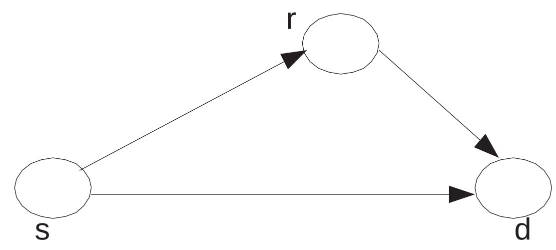
Cooperative Communications and Networking

Chapter 8

Multi-Node Cooperative Communications with Relay-Selection: When to Cooperate and Whom to Cooperate with?

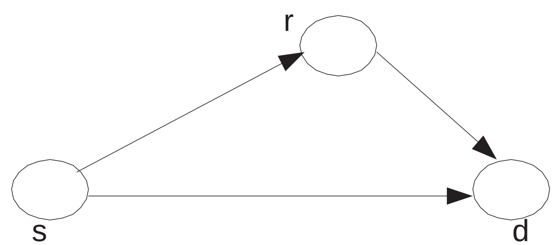
Conventional Single-Relay DF Cooperative Scenario



- Full Diversity Order

- Transmission Rate=$\frac{1}{2}$ SPCU !!

Conventional Single-Relay Cooperative Scenario

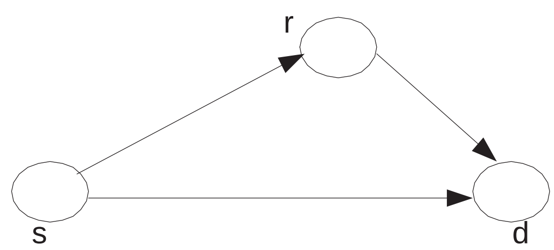


- Full Diversity Order

- Transmission Rate=$\frac{1}{2}$ SPCU !!

- How to increase the transmission rate, while guaranteeing full diversity order?

Relay-Selection Idea



- The source asks for the relay's help only if it needs it. Otherwise, it uses direct transmission in one phase only.

- Need to answer the question "When to Cooperate?" to achieve
  - $\frac{1}{2} \leq$ Transmission Rate < 1 SPCU
  - Full diversity

Outline

- Motivation for Relay-Selection Criterion
- System Model
- Performance Analysis
- Multi-node Scenario: When to Cooperate and Whom to Cooperate with?
- Data Rate-SER Tradeoff
- Simulation Results
- Conclusion

- The SER for M-PSK signalling can be upper bounded as

$$Pr(e) \leq \frac{N_0^2}{b^2\ \delta_{s,d}^2\ P\ P_1} \left(\frac{A^2}{r\ \delta_{s,r}^2} + \frac{B}{(1-r)\ \delta_{r,d}^2} \right) , \qquad (1)$$

where $b = \sin^2(\pi/M)$, $A = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \sin^2\theta\ d\theta = \frac{M-1}{2M} + \frac{\sin(\frac{2\pi}{M})}{4\pi}$, $B = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \sin^4\theta\ d\theta = \frac{3(M-1)}{8M} + \frac{\sin(\frac{2\pi}{M})}{4\pi} - \frac{\sin(\frac{4\pi}{M})}{32\pi}$, and $r = \frac{P_1}{P}$ is referred to as *power ratio*.

- A metric which gives a measure, in some sense, about how much help a relay can provide to the source, can be written as

$$m = \frac{A^2}{r\ \delta_{s,r}^2} + \frac{B}{(1-r)\ \delta_{r,d}^2} . \qquad (2)$$

Motivation for Relay-Selection Criterion

- We can formulate (2) on a standard harmonic mean function as

$$m' = \frac{2\ q_1\ q_2}{m} = \frac{2\ q_1\ q_2\ \delta_{s,r}^2\ \delta_{r,d}^2}{q_1\ \delta_{r,d}^2 + q_2\ \delta_{s,r}^2} = \mu_H(q_1\ \delta_{r,d}^2, q_2\ \delta_{s,r}^2)\ . \quad (3)$$

where $q_1 = \frac{A^2}{r}$ and $q_2 = \frac{B}{(1-r)}$.

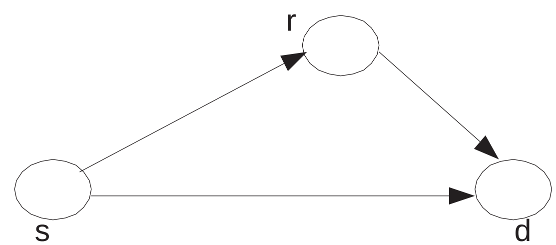
- The instantaneous harmonic mean function can be defined as

$$\beta_m = \mu_H(q_1\ \beta_{r,d}, q_2\ \beta_{s,r}) = \frac{2\ q_1\ \ q_2\ \beta_{s,r}\ \beta_{r,d}}{q_1\ \beta_{r,d} + q_2\ \beta_{s,r}}\ , \quad (4)$$

where $\beta_{s,r} = |h_{s,r}|^2$ and $\beta_{r,d} = |h_{r,d}|^2$.

- The instantaneous value of (4) can be used to give instantaneous indication about the relay's capability to help.

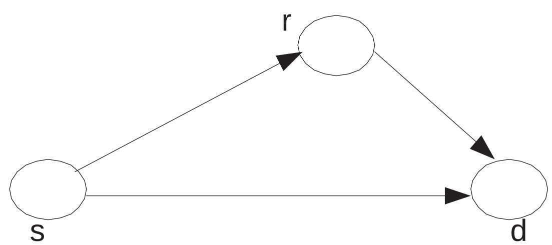
Main Idea: When to Cooperate?



