```
In[73]:= Needs["DiscreteMath`RSolve`"]

In[74]:= Needs["Graphics`Graphics3D`"]

In[75]:= Needs["Graphics`PlotField3D`"]
```

à Question 8

Although this question asks to use a spreadsheet, we shall here use *Mathematica* to plot the trajectories. First we solve for the equilibrium values of the instrumental variables, *r* and *g*.

```
In[76]:= Solve[{r == -3.925 + 0.5 g, r == 7.958 + 0.186 g}, {r, g}]

Out[76]= {{r → 14.997, g → 37.8439}}
```

First we shall simply list the computations for g[t] and r[t] for each of the initial conditions. This requires us to solve the systems of difference equations for each initial condition. We do this for ten periods.

(a)

```
In[77]:= sol81 = RSolve[
           {g[t + 1] == g[t] - 0.5 (g[t] - 37.8439), r[t + 1] == r[t] - 0.75 (r[t] - 14.997),
            g[0] == 20, r[0] == 12}, {g[t], r[t]}, t]

         General::spell1 :
          Possible spelling error: new symbol name "sol81" is similar to existing symbol "sol1".
```

Out[77]= $\{\{g[t] \to 2.\ (-8.92195\ 0.5^{t} + 18.922\ 1.^{t}),$
$r[t] \to 4.\ (-0.74925\ 0.25^{t} + 3.74925\ 1.^{t})\}\}$

```
In[78]:= path81 = Table[{t, sol81[[1, 1, 2]], sol81[[1, 2, 2]]}, {t, 0, 10}];
         TableForm[path81, TableHeadings -> {{}, {"t", "g[t]", "r[t]"}}]

         General::spell1 :
          Possible spelling error: new symbol name "path81" is similar to existing symbol "path1".
```

Out[79]//TableForm=

t	g[t]	r[t]
0	20.	12.
1	28.922	14.2478
2	33.3829	14.8097
3	35.6134	14.9502
4	36.7287	14.9853
5	37.2863	14.9941
6	37.5651	14.9963
7	37.7045	14.9968
8	37.7742	14.997
9	37.809	14.997
10	37.8265	14.997

(b)

```
In[80]:= sol82 = RSolve[
           {g[t + 1] == g[t] - 0.5 (g[t] - 37.8439), r[t + 1] == r[t] - 0.75 (r[t] - 14.997),
            g[0] == 20, r[0] == 20}, {g[t], r[t]}, t]

         General::spell1 :
          Possible spelling error: new symbol name "sol82" is similar to existing symbol "sol2".
```

Out[80]= $\{\{g[t] \to 2.\ (-8.92195\ 0.5^{t} + 18.922\ 1.^{t}),\ r[t] \to 4.\ (1.25075\ 0.25^{t} + 3.74925\ 1.^{t})\}\}$

```
In[81]:= path82 = Table[{t, sol82[[1, 1, 2]], sol82[[1, 2, 2]]}, {t, 0, 10}];
         TableForm[path82, TableHeadings -> {{}, {"t", "g[t]", "r[t]"}}]

         General::spell1 :
          Possible spelling error: new symbol name "path82" is similar to existing symbol "path2".
```

Out[82]//TableForm=

t	g[t]	r[t]
0	20.	20.
1	28.922	16.2477
2	33.3829	15.3097
3	35.6134	15.0752
4	36.7287	15.0165
5	37.2863	15.0019
6	37.5651	14.9982
7	37.7045	14.9973
8	37.7742	14.9971
9	37.809	14.997
10	37.8265	14.997

(c)

```
In[83]:= sol83 = RSolve[
           {g[t + 1] == g[t] - 0.5 (g[t] - 37.8439), r[t + 1] == r[t] - 0.75 (r[t] - 14.997),
            g[0] == 50, r[0] == 10}, {g[t], r[t]}, t]

         General::spell1 :
          Possible spelling error: new symbol name "sol83" is similar to existing symbol "sol3".
```

Out[83]= $\{\{g[t] \to 2.\ (6.07805\ 0.5^{t} + 18.922\ 1.^{t}),\ r[t] \to 4.\ (-1.24925\ 0.25^{t} + 3.74925\ 1.^{t})\}\}$

```
In[84]:= path83 = Table[{t, sol83[[1, 1, 2]], sol83[[1, 2, 2]]}, {t, 0, 10}];
         TableForm[path83, TableHeadings -> {{}, {"t", "g[t]", "r[t]"}}]

         General::spell1 :
          Possible spelling error: new symbol name "path83" is similar to existing symbol "path3".
```

Out[85]//TableForm=

t	g[t]	r[t]
0	50.	10.
1	43.9219	13.7477
2	40.8829	14.6847
3	39.3634	14.9189
4	38.6037	14.9775
5	38.2238	14.9921
6	38.0338	14.9958
7	37.9389	14.9967
8	37.8914	14.9969
9	37.8676	14.997
10	37.8558	14.997

(d)

```
In[86]:= sol84 = RSolve[
           {g[t + 1] == g[t] - 0.5 (g[t] - 37.8439), r[t + 1] == r[t] - 0.75 (r[t] - 14.997),
            g[0] == 50, r[0] == 20}, {g[t], r[t]}, t]

         General::spell1 :
          Possible spelling error: new symbol name "sol84" is similar to existing symbol "sol4".

Out[86]= {{g[t] → 2. (6.07805 0.5^t + 18.922 1.^t), r[t] → 4. (1.25075 0.25^t + 3.74925 1.^t)}}

In[87]:= path84 = Table[{t, sol84[[1, 1, 2]], sol84[[1, 2, 2]]}, {t, 0, 10}];
         TableForm[path84, TableHeadings -> {{}, {"t", "g[t]", "r[t]"}}]

         General::spell1 :
          Possible spelling error: new symbol name "path84" is similar to existing symbol "path4".

Out[88]//TableForm=
```

t	g[t]	r[t]
0	50.	20.
1	43.9219	16.2477
2	40.8829	15.3097
3	39.3634	15.0752
4	38.6037	15.0165
5	38.2238	15.0019
6	38.0338	14.9982
7	37.9389	14.9973
8	37.8914	14.9971
9	37.8676	14.997
10	37.8558	14.997

It is already apparent from each of these tables that the trajectories converge on the equilibrium values. To show this more clearly we place each of the trajectories in the (g,r)-space along with the "target lines".

