674
7 Other	239	1,349	2,039	48,835	35,110	83,322	36,277

US Make Table fo 2003	1	2	3	4	5	6	7
1 Agriculture	273,244	0	0	67	0	1,748	0
2 Mining	0	232,387	0	10,843	0	0	0
3 Construction	0	0	1,063,285	0	0	0	0
4 Manufacturing	0	0	0	3,856,583	0	30,555	3,278
5 Trade, Transport & Utils	0	570	0	0	2,855,126	41	957
6 Services	0	475	0	0	133	9,136,001	3,278
7 Other	3,359	896	0	3,936	104,957	323,996	1,827,119

Commodity Final Demands for U.S. 2003 Input-Output Tables

Commodity\Final Demand	Personal consumption expenditures	Private fixed investment	Change in private inventories	Exports of goods and services	Imports of goods and services	Government consumption expenditures and gross investment	Total Final Demand
Agriculture	47,922	-	175	24,859	(26,769)	(1,136)	45,050
Mining	72	35,698	1,912	4,739	(125,508)	702	(82,384)
Construction	-	704,792	-	71	-	224,468	929,331
Manufacturing	1,301,616	573,197	8,983	506,780	(1,075,128)	94,705	1,410,152
Trade, Transportation & Utilities	1,549,792	125,271	2,994	131,884	8,065	10,289	1,828,294
Services	4,780,516	303,426	461	175,546	(44,060)	30,256	5,246,145
Other	80,963	(75,404)	(15,748)	98,989	(177,578)	1,716,238	1,627,459
Total	7,760,881	1,666,980	(1,224)	942,868	(1,440,979)	2,075,522	11,004,047

Chapter 6, Multipliers: Exercise Problems

6.1 Rank sectors in terms of their importance as measured by output multipliers in each of the economies represented by the data in problems 2.1 through 2.9 (include problem 2.10 if you did it.)

6.2 Consider one (or more) of the problems in Chapter 2. Using output multipliers, from problem 6.1, in conjunction with the new final demands in the problem in Chapter 2, derive the total value of output (across all sectors) associated with the new final demands. Compare your results with the total output obtained by summing the elements in the gross output vector which you found as the solution to the problem in Chapter 2. [In matrix notation, this is comparing $\mathbf{m}(o)\Delta\mathbf{f}$ with $\mathbf{i}'\Delta\mathbf{x} = \mathbf{i}'\mathbf{L}\Delta\mathbf{f}$; we know that they must be equal, since output multipliers are the column sums of the Leontief inverse— $\mathbf{m}(o) = \mathbf{i}'\mathbf{L}$.]

6.3 Using the data in problem 2.3, find output multipliers and both type I and type II income multipliers for the two sectors. Check that the ratio of the type II to the type I income multiplier is the same for both sectors.

6.4 You have assembled the following facts about the two sectors that make up the economy of a small country that you want to study (data pertain to the most recent quarter). Total interindustry inputs were $50 and $100, respectively, for Sectors 1 and 2. Sector 1's sales to final demand were $60 and Sector 1's total output was $100. Sector 2's sales to Sector 1 were $30 and this represented 10 percent of Sector 2's total output. After national elections are held, it may turn out that different government policy will be forthcoming during the first quarter of the coming year.

(a) In which of the two sectors does an increase of $100 in government purchases have the larger effect?
(b) How much larger is it than if the $100 were spent on purchases of the other sector?

6.5 Consider an input output economy defined by $\mathbf{Z} = \begin{bmatrix} 140 & 350 \\ 800 & 50 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1{,}000 \\ 1{,}000 \end{bmatrix}$.

(a) In the situation depicted in that question, if you were asked to design an advertising campaign to stimulate export sales of one of the goods produced in the country, would you concentrate your efforts on the product of sector 1 or of sector 2 or on some combination of the two? Why?
(b) If labor input coefficients for the two sectors in the region were found to be $a_{31} = 0.1$ and $a_{32} = 0.18$, how might your answer to part (a) of this question be changed, if at all?

6.6 Using the elements in the full two-region interregional Leontief inverse from problem 3.2, find:

(a) Simple intraregional output multipliers for sectors 1 and 2 [$\mathbf{m}(o)^{rr}$ and $\mathbf{m}(o)^{ss}$, as in (6.25) and (6.26)];

(b) Simple national (total) output multipliers for sectors 1 and 2 [$\mathbf{m}(o)^{r}$ and $\mathbf{m}(o)^{s}$, as was done in (6.30) in the text];

(c) Sector-specific simple national output multipliers for sectors 1 and 2 in regions r and s. This means finding the four multipliers in $\mathbf{m}(o)^{\bullet r} = \left[m(o)_{11}^{\bullet r} \quad m(o)_{21}^{\bullet r} \quad m(o)_{12}^{\bullet r} \quad m(o)_{22}^{\bullet r} \right]$ and $\mathbf{m}(o)^{\bullet s}$, defined similarly.

6.7 On the basis of the results in problem 6.6, above:

(a) For which sector's output does new final demand produce the largest total intraregional output stimulus in region r? In region s?

(b) For which sector in which region does an increase in final demand have the largest national (two-region) impact?

(c) To increase the output of sector 1 nationally (i.e., in both regions), would it be better to institute policies that would increase household demand in region r or in region s?

(d) Answer question (c) if the objective is now to increase sector 2 output nationally.

6.8 Answer problems 6.6 and 6.7, above, for the multiregional case, using the elements in $(\mathbf{I}-\mathbf{CA})^{-1}\mathbf{C}$ from problem 3.3.

6.9 The government in problem 3.4 is interested in starting an overseas advertising and promotion campaign aimed at increasing export sales of the products of the country. There is specialization of production in the regions of the country; in particular, the products are shown in the table below:

	Region A	Region B	Region C
Manufacturing	Scissors	Cloth	Pottery
Agriculture	Oranges	Walnuts	None

For which product (or products) would an increase in export sales cause the greatest stimulation of the national economy?

6.10 If you have software (or patience), find $|(\mathbf{I}-\bar{\mathbf{A}})| / |(\mathbf{I}-\mathbf{A})|$ for our numerical example in which $\mathbf{A} = \begin{bmatrix} .15 & .25 \\ .20 & .05 \end{bmatrix}$ and $\bar{\mathbf{A}} = \left[\begin{array}{cc|c} .15 & .25 & .05 \\ .20 & .05 & .40 \\ \hline .30 & .25 & .05 \end{array}\right]$, demonstrating that it is equal to $(1/g) =$ 1.29, as in Appendix 6.2.

Chapter 7, Supply, Linkages, Important Coefficients: Exercise Problems

7.1 The centrally planned economy of Czaria is involved in its planning for the next fiscal year. The technical coefficients and total industry outputs for Czaria are given below:

	1	2	3	4	Total Output
1. Agriculture	0.168	0.155	0.213	0.212	12,000
2. Mining	0.194	0.193	0.168	0.115	15,000
3. Military Manufacturing	0.105	0.025	0.126	0.124	12,000
4. Civilian Manufacturing	0.178	0.101	0.219	0.186	16,000

(a) Compute the output inverse for this economy.
(b) If next year's value-added inputs for agriculture, mining, military manufactured products and civilian manufacturing in Czaria are projected to be \$4,558 million, \$5,665 million, \$2,050 million and \$5,079 million, respectively, compute the projected GDP for Czaria next year.
(c) Compute the new total gross production for each economic sector. Note that this is the "old view" of the Ghosh model as described in section 7.1.1.

7.2 Consider a case where $\mathbf{Z} = \begin{bmatrix} 13 & 75 & 45 \\ 53 & 21 & 48 \\ 67 & 68 & 93 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 130 \\ 150 \\ 220 \end{bmatrix}$ for a base year.

(a) If final demands for the next year are projected to be $\mathbf{f}^1 = \begin{bmatrix} 200 \\ 300 \\ 500 \end{bmatrix}$ and the change in interindustry transactions is expected to be $\Delta\mathbf{Z} = \begin{bmatrix} 0 & 5 & 0 \\ 10 & 0 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ what is the mean absolute percentage difference (MAPD) between the output coefficients for the base year and next year?
(b) Now compute MAPD between the corresponding output inverses.

7.3 For input-output transactions matrix of $\mathbf{Z} = \begin{bmatrix} 384 & 520 & 831 \\ 35 & 54 & 530 \\ 672 & 8 & 380 \end{bmatrix}$ and total outputs of $\mathbf{x} = \begin{bmatrix} 2,500 \\ 1,200 \\ 3,000 \end{bmatrix}$ for a base year, if additional growth in value added for the next year is projected to result in $\mathbf{v}^{new} = \begin{bmatrix} 2,000 \\ 1,000 \\ 1,500 \end{bmatrix}$, what are the price changes of output for the three industries for the new year relative to the base year?

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7.4 For the economy shown in problem 7.3, compute the value-added coefficients for next year using the supply model. Compute **L** and show that the Leontief price model from Chapter 2 produces the same relative price changes of industrial output for the new year relative to the base year as found in problem 7.3.

7.5 Consider the case of $\mathbf{Z} = \begin{bmatrix} 418 & 687 & 589 & 931 \\ 847 & 527 & 92 & 654 \\ 416 & 702 & 911 & 763 \\ 263 & 48 & 737 & 329 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 2,000 \\ 3,000 \\ 2,500 \\ 1,500 \end{bmatrix}$.

(a) Compute the direct and total backward linkages.
(b) Compute the direct and total forward linkages.

7.6 Consider the three region IRIO table for Japan given in problem in Table A4.1.1 of Appendix S4.1. Using the measure of spatial backward linkage of $B(d)^{rr} = (1/n)\mathbf{i}'\mathbf{A}^{rr}\mathbf{i}$ (and analogous measures for direct forward and total backward and forward linkage), which of the three regions is the "least backward linked" to the other regions and, similarly, which region is the least "forward linked"?