- If $\frac{\beta_{s,d}}{\beta_m} \geq \alpha$, where α is called *cooperation threshold*, then the source does not utilize the relay
  - The resultant instantaneous data rate is 1 SPCU

- If $\frac{\beta_{s,d}}{\beta_m} < \alpha$, then the source employs the relay to forward its information to the destination in two consecutive phases
  - The resultant instantaneous data rate is 1/2 SPCU

System Model



- In the first phase, if $\frac{\beta_{s,d}}{\beta_m} \geq \alpha$, then the source decides to use direct transmission only
  - This mode is referred to as the *direct-transmission* mode
  - The received symbol at the destination can be modeled as

$$y^{\phi}_{s,d} = \sqrt{P}\, h_{s,d}\, x + \eta_{s,d}, \tag{5}$$

where $\phi = \{\ \beta_{s,d} \geq \alpha\ \beta_m\ \}$ denotes the event of direct transmission.

System Model

- If $\frac{\beta_{s,d}}{\beta_m} < \alpha$, then the source employs the relay to transmit its information
  - This mode is denoted by *relay-cooperation* mode
  - In the first phase,

$$y_{s,d}^{\phi^c} = \sqrt{P_1}\, h_{s,d}\, x + \eta_{s,d}, \qquad y_{s,r}^{\phi^c} = \sqrt{P_1}\, h_{s,r}\, x + \eta_{s,r}, \quad (6)$$

  - In the second phase,

$$y_{r,d}^{\phi^c} = \sqrt{\tilde{P}_2}\, h_{r,d}\, x + \eta_{r,d}, \quad (7)$$

    where $\tilde{P}_2 = P_2$ if the relay decodes the symbol correctly, otherwise $\tilde{P}_2 = 0$.

  - Power is distributed between the source and the relay subject to the power constraint $P_1 + P_2 = P$.

- Let $\beta_{i,j} = |h_{i,j}|^2$ that is exponentially distributed with parameter $1/\delta_{i,j}^2$

- The PDF and the CDF of β_m can be written as

$$p_{\beta_m}(\beta_m) = \frac{\beta_m}{2\,t_1^2}\,\exp(-\frac{t_2}{2}\,\beta_m)\left(t_1\,t_2\,K_1(\frac{\beta_m}{t_1}) + 2\,K_0(\frac{\beta_m}{t_1})\right) U\,(\beta$$

$$P_{\beta_m}(\beta_m) = 1 - \frac{\beta_m}{t_1}\,\exp(-\frac{t_2}{2}\,\beta_m)\,K_1(\frac{\beta_m}{t_1})\,,$$

where

$$t_1 = \sqrt{q_1 q_2 \delta_{s,r}^2 \delta_{r,d}^2}\,, \quad t_2 = \frac{1}{q_2\delta_{s,r}^2} + \frac{1}{q_1\delta_{r,d}^2}\,. \qquad (9)$$

- The probability of direct-transmission mode can be obtained as

$$\begin{aligned} Pr(\phi) &= Pr\left(\beta_{s,d} \geq \alpha\, \beta_m\right) \\ &\approx \frac{t_2\, \delta_{s,d}^2}{2\alpha + t_2\, \delta_{s,d}^2}\,, \end{aligned} \tag{10}$$

where we approximated $K_1(.)$ as $K_1(x) \approx \frac{1}{x}$

- The probability of the relay-cooperation mode is

$$Pr(\phi^c) \;=\; 1 - Pr(\phi) \approx \frac{2\alpha}{2\alpha + t_2\, \delta_{s,d}^2}\,. \tag{11}$$

- The average data rate can be written as

$$R = Pr(\phi) + \frac{1}{2}\, Pr(\phi^c)\,. \tag{12}$$

Average Data Rate

Theorem 1 *The data rate of the relay-selection decode-and-forward cooperative scenario, utilizing single relay, can be approximated as*

$$R \approx \frac{\alpha + (\frac{1-r}{B\,\delta_{s,r}^2} + \frac{r}{A^2\,\delta_{r,d}^2})\,\delta_{s,d}^2}{2\alpha + (\frac{1-r}{B\,\delta_{s,r}^2} + \frac{r}{A^2\,\delta_{r,d}^2})\,\delta_{s,d}^2} \quad SPCU.$$

SER Analysis

- The probability of symbol error is defined as

$$Pr(e) = Pr(e/\phi) \cdot Pr(\phi) + Pr(e/\phi^c) \cdot Pr(\phi^c) \tag{13}$$

- The SER of the direct-transmission mode can be calculated as follows.
  - The instantaneous direct-transmission SNR is $\gamma^\phi = \frac{P\,\beta_{s,d}}{N_0}$
  - The conditional direct-transmission SER can be written as

$$Pr(e/\phi, \beta_{s,d}) = \Psi(\gamma^\phi) \triangleq \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{b\,\gamma^\phi}{\sin^2\theta}\right) d\theta, \tag{14}$$

where $b = \sin^2(\pi/M)$

- We can obtain

$$Pr(e/\phi)\ Pr(\phi) = \int_0^{\infty} Pr(e/\phi, \beta_{s,d})\ Pr(\phi/\beta_{s,d})\ p_{\beta_{s,d}}(\beta_{s,d})\ d\beta_{s,d}$$

$$\approx\ \frac{t_2\ \delta_{s,d}^2}{2\alpha}\ F_1\Big(\ (1 + \frac{b\ P\ \delta_{s,d}^2}{N_0\ \sin^2\theta})\ (1 + \frac{t_2\ \delta_{s,d}^2}{2\alpha} + \frac{b\ P\ \delta_{s,d}^2}{N_0\ \sin^2\theta})\ \Big)\ , \quad (15)$$

where $F_1\Big(x(\theta)\Big) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{x(\theta)}\ d\theta$

- We obtain the SER of the relay-cooperation mode as follows
  - The output of the MRC can be written as

$$y^{\phi^c} = \frac{\sqrt{P_1}\ h_{s,d}^*}{N_0}\ y_{s,d}^{\phi^c} + \frac{\sqrt{\tilde{P}_2}\ h_{r,d}^*}{N_0}\ y_{r,d}^{\phi^c}\ . \qquad (16)$$

- The instantaneous SNR of the MRC output can be written as $\gamma^{\phi^c} = \frac{P_1\beta_{s,d}+\tilde{P}_2\beta_{r,d}}{N_0}$.

