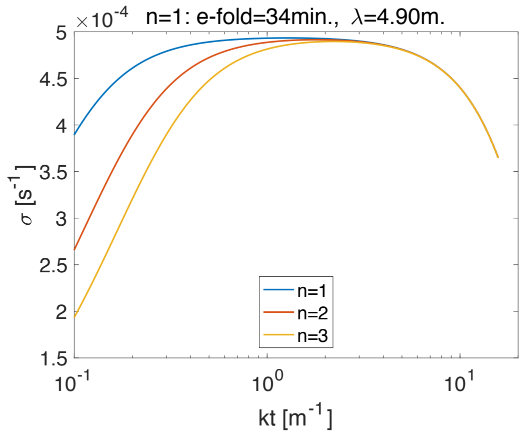

Figure 1: Growth rate versus horizontal wavenumber for convective modes $n = 1,2,3$.

5: A convective mixed layer

The Rayleigh number is 4.5×10^{12}. The easiest way to compute the fastest-growing mode is by plugging the given parameter values into the quadratic (**??**) and solving numerically. Compute σ over a range of wavenumbers, plot and check that your range includes the maximum growth rate, then identify the corresponding wavenumber.

We get a horizontal wavelength of about 5m. The growth rate is $4.5 \times 10^{-4} s^{-1}$, and the e-folding time, σ^{-1}, is about 34 minutes. The point here is that the e-folding time is short compared with the time over which conditions for instability persist, so that the instability has enough time to grow. If the e-folding time was longer than a few hours, the sun would be up before convection really had a chance to get started.

Here is the code we used:

```
clear
close all

% plotting parameters
fs=20;
lw=1.5;
set(0,'DefaultAxesFontSize',fs)

% problem parameters
nu=1e-6;         % viscosity
kappa=1.4e-7;    % diffusivity
H=40;            % layer thickness
drho=1e-6;       % density difference
g=9.81;          % gravity

% computed parameters
Bz=-g*drho/H;            % buoyancy gradient
Pr=nu/kappa;             % Prandtl number
Ra=-Bz*H^4/(nu*kappa)    % Rayleigh number

kt_sts=10.^[.5 :.01 :2.8];  %wavenumber values
```

```
for n=1:3
    for i=1:length(kt_sts)
        kt_st=kt_sts(i); % choose one value of kt_st
        sigma_st=RB_convection(kt_st,n,1,-Ra*Pr,Pr,1);
        sr_st=max(real(sigma_st));        % pick the larger root
        sr(n,i)=sr_st*kappa/H^2;          % redimensionalize growth rate
    end

    i0=find(sr(n,:)==max(sr(n,:))); % find index for FGM
    kt0=kt_sts(i0)/H;               % redimensionalize wavenumber of FGM
    efold_min(n)=1/sr(n,i0)/60;     % e-folding time of FGM in minutes
    lambda(n)=2*pi/kt0;             % wavelength of FGM in m
end

% plot
figure
semilogx(kt_sts/H,sr,'linewidth',lw);
xlim([.1 20])
h=legend('n=1','n=2','n=3','location','south');
xlabel('kt [m^{-1}]');
ylabel('\sigma [s^{-1}]');
title(sprintf('n=1: e-fold=%.0fmin., \\lambda=%.2fm.',...
    efold_min(1),lambda(1)),'fontweight','normal');
set(gca,'fontsize',fs-2)
```

and subroutine

```
function sigma=RB_convection(kt,n,H,Bz,nu,kappa)
%
% Calculate growth rate for Rayleigh-Benard convection.
% Inputs:
% kt = horizontal projection of wave vector
% n = vertical quantization number
% H = layer thickness
% Bz = buoyancy gradient
% nu = viscosity
% kappa = diffusivity
%
% Outputs
% sigma = growth rate (two values)
%

K2=kt^2+(n*pi/H)^2; % total wave vector length (squared)
% coefficients for quadratic equation
B=(nu+kappa)*K2;
C=nu*kappa*K2.^2+Bz*kt^2./K2;
D=B.^2-4*C; % discriminant
```

```
sigma(1)=.5*( -B+sqrt(D));
sigma(2)=.5*( -B-sqrt(D));
return
```

Results are shown in figure 1.