13: The fourth derivative matrix

(**a**) Here is the 2nd-order, centered finite-difference approximation to the fourth derivative:

$$f_i^{(4)} = f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1} + f_{i+2}.$$

(**b**) Boundary conditions are implemented as follows:

- for rigid boundaries,

$$f_1^{(4)} = 7f_1 - 4f_2 + 7f_3\ ; \quad f_N^{(4)} = f_{N-2} - 4f_{N-1} + 7f_N,$$

- for frictionless boundaries,

$$f_1^{(4)} = 5f_1 - 4f_2 + 7f_3\ ; \quad f_N^{(4)} = f_{N-2} - 4f_{N-1} + 5f_N,$$

- and for both boundary types,

$$f_2^{(4)} = -4f_1 + 6f_2 - 4f_3 + f_4\ ; \quad f_{N-1}^{(4)} = f_{N-3} - 4f_{N-2} + 6f_{N-1} - 4f_N.$$