- The conditional SER of the relay-cooperation mode is

$$
\begin{aligned}
Pr(e/\phi^c, \beta_{s,d}, \beta_{s,r}, \beta_{r,d}) = {} & \Psi(\gamma^{\phi^c})|_{\tilde{P}_2=0}\, \Psi(\frac{P_1\beta_{s,r}}{N_0}) \\
& +\Psi(\gamma^{\phi^c})|_{\tilde{P}_2=P_2} \left(1 - \Psi(\frac{P_1\beta_{s,r}}{N_0})\right) . \qquad (17)
\end{aligned}
$$

SER Upper Bound

- An upper bound for the total SER can be obtained as

$$Pr(e) \leq \frac{t_2\ \delta_{s,d}^2}{2\alpha}\ F_1\Big(\ (1+\frac{b\ P\ \delta_{s,d}^2}{N_0\ \sin^2\theta})\ (1+\frac{t_2\ \delta_{s,d}^2}{2\alpha}+\frac{b\ P\ \delta_{s,d}^2}{N_0\ \sin^2\theta})\ \Big)$$
$$+\frac{1}{\pi}\int_{\theta_1=0}^{\frac{(M-1)\pi}{M}} \frac{\left(M_{\beta_m}\left(\frac{bP_2}{2q_1N_0\sin^2\theta_1}\right)+\frac{1}{\pi}\int_{\theta_2=0}^{\frac{(M-1)\pi}{M}} M_{\beta_m}\left(\frac{bP_1}{2q_2N_0\sin^2\theta_2}\right)d\theta_2\right)}{1+\frac{b\ P_1\ \delta_{s,d}^2}{N_0\ \sin^2\theta_1}}$$

- An approximation to the MGF of two independent exponential random variables at high enough SNR as

$$M_{\beta_m}(\gamma) \approx \frac{q_1\ \delta_{r,d}^2 + q_2\ \delta_{s,r}^2}{2\ \gamma}\ . \tag{18}$$

SER Upper Bound

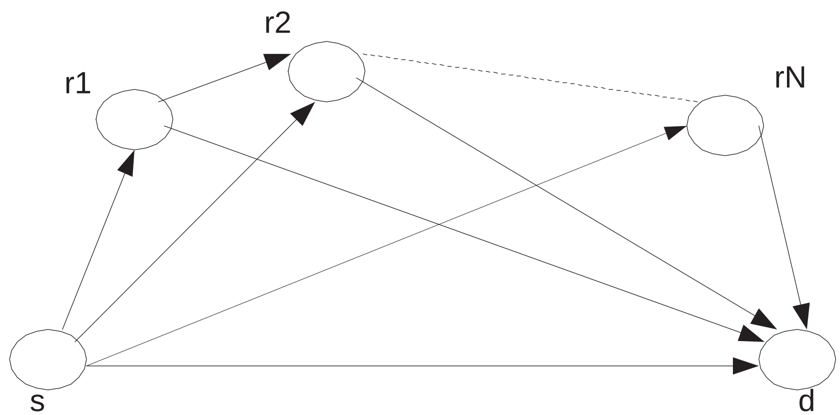
Theorem 2 *At high SNR, $\gamma = \frac{P}{N_0}$, the SER of the single-relay relay-selection decode-and-forward cooperative scheme is upper bounded as*

$$Pr(e) \leq (CG \cdot \gamma)^{-2} \,,$$

where CG denotes the coding gain and is equal to

$$CG = \sqrt{\frac{b^2\,\delta_{s,d}^2}{B\left(\frac{\frac{1-r}{B\,\delta_{s,r}^2}+\frac{r}{A^2\,\delta_{r,d}^2}}{2\,\alpha} + \frac{2\,A^2\left(\frac{A^2\,\delta_{r,d}^2}{r}+\frac{B\,\delta_{s,r}^2}{1-r}\right)}{r^2\,(1-r)}\right)}}\,.$$

Multi-node Scenario

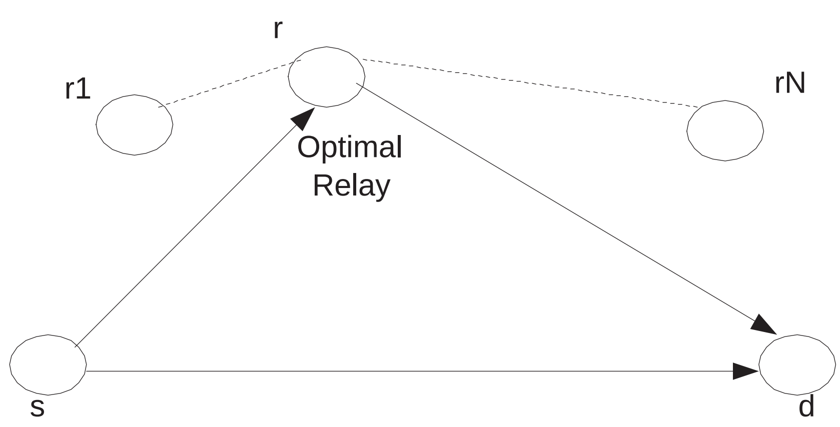


- For the conventional protocol
  - Full diversity order is achieved
  - The data rate is $R = \frac{1}{N+1}$ SPCU

- How to increase the transmission rate, while guaranteeing full diversity order?