(a)

```
In[89]:= points81 = Table[{sol81[[1, 1, 2]], sol81[[1, 2, 2]]}, {t, 0, 10}];

In[90]:= traj81 = ListPlot[points81, PlotJoined -> True, DisplayFunction -> Identity];
```

(b)

```
In[91]:= points82 = Table[{sol82[[1, 1, 2]], sol82[[1, 2, 2]]}, {t, 0, 10}];

In[92]:= traj82 = ListPlot[points82, PlotJoined -> True, DisplayFunction -> Identity];
```

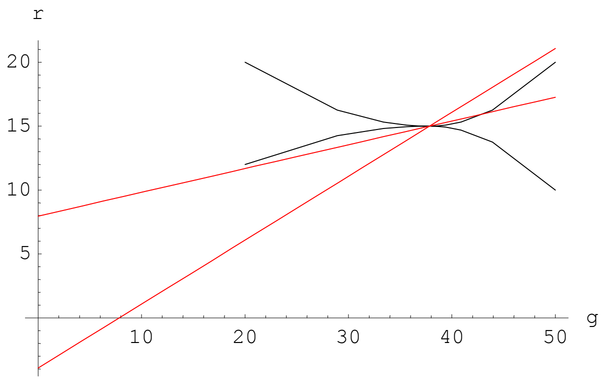
(c)

```
In[93]:= points83 = Table[{sol83[[1, 1, 2]], sol83[[1, 2, 2]]}, {t, 0, 10}];

In[94]:= traj83 = ListPlot[points83, PlotJoined -> True, DisplayFunction -> Identity];
```

(d)

```
In[95]:= points84 = Table[{sol84[[1, 1, 2]], sol84[[1, 2, 2]]}, {t, 0, 10}];

In[96]:= traj84 = ListPlot[points84, PlotJoined -> True, DisplayFunction -> Identity];

In[97]:= targetlines = Plot[{-3.925 + 0.5 g, 7.958 + 0.186 g}, {g, 0, 50},
              DisplayFunction -> Identity, PlotStyle -> {RGBColor[1, 0, 0]}];
```

```
In[98]:= Show[{traj81, traj82, traj83, traj84, targetlines},
           AxesLabel -> {"g", "r"}, DisplayFunction -> $DisplayFunction];
```



à Question 9

If interest rates are set to achieve internal balance and government spending to achieve external balance, then the set of equations are:

$$g_{t+1} = -21.3925 + 0.5\,g_t + 2.688\,r_t$$
$$r_{t+1} = -2.94375 + 0.375\,g_t + 0.25\,r_t$$

■ (a) Four initial points, one in each quadrant.

We consider four initial points as follows:

$(g_0, r_0) = (20, 20)$ Quadrant I
$(g_0, r_0) = (50, 20)$ Quadrant II
$(g_0, r_0) = (20, 40)$ Quadrant III
$(g_0, r_0) = (20, 10)$ Quadrant IV

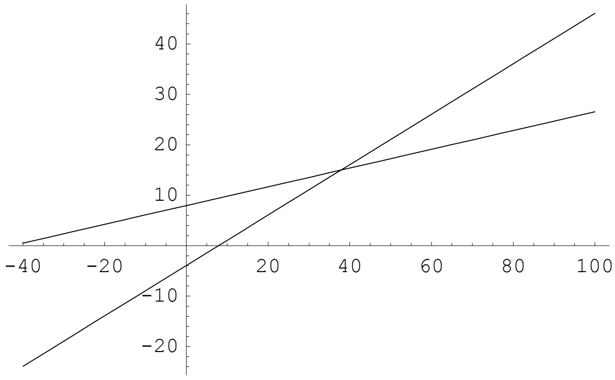
which are determined by the following two target lines:

$$r = -3.925 + 0.5\,g$$
$$g = -42.785 + 5.376\,r$$

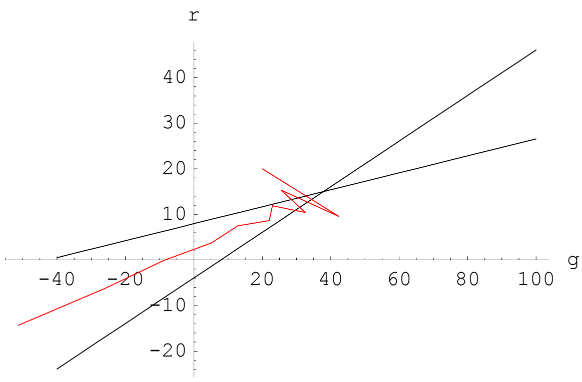
and equilibrium points:

```
In[99]:= Solve[{r == -3.925 + 0.5 g, g == -42.785 + 5.376 r}, {g, r}]

Out[99]= {{g → 37.847, r → 14.9985}}
```

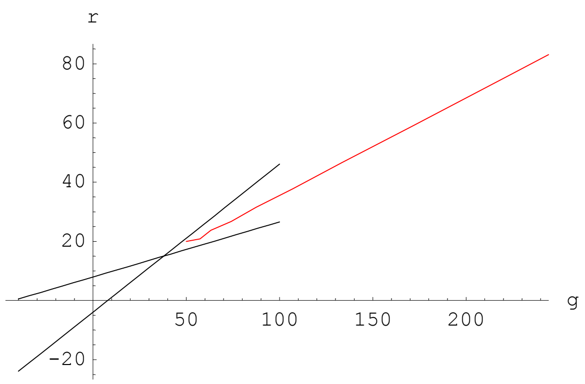
```
In[100]:= targetlines =
            Plot[{-3.925 + 0.5 g, (42.785 / 5.376) + (1 / 5.376) g}, {g, -40, 100}];
```



```
In[101]:= result1 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
              -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 20, r[0] == 20}, {g[t], r[t]}, t]

Out[101]= {{g[t] →
            -20. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                  2.45736 1.38674^t, 0]) - 7.3675 (-1. If[t == 1, 1, 0] +
                If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
             51.3247 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                  1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
            -1.1325 (-20. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
               If[t ≥ 1, -3.23956 (-0.636744)^t - 41.1426 1.^t + 44.3821 1.38674^t, 0])}}

In[102]:= points91 = Table[{result1[[1, 1, 2]], result1[[1, 2, 2]]}, {t, 0, 10}];

In[103]:= traj91 = ListPlot[points91, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[1, 0, 0]}];

In[104]:= Show[{targetlines, traj91},
            DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```

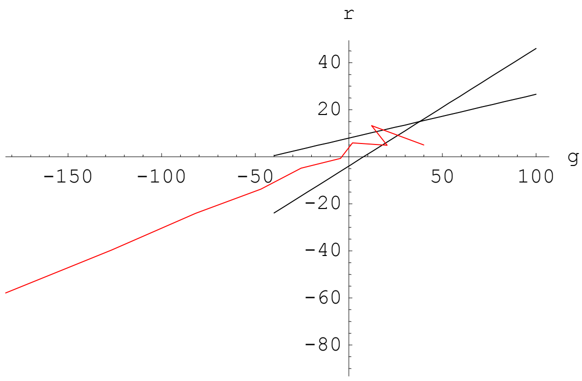


