9: Transforming the Rayleigh equation

(a) Linearize the material derivative as in (3.6). Since there is no mean displacement, write η as $\varepsilon \eta'$, then apply your expression for the material derivative. You should get

$$\frac{\partial \eta'}{\partial t} + U \frac{\partial \eta'}{\partial x} = w'.$$

Now substitute the normal mode form (3.15) to get:

$$\iota k(U-c)\hat{\eta} = \hat{w}.$$

(b) Differentiate the result of (a) twice and substitute into the Rayleigh equation:

$$\hat{w}_{zz} = \left(\frac{U_{zz}}{U-c} + \tilde{k}^2\right)\hat{w}.$$

Now simplify to get the desired result:

$$[(U-c)^2\hat{\eta}_z]_z = \tilde{k}^2(U-c)^2\hat{\eta}.$$