Multi-node Scenario: Relay-Selection Idea

- Answer two questions: "When to Cooperate?" and "Whom to Cooperate with?" to achieve
  - $\frac{1}{2} \leq$ Transmission Rate $<$ 1 SPCU
  - Full diversity order
- The main idea is to choose 1 relay only to cooperate with the source, in case of cooperation



- Whom to cooperate with?

  The optimal relay is the relay with the maximum scaled harmonic mean function of its source-relay and relay-destination channels gain among all the N relays

- When to cooperate?

  Same as in the single-relay case described before

- The system model of the multi-node relay-selection cooperative scenario is the same as that of the single relay scenario utilizing the optimal relay

- The metric for each relay is

$$\beta_i = \mu_H(q_1\ \beta_{r_i,d}, q_2\ \beta_{s,r_i}) = \frac{2\ q_1\ \ q_2\ \beta_{r_i,d}\ \beta_{s,r_i}}{q_1\ \beta_{r_i,d} + q_2\ \beta_{s,r_i}}\ ,$$
$$for\ i = 1, 2, \cdots, N\ . \qquad (19)$$

- The optimal relay has a metric equal to

$$\beta_{max} = \max\{\ \beta_1, \beta_2, \ldots, \beta_N\ \}\ . \qquad (20)$$

- The CDF of β_{max} can be written as

$$\begin{aligned} P_{\beta_{max}}(\beta) &= Pr(\ \beta_1 \leq \beta\ , \beta_2 \leq \beta\ , \ldots, \beta_N \leq \beta\ ) = \prod_{i=1}^{N}\ P_{\beta_i}(\beta) \\ &= \prod_{i=1}^{N} \left(1 - \frac{\beta_i}{t_{1,i}}\ \exp(-\frac{t_{2,i}}{2}\,\beta)\ K_1(\frac{\beta}{t_{1,i}}) \right)\ , \qquad (21) \end{aligned}$$

- The PDF of β_{max} is written as

$$p_{\beta_{max}}(\beta) = \sum_{j=1}^{N} p_{\beta_j}(\beta) \left(\prod_{i=1, i\neq j}^{N} \Big(1 - \exp(-\frac{t_{2,i}}{2}\,\beta) \Big) \right). \quad (22)$$

- For mathematical simplicity, we consider the symmetric scenario where $\delta^2_{s,r_i} = \delta^2_{s,r}$ and $\delta^2_{r_i,d} = \delta^2_{r,d}$ for $i = 1, 2, \ldots, N$.

- The CDF and PDF of β_{max} can be written as

$$\begin{aligned} P_{\beta_{max}}(\beta) &= \left(1 - \frac{\beta}{t_1}\, \exp(-\frac{t_2}{2}\,\beta)\, K_1(\frac{\beta}{t_1}) \right)^N \\ p_{\beta_{max}}(\beta) &= N \left(1 - \frac{\beta}{t_1}\, \exp(-\frac{t_2}{2}\,\beta)\, K_1(\frac{\beta}{t_1}) \right)^{N-1} p_{\beta_m}(\beta)\,, \end{aligned}$$

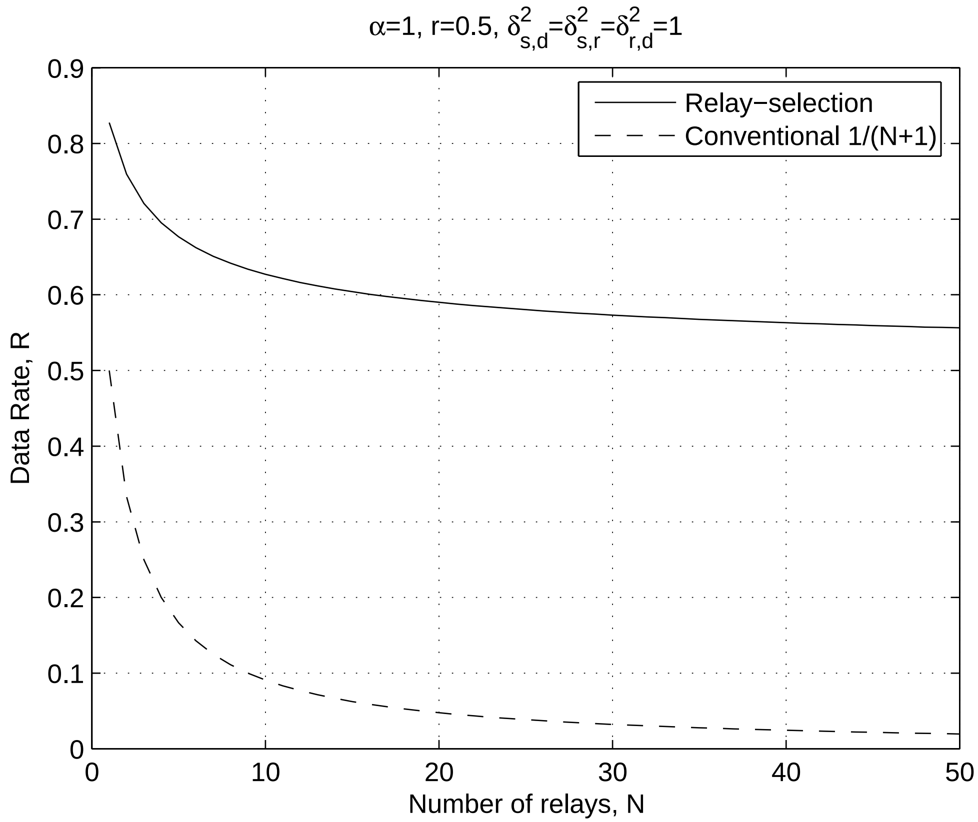
- The probability of the direct-transmission mode can be obtained as

$$\begin{aligned} Pr(\phi) &= Pr(\beta_{s,d} \geq \alpha\ \beta_{max}) \\ &\approx \sum_{n=0}^{N} \binom{N}{n} (-1)^n \frac{2\alpha}{2\alpha + t_2\ \delta_{s,d}^2\ n}\ , \end{aligned} \tag{23}$$