```
In[105]:= result2 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
            -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 50, r[0] == 20}, {g[t], r[t]}, t]

Out[105]= {{g[t] →
            -50. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                  2.45736 1.38674^t, 0]) + 30.1325 (-1. If[t == 1, 1, 0] +
                If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
             43.8247 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                  1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
            -1.1325 (-20. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
               If[t ≥ 1, 1.66967 (-0.636744)^t - 41.1426 1.^t + 39.4729 1.38674^t, 0])}}

In[106]:= points92 = Table[{result2[[1, 1, 2]], result2[[1, 2, 2]]}, {t, 0, 10}];

In[107]:= traj92 = ListPlot[points92, PlotJoined -> True, DisplayFunction -> Identity,
            PlotStyle -> {RGBColor[1, 0, 0]}];

In[108]:= Show[{targetlines, traj92},
          DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```

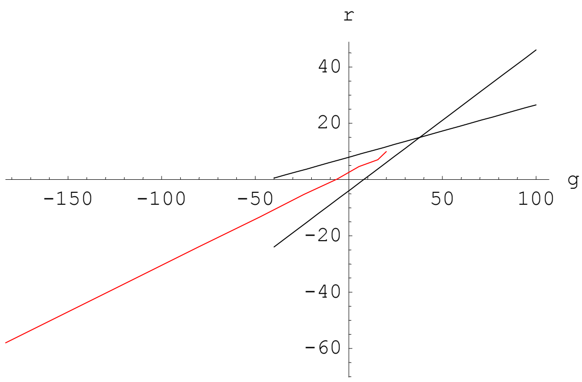



```
In[109]:= result3 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
            -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 40, r[0] == 5}, {g[t], r[t]}, t]

Out[109]= {{g[t] → -40. (-1. If[t == 0, 1, 0] +
                If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t - 2.45736 1.38674^t, 0]) +
             57.9525 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.192258 (-0.636744)^t +
                  1.57978 1.^t - 1.77204 1.38674^t, 0]) + 6.00467 (-1. If[t == 2, 1, 0] +
                If[t ≥ 3, -0.301939 (-0.636744)^t + 1.57978 1.^t - 1.27784 1.38674^t, 0]),
           r[t] → -1.1325 (-5. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
               If[t ≥ 1, 5.85253 (-0.636744)^t - 20.2184 1.^t + 14.3659 1.38674^t, 0])}}

In[110]:= points93 = Table[{result3[[1, 1, 2]], result3[[1, 2, 2]]}, {t, 0, 10}];

In[111]:= traj93 = ListPlot[points93, PlotJoined -> True, DisplayFunction -> Identity,
            PlotStyle -> {RGBColor[1, 0, 0]}];
```

```
In[112]:= Show[{targetlines, traj93},
            DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```




```
In[113]:= result4 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
              -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 20, r[0] == 10}, {g[t], r[t]}, t]

Out[113]= {{g[t] →
             -20. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                  2.45736 1.38674^t, 0]) + 19.5125 (-1. If[t == 1, 1, 0] +
                If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
              24.4447 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                  1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
             -1.1325 (-10. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
                If[t ≥ 1, 0.639955 (-0.636744)^t - 27.1931 1.^t + 26.5532 1.38674^t, 0])}}

In[114]:= points94 = Table[{result4[[1, 1, 2]], result4[[1, 2, 2]]}, {t, 0, 10}];

In[115]:= traj94 = ListPlot[points94, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[1, 0, 0]}];

In[116]:= Show[{targetlines, traj94},
            DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```



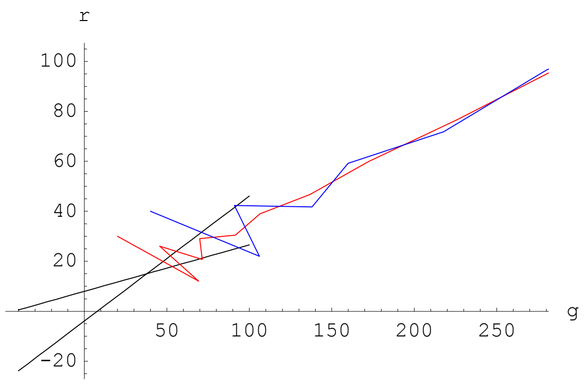
■ (b) Sectors I and III

We shall consider just two points in each of the two sectors as follows:

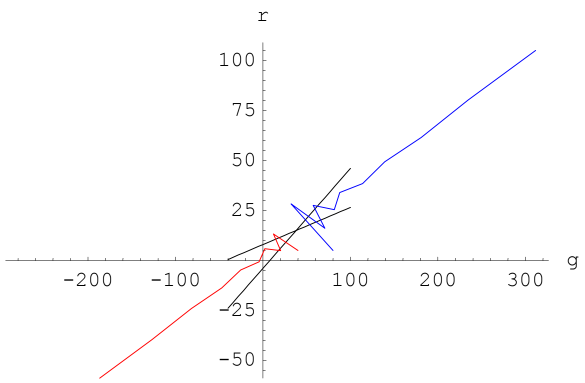
$(g_0, r_0) = (20, 30)$ $(g_0, r_0) = (40, 40)$ for Sector I

$(g_0, r_0) = (40, 5)$ $(g_0, r_0) = (80, 5)$ for Sector III

```
In[117]:= result5 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
              -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 20, r[0] == 30}, {g[t], r[t]}, t]

Out[117]= {{g[t] →
             -20. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                   2.45736 1.38674^t, 0]) - 34.2475 (-1. If[t == 1, 1, 0] +
                 If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
              78.2047 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                   1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
             -1.1325 (-30. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
                If[t ≥ 1, -7.11907 (-0.636744)^t - 55.092 1.^t + 62.2111 1.38674^t, 0])}}

In[118]:= points95 = Table[{result5[[1, 1, 2]], result5[[1, 2, 2]]}, {t, 0, 10}];

In[119]:= traj95 = ListPlot[points95, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[1, 0, 0]}];

In[120]:= result6 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
              -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 40, r[0] == 40}, {g[t], r[t]}, t]

Out[120]= {{g[t] →
             -40. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                   2.45736 1.38674^t, 0]) - 36.1275 (-1. If[t == 1, 1, 0] +
                 If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
              100.085 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                   1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
             -1.1325 (-40. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
                If[t ≥ 1, -7.72576 (-0.636744)^t - 69.0415 1.^t + 76.7672 1.38674^t, 0])}}

In[121]:= points96 = Table[{result6[[1, 1, 2]], result6[[1, 2, 2]]}, {t, 0, 10}];

In[122]:= traj96 = ListPlot[points96, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[0, 0, 1]}];

In[123]:= Show[{targetlines, traj95, traj96},
            DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```