7.7 Consider the 2005 U.S. input-output table shown below.

(a) If the agriculture sector were hypothetically extracted from the economy, what would be the decrease in total output of the economy?
(b) Which of the sectors would create the largest decrease in total output if it were hypothetically extracted?

US Technical Coefficients 2005	1	2	3	4	5	6	7	Tot. Output
1 Agriculture	0.2258	0.0000	0.0015	0.0384	0.0001	0.0017	0.0007	312,754
2 Mining	0.0027	0.1432	0.0075	0.0675	0.0367	0.0004	0.0070	396,563
3 Construction	0.0051	0.0002	0.0010	0.0018	0.0037	0.0071	0.0215	1,302,388
4 Manufacturing	0.1955	0.0877	0.2591	0.3222	0.0547	0.0566	0.1010	4,485,529
5 Trade, Transport & Utilities	0.0819	0.0422	0.1011	0.0994	0.0704	0.0334	0.0487	3,355,944
6 Services	0.0843	0.1276	0.1225	0.1172	0.1760	0.2783	0.2026	10,477,640
7 Other	0.0099	0.0095	0.0093	0.0219	0.0215	0.0188	0.0240	2,526,325

7.8 Consider an economy with $\mathbf{Z} = \begin{bmatrix} 8 & 64 & 89 \\ 28 & 44 & 77 \\ 48 & 24 & 28 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 300 \\ 250 \\ 200 \end{bmatrix}$. Examine element a_{13} for "inverse importance" if the criteria are:

(a) $\alpha = 30$ and $\beta = 5$—that is, if a 30 percent change in a_{13} generates a 5 percent change in one or more elements in the associated Leontief inverse.

(b) $\alpha = 20$ and $\beta = 10$.

(c) $\alpha = 10$ and $\beta = 10$.

This illustrates the sensitivity of the results to the values of α and β specified by the analyst.

7.9 Create a supply-driven model for the U.S. economy for 2005 using the data that are presented in problem 7.7. Determine the sensitivity of the national economy to an interruption in a scarce-factor input—for example, a strike—in one of the sectors.

7.10 Using the input-output data for the United States presented in the Supplemental Resources to this text (Appendix SD1, described in Appendix B), find both the direct and the direct and indirect forward and backward linkages for the sectors in the U.S. economy and examine how these linkages may have changed over time.

Chapter 8, Structural Decomposition Analysis: Exercise Problems

8.1 Consider an input-output economy specified at two points in time, t^0 and t^1 by

$$\mathbf{Z}^0 = \begin{bmatrix} 10 & 20 & 30 \\ 5 & 2 & 25 \\ 20 & 40 & 60 \end{bmatrix},\ \mathbf{f}^0 = \begin{bmatrix} 60 \\ 40 \\ 55 \end{bmatrix},\ \mathbf{Z}^1 = \begin{bmatrix} 15 & 25 & 40 \\ 12 & 7.5 & 30 \\ 10 & 30 & 40 \end{bmatrix},\ \mathbf{f}^1 = \begin{bmatrix} 75 \\ 55 \\ 40 \end{bmatrix}$$

We seek to measure how the economy has changed in structure over the period. Compute for each sector the change in total output between the two years that was attributable to changing final demand or to changing technology.

8.2 Consider input-output data for the U.S. economy provided below for the years 1972 and 2002. Compute the changes in total output between 1972 and 2002 for all sectors attributed to changes in final demand and to changes in technology.

A and x for US, 2002		1	2	3	4	5	6	7	Tot. Output
1	Agriculture	0.2637	0.0020	0.0028	0.0374	0.0007	0.0008	0.0008	270,514
2	Mining	0.0032	0.0467	0.0097	0.0377	0.0226	0.0005	0.0040	184,516
3	Construction	0.0040	0.0336	0.0007	0.0030	0.0053	0.0078	0.0186	967,568
4	Manufacturing	0.1502	0.0942	0.2399	0.3464	0.0645	0.0464	0.0939	3,850,417
5	Trade, Transport & Utils	0.0868	0.0676	0.0960	0.0920	0.0816	0.0302	0.0475	2,811,865
6	Services	0.1310	0.2416	0.1436	0.1349	0.1813	0.2640	0.1954	8,948,582
7	Other	0.0098	0.0159	0.0083	0.0160	0.0276	0.0179	0.0203	2,146,282

A and X for US, 1972		1	2	3	4	5	6	7	Tot. Output
1	Agriculture	0.3141	0.0003	0.0028	0.0542	0.0010	0.0053	0.0012	83,955
2	Mining	0.0019	0.0542	0.0091	0.0296	0.0160	0.0002	0.0020	30,386
3	Construction	0.0069	0.0282	0.0003	0.0043	0.0156	0.0263	0.0166	165,998
4	Manufacturing	0.1436	0.0943	0.3522	0.3771	0.0407	0.0892	0.0078	761,194
5	Trade, Transport & Utils	0.0616	0.0481	0.1043	0.0786	0.0980	0.0442	0.0202	377,389
6	Services	0.0865	0.1471	0.0686	0.0591	0.1157	0.1621	0.0105	522,215
7	Other	0.0023	0.0063	0.0042	0.0117	0.0118	0.0096	0.0033	161,207

8.3 Consider an input-output economy specified by

$$\mathbf{Z}^0 = \begin{bmatrix} 10 & 20 & 25 \\ 15 & 5 & 30 \\ 30 & 40 & 5 \end{bmatrix},\ \mathbf{f}^0 = \begin{bmatrix} 45 \\ 30 \\ 25 \end{bmatrix},\ \mathbf{Z}^1 = \begin{bmatrix} 15 & 30 & 37.5 \\ 22.5 & 7.5 & 45 \\ 45 & 60 & 7.5 \end{bmatrix},\ \mathbf{f}^1 = \begin{bmatrix} 67.5 \\ 45 \\ 37.5 \end{bmatrix}$$

Using any of the basic structural decomposition formulations, determine how the economy has changed in structure from year 0 to year 1.

8.4 Again, consider two observations on an input-output economy specified by

$$\mathbf{Z}^0 = \begin{bmatrix} 12 & 15 & 35 \\ 24 & 11 & 30 \\ 36 & 50 & 8 \end{bmatrix}, \ \mathbf{f}^0 = \begin{bmatrix} 50 \\ 35 \\ 26 \end{bmatrix}, \ \mathbf{Z}^1 = \begin{bmatrix} 20 & 30 & 45 \\ 35 & 23 & 50 \\ 50 & 65 & 24 \end{bmatrix}, \ \mathbf{f}^1 = \begin{bmatrix} 55 \\ 50 \\ 60 \end{bmatrix}$$

Compute the alternative structural decompositions in Table 8.1 for the change between year 0 to year 1 for this economy.

8.5 Using the input-output economy specified in problem 8.4, assume that the final demand vectors can be specified with two components: $\mathbf{F}^0 = \begin{bmatrix} \mathbf{f}_1^0 \ \mathbf{f}_2^0 \end{bmatrix} = \begin{bmatrix} 20 & 30 \\ 15 & 20 \\ 12 & 14 \end{bmatrix}$ and $\mathbf{F}^1 = \begin{bmatrix} \mathbf{f}_1^1 \ \mathbf{f}_2^1 \end{bmatrix} = \begin{bmatrix} 25 & 30 \\ 30 & 20 \\ 35 & 25 \end{bmatrix}$. Provide the sector-specific and economy-wide decomposition results with additional details for sectoral technology and final-demand decomposition of level, mix and distribution.

Chapter 9, Nonsurvey Methods, Fundamentals: Exercise Problems

9.1 Consider the following U.S. input-output tables for 1997[1], 2003 and 2005.

US Use 1997	1	2	3	4	5	6	7	Imports
1 Agriculture	74,938	15	1,121	150,341	2,752	13,400	11	(23,123)
2 Mining	370	19,461	4,281	112,513	53,778	5,189	30	(64,216)
3 Construction	1,122	29	832	7,499	11,758	50,631	27	-
4 Manufactuirng	49,806	19,275	178,903	1,362,660	169,915	418,412	1,914	(765,454)
5 Trade Transport & Utilities	21,650	11,125	76,056	380,272	199,004	224,271	612	6,337
6 Services	32,941	45,234	107,723	483,686	545,779	1,592,426	3,801	(16,942)
7 Other	63	781	422	33,905	19,771	26,730	-	(126,350)

US Make 1997	1	2	3	4	5	6	7	Industry Output
1 Agriculture	284,511	-	65	356	455	1,152	-	286,539
2 Mining	-	158,239	109	9,752	295	258	-	168,653
3 Construction	-	-	670,210	-	-	-	-	670,210
4 Manufactuirng	-	727	1,258	3,703,275	39,720	36,034	3,669	3,784,683
5 Trade Transport & Utilities	556	381	21,393	15,239	2,201,532	141,674	-	2,380,776
6 Services	-	410	54,850	1,306	109,292	6,444,098	1,821	6,611,778
7 Other	-	-	6,206	-	-	7,010	947,023	960,238
Commodity Output	285,067	159,757	754,091	3,729,928	2,351,295	6,630,226	952,513	14,862,876

US Use 2003	1	2	3	4	5	6	7	Imports
1 Agriculture	61,946	1	1,270	147,559	231	18,453	2,093	(26,769)
2 Mining	441	33,299	6,927	174,235	89,246	1,058	11,507	(125,508)
3 Construction	942	47	1,278	8,128	10,047	65,053	48,460	-
4 Manufactuirng	47,511	22,931	265,115	1,249,629	132,673	516,730	226,689	(1,075,128)
5 Trade Transport & Utilities	24,325	13,211	100,510	382,630	190,185	297,537	123,523	8,065
6 Services	25,765	42,276	147,876	509,084	490,982	2,587,543	442,674	(44,060)
7 Other	239	1,349	2,039	48,835	35,110	83,322	36,277	(177,578)