Theorem 3 *The data rate of the multi-node relay-selection decode-and-forward symmetric cooperative scenario, employing N relays, is approximated as*

$$R \approx \frac{1}{2} \cdot \left(1 + \sum_{n=0}^{N} \binom{N}{n} (-1)^n \frac{2\alpha}{2\alpha + \left(\frac{1-r}{B\ \delta_{s,r}^2} + \frac{r}{A^2\ \delta_{r,d}^2}\right)\ \delta_{s,d}^2\ n} \right) \quad SPCU\ .$$

Multi-node Scenario: Average Data Rate



The data rate decreases down to 0.5 as N increases, because the probability of the direct-transmission mode decreases down to 0 as N goes to ∞.

Multi-node Scenario: SER Analysis

- The SER derivation is similar to the single-relay case with using the PDF and CDF of β_{max} instead of the PDF and CDF of β_m

- The MGF of β_{max} is

$$M_{\beta_{max}}(\gamma) = N \sum_{n=0}^{N-1} \binom{N-1}{n} (-1)^n M_{\beta_m}(\gamma + \frac{n\, t_2}{2}) \,, \quad (24)$$

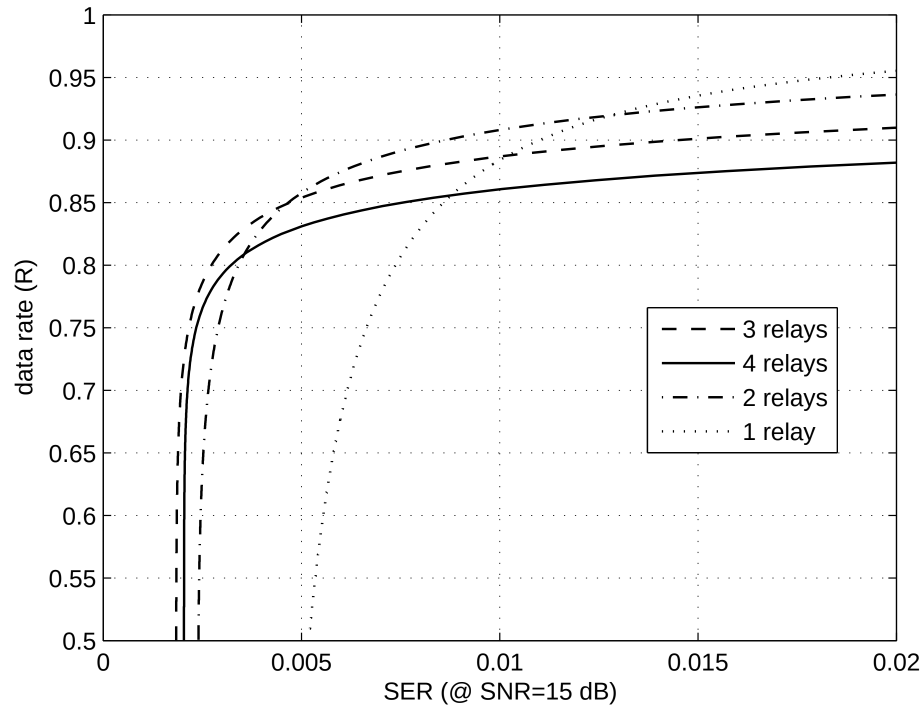
Theorem 4 *At high SNR* $\gamma = \frac{P}{N_0}$*, the SER of the multi-node relay-selection decode-and-forward symmetric cooperative scenario, utilizing* N *relays, is upper bounded by* $Pr(e) \leq (CG \cdot \gamma)^{-(N+1)}$*, where*

$$CG^{-(N+1)} = \left(\frac{N!\ (\frac{1-r}{B\ \delta_{s,r}^2} + \frac{r}{A^2\ \delta_{r,d}^2})^{N-1}}{b^{N+1}\ \delta_{s,d}^2} \right)$$

$$\left(\frac{(\frac{1-r}{B\delta_{s,r}^2} + \frac{r}{A^2\delta_{r,d}^2}) I(2N+2)}{(2\alpha)^N} + \frac{\left(\frac{A^2\delta_{r,d}^2}{r} + \frac{B\delta_{s,r}^2}{1-r} \right) \left(A^{2N} I(2\,N+2) + B^N A I(2N) \right)}{r^{N+1}(1-r)^N} \right)$$

where $I(p) = \frac{1}{\pi} \int_{\theta=0}^{\frac{(M-1)\pi}{M}} \sin^p \theta\ d\theta$.