```
In[124]:= result7 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
           -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 40, r[0] == 5}, {g[t], r[t]}, t]

Out[124]= {{g[t] → -40. (-1. If[t == 0, 1, 0] +
               If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t - 2.45736 1.38674^t, 0]) +
           57.9525 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.192258 (-0.636744)^t +
                 1.57978 1.^t - 1.77204 1.38674^t, 0]) + 6.00467 (-1. If[t == 2, 1, 0] +
               If[t ≥ 3, -0.301939 (-0.636744)^t + 1.57978 1.^t - 1.27784 1.38674^t, 0]),
         r[t] → -1.1325 (-5. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
              If[t ≥ 1, 5.85253 (-0.636744)^t - 20.2184 1.^t + 14.3659 1.38674^t, 0])}}

In[125]:= points97 = Table[{result7[[1, 1, 2]], result7[[1, 2, 2]]}, {t, 0, 10}];

In[126]:= traj97 = ListPlot[points97, PlotJoined -> True, DisplayFunction -> Identity,
           PlotStyle -> {RGBColor[1, 0, 0]}];

In[127]:= result8 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t], r[t + 1] ==
           -2.94375 + 0.375 g[t] + 0.25 r[t], g[0] == 80, r[0] == 5}, {g[t], r[t]}, t]

Out[127]= {{g[t] → -80. (-1. If[t == 0, 1, 0] +
               If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t - 2.45736 1.38674^t, 0]) +
           107.953 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.192258 (-0.636744)^t +
                 1.57978 1.^t - 1.77204 1.38674^t, 0]) - 3.99533 (-1. If[t == 2, 1, 0] +
               If[t ≥ 3, -0.301939 (-0.636744)^t + 1.57978 1.^t - 1.27784 1.38674^t, 0]),
         r[t] → -1.1325 (-5. (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
              If[t ≥ 1, 12.3982 (-0.636744)^t - 20.2184 1.^t + 7.82026 1.38674^t, 0])}}

In[128]:= points98 = Table[{result8[[1, 1, 2]], result8[[1, 2, 2]]}, {t, 0, 10}];

In[129]:= traj98 = ListPlot[points98, PlotJoined -> True, DisplayFunction -> Identity,
           PlotStyle -> {RGBColor[0, 0, 1]}];

In[130]:= Show[{targetlines, traj97, traj98},
          DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```



■ (c) A point on the stable arm

The stable and unstable arms of the saddle point can be obtained from the Eigenvectors of the system defined by the following matrix:

$$\begin{pmatrix} 0.5 & 2.688 \\ 0.375 & 0.25 \end{pmatrix}$$

```
In[131]:= matrixA9 = ( 0.5    2.688 )
                     ( 0.375  0.25  )

Out[131]= {{0.5, 2.688}, {0.375, 0.25}}

In[132]:= Eigenvalues[matrixA9]

Out[132]= {1.38674, -0.636744}

In[133]:= Eigenvectors[matrixA9]

Out[133]= {{0.94966, 0.313283}, {-0.921027, 0.389498}}
```

Taking a unit value in the *g*-direction, we obtain the following values for *r.* These values represent points on the stable and unstable arms, where the positive value is on the unstable arm and the negative value on the stable arm, as can be seen in terms of Figure 5.17 (p.242) of the text.

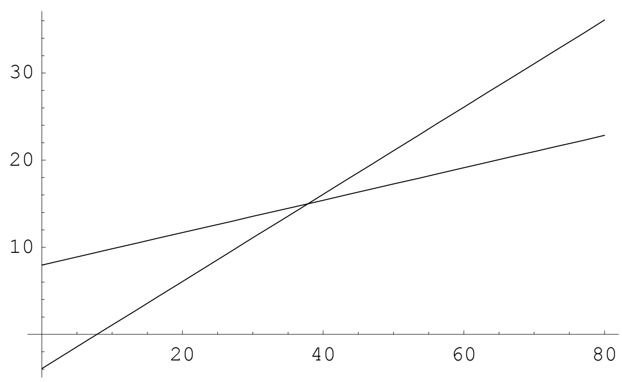
```
In[134]:= {{1, 0.313283 / 0.94966}, {1, -0.389498 / 0.921027}}

Out[134]= {{1, 0.32989}, {1, -0.422895}}
```