US Make 2003	1	2	3	4	5	6	7	Output
1 Agriculture	273,244	-	-	67	-	1,748	-	275,058
2 Mining	-	232,387	-	10,843	-	-	-	243,231
3 Construction	-	-	1,063,285	-	-	-	-	1,063,285
4 Manufactuirng	-	-	-	3,856,583	-	30,555	3,278	3,890,416
5 Trade Transport & Utilities	-	570	-	-	2,855,126	41	957	2,856,693
6 Services	-	475	-	-	133	9,136,001	3,278	9,139,886
7 Other	3,359	896	-	3,936	104,957	323,996	1,827,119	2,264,263
Commodity Output	276,602	234,328	1,063,285	3,871,429	2,960,216	9,492,341	1,834,631	19,732,832

US Use 2005	1	2	3	4	5	6	7	Imports
1 Agriculture	71,682	1	1,969	174,897	335	18,047	1,671	(31,248)
2 Mining	524	57,042	8,045	297,601	123,095	1,290	16,570	(226,059)
3 Construction	1,597	74	1,329	7,886	12,449	74,678	54,282	-
4 Manufactuirng	61,461	34,860	339,047	1,452,738	183,135	589,452	255,456	(1,372,424)
5 Trade Transport & Utilities	26,501	17,197	136,193	460,348	244,153	362,324	127,266	6,790
6 Services	27,274	52,297	165,179	543,690	610,978	3,017,728	529,779	(50,588)
7 Other	240	1,323	2,021	61,316	44,561	90,071	39,656	(208,971)

US Make 2005	1	2	3	4	5	6	7	Output
1 Agriculture	310,868	-	-	65	-	1,821	-	312,754
2 Mining	-	373,811	-	22,752	-	-	-	396,563
3 Construction	-	-	1,302,388	-	-	-	-	1,302,388
4 Manufactuirng	-	-	-	4,454,957	-	26,106	4,467	4,485,529
5 Trade Transport & Utilities	-	808	-	-	3,354,043	47	1,046	3,355,944
6 Services	-	556	-	-	152	10,473,161	3,771	10,477,640
7 Other	4,657	1,410	-	4,111	115,428	339,582	2,061,136	2,526,325
Commodity Output	315,525	376,586	1,302,388	4,481,885	3,469,622	10,840,717	2,070,419	22,857,143

[1] These tables differ from those provided in the supplemental resources for this text (described in Appendix B) in that they reflect data assembled "before redefinitions" as discussed in Chapter 4.

Produce industry-by-industry transactions tables using the assumption of industry-based technology for these three years. Suppose historical price indices for these tables are given in the following table (price indices in percent relative to some arbitrary earlier year):

	1997	2003	2005
Agriculture	100	113.5	122.7
Mining	96.6	131.3	201
Construction	181.6	188.9	209.9
Manufactuirng	133.7	150.8	156.9
Trade, Transport & Utilities	200.4	205.7	217.1
Services	129.3	151.6	219.8
Other	140	144.7	161.4

Produce a set of constant price input-output tables for the same years using 2005 as the base year for prices.

9.2 For the constant price tables constructed in problem 9.1, suppose we measure year-to-year change as the average of the absolute value of differences between the column sums of **A** for the same industry sectors in two different years. Which three sectors exhibited the most change from 1997 to 2005? How does that compare with the three sectors most changed sectors measured in nominal dollars rather than constant dollars? Why are they different?

9.3 Using the current price tables constructed in problem 9.1, compute the marginal input coefficients between the years 1997 and 2005.

9.4 Consider the following interindustry transactions and total outputs two-sector input-output economy for the year 2020:

2020	A	B	Total Output
A	1	2	10
B	3	4	10

Suppose estimates are generated for the year 2030 for the vectors of total final demand, total value-added, and total output in the following table.

2030	Final Demand	Value Added	Total Output
A	12	10	25
B	6	8	20

Using the 2020 table as a base and using the 2030 projections for final demand, value-added and total output, compute an estimate of the 2030 technical coefficients table using the RAS technique.

9.5 Using the 1997 input-output table expressed in 1997 dollars constructed in problem 9.1 and the vectors of intermediate inputs, intermediate outputs, and total outputs from the corresponding input-output table for 2005, compute an RAS estimate of the 2005 table using the 1997 table as a base. Compute the mean absolute percentage error (MAPE) of the RAS-estimated table for 2005 compared with the "real" 2005 table.

9.6 Suppose we have a baseline transactions matrix defined as $\mathbf{Z}(0) = \begin{bmatrix} 100 & 55 & 25 \\ 50 & 75 & 45 \\ 25 & 10 & 110 \end{bmatrix}$. We are provided with estimates of intermediate inputs and outputs, $\mathbf{v}(1) = \begin{bmatrix} 265 \\ 225 \\ 325 \end{bmatrix}$ and $\mathbf{u}(1) = \begin{bmatrix} 325 \\ 235 \\ 255 \end{bmatrix}$, respectively.

(a) Compute an estimate of the transactions table for the next year, $\tilde{\mathbf{Z}}^Z(1)$ using $\mathbf{Z}(0)$, $\mathbf{v}(1)$ and $\mathbf{u}(1)$, using the RAS technique.

(b) Suppose we know the vector of total outputs, $\mathbf{x}(0) = \begin{bmatrix} 750 \\ 500 \\ 1{,}000 \end{bmatrix}$, corresponding to $\mathbf{Z}(0)$, and we also have an estimate of total outputs for next year, $\mathbf{x}(1) = \begin{bmatrix} 1{,}000 \\ 750 \\ 1{,}500 \end{bmatrix}$. Compute $\mathbf{A}(0)$ and use it along with $\mathbf{v}(1)$ and $\mathbf{u}(1)$ to generate an estimate of the technical coefficients matrix for next year $\tilde{\mathbf{A}}^A(1)$. Finally, compute $\tilde{\mathbf{A}}^Z(1) = \tilde{\mathbf{Z}}^Z(1)\hat{\mathbf{x}}(1)^{-1}$. Is $\tilde{\mathbf{A}}^A(1) = \tilde{\mathbf{A}}^Z(1)$? Why or why not?

9.7 For the economy in problem 9.6, suppose we acquire a survey-based table of technical coefficients next year of $\mathbf{A}(1) = \begin{bmatrix} .2 & .1 & .033 \\ .035 & .167 & .05 \\ .03 & .033 & .133 \end{bmatrix}$. At the beginning of the survey we know only $a(1)_{32} = .033$ and we use that along with $\mathbf{A}(0)$, $\mathbf{v}(1)$ and $\mathbf{u}(1)$ to generate an intermediate estimate of the entire matrix of coefficients, $\underset{\sim}{\tilde{\mathbf{A}}}(1)$. If we measure difference between two matrices as MAPE, which estimate of $\mathbf{A}(1)$ is better— $\tilde{\mathbf{A}}(1)$ or $\underset{\sim}{\tilde{\mathbf{A}}}(1)$? Suppose early in the survey period we determine $a(1)_{11} = .2$ instead of knowing $a(1)_{32}$. Which estimate of $\mathbf{A}(1)$ is better— $\tilde{\mathbf{A}}(1)$ or $\underset{\sim}{\tilde{\mathbf{A}}}(1)$? How does this case differ from the case where $a(1)_{32}$ is known?

9.8 Consider the transactions matrix $\mathbf{Z}(0) = \begin{bmatrix} 100 & 55 & 25 \\ 0 & 75 & 25 \\ 25 & 10 & 110 \end{bmatrix}$ and projected vectors of intermediate inputs and outputs, $\mathbf{v}(1) = \begin{bmatrix} 125 \\ 140 \\ 160 \end{bmatrix}$ and $\mathbf{u}(1) = \begin{bmatrix} 180 \\ 100 \\ 145 \end{bmatrix}$, respectively. Compute the

RAS estimate, $\tilde{\mathbf{Z}}(1)$. Suppose we learn that $v_1(0) = 100$ instead of 125. Is it possible to compute $\tilde{\mathbf{Z}}(1)$ via the RAS technique? Why or why not?

9.9 For the U.S. input-output tables for 1997 and 2005 (from problem 9.1, expressed in current dollars rather than constant dollars), compute the RAS estimate $\tilde{\mathbf{A}}(2005)$ using $\mathbf{A}(1997)$, $\mathbf{v}(2005)$, and $\mathbf{u}(2005)$. Compute the MAPE for $\tilde{\mathbf{A}}(2005)$ compared with $\mathbf{A}(2005)$. How does that error compare with the MAPE for $\tilde{\mathbf{L}}(2005) = [\mathbf{I} - \tilde{\mathbf{A}}(2005)]^{-1}$ when compared with $\mathbf{L}(2005)$?

Chapter 10, Nonsurvey Methods, Extensions: Exercise Problems

10.1 The economy of the Land of Lilliput is described by the following input-output table:

L	Interindustry Transactions		Total Outputs
	A	*B*	
A	1	6	20
B	4	2	15

Land of Brobdingnag is described by another input-output table:

B	Interindustry Transactions		Total Outputs
	A	*B*	
A	7	4	35
B	1	5	15

The economy of the distant land of the Houyhnhnms is described by yet another input-output table:

H	Interindustry Transactions		Total Outputs
	A	*B*	
A	20	30.67	100
B	2.86	38.3	115

(a) Compute the vectors of value-added, intermediate inputs, final-demand, and intermediate outputs for each economy.
(b) A Lilliputian economist is interested in examining the structure of the Brobdingnagian economy. Likewise, a Brobdingnagian economist is interested in examining the structure of the Lilliputian economy. However, each economist only has available to him the value-added, final-demand, and total-output vectors for the foreign economy. Each economist knows the RAS modification procedure and uses it with the technical coefficients matrix of his own economy serving as the base **A** matrix. Which of the two economists calculates a better estimate of the foreign economy's technical coefficients matrix in terms of mean absolute deviation (all elements of **A**)?
(c) An economist in the distant land of the Houyhnhnms learned of the two other economies from a world traveler. He becomes interested in the economic structures of these foreign lands but is only able to obtain the final-demand, value-added, and total-output vectors for each economy from the world traveler. The economist uses RAS with her own country's **A** matrix as a base to estimate the interindustry structure of

the two distant lands. Which economy does he estimate more accurately in terms of a mean absolute deviation? Do you notice anything peculiar about the comparative structures of the Lilliputian, Brobdingnagian and Houyhnhnm economies?

