Every diffraction angle is required to be within the range between  $-90^{\circ}$  and  $90^{\circ}$ , i.e.,  $-1 \le \sin \theta_{1q} \le 1$  and  $-1 \le \sin \theta_{2q} \le 1$ .

On the air side,  $n_1 = 1$ ; thus

$$-1 \le \sin \theta_{1q} = \frac{q\lambda}{n_1 \Lambda} \le 1 \quad \Rightarrow \quad 0 \le |q| \le \frac{n_1 \Lambda}{\lambda} = \frac{1 \times 2 \times 10^{-6}}{850 \times 10^{-9}} = 2.35.$$

There are five diffraction orders on the air side for q = -2, -1, 0, 1, 2. The diffraction angles with respect to the surface normal are

$$\theta_{1q} = \sin^{-1} \frac{q\lambda}{n_1 \Lambda} = \sin^{-1} \frac{q \times 850 \times 10^{-9}}{1 \times 2 \times 10^{-6}}$$
  
$$\Rightarrow \quad \theta_{1q} = -58.21^\circ, \ -25.15^\circ, 0^\circ, 25.15^\circ, 58.21^\circ$$

On the glass side,  $n_2 = 1.5$ ; thus

$$-1 \le \sin \theta_{2q} = \frac{q\lambda}{n_2\Lambda} \le 1 \quad \Rightarrow \quad 0 \le |q| \le \frac{n_2\Lambda}{\lambda} = \frac{1.5 \times 2 \times 10^{-6}}{850 \times 10^{-9}} = 3.52.$$

There are seven diffraction orders on the glass side for q = -3, -2, -1, 0, 1, 2, 3. The diffraction angles with respect to the surface normal are

$$\theta_{2q} = \sin^{-1} \frac{q\lambda}{n_2 \Lambda} = \sin^{-1} \frac{q \times 850 \times 10^{-9}}{1.5 \times 2 \times 10^{-6}}$$
  
$$\Rightarrow \quad \theta_{2q} = -58.21^\circ, -34.52^\circ, -16.46^\circ, 0^\circ, 16.46^\circ, 34.52^\circ, 58.21^\circ.$$

## 5.2.4 Surface Grating–Waveguide Coupling

A grating fabricated on the surface of a waveguide can couple a radiation field that propagates in the homogeneous space on one side of the waveguide into a waveguide mode. In reverse operation, it can also couple a waveguide mode into a radiation field from the surface of the waveguide. These concepts are illustrated in Fig. 5.15.

For this purpose, it is necessary to phase match the radiation field with the waveguide mode in the longitudinal direction of the waveguide, which is taken to be the z direction. For coupling



**Figure 5.15** Surface grating for (a) input coupling and (b) output coupling of a waveguide mode.