The equation for the stable arm is then,

$$14.9985 - 0.422895\,(g - 37.847)$$

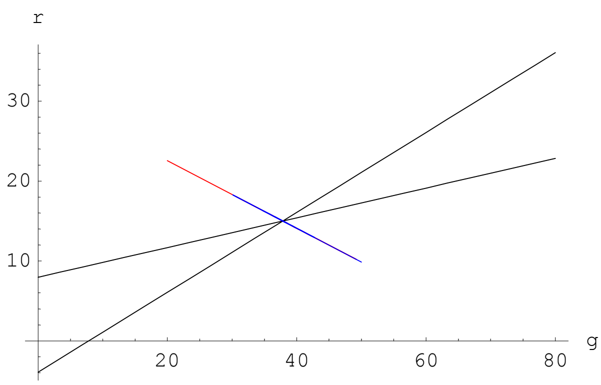
We shall consider two points, one in quadrant I and a second in quadrant III. The first is determined by $g = 20$ and the second by $g = 50$

```
In[135]:= targetlines2 =
            Plot[{-3.925 + 0.5 g, (42.785 / 5.376) + (1 / 5.376) g}, {g, 0, 80}];
```




```
In[136]:= {{20, 14.9985 - 0.422895 (20 - 37.847)}, {50, 14.9985 - 0.422895 (50 - 37.847)}}

Out[136]= {{20, 22.5459}, {50, 9.85906}}

In[137]:= result9 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t],
             r[t + 1] == -2.94375 + 0.375 g[t] + 0.25 r[t],
             g[0] == 20, r[0] == 22.5459}, {g[t], r[t]}, t]

Out[137]= {{g[t] →
            -20. (-1. If[t == 0, 1, 0] + If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t -
                  2.45736 1.38674^t, 0]) - 14.2109 (-1. If[t == 1, 1, 0] +
                If[t ≥ 2, 0.192258 (-0.636744)^t + 1.57978 1.^t - 1.77204 1.38674^t, 0]) +
             58.1681 (-1. If[t == 2, 1, 0] + If[t ≥ 3, -0.301939 (-0.636744)^t +
                  1.57978 1.^t - 1.27784 1.38674^t, 0]), r[t] →
            -1.1325 (-22.5459 (0.108096 (-0.636744)^t - 1.39494 1.^t + 2.16985 1.38674^t) +
               If[t ≥ 1, -4.22724 (-0.636744)^t - 44.694 1.^t + 48.9212 1.38674^t, 0])}}
```

```
In[138]:= points99 = Table[{result9[[1, 1, 2]], result9[[1, 2, 2]]}, {t, 0, 10}];

In[139]:= traj99 = ListPlot[points99, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[1, 0, 0]}];

In[140]:= result10 = RSolve[{g[t + 1] == -21.3925 + 0.5 g[t] + 2.688 r[t],
             r[t + 1] == -2.94375 + 0.375 g[t] + 0.25 r[t],
             g[0] == 50, r[0] == 9.85906}, {g[t], r[t]}, t]

Out[140]= {{g[t] → -50. (-1. If[t == 0, 1, 0] +
                 If[t ≥ 1, -0.122419 (-0.636744)^t + 1.57978 1.^t - 2.45736 1.38674^t, 0]) +
              57.3913 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.192258 (-0.636744)^t +
                   1.57978 1.^t - 1.77204 1.38674^t, 0]) + 16.5658 (-1. If[t == 2, 1, 0] +
                 If[t ≥ 3, -0.301939 (-0.636744)^t + 1.57978 1.^t - 1.27784 1.38674^t, 0]),
            r[t] → -1.1325 (4.53813 (-0.636744)^t - 13.2437 1.^t + 9.28217×10^-6 1.38674^t)}}

In[141]:= points910 = Table[{result10[[1, 1, 2]], result10[[1, 2, 2]]}, {t, 0, 10}];

In[142]:= traj910 =
             ListPlot[points910, PlotJoined -> True, DisplayFunction -> Identity,
              PlotStyle -> {RGBColor[0, 0, 1]}];

In[143]:= Show[{targetlines2, traj99, traj910},
             DisplayFunction -> $DisplayFunction, AxesLabel -> {"g", "r"}];
```



à Question 10

We have the system of equations:

$$g_{t+1} = -1.875 + 0.25\, g_t + 0.375\, S_t$$
$$S_{t+1} = 10 - 2\, g_t + 0.2\, S_t$$

and we have the target equilibrium lines:

$$g_t^* = -2.5 + 0.5\, S_t$$
$$S_t^* = 20 - 4\, g_t$$

(a)

```
In[144]:= result10 = RSolve[{g[t + 1] == -1.875 + 0.25 g[t] + 0.375 s[t],
              s[t + 1] == 10 - 2 g[t] + 0.2 s[t], g[0] == 2.5, s[0] == 12}, {g[t], s[t]}, t]

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        General::stop :
         Further output of $RecursionLimit::reclim will be suppressed during this calculation.

        $IterationLimit::itlim : Iteration limit of 4096 exceeded.

Out[144]= {{g[t] → -2.5 (-1. If[t == 0, 1, 0] +
                 If[t ≥ 1, (-0.35463 + 0.0276228 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) - (0.35463 +
                      0.0276228 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) - 0.740741 1.^(-1+t), 0. + 0. ⅈ]) +
              0.375 (-1. If[t == 1, 1, 0] + If[t ≥ 2, (-0.35463 + 0.0276228 ⅈ)
                    (0.225 - 0.865664 ⅈ)^(-2+t) - (0.35463 + 0.0276228 ⅈ)
                    (0.225 + 0.865664 ⅈ)^(-2+t) - 0.740741 1.^(-2+t), 0. + 0. ⅈ]) -
              0.125 (-1. If[t == 2, 1, 0] + If[t ≥ 3, (-0.35463 + 0.0276228 ⅈ)
                    (0.225 - 0.865664 ⅈ)^(-3+t) - (0.35463 + 0.0276228 ⅈ)
                    (0.225 + 0.865664 ⅈ)^(-3+t) - 0.740741 1.^(-3+t), 0. + 0. ⅈ]),
            s[t] → 1.25 (-12. ((-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^t -
                   (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^t - 0.592593 1.^t) +
                10. If[t ≥ 1, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) -
                   (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) -
                   0.592593 1.^(-1+t), 0. + 0. ⅈ] -
                9.25 If[t ≥ 2, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-2+t) -
                   (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-2+t) -
                   0.592593 1.^(-2+t), 0. + 0. ⅈ])}}

In[145]:= points101 = Table[{result10[[1, 1, 2]], result10[[1, 2, 2]]}, {t, 0, 10}];

In[146]:= traj101 =
            ListPlot[points101, PlotJoined -> True, DisplayFunction -> Identity];
```

(b)