(d) The Land of Lilliput plans to build a new power plant which will require the following value of output (in millions of dollars) from each of the economy's industries (directly, so it can be thought of as a final demand presented to the Lilliputian economy) of $\mathbf{f} = \begin{bmatrix} 100 & 150 \end{bmatrix}'$. How accurate, measured as an average mean absolute deviation, is the Houyhnhnms' estimate of the total industrial activity (output) in the Lilliputian economy required to construct this power plant?

10.2 Suppose the economies given in problem 10.1 are really three-sector economies where the economy of the Land of Lilliput is described by the following input-output table:

	Interindustry Transactions			Total Outputs
	A	*B*	*C*	
A	1	6	6	20
B	4	2	1	15
C	4	1	1	12

The economy of the neighboring land of Brobdingnag is described by another input-output table:

	Interindustry Transactions			Total Outputs
	A	*B*	*C*	
A	7	4	8	35
B	1	5	1	15
C	6	2	7	30

The economy of the distant land of Houyhnhnms is described by yet another input-output table:

	Interindustry Transactions			Total Outputs
	A	*B*	*C*	
A	5.5	33	33	1,101
B	22	11	5.5	82.5
C	22	5.5	5.5	66

Solve parts a, b, and c of problem 10.1 for these new economies.

10.3 Consider the following input-output table for Region 1:

	A	*B*	Total Output
A	1	2	10
B	3	4	10

We are interested in determining the impact of a particular final demand in another region (Region 2). Suppose we have the following data concerning Region 2.

	Value Added	Final Demand	Total Outputs
A	10	11	15
B	13	12	20

Suppose that the cost of computing an RAS estimate of Region 2's input-output table using Region 1's **A** matrix as a base table is given by nc_1, where n is the number of RAS iterations. One iteration is defined by one row and one column adjustment, that is, $\mathbf{A}^k = \hat{\mathbf{r}}\mathbf{A}^{k-1}\hat{\mathbf{s}}$ (a row adjustment alone as the last iteration would also be counted as an iteration). We ultimately wish to compute the impact of a new final demand in Region 2. This impact (the total outputs required to support the new final demand) can be computed exactly or by using the round-by-round approximation of the inverse. We know that: (1) The cost of computing the inverse exactly on a computer is c_1 and the cost of using this inverse in impact analysis is c_2 (let us assume that $c_2 = 10c_1$, that is, the cost of computing the inverse is ten times the cost of using it in impact analysis.) (2) The cost of a round-by-round approximation of impact analysis is mc_1, where m is the order of the round-by-round approximation, that is, $\mathbf{f} + \mathbf{Af} + \mathbf{A}^2\mathbf{f} + \cdots + \mathbf{A}^m\mathbf{f}$.

(a) Assuming that a fourth-order round-by-round approximation is sufficiently accurate ($m = 4$), which method of impact analysis should we use to minimize cost—(1) or (2)?
(b) What is the total cost of performing impact analysis, including the cost of the RAS approximation (tolerance of 0.01) and of the impact analysis scheme you chose in a?
(c) If the budget for the entire impact-analysis calculation is $7c_1$, what level of tolerance can you afford: 0.01, 0.001, 0.0001, 0.00001, or 0.000001?

10.4 Examine the behavior of the adjustment term that converts location-quotient approach *FLQ* to *FLQA*, $\lambda = \{\log_2[1 + (x^r / x^n)]\}^\delta$ for values of $x^r / x^n = .01, .1, .25, .5, .75$ and 1 cross tabulated with values of $\delta =$ 0, .1, .3, .5 and 1.

10.5 The matrix of technical coefficients for a national economy, $\mathbf{A}^N$, and the vector of total outputs, $\mathbf{x}^N$, as well as the corresponding values for a target region, $\mathbf{A}^R$ and $\mathbf{x}^R$, are

$$\mathbf{A}^N = \begin{bmatrix} .1830 & .0668 & .0087 \\ .1377 & .3070 & .0707 \\ .2084 & .2409 & .2999 \end{bmatrix} \quad \mathbf{x}^N = \begin{bmatrix} 518,288.6 \\ 4,953,700.6 \\ 14,260,843.0 \end{bmatrix}$$

$$\mathbf{A}^R = \begin{bmatrix} .1092 & .0324 & .0036 \\ .0899 & .0849 & .0412 \\ .1603 & .1170 & .2349 \end{bmatrix} \quad \mathbf{x}^R = \begin{bmatrix} 8,262.7 \\ 95,450.8 \\ 170,690.3 \end{bmatrix}$$

Compute the matrix of simple location quotients (SLQ) and the estimate of the matrix of regional technical coefficients using the SLQ.

10.6 For the national and regional data specified in problem 10.5, compute the matrix of Cross-Industry Quotients (CIQ) and the estimate of the matrix of regional technical coefficients using the CIQ.

10.7 Consider once again the national and regional data specified in problem 10.5, estimate of the matrix of regional technical coefficients using the RAS technique.

10.8 Compare the estimated regional matrices of technical coefficients computed in problems 10.5, 10.6 and 10.7. In terms of mean absolute deviation from the actual regional technical coefficients, which technique provides the most accurate estimate?

10.9 Using the three-sector, three-region Chinese MRIO data for 2000 specified in problem 3.6, create estimates of the intraregional input coefficients and their associated Leontief inverses for regions 2 (South China) and 3 (Rest of China), using the same reduction techniques and measures of difference that appear in Table 10.1 for the China 2012 case.

10.10 The following are the 1997 matrix of technical coefficients and vector of total outputs for the State of Washington as well as the 2003 matrix of technical coefficients for the United States, where the sectors are defined as (1) agriculture, (2) mining, (3) construction, (4) manufacturing, (5) trade, transportation and utilities, (6) services, and (7) other:

$$\mathbf{A}^W = \begin{bmatrix} .1154 & .0012 & .0082 & .0353 & .0019 & .0033 & .0016 \\ .0008 & .0160 & .0057 & .0014 & .0022 & .0002 & .0001 \\ .0072 & .0084 & .0066 & .0043 & .0074 & .0196 & .0133 \\ .0868 & .0287 & .0958 & .0766 & .0289 & .0244 & .0205 \\ .0625 & .0278 & .0540 & .0525 & .0616 & .0317 & .0480 \\ .0964 & .1207 & .0704 & .0596 & .1637 & .1991 & .2224 \\ .0020 & .0031 & .0056 & .0019 & .0045 & .0051 & .0066 \end{bmatrix} \quad \mathbf{x}^W = \begin{bmatrix} 7,681.0 \\ 581.7 \\ 17,967.1 \\ 77,483.7 \\ 56,967.2 \\ 109,557.6 \\ 4,165.5 \end{bmatrix}$$

$$\mathbf{A}^{US} = \begin{bmatrix} .2225 & .0000 & .0012 & .0375 & .0001 & .0020 & .0010 \\ .0021 & .1360 & .0072 & .0453 & .0311 & .0003 & .0053 \\ .0034 & .0002 & .0012 & .0021 & .0035 & .0071 & .0214 \\ .1724 & .0945 & .2488 & .3204 & .0468 & .0572 & .1004 \\ .0853 & .0527 & .0912 & .0950 & .0643 & .0314 & .0526 \\ .0902 & .1676 & .1339 & .1261 & .1655 & .2725 & .1882 \\ .0101 & .0140 & .0103 & .0214 & .0206 & .0200 & .0247 \end{bmatrix}$$

Using the RAS technique, estimate the Washington State table using the U.S. matrix of technical coefficients as a starting point. Compute the mean absolute deviation of the estimated state technical coefficients matrix from the actual state matrix.

10.11 Suppose in problem 10.10, that we do not know all of the technical coefficients for the Washington State economy, $\mathbf{A}^W$, but we do know several, namely a_{11}^W, a_{62}^W and a_{65}^W. Using