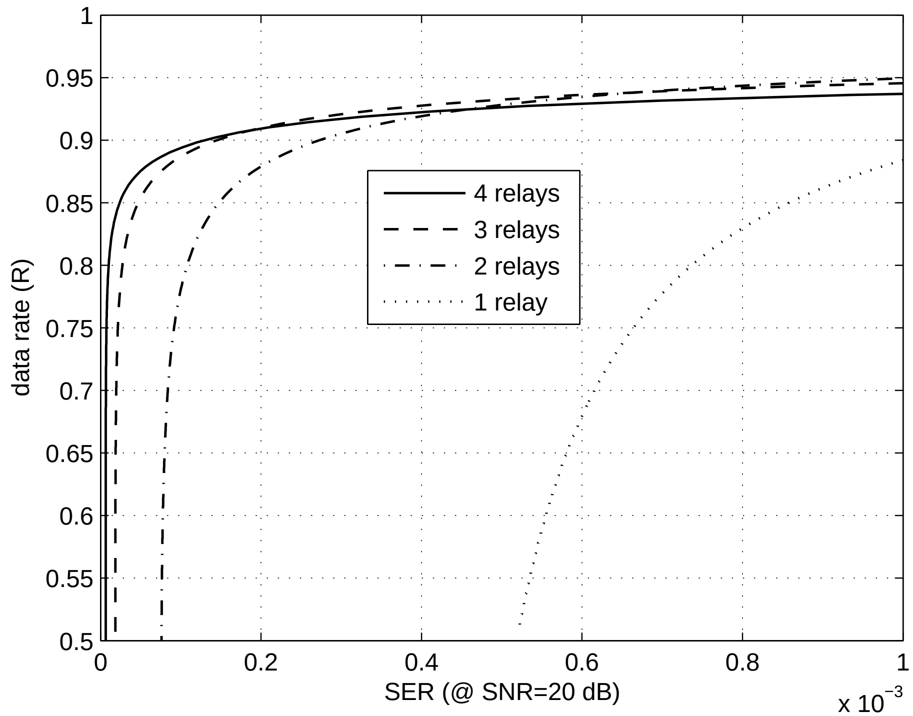
Data Rate-SER Tradeoff



- At SNR=15 dB, the tradeoff achieved using three relays is the best at low SER region.

- The SER is almost constant at 0.002 while the data rate increases from 0.5 to 0.8 SPCU for 2,3, and 4 relays.

Data Rate-SER Tradeoff



- At SNR=20 dB, the tradeoff achieved by four relays is the best in the low SER region.

- As we increase the SNR, the effect of the diversity order becomes more significant

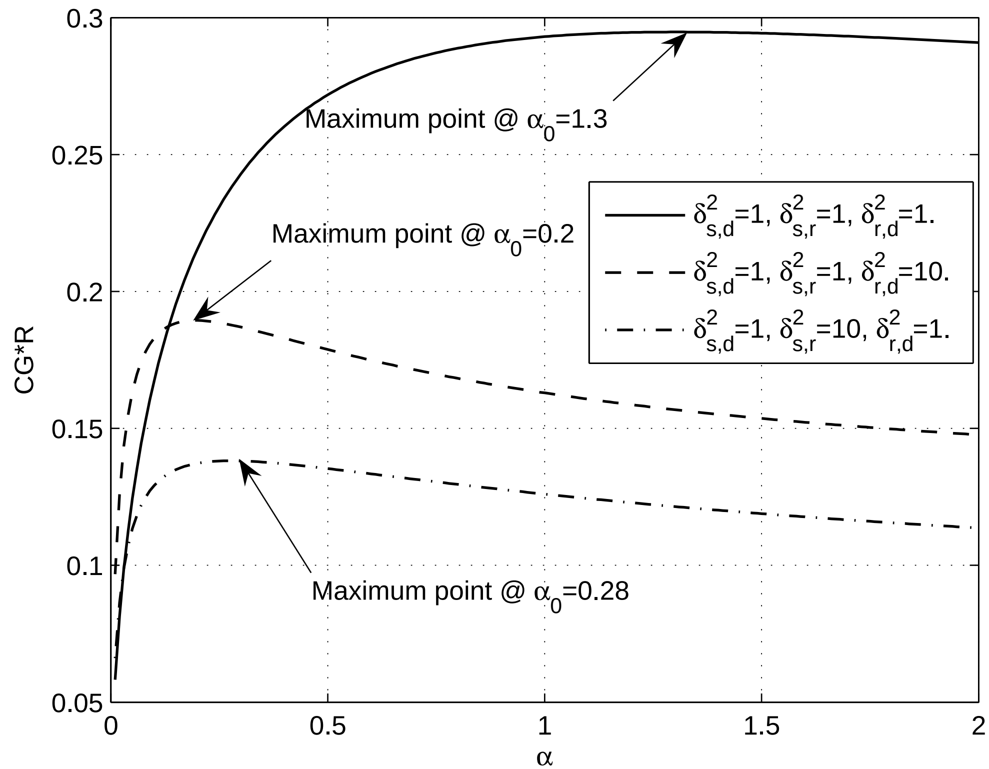
Data Rate-SER Tradeoff

- We conclude that increasing the number of relays does not necessarily lead to a better data rate-SER tradeoff curve

- As an example of choosing the optimum cooperation threshold, we choose an optimization metric as