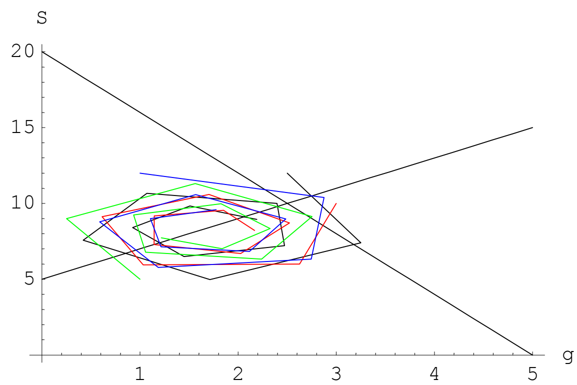
```
In[147]:= result102 = RSolve[{g[t + 1] == -1.875 + 0.25 g[t] + 0.375 s[t],
            s[t + 1] == 10 - 2 g[t] + 0.2 s[t], g[0] == 3, s[0] == 10}, {g[t], s[t]}, t]

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        General::stop :
         Further output of $RecursionLimit::reclim will be suppressed during this calculation.

        $IterationLimit::itlim : Iteration limit of 4096 exceeded.

Out[147]= {{g[t] → -3. (-1. If[t == 0, 1, 0] +
                If[t ≥ 1, (-0.35463 + 0.0276228 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) - (0.35463 +
                     0.0276228 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) - 0.740741 1.^(-1+t), 0. + 0. ⅈ]) +
             1.725 (-1. If[t == 1, 1, 0] + If[t ≥ 2, (-0.35463 + 0.0276228 ⅈ)
                   (0.225 - 0.865664 ⅈ)^(-2+t) - (0.35463 + 0.0276228 ⅈ)
                   (0.225 + 0.865664 ⅈ)^(-2+t) - 0.740741 1.^(-2+t), 0. + 0. ⅈ]) -
             0.975 (-1. If[t == 2, 1, 0] + If[t ≥ 3, (-0.35463 + 0.0276228 ⅈ)
                   (0.225 - 0.865664 ⅈ)^(-3+t) - (0.35463 + 0.0276228 ⅈ)
                   (0.225 + 0.865664 ⅈ)^(-3+t) - 0.740741 1.^(-3+t), 0. + 0. ⅈ]),
           s[t] → 1.25 (-10. ((-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^t -
                  (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^t - 0.592593 1.^t) +
                8.5 If[t ≥ 1, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) -
                  (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) -
                  0.592593 1.^(-1+t), 0. + 0. ⅈ] -
                9.75 If[t ≥ 2, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-2+t) -
                  (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-2+t) -
                  0.592593 1.^(-2+t), 0. + 0. ⅈ])}}

In[148]:= points102 = Table[{result102[[1, 1, 2]], result102[[1, 2, 2]]}, {t, 0, 10}];

In[149]:= traj102 = ListPlot[points102, PlotJoined -> True,
            DisplayFunction -> Identity, PlotStyle -> {RGBColor[1, 0, 0]}];
```

(c)

```
In[150]:= result103 = RSolve[{g[t + 1] == -1.875 + 0.25 g[t] + 0.375 s[t],
            s[t + 1] == 10 - 2 g[t] + 0.2 s[t], g[0] == 1, s[0] == 5}, {g[t], s[t]}, t]

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        $RecursionLimit::reclim : Recursion depth of 256 exceeded.

        General::stop :
         Further output of $RecursionLimit::reclim will be suppressed during this calculation.

        $IterationLimit::itlim : Iteration limit of 4096 exceeded.

Out[150]= {{g[t] → -1. (-1. If[t == 0, 1, 0] +
                If[t ≥ 1, (-0.35463 + 0.0276228 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) - (0.35463 +
                      0.0276228 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) - 0.740741 1.^(-1+t), 0. + 0. ⅈ]) +
             1.2 (-1. If[t == 1, 1, 0] + If[t ≥ 2, (-0.35463 + 0.0276228 ⅈ)
                    (0.225 - 0.865664 ⅈ)^(-2+t) - (0.35463 + 0.0276228 ⅈ)
                    (0.225 + 0.865664 ⅈ)^(-2+t) - 0.740741 1.^(-2+t), 0. + 0. ⅈ]) -
             2.45 (-1. If[t == 2, 1, 0] + If[t ≥ 3, (-0.35463 + 0.0276228 ⅈ)
                    (0.225 - 0.865664 ⅈ)^(-3+t) - (0.35463 + 0.0276228 ⅈ)
                    (0.225 + 0.865664 ⅈ)^(-3+t) - 0.740741 1.^(-3+t), 0. + 0. ⅈ]),
           s[t] → 1.25 (-5. ((-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^t -
                  (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^t - 0.592593 1.^t) -
                1.75 If[t ≥ 1, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-1+t) -
                   (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-1+t) -
                   0.592593 1.^(-1+t), 0. + 0. ⅈ] -
                4.5 If[t ≥ 2, (-0.103704 - 0.300775 ⅈ) (0.225 - 0.865664 ⅈ)^(-2+t) -
                   (0.103704 - 0.300775 ⅈ) (0.225 + 0.865664 ⅈ)^(-2+t) -
                   0.592593 1.^(-2+t), 0. + 0. ⅈ])}}

In[151]:= points103 = Table[{result103[[1, 1, 2]], result103[[1, 2, 2]]}, {t, 0, 10}];

In[152]:= traj103 = ListPlot[points103, PlotJoined -> True,
            DisplayFunction -> Identity, PlotStyle -> {RGBColor[0, 1, 0]}];
```

(d)

```
In[153]:= result104 = RSolve[{g[t + 1] == -1.875 + 0.25 g[t] + 0.375 s[t],
             s[t + 1] == 10 - 2 g[t] + 0.2 s[t], g[0] == 1, s[0] == 12}, {g[t], s[t]}, t]

    $RecursionLimit::reclim : Recursion depth of 256 exceeded.

    $RecursionLimit::reclim : Recursion depth of 256 exceeded.

    $RecursionLimit::reclim : Recursion depth of 256 exceeded.

    General::stop :
     Further output of $RecursionLimit::reclim will be suppressed during this calculation.

    $IterationLimit::itlim : Iteration limit of 4096 exceeded.

Out[153]= {{g[t] → -1. (-1. If[t == 0, 1, 0] +
              If[t ≥ 1, (-0.35463 + 0.0276228 i) (0.225 - 0.865664 i)^(-1+t) - (0.35463 +
                  0.0276228 i) (0.225 + 0.865664 i)^(-1+t) - 0.740741 1.^(-1+t), 0. + 0. i]) -
            1.425 (-1. If[t == 1, 1, 0] + If[t ≥ 2, (-0.35463 + 0.0276228 i)
                  (0.225 - 0.865664 i)^(-2+t) - (0.35463 + 0.0276228 i)
                  (0.225 + 0.865664 i)^(-2+t) - 0.740741 1.^(-2+t), 0. + 0. i]) +
            0.175 (-1. If[t == 2, 1, 0] + If[t ≥ 3, (-0.35463 + 0.0276228 i)
                  (0.225 - 0.865664 i)^(-3+t) - (0.35463 + 0.0276228 i)
                  (0.225 + 0.865664 i)^(-3+t) - 0.740741 1.^(-3+t), 0. + 0. i]),
          s[t] → 1.25 (-12. ((-0.103704 - 0.300775 i) (0.225 - 0.865664 i)^t -
                  (0.103704 - 0.300775 i) (0.225 + 0.865664 i)^t - 0.592593 1.^t) +
              7. If[t ≥ 1, (-0.103704 - 0.300775 i) (0.225 - 0.865664 i)^(-1+t) -
                  (0.103704 - 0.300775 i) (0.225 + 0.865664 i)^(-1+t) -
                  0.592593 1.^(-1+t), 0. + 0. i] -
              6.25 If[t ≥ 2, (-0.103704 - 0.300775 i) (0.225 - 0.865664 i)^(-2+t) -
                  (0.103704 - 0.300775 i) (0.225 + 0.865664 i)^(-2+t) -
                  0.592593 1.^(-2+t), 0. + 0. i])}}

In[154]:= points104 = Table[{result104[[1, 1, 2]], result104[[1, 2, 2]]}, {t, 0, 10}];

In[155]:= traj104 = ListPlot[points104, PlotJoined -> True,
             DisplayFunction -> Identity, PlotStyle -> {RGBColor[0, 0, 1]}];

In[156]:= targetlines10 =
            Plot[{5 + 2 g, 20 - 4 g}, {g, 0, 5}, DisplayFunction -> Identity];

In[157]:= Show[{targetlines10, traj101, traj102, traj103, traj104},
            AxesLabel -> {"g", "S"}, DisplayFunction -> $DisplayFunction];
```



This plot readily shows all trajectories converging on the equilibrium - but very slowly.