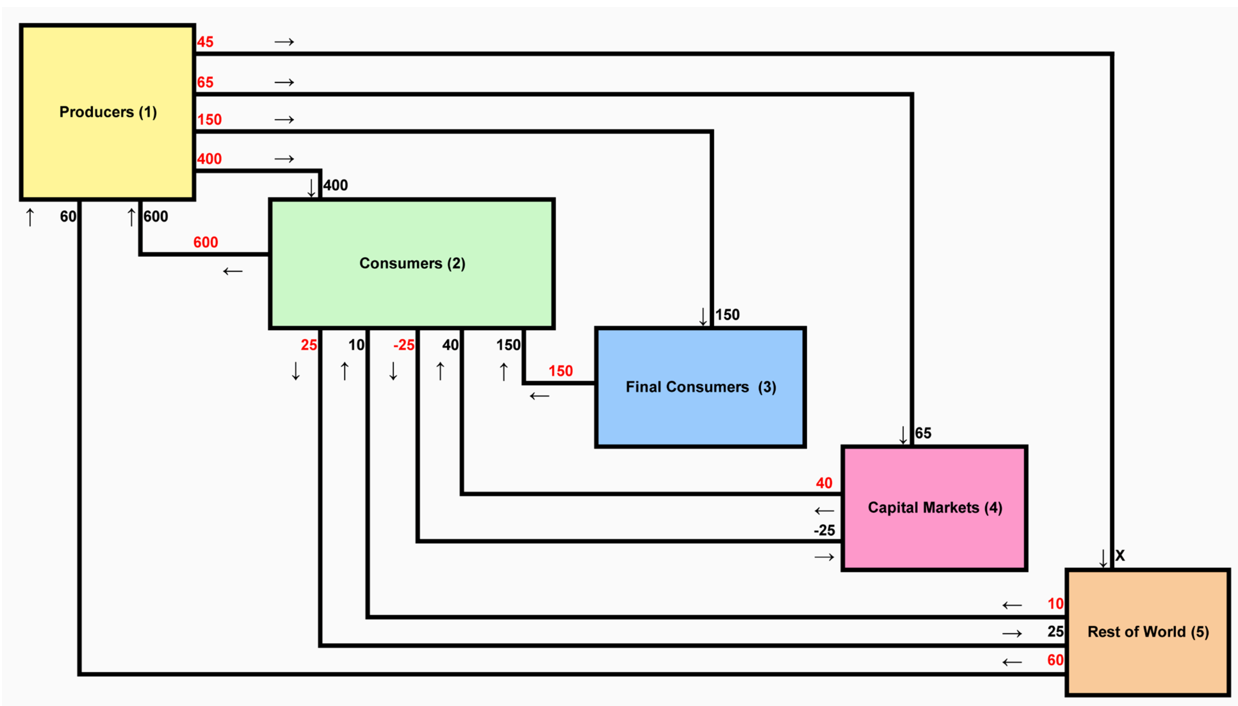
RAS as an estimating procedure, how do we incorporate knowing these coefficients only into the process of estimating the balance of the Washington State technical coefficients using $\mathbf{A}^{US}$ as the initial estimate using the total outputs and intermediate inputs and outputs that you found in problem 10.10. Compute a revised estimate of the Washington State Economy. How does it compare with the original estimate you found in problem 10.10?

10.12 Suppose in problem 10.11 you determine from exogenous sources some alternative technical coefficients, namely a_{67}^{W}, a_{42}^{W} and a_{54}^{W}. Compute a revised estimate of the Washington State matrix of technical coefficients using these known coefficients. Compute another estimate using both these and the previously identified known coefficients (from problem 10.11). How does this yet again revised estimate of the Washington State matrix of technical coefficients compare with the estimates you found in problems 10.10 and 10.11?

Chapter 11, Social Accounting Matrices: Exercise Problems

11.1 Consider a macro economy represented by the figure below. Construct a "Macro-SAM" representation of this economy. What is the missing value for sales of exports, *X*? Show the SAM in two forms: (a) with the Final Consumers sector included as part of the Consumers sector and (2) with the Final Consumers sector included as a separately defined sector.

Sample Macroeconomy



11.2 For the economy depicted in problem 11.1, suppose the following input-output accounts are collected:

		Commodities: Manuf.	Commodities: Services	Industries: Manuf.	Industries: Services	Final Demand	Totals	Grand Total
Commodities	Manuf.			94	96	110	300	660
	Services			94	117	148	360	
Industries	Manuf.	295	0				295	660
	Services	5	360				365	
Value Added				106	152	260		
Totals		300	360	295	365			

Grand Total	660	660	

Construct the "fully articulated" SAM, i.e., including the interindustry detail provided by these input-output accounts. Allocate final demand as part of consumer demand and assign commodity imports to v_{ii} for competitive imports to industry *i*. There is no unique solution.

11.3 For the fully articulated SAM in problem 11.2 expand the SAM to include sectors defined for consumer demand and exports. Again, there is no unique solution, but the SAM must be balanced, i.e., row and column sums equal.

11.4 For the SAM developed in problem 11.3:

(a) Compute the matrix of total expenditure shares.
(b) Assume final demand and value added sectors are considered exogenous transactions to this economy. Compute the SAM coefficient matrix.
(c) Compute the "direct effect" for this SAM.

11.5 Consider the following SAM for the developing nation of Sri Lanka:[2]

Sri Lanka SAM 1970	Value Added	Insti-tutions	Indirect Taxes	Surplus/ Deficit	Pro-duction	Rest of World	Total
Value Added					11473		11473
Institutions	11360	2052	1368			3	14783
Indirect Taxes		389			885	94	1368
Surplus/Deficit		-425				425	0
Production		11312			4660	2113	18085
Rest of World	113	1455			1067		2635
Total	11473	14783	1368	0	18085	2635	

If we consider Surplus/Deficit and Rest of World as external to the SAM, compute the direct, indirect, cross and total multipliers in the additive form.

11.6 Consider the unbalanced SAM given in the table below. Independent analysis indicates the total output of each sector; these are given in the additional column specified in the table. Use biproportional scaling to produce a balanced SAM with rows and columns both summing to the independent sector output estimates.

	Prod.	Cons.	Capital	ROW	Totals	Estimated Totals
Producers	0	600	65	45	710	660
Consumers	700	0	-25	15	690	600
Capital	0	40	0	0	40	40
Rest of World	50	10	0	0	60	60
Totals	750	650	40	60	1500	1360

[2] Adapted from Pyatt and Round (1979), pp. 852-853.

11.7 For the unbalanced SAM given in problem 11.6, if in addition to the estimated totals we become aware that the elements $z_{23} = -25$, $z_{24} = 15$ and $z_{42} = 10$ in the balanced SAM are fixed, use biproportional scaling with some fixed exogenous data for these elements to produce a balanced SAM.

11.8 Consider the following "macro-SAM" for the U.S. economy for 1988[3]:

US SAM 1988	Prod.	Comm.	Labor	Property	Enter-prises	House-holds	Govt.	Capital	Rest of World	Taxes	Errors	Total
Production		4831										4831
Commodities						3235	970	750	431			5386
Labor	2908											2908
Property	1556								117			1673
Enterprises				1589		95	93					1777
Households			2463		1045		556					4064
Government	377		445		138	587		96		18		1661
Capital					594	145			117		-10	846
Rest of World		537		84		2	42					665
Taxes		18										18
Errors & Omissions	-10											-10
Total	4831	5386	2908	1673	1777	4064	1661	846	665	18	-10	

If we consider the first five sectors as the endogenous sectors, compute the direct, indirect, cross and total multipliers in their multiplicative form.

11.9 For the macro-SAM specified in problem 11.8, compute the direct, indirect and total multipliers in their additive form. What do you notice about the direct multipliers compared in the additive form compared with the direct multipliers in their multiplicative form?

11.10 Consider the SAM for the U.S. (1988) expanded with interindustry detail shown in Table P11.10. If we consider the first nine sectors as the endogenous sectors, compute the total multipliers.

[3] As reported in Reinert and Roland-Holst (1992), pp. 173-187.

Table P11.10

SAM with Expanded Interindustry Detail for United States, 1988[4]

US SAM 1988 ($ billions)	1 Agric.	2 Mining	3 Const.	4 Nondur. Manuf	5 Durable Manuf.	6 Transp. & Util	7 Trade	8 Finance	9 Services	10 Labor	11 Propty	12 Enter-prises	13 House-holds	14 Govt.	15 Capital	16 Rest of World	17 Tariffs	18 Errors	TOTAL
1 Agriculture	42	0	2	98	8	0	3	8	7	0	0	0	18	7	1	22	0	0	214
2 Mining	0	10	2	82	8	35	0	0	0	0	0	0	1	0	2	8	0	0	148
3 Construction	2	12	1	7	9	21	6	36	18	0	0	0	0	134	358	0	0	0	602
4 Nondurable Manuf.	30	1	35	370	83	37	24	14	149	0	0	0	453	38	4	93	0	0	1332
5 Durable Manuf.	4	3	175	55	480	19	7	4	81	0	0	0	236	97	296	187	0	0	1643
6 Transport & Utilities	5	1	17	66	65	78	46	31	84	0	0	0	310	34	13	26	0	0	774
7 Trade	8	1	72	57	73	11	14	7	50	0	0	0	529	11	56	43	0	0	932
8 Finance	10	3	10	18	25	14	52	20	79	0	0	0	771	16	22	25	0	0	1065
9 Services	5	1	53	68	74	31	124	93	214	0	0	0	917	632	0	27	0	0	2240
10 Labor	33	18	197	218	430	212	385	217	1198	0	0	0	0	0	0	0	0	0	2908
11 Property	60	56	32	142	69	207	147	511	332	0	0	0	0	0	0	117	0	0	1673
12 Enterprise	0	0	0	0	0	0	0	0	0	0	1589	0	96	92	0	0	0	0	1778
13 Households	0	0	0	0	0	0	0	0	0	2463	0	1046	0	556	0	0	0	0	4064
14 Government	8	12	7	28	18	35	127	113	30	445	0	138	587	0	96	0	16	0	1659
15 Capital	0	0	0	0	0	0	0	0	0	0	0	594	145	0	0	117	0	-10	846
16 Rest of World	8	31	0	115	295	75	0	12	2	0	83	0	2	42	0	0	0	0	665
17 Tariffs	0	0	0	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	16
18 Errors & Omissions	0	0	-1	-1	-1	-1	-1	-2	-2	0	0	0	0	0	0	0	0	0	-10
TOTAL	214	148	602	1332	1643	774	932	1065	2240	2908	1673	1778	4064	1659	846	665	16	-10	

[4] As reported in Reinert and Rolad-Holst (1992).

Chapter 12, Energy: Exercise Problems

12.1 Consider the following three-sector input-output economy; two sectors are energy sectors (oil is the primary energy sector and refined petroleum is the secondary energy sector):

Interindustry Transactions ($\$10^6$)	Oil	Refined Petroleum	Manuf.	Final Demand	Total Output
Oil	0	20	0	0	20
Refined Petroleum	2	2	2	24	30
Manufacturing	0	0	0	20	20

The energy sector transactions are also measured in quadrillions of Btus in the following table:

Energy Sector Transactions (10^{15} Btus)	Oil	Refined Petroleum	Manuf.	Final Demand	Total Output
Oil	0	20	0	0	20
Refined Petroleum	1	1	1	17	20

Given this information, do the following:

(a) Compute (1) the matrix of implied inverse energy prices, (2) the direct energy requirements matrix, and (3) the total energy requirements matrix (including an accounting for energy consumed by final demand) using the method developed in Appendix S12.1. Do you notice anything peculiar about the total energy requirements matrix?