$$\max_{\alpha} \; CG \cdot R \,, \tag{25}$$

Single-Relay Data Rate-SER Tradeoff



- Single-relay optimum values using the (CG*R) optimization criterion

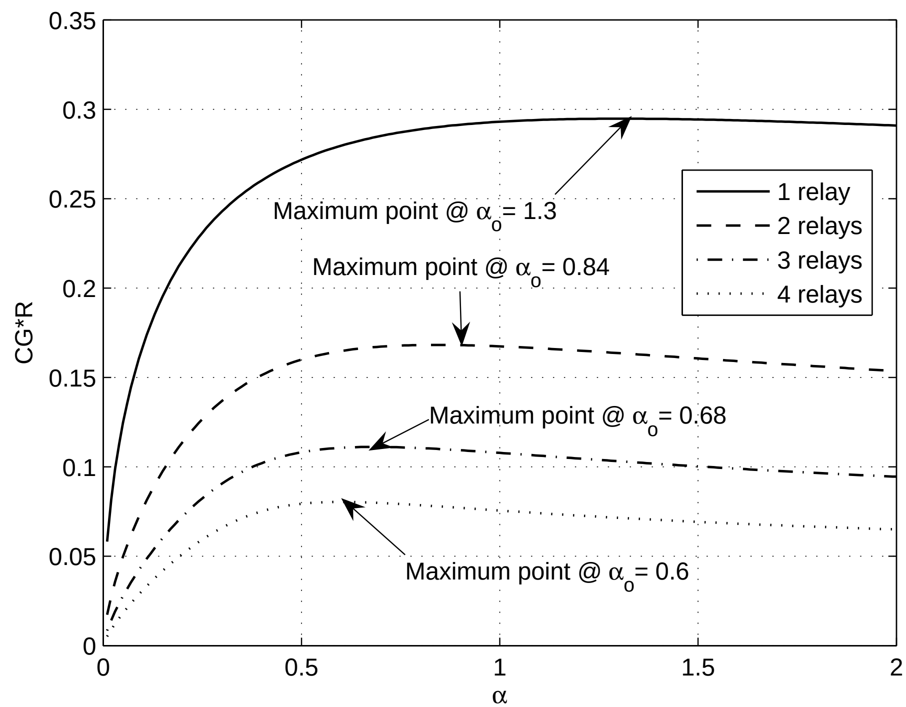
Single-Relay Data Rate-SER Tradeoff

$\delta^2_{s,d}$	$\delta^2_{s,r}$	$\delta^2_{r,d}$	r	α	R	CG
1	1	1	0.57447	1.3	0.8018	0.3676
1	1	10	0.6819	0.2	0.8759	0.2162
1	10	1	0.5129	0.28	0.9119	0.1515

- Single-relay optimum values

- The cooperation threshold is almost the same for cases 2 and 3, i.e., it treats the source-relay and relay-destination in almost a symmetric way

Multi-node Data Rate-SER Tradeoff



- Multi-node optimum values using unity-channel variances

- increasing the number of relays affects the optimum co-operation threshold values according to the CG*R optimization criterion

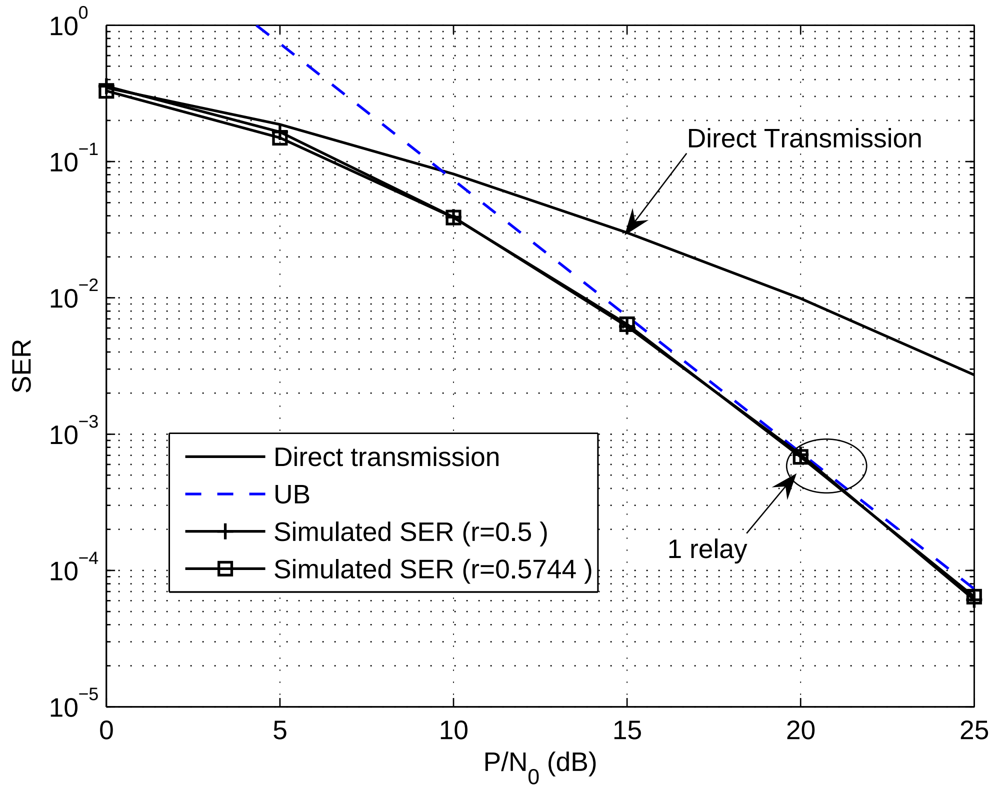
Multi-node Data Rate-SER Tradeoff

N	r	α	R	CG
1	0.5744	1.3	0.8018	0.3676
2	0.5528	0.84	0.7880	0.2135
3	0.5409	0.68	0.7824	0.1421
4	0.5334	0.6	0.7781	0.1033

- The optimum power ratio is slightly decreasing with the number of relays. Because, increasing the number of relays will increase the probability of finding a better relay, which can receive the symbols from the source more correctly. Thus, it can send with almost equal power with the source.