à Question 11

```
In[158]:= mA := {{2, 3}, {1, -2}}

In[159]:= mB := {{4, -2}, {1, -1}}

In[160]:= mC := {{3, 2, 1}, {-1, 0, 3}}

In[161]:= mA.mB // MatrixForm
```

Out[161]//MatrixForm=

$$\begin{pmatrix} 11 & -7 \\ 2 & 0 \end{pmatrix}$$

```
In[162]:= Tr[mA.mB]
```

Out[162]= 11

```
In[163]:= Det[mA.mB]
```

Out[163]= 14

```
In[164]:= Transpose[mA.mC] // MatrixForm
```

Out[164]//MatrixForm=

$$\begin{pmatrix} 3 & 5 \\ 4 & 2 \\ 11 & -5 \end{pmatrix}$$

```
In[165]:= Inverse[mB] // MatrixForm
```

Out[165]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & -2 \end{pmatrix}$$

```
In[166]:= Eigenvalues[mA]
```

Out[166]= $\{-\sqrt{7}, \sqrt{7}\}$

```
In[167]:= Eigenvalues[mB]
```

Out[167]= $\left\{\frac{1}{2}\left(3-\sqrt{17}\right), \frac{1}{2}\left(3+\sqrt{17}\right)\right\}$

```
In[168]:= Eigenvectors[mA]
```

Out[168]= $\{\{2-\sqrt{7}, 1\}, \{2+\sqrt{7}, 1\}\}$

```
In[169]:= Eigenvectors[mB]
```

Out[169]= $\left\{\left\{\frac{1}{2}\left(5-\sqrt{17}\right), 1\right\}, \left\{\frac{1}{2}\left(5+\sqrt{17}\right), 1\right\}\right\}$

```
In[170]:= CharacteristicPolynomial[mA, λ]
```

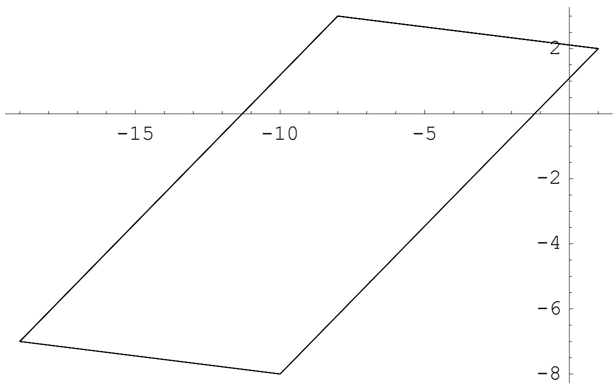
Out[170]= $-7+\lambda^2$

à Question 12

Solve the following system

$$x_{t+1} = -5 + x_t - 2\,y_t$$
$$y_{t+1} = 4 + x_t - y_t$$
$$x_0 = 1,\ \ y_0 = 2$$

```
In[171]:= Solve[{x == -5 + x - 2 y, y == 4 + x - y}, {x, y}]

Out[171]= {{x → -9, y → -5/2}}

In[172]:= sol = RSolve[{x[t + 1] == -5 + x[t] - 2 y[t],
             y[t + 1] == 4 + x[t] - y[t], x[0] == 1, y[0] == 2}, {x[t], y[t]}, t]

Out[172]= {{x[t] → -9 + (5 + ⅈ/2) (-ⅈ)^t + (5 - ⅈ/2) ⅈ^t,
            y[t] → (1/4 + ⅈ/4) ((-5 + 5 ⅈ) + (10 + ⅈ) (-ⅈ)^t - (1 + 10 ⅈ) ⅈ^t)}}

In[173]:= solx = -9 + (5 + ⅈ/2) (-ⅈ)^t + (5 - ⅈ/2) ⅈ^t

        General::spell1 :
         Possible spelling error: new symbol name "solx" is similar to existing symbol "sol".

Out[173]= -9 + (5 + ⅈ/2) (-ⅈ)^t + (5 - ⅈ/2) ⅈ^t

In[174]:= soly = (1/4 + ⅈ/4) ((-5 + 5 ⅈ) + (10 + ⅈ) (-ⅈ)^t - (1 + 10 ⅈ) ⅈ^t)

        General::spell :
         Possible spelling error: new symbol name "soly" is similar to existing symbols {sol, solx}.

Out[174]= (1/4 + ⅈ/4) ((-5 + 5 ⅈ) + (10 + ⅈ) (-ⅈ)^t - (1 + 10 ⅈ) ⅈ^t)

In[175]:= dataset = Table[{solx, soly}, {t, 0, 10}]

Out[175]= {{1, 2}, {-8, 3}, {-19, -7}, {-10, -8}, {1, 2},
           {-8, 3}, {-19, -7}, {-10, -8}, {1, 2}, {-8, 3}, {-19, -7}}

In[176]:= ListPlot[dataset, PlotJoined → True];
```



à Question 13

```
In[177]:= mA := {{1, -2}, {1, -1}}

In[178]:= {V, J} = JordanDecomposition[{{1, -2}, {1, -1}}]

Out[178]= {{{1 - i, 1 + i}, {1, 1}}, {{-i, 0}, {0, i}}}

In[179]:= MatrixForm /@ {V, J}
```

Out[179]= $\left\{\begin{pmatrix} 1-i & 1+i \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\right\}$

```
In[180]:= Inverse[V].mA.V

Out[180]= {{-i, 0}, {0, i}}
```

à Question 14

We have already solved for the fixed point in question 12 and shown that for this model the system is cyclical. The question is specifically to set up the problem on a spreadsheet to show that the same cyclical pattern results.