(b) Reformulate this problem as a hybrid-units input-output model; that is, recompute the technical coefficients and Leontief inverse using value terms for nonenergy sectors and energy units (Btus) for energy sector. Does this model conform to the conditions of energy conservation?

12.2 Consider the following input-output transactions table in value terms (millions of dollars) for two industries—*A* and *B*:

	A	*B*	Total Output
A	2	4	100
B	6	8	100

Suppose we have a direct energy requirements matrix for this economy that is given by:

$$\mathbf{D} = \begin{bmatrix} .2 & .3 \\ .1 & .4 \end{bmatrix} \begin{array}{l} 10^{15} \text{ Btus of oil per million dollars of output} \\ 10^{15} \text{ Btus of coal per million dollars of output} \end{array}$$

(a) Compute the total energy requirements matrix (neglecting energy consumption by final demand).
(b) Suppose that the final demands for industries *A* and *B* are projected to be \$200 million and \$100 million respectively for the next year. What is the net increase in energy (both oil and gas) required to support this new final demand (neglect energy consumed directly by final demand, since you do not have the information to do this calculation anyway)? What fraction of this net increase is a direct energy requirement and what fraction is indirect (total minus direct)?
(c) Suppose an energy conservation measure in industry *B* caused the direct energy requirement of that industry for coal to be reduced from 0.4 to 0.3 (10^{15} Btus of coal per dollar of output of industry *B*). How does this change the direct and total energy requirements needed to support the new final demand given in b?

12.3 Consider the following input-output table ($\$10^6$)

	Transactions			Total Output
	Autos	Oil	Electricity	
Autos	2	6	1	10
Oil	0	0	20	20
Electricity	3	2	1	30

Assume that the implied inverse energy price matrix for this economy is given by the following (in dollars per billion Btu)

	Autos	Oil	Electricity	Final Demand
Oil	0	0	0.4082	0
Electricity	0.3333	0.2857	0.5	1.2912

(a) Compute the current energy flows matrix, that is, the distribution of each energy type among the industries in the economy measured in Btus.
(b) Compute the direct energy coefficients matrix.
(c) If a final demand vector of \$2 million worth of autos and 18 quadrillion Btus of electricity is presented to this economy, what would be the total amount of energy (of each type) required to support this final demand?
(d) Compute the total energy requirement using the alternative method of Appendix S12.1.

12.4 Recall that the conditions for energy conservation in an input-output model can be expressed as $\boldsymbol{\alpha}\hat{\mathbf{x}} = \boldsymbol{\alpha}\mathbf{Z} + \mathbf{G}$ where $\boldsymbol{\alpha}$ is the matrix of total energy coefficients, $\mathbf{Z}$ is the matrix of interindustry transactions, $\mathbf{x}$ is the vector of total outputs, and $\mathbf{G}$ is the matrix of primary energy outputs.

(a) Show that the hybrid-units formulation of the energy input-output model—that is, where $\mathbf{x}$ is replaced by $\mathbf{x}^*$ and $\mathbf{Z}$ is replaced by $\mathbf{Z}^*$—satisfies these conditions in general.
(b) Given the following two tables of total energy coefficients, explain which of them satisfy the conditions of energy conservation and why. Use the convention that crude

oil is a primary energy sector and refined petroleum and electricity are secondary energy sectors.

Case 1	Crude Oil	Refined Petroleum	Electricity	Autos
Crude Oil	0	.6	.5	.3
Refined Petroleum	0	.4	.5	.2
Electricity	0	.2	0	.1

Case 2	Crude Oil	Refined Petroleum	Electricity	Autos
Crude Oil	0	.6	.5	.3
Refined Petroleum	0	.4	.2	.1
Electricity	0	.2	0	.1

12.5 An energy input-output model is defined (in \$$10^6$ units) by $\mathbf{Z} = \begin{bmatrix} 0 & 10 & 0 \\ 5 & 5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} 0 \\ 25 \\ 20 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 10 \\ 40 \\ 20 \end{bmatrix}$. Industries I and II are energy industries with patterns of output allocation expressed in energy terms (10^{15} Btus) by $\mathbf{E} = \begin{bmatrix} 0 & 40 & 0 \\ 5 & 5 & 15 \end{bmatrix}$ and $\mathbf{g} = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$.

(a) Compute $\boldsymbol{\varepsilon}$, the total energy requirements matrix (via the traditional method outlined in Appendix S12.1).
(b) Now compute $\boldsymbol{\alpha}$, the hybrid-units total energy requirements matrix.

12.6 Consider the following hybrid units transactions matrix and vector of total outputs, i.e., the first three rows of the energy sectors (oil, coal, and electricity) are measured in millions of Btu and the last row, manufacturing, is measured in millions of dollars: $\mathbf{Z}^* = \begin{bmatrix} 0 & 0 & 40 & 0 \\ 0 & 0 & 60 & 0 \\ 2 & 3 & 12 & 48 \\ 15 & 20 & 30 & 40 \end{bmatrix}$ and $\mathbf{x}^* = \begin{bmatrix} 40 \\ 60 \\ 100 \\ 200 \end{bmatrix}$. Suppose final demand for manufactured goods increased by \$200 billion. What would be the increase in total primary energy used to satisfy this growth in final demand?

12.7 For the economy specified in problem 12.6, two alternative technologies are proposed for generating electric power, which involve alternative new specifications for the matrix of technical coefficients depicting different "recipes" for electric power production in the economy, $\mathbf{A}^{*(I)}$ and $\mathbf{A}^{*(II)}$. The original electric power generation column of the matrix of technical coefficients is given by $\mathbf{A}^*$. Suppose the two alternative changed columns of the matrix of technical coefficients corresponding to the alternative technologies are given by

$\mathbf{A}_{\bullet 3}^{*(I)} = \begin{bmatrix} 0.2 \\ 0.7 \\ 0.1 \\ 0.4 \end{bmatrix}$ and $\mathbf{A}_{\bullet 3}^{*(II)} = \begin{bmatrix} 0.5 \\ 0.4 \\ 0.12 \\ 0.4 \end{bmatrix}$. A new vector of changes in final demand is given as $\Delta\mathbf{f}^* = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 30 \end{bmatrix}$. Which economy [matrix incorporating the specifications $\mathbf{A}^*$, $\mathbf{A}^{*(I)}$, or $\mathbf{A}^{*(II)}$] reflects the most energy intensive manufacturing, i.e., which one of the two new technologies consumes the least primary energy per unit of final demand of manufacturing and how much less primary energy does that technology consume than the other to support the change in final demand, $\Delta\mathbf{f}^*$?

12.8 For the original energy-economy defined in problem 12.6 ($\mathbf{A}^*$), suppose an energy conserving manufacturing process is developed that can be depicted as a new column of the matrix of technical coefficients for manufacturing, given by $\mathbf{A}_{\bullet 4}^{*(new)} = \begin{bmatrix} 0 \\ 0 \\ 0.12 \\ 0.20 \end{bmatrix}$. If this new process were adopted, how much primary energy would be saved in the economy, both directly in terms of fuel used directly in manufacturing, and indirectly in the energy embodied the inputs to manufacturing?

12.9 Suppose the original energy economy used in problem 12.6 is faced with an oil supply shortage of a ten percent reduction in total input of oil available in the economy. What would be the corresponding reduction in GDP? To do this calculation you will need to know the energy prices to final demand, which are given by $\mathbf{p}_f = [p_{kf}] = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. If new electric power generation technology *I* from problem 12.7 and the energy conserving manufacturing process from problem 12.8 were both incorporated into the economy, what would be the change in GDP with the same oil shortage?

12.10 Below are 9-sector 1963 and 1980 input-output tables for the United States expressed in hybrid units (quadrillions of Btus for energy sectors and millions of dollars for non-energy sectors). The first five sectors are energy sectors: (1) coal, (2) oil, (3) refined petroleum products, (4) electricity, and (5) natural gas. The remaining four sectors are non-energy sectors: (6) natural resources, (7) manufacturing, (8) transportation, and (9) services. Using the approach derived in section 12.2 of the text, determine the amounts of the change in total energy use of each energy type between 1963 and 1980 and the components of that change that are attributable to change in production functions, to change in final demand, and to the interaction between the changes in production functions and final demand.