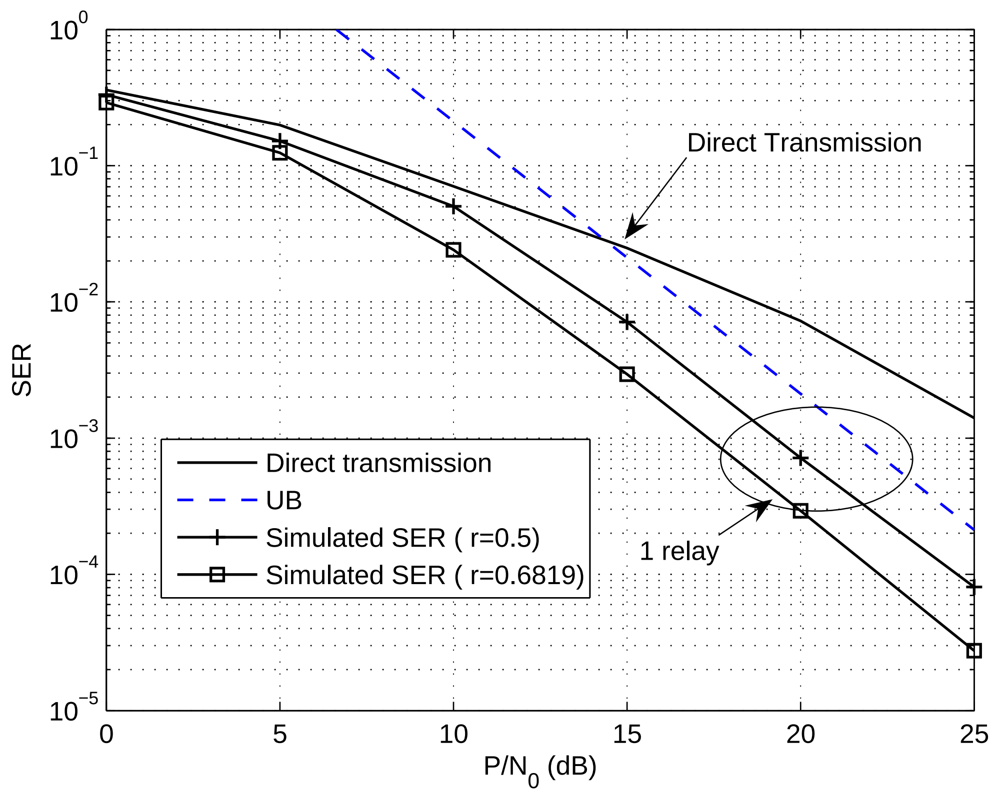
- The data rate is slightly decreasing with increasing the number of relays, because the probability that the source-destination channel is better than all the relay's metrics goes to 0 as N goes to ∞.

Simulation Results: Single-Relay



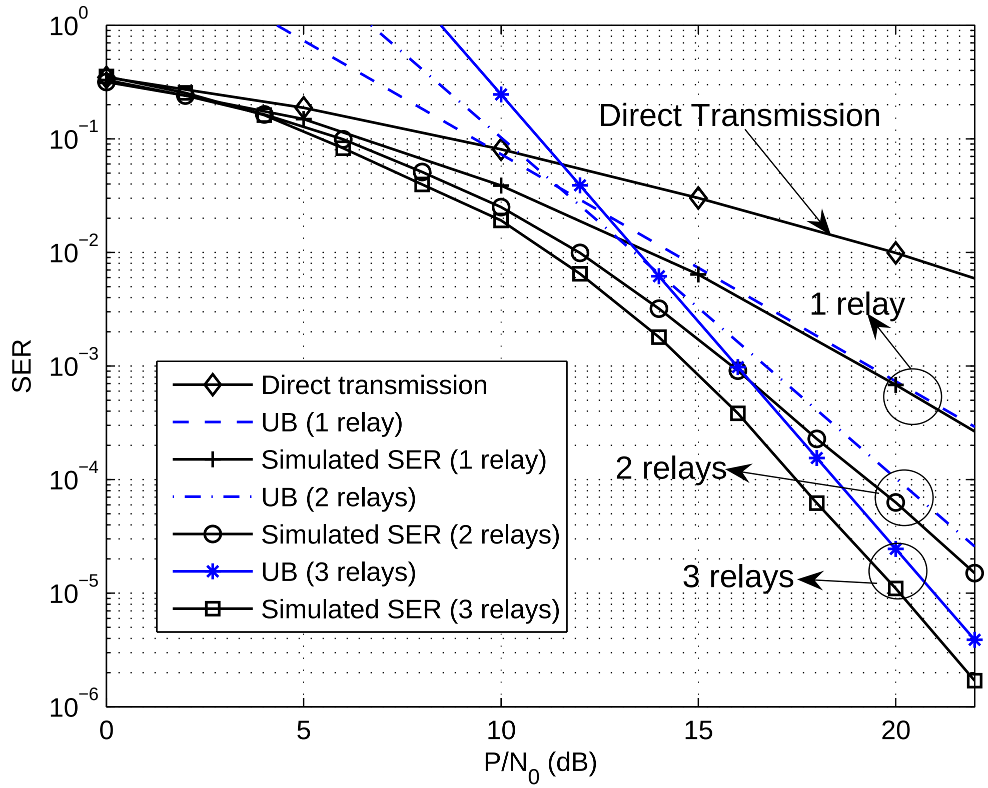
- Unity channel variances

Simulation Results: Single-Relay



- $\delta^2_{r,d} = 10$

Simulation Results: Multi-node



- Unity channel variances

Conclusion

- We have proposed new single-relay and multi-node relay-selection decode-and-forward cooperative scenarios, which utilize the partial CSI available at the source and the relays

- The main objective of this work is to achieve higher data rate and guarantee full diversity order

- The relay's capability to help is based on the scaled harmonic mean function of its source-relay and relay-destination channels gain

- As for the optimum cooperation threshold, we have shown the data rate-SER tradeoff curve, which determines the optimal cooperation threshold