à Question 15

```
In[181]:= Clear[mA, V, J]

In[182]:= Solve[{x == 5.6 - 0.4 x, y == 3.5 + 0.4 x - 0.5 y}, {x, y}]

Out[182]= {{x → 4., y → 3.4}}

In[183]:= mA := {{-0.4, 0}, {0.4, -0.5}}

In[184]:= {V, J} = JordanDecomposition[mA]

Out[184]= {{{-4., 4.12311}, {1., 0.}}, {{-0.5, 0}, {0, -0.4}}}

In[185]:= MatrixForm /@ {V, J}
```

Out[185]= $\left\{\begin{pmatrix} -4. & 4.12311 \\ 1. & 0. \end{pmatrix}, \begin{pmatrix} -0.5 & 0 \\ 0 & -0.4 \end{pmatrix}\right\}$

```
In[186]:= Inverse[V].mA.V

Out[186]= {{-2.1, 1.64924}, {-1.64924, 1.2}}
```

Even if we evaluate

```
In[187]:= V.mA.Inverse[V]

Out[187]= {{-0.5, 1.24924}, {0., -0.4}}
```

We do not arrive at the matrix J. There appears to be a flaw in the JordanDecomposition programme of *Mathematica*. The correct matrix V is

```
In[188]:= newV := {{1, 0}, {4, -4}}
```

To verify this

```
In[189]:= Inverse[newV].mA.newV

Out[189]= {{-0.4, 0.}, {0., -0.5}}
```

We shall pursue the answer with newV.

```
In[190]:= u0 := {{2}, {1}}

In[191]:= Inverse[newV].u0
```

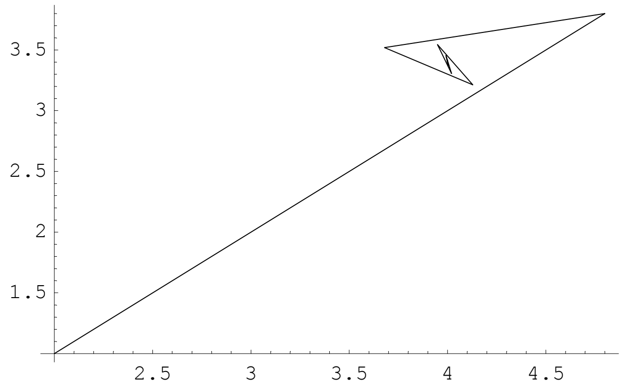
Out[191]= $\{\{2\}, \{\frac{7}{4}\}\}$

```
In[192]:= Eigenvalues[mA]

Out[192]= {-0.5, -0.4}

In[193]:= ustar := {{4}, {3.4}}

    General::spell :
     Possible spelling error: new symbol name "ustar" is similar to existing symbols {xstar, ystar}.

In[194]:= u0Minusustar = u0 - ustar

Out[194]= {{-2}, {-2.4}}

In[195]:= Inverse[newV].u0Minusustar

Out[195]= {{-2.}, {-1.4}}
```

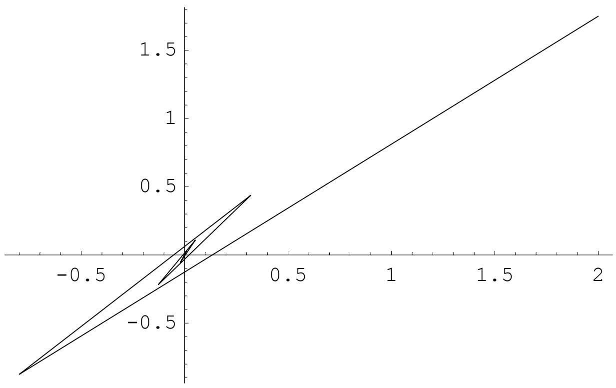
Although the question requires the problem to be set up on a spreadsheet, we shall here use *Mathematica* to set out the plots of the original system and its canonical form.

```
In[196]:= sol15 = RSolve[{x[t] == 5.6 - 0.4 x[t - 1],
             y[t] == 3.5 + 0.4 x[t - 1] - 0.5 y[t - 1], x[0] == 2, y[0] == 1}, {x[t], y[t]}, t]

Out[196]= {{x[t] → -2.5 (0.8 (-0.4)^t - 1.6 1.^t),
             y[t] → -5. (2.13333 (-0.5)^t - 1.88571 (-0.4)^t - 0.447619 1.^t +
                 2.44 If[t ≥ 2, -1.33333 (-0.5)^t + 1.42857 (-0.4)^t - 0.0952381 1.^t, 0])}}

In[197]:= solx = -2.5`
             (0.7999999999999998` (-0.4`)^t - 1.5999999999999996` 0.9999999999999999`^t)

Out[197]= -2.5 (0.8 (-0.4)^t - 1.6 1.^t)

In[198]:= soly = -5.` (2.133333333333333` (-0.5000000000000001`)^t -
              1.885714285714285` (-0.4`)^t - 0.44761904761904764` 1.`^t +
              2.44` If[t ≥ 2, -1.3333333333333324` (-0.5000000000000001`)^t +
                 1.4285714285714277` (-0.4`)^t - 0.09523809523809523` 1.`^t, 0])

Out[198]= -5. (2.13333 (-0.5)^t - 1.88571 (-0.4)^t - 0.447619 1.^t +
            2.44 If[t ≥ 2, -1.33333 (-0.5)^t + 1.42857 (-0.4)^t - 0.0952381 1.^t, 0])
```

```
In[199]:= dataset1 = Table[{solx, soly}, {t, 0, 10}]

Out[199]= {{2., 1.}, {4.8, 3.8}, {3.68, 3.52}, {4.128, 3.212}, {3.9488, 3.5452},
           {4.02048, 3.30692}, {3.99181, 3.45473}, {4.00328, 3.36936},
           {3.99869, 3.41663}, {4.00052, 3.39116}, {3.99979, 3.40463}}

In[200]:= ListPlot[dataset1, PlotJoined → True];
```




```
In[201]:= dataset2 = Table[{(-0.4)^t 2, (-0.5)^t 1.75}, {t, 0, 10}]

Out[201]= {{2, 1.75}, {-0.8, -0.875}, {0.32, 0.4375}, {-0.128, -0.21875},
           {0.0512, 0.109375}, {-0.02048, -0.0546875}, {0.008192, 0.0273438},
           {-0.0032768, -0.0136719}, {0.00131072, 0.00683594},
           {-0.000524288, -0.00341797}, {0.000209715, 0.00170898}}

In[202]:= ListPlot[dataset2, PlotJoined → True];
```



Both the original plot and the canonical form indicates that the system is asmptotically stable.