1980	1	2	3	4	5	6	7	8	9	Total Output
1	0.0012	0.0000	0.0007	1.5464	0.0000	0.0000	0.0002	0.0000	0.0000	18,597
2	0.0001	0.0319	0.8960	0.0001	0.8707	0.0000	0.0001	0.0000	0.0000	36,842
3	0.0063	0.0024	0.0612	0.3344	0.0008	0.0005	0.0002	0.0023	0.0002	31,215
4	0.0026	0.0021	0.0035	0.0822	0.0020	0.0000	0.0001	0.0000	0.0001	7,827
5	0.0006	0.0461	0.0301	0.4856	0.0720	0.0001	0.0003	0.0000	0.0001	19,244
6	0.2092	1.4027	0.5040	7.8254	0.4350	0.0896	0.0628	0.0355	0.0289	6,194,571
7	2.6323	0.8480	2.4090	3.5155	0.1804	0.2672	0.3780	0.0493	0.0626	18,081,173
8	0.1773	0.0806	2.1831	4.8195	0.0794	0.0199	0.0251	0.1289	0.0141	2,240,904
9	1.8576	2.6159	2.7945	8.5173	1.2302	0.1831	0.1238	0.1224	0.2027	23,803,723
1963										
1	0.0019	0.0000	0.0008	1.7415	0.0010	0.0000	0.0004	0.0001	0.0000	12,476
2	0.0000	0.0423	0.7996	0.0007	0.9308	0.0000	0.0003	0.0000	0.0000	30,384
3	0.0015	0.0011	0.0600	0.1973	0.0031	0.0004	0.0003	0.0021	0.0002	19,878
4	0.0015	0.0007	0.0018	0.0963	0.0002	0.0000	0.0001	0.0000	0.0000	3,128
5	0.0001	0.0035	0.0330	0.7046	0.0919	0.0000	0.0003	0.0001	0.0001	13,194
6	0.0456	0.4582	0.5926	7.9623	0.6565	0.1111	0.0835	0.0415	0.0426	4,865,092
7	0.8684	0.4081	1.1700	1.0933	0.0937	0.2340	0.4035	0.0498	0.0496	11,333,710
8	0.1105	0.0655	1.1964	4.5632	0.3965	0.0231	0.0256	0.0863	0.0121	1,131,226
9	0.4794	2.2388	1.9461	8.0643	1.1016	0.1121	0.0881	0.1203	0.1721	10,588,385

Chapter 13, Environment: Exercise Problems

13.1 Assume that we have the following direct coefficient matrices for energy, air pollution, and employment ($\mathbf{D}^e$, $\mathbf{D}^v$ and $\mathbf{D}^l$, respectively) for two industries, 1 and 2:

$\mathbf{D}^e = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$, $\mathbf{D}^v = \begin{bmatrix} 0.2 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}$ and $\mathbf{D}^l = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}$. Notice that industry 2 is both a high-polluting and high-employment industry. Suppose that the local government has an opportunity to spend a total of \$10 million on a regional development project. Two projects are candidates: (1) Project 1 would spend appropriated dollars in the ratio of 60 percent to industry 1 and 40 percent to industry 2; the minimum size of this project is \$4 million; (2) Project 2 would spend appropriated dollars in the ratio of 30 percent to industry 1 and 70 percent to industry 2; the minimum size of this project is \$2 million. The government can adopt either project or a combination of the two projects (as long as the minimum size of each project is at least maintained and the total budget is not overrun). In other words, we might describe the options available to the government as:

$$\begin{bmatrix} \beta_a \\ \beta_b \end{bmatrix} = \alpha_1 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix}$$

where α_1 and α_2 are budgets allocated to projects 1 and 2, respectively. β_a and β_b are the total final demands presented to the regional economy by the combination of projects for industries *A* and *B*, respectively. Suppose that four alternative compositions of these projects are being considered (1) $\begin{cases} \alpha_1 = 4 \\ \alpha_2 = 2 \end{cases}$, (2) $\begin{cases} \alpha_1 = 5 \\ \alpha_2 = 5 \end{cases}$, (3) $\begin{cases} \alpha_1 = 10 \\ \alpha_2 = 0 \end{cases}$ and (4) $\begin{cases} \alpha_1 = 0 \\ \alpha_2 = 10 \end{cases}$.

The following table of constraints describes the local regulation on energy consumption and environmental pollution in the region:

	Maximum Allowable Changes Collectively by All Industries
Oil Consumption (10^{15} Btus)	3.0
Coal Consumption (10^{15} Btus)	no limit
SO_2 Emissions (tons)	14.5
NO_x Emissions (tons)	10

Finally, suppose that the regional economy is currently described by the following input-output transactions table (in millions of dollars):

	A	*B*	Total Output
A	1	3	10
B	5	1	10

(a) Which of the proposed combinations of projects (1), (2), (3) and (4) permit the region to operate within the above constraints on energy consumption and air pollution emission and within the established budget constraint?
(b) Which of these "legal" projects that you identified in (a) should be adopted to maximize the employment in the region?

13.2 Assume that a regional economy has two primary industries: *A* and *B*. In producing these two products it was observed last year that air pollution emissions associated with this industrial activity included 3 pounds of SO_2 and 1 pound of NO_x emitted per dollar's worth of output of industry *A*, and 5 pounds of SO_2 and 2 pounds of NO_x emitted per dollar's worth of output of industry *B*. It was also observed that industries *A* and *B* consumed 1×10^6 tons and 6×10^6 tons of coal respectively during that year. Industry *A* also consumed 2×10^6 barrels of oil. Total employment in the region was 100,000 (40 percent of which were employed by industry *A* and the rest by industry *B*). The regional planning agency has constructed the following input-output table of interindustry activity in the region (in $\$10^6$):

	A	*B*	Total Output
A	2	6	10
B	6	12	10

Assume that with growth in the region during the next year the new final-demand vector will be $\begin{bmatrix} 15 & 25 \end{bmatrix}'$. Using what you know about constructing a generalized input-output model, determine the following: (a) the total consumption of each energy type (coal and oil) during the next year, (b), the total pollution emission (of each type) during the next year and (c) the level of total employment during the next year.

13.3 A regional planning agency initiates a regional development plan. Four projects are being considered that would represent government purchases of regionally produced products, that is, final demands presented to the regional economy (see table).

Regional Industry	Project Expenditure (millions of dollars)			
	Project 1	Project 2	Project 3	Project 4
A	2	4	2	2
B	2	0	0	2
C	2	2	4	3

You are given additional information. The matrix of technical coefficients is:
$\mathbf{A} = \begin{bmatrix} 0.04 & 0.23 & 0.38 \\ 0.33 & 0.52 & 0.47 \\ 0 & 0 & 0.1 \end{bmatrix}$. The relationships between the following quantities and total output are also known:

	Industry		
	A	B	C
Pollution emission (grams/$ output)	4.2	7	9.1
Energy Consumption (bbls oil/$ output)	7.6	2.6	0.5
Employment (workers/ $ output)	0.73	0.33	0.63

(a) Which of the four projects contributes most to gross regional output?
(b) Which of the projects causes regional consumption of energy to increase the most?
(c) Which of the projects contributes most to regional employment?

13.4 Consider an input output economy defined by $\mathbf{Z} = \begin{bmatrix} 140 & 350 \\ 800 & 50 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1{,}000 \\ 1{,}000 \end{bmatrix}$. Suppose this is an economy in deep economic trouble. The federal government has at its disposal policy tools that can be implemented to stimulate demand for goods from one sector or the other. Also suppose that the plants in sector 1 discharge 0.3 lbs. of airborne particulate substances for every dollar of output (0.3 lbs/$ output), while sector 2 pollutes at 0.5 lbs/$ output. Finally, let labor input coefficients be 0.005 and 0.07 for sectors 1 and 2, respectively.

(a) Would a conflict of interest arise between unions and environmentalists in determining the sector toward which the government should direct its policy effort? (You need not close the matrix with respect to either households or pollution generation to answer this question.)
(b) Can you think of a technological reason why or why not a dispute might arise?

13.5 Consider the following interindustry transactions table:

		Purchasing Sector		Total Output
		1	2	
Selling Sector	1	140	350	2,000
	2	800	50	1,850

An amount of pollution generated by sector 1 is 10 units and by sector 2 is 25 units. Pollution abatement reduced pollution by 5 units in sector 1 and 12 units in sector 2. Total pollution permitted by local regulation is 12 units. Using a pollution-activity-augmented Leontief formulation, what is the level of output for each industry and the total pollution generated if final demands for both sectors increase by 100?

13.6 In problems 10.5 and 10.6 national and regional input-output tables are defined with three sectors (natural resources, manufacturing, and services) with the following matrices of technical coefficients and vectors of total outputs, respectively,

$$\mathbf{A}^N = \begin{bmatrix} .1830 & .0668 & .0087 \\ .1377 & .3070 & .0707 \\ .1603 & .2409 & .2999 \end{bmatrix},\ \mathbf{x}^N = \begin{bmatrix} 518{,}288.6 \\ 4{,}953{,}700.6 \\ 14{,}260{,}843.0 \end{bmatrix},\ \mathbf{A}^R = \begin{bmatrix} .1092 & .0324 & .0036 \\ .0899 & .0849 & .0412 \\ .1603 & .1170 & .2349 \end{bmatrix} \text{ and}$$

$\mathbf{x}^R = \begin{bmatrix} 8{,}262.7 \\ 95{,}450.8 \\ 170{,}690.3 \end{bmatrix}$. The following table of energy use, pollution, and employment coefficients are defined that apply to both the regional and national economies:

	Industry		
	Nat. Res.	Manuf.	Services
Pollution Emission (grams/$ output)	4.2	7	9.1
Energy Consumption (Btusl/$ output)	7.6	2.6	0.5
Employment (person-hrs/ $ output	7.3	3.3	6.3

Suppose a major new public works initiative by the federal government is characterized by the following vector of increases in federal spending: $\Delta\mathbf{f}' = \begin{bmatrix} 250 & 3{,}000 & 7{,}000 \end{bmatrix}$, of which 20 percent will be spent in the region. How do the percentage changes in total impacts on pollution, energy use, employment and total industrial output of each industry sector for the region compare with those of the nation as a whole?

13.7 For the regional economy described in problem 13.6 prior to the projected final demand, if there were a 10 percent shortfall in the availability of energy, what would be the corresponding impacts on GDP?

13.8 An input-output economy is specified by $\mathbf{A} = \begin{bmatrix} .3 & .1 \\ .2 & .5 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Write the linear programming (LP) formulation for finding **x**, the vector of total outputs. Solve this LP problem graphically. Suppose also that pollution is generated at a rate of 2.5 units per dollar of output of industry 1 and 2 units per dollar of output of industry 2. Replace the objective function for the LP problem above with minimizing pollution emissions. Solve this LP problem graphically and compare the solution with the first LP problem.

13.9 For the economy specified in problem 13.8, suppose that employment is generated at a rate of 6 and 3 units per dollar's worth of output for industries 1 and 2, respectively, and that there is a very high priority employment target of 7.5 units for industry 2. Find the vector of total outputs that meets the employment target for industry 2 as the highest priority, then as the next highest priority meets final demand requirements while minimizing pollution generation to the extent possible and if possible to a total level of 10 units of pollution between the two industries.

13.10 For the 1997 U.S. input-output table (industry-by-industry assuming industry-based technology, after redefinitions) provided in Appendix SD1, suppose the vector of units of carbon dioxide emissions generated per dollar of total output is given by $\mathbf{d} = [2 \quad 3 \quad 4 \quad 7 \quad 10 \quad 5 \quad 4]'$. Presume that the availability of new technology reduces the emissions per dollar of output in the year 2007 for the manufacturing sector by 10 percent and the construction sector by 15 percent. The input-output table for 2007 is also provided in Appendix SD1. For this case how much do total emissions of carbon dioxide increase or decrease in the United States in 2007 relative to 1997 levels?

13.11 Consider the 3 region 2 sector IRIO interindustry technical coefficients matrix defined by

$$\mathbf{A} = \left[\begin{array}{cc:cc:cc} .222 & .121 & .027 & .023 & .007 & .014 \\ .217 & .028 & .014 & .015 & .021 & .012 \\ \hdashline .02 & .025 & .126 & .088 & .019 & .019 \\ .002 & .025 & .06 & .141 & .002 & .019 \\ \hdashline .012 & .02 & .019 & .005 & .192 & .179 \\ .022 & .017 & .005 & .013 & .195 & .164 \end{array}\right].$$ The corresponding Leontief inverse is then

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \left[\begin{array}{cc:cc:cc} 1.335 & .17 & .048 & .045 & .025 & .033 \\ .3 & .1069 & .029 & .031 & .039 & .03 \\ \hdashline .043 & .04 & 1.155 & .125 & .038 & .039 \\ .018 & .037 & .085 & 1.217 & .015 & .034 \\ \hdashline .04 & .038 & .033 & .018 & 1.308 & .282 \\ .051 & .036 & .019 & .025 & .307 & 1.265 \end{array}\right].$$ We define a vector of

CO_2 emission coefficients as $\mathbf{g}' = \left[\begin{array}{cc:cc:cc} .9 & .4 & .3 & 1.0 & .2 & .7 \end{array}\right]$. Consider a new vector . of

final demands presented to this IRIO economy, defined by $$\mathbf{f}^{new} = \left[\begin{array}{c} 1500 \\ 2000 \\ \hdashline 55 \\ 40 \\ \hdashline 5 \\ 3 \end{array}\right].$$ Calculate the

vector of the total CO_2 emissions associated with the total economic production for each sector in each region, first attributed to where the pollution is generated and second attributed to where the consumption occurs that generates the demand for the production that generates the emissions.

13.12 Consider the Global IRIO transactions tables aggregated to 3 regions (the US, China, and Rest of World) and 3-Sector industry sectors (Agriculture and Mining, Manufacturing, and Services & Utilities) for the years 2005 and 2015 given in Appendix SD2. Suppose we estimate CO_2 emission indices as $\mathbf{g} = \left[\begin{array}{ccc:ccc:ccc} .2 & .3 & .1 & .3 & .5 & .2 & .1 & .2 & .1 \end{array}\right]$ per million US dollars and, for simplicity, we assume these indices do not change between 2005 and 2015. What are the vectors of total global emissions for 2005 and 2015 and for each global region individually if we attribute emissions to the producing sectors and if we attribute the emissions to consuming sectors? What is the percentage shift (increase or decrease) in each region if we attribute emissions to consumption rather than production?

Chapter 14, Mixed and Dynamic Models: Exercise Problems

Dynamic Models

14.1 Consider an input-output economy with technical coefficients defined as $\mathbf{A}=\begin{bmatrix}0.3 & 0.1\\ 0.2 & 0.5\end{bmatrix}$ and capital coefficients defined as $\mathbf{B}=\begin{bmatrix}.01 & .003\\ .005 & .020\end{bmatrix}$. Current final demand is $\mathbf{f}^0=\begin{bmatrix}100\\ 100\end{bmatrix}$ and the projections for the next three years for final demand are given by $\mathbf{f}^1=\begin{bmatrix}125\\ 160\end{bmatrix}$, $\mathbf{f}^2=\begin{bmatrix}150\\ 175\end{bmatrix}$ and $\mathbf{f}^3=\begin{bmatrix}185\\ 200\end{bmatrix}$. We are not interested in total output for beyond the projection three years, but what would be the projections of total output for this economy in the next three years?

14.2 Consider the following closed dynamic input-output model, $\mathbf{Ax}+\mathbf{B}(\mathbf{x}^t-\mathbf{x})=\mathbf{x}$ where: $\mathbf{x}^t$ = future outputs, $\mathbf{x}$ = current outputs, and where $\mathbf{A}=\begin{bmatrix}0.5 & 0.1\\ 0.1 & 0.5\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}$. Assume that $\mathbf{x}^t=\lambda\mathbf{x}$, where λ is some scalar (the turnpike growth rate); compute λ.

14.3 Given the closed dynamic input-output model $\mathbf{Ax}+\mathbf{B}(\mathbf{x}^t-\mathbf{x})=\mathbf{x}$, where

$$\mathbf{A}=\begin{bmatrix}0.1 & 0.2\\ 0.3 & 0.4\end{bmatrix} \text{ and } \mathbf{B}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

(a) Compute the turnpike growth rate for this example.
(b) If both the capital coefficients for the first industry (the first column of **B**) are changed to 0.1, then what is the new turnpike growth rate and what has happened to the apparent "health" of the economy?

14.4 Consider an input-output economy with technical coefficients defined as $\mathbf{A}=\begin{bmatrix}0.2 & 0.1\\ 0.3 & 0.5\end{bmatrix}$ and capital coefficients defined as $\mathbf{B}=\begin{bmatrix}.02 & .002\\ .003 & .01\end{bmatrix}$. Current final demand is $\mathbf{f}^0=\begin{bmatrix}185\\ 200\end{bmatrix}$ and final demands for previous three years are given by $\mathbf{f}^{-1}=\begin{bmatrix}150\\ 175\end{bmatrix}$, $\mathbf{f}^{-2}=\begin{bmatrix}125\\ 160\end{bmatrix}$, and $\mathbf{f}^{-3}=\begin{bmatrix}100\\ 100\end{bmatrix}$. Compute the "dynamic" multipliers for this economy that show how direct and indirect input requirements for final demands in period 0 are distributed backward over time for previous three years.

14.5 Consider an input-output economy specified by $\mathbf{Z} = \begin{bmatrix} 14 & 76 & 46 \\ 54 & 22 & 5 \\ 68 & 71 & 94 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 100 \\ 200 \\ 175 \end{bmatrix}$ where the three industrial sectors are manufacturing, oil and electricity.

(a) Suppose the economic forecasts determine that total domestic output for oil and electricity will remain unchanged in the next year and final demand for manufactured goods will increase by 30 percent. What would be the input-output projections of final demand for oil and electricity and the total output of manufacturing?
(b) If instead the final demand for manufactured goods increased by 50 percent instead of 30 percent, what are the new projections of final demand for oil and electricity and the total output of manufacturing?

14.6 Consider the impact on the economy of problem 2.1 of the establishment of a new economic sector, finance and insurance (sector 3).

(a) Suppose you know that the total output of this new sector will be $900 during the current year (its first year of operation), and that its needs for agricultural and manufactured goods are represented by $a_{13} = 0.001$ and $a_{23} = 0.07$. In the absence of any further information, what would you estimate to be the impact of this new sector on the economy?
(b) You later learn (1) that the agriculture and manufacturing sectors bought $20 and $40 in finance and insurance services last year from foreign firms (i.e., that they imported these inputs), and (2) that sector 3 will use $15 of its own product for each $100 worth of its output. Assuming that they will now buy from the domestic sector, how might you now assess the impact of the new sector on this economy?

14.7 Recall the Czaria economy from problem 7.1. Next year's projected total outputs in millions of dollars for agriculture, mining and civilian manufacturing in Czaria are 4,558, 5,665 and 5,079, respectively, and final demand of military manufactured products is projected to be $2,050 million. Compute the GDP and total gross production of the economy next year.

14.8 Consider the 2005 U.S. input-output data specified below, including industry based technical coefficients and the make matrix. Suppose our economic forecast projects for 2010 a 10 percent growth in final demand for agriculture, mining, and construction, a 5 percent growth in final demand for manufactured goods, and a 6 percent growth in total output for the trade, transportation, utilities, services and other economic sectors. What are the corresponding input-output estimates of total output for agriculture, mining, construction and manufacturing as well as the estimates of final demand for trade, transportation, utilities, services and other economic sectors?

A	1	2	3	4	5	6	7
1	0.2258	0.0000	0.0015	0.0384	0.0001	0.0017	0.0007
2	0.0027	0.1432	0.0075	0.0675	0.0367	0.0004	0.0070
3	0.0051	0.0002	0.0010	0.0018	0.0037	0.0071	0.0215
4	0.1955	0.0877	0.2591	0.3222	0.0547	0.0566	0.1010
5	0.0819	0.0422	0.1011	0.0994	0.0704	0.0334	0.0487
6	0.0843	0.1276	0.1225	0.1172	0.1760	0.2783	0.2026
7	0.0099	0.0095	0.0093	0.0219	0.0215	0.0188	0.0240

V	1	2	3	4	5	6	7
1	310,868	0	0	65	0	1,821	0
2	0	373,811	0	22,752	0	0	0
3	0	0	1,302,388	0	0	0	0
4	0	0	0	4,454,957	0	26,106	4,467
5	0	808	0	0	3,354,043	47	1,046
6	0	556	0	0	152	10,473,161	3,771
7	4,657	1,410	0	4,111	115,428	339,582	2,061,136