

RELATIVITY, GRAVITATION AND COSMOLOGY

Introduction	9
---------------------	----------

Chapter I Special relativity and spacetime	11
---	-----------

Introduction	11
1.1 Basic concepts of special relativity	12
1.1.1 Events, frames of reference and observers	12
1.1.2 The postulates of special relativity	14
1.2 Coordinate transformations	16
1.2.1 The Galilean transformations	16
1.2.2 The Lorentz transformations	18
1.2.3 A derivation of the Lorentz transformations	21
1.2.4 Intervals and their transformation rules	23
1.3 Consequences of the Lorentz transformations	24
1.3.1 Time dilation	24
1.3.2 Length contraction	26
1.3.3 The relativity of simultaneity	27
1.3.4 The Doppler effect	28
1.3.5 The velocity transformation	29
1.4 Minkowski spacetime	31
1.4.1 Spacetime diagrams, lightcones and causality	31
1.4.2 Spacetime separation and the Minkowski metric	35
1.4.3 The twin effect	38

Chapter 2 Special relativity and physical laws	45
---	-----------

Introduction	45
2.1 Invariants and physical laws	46
2.1.1 The invariance of physical quantities	46
2.1.2 The invariance of physical laws	47
2.2 The laws of mechanics	49
2.2.1 Relativistic momentum	49
2.2.2 Relativistic kinetic energy	52
2.2.3 Total relativistic energy and mass energy	54
2.2.4 Four-momentum	56
2.2.5 The energy–momentum relation	58
2.2.6 The conservation of energy and momentum	60
2.2.7 Four-force	61
2.2.8 Four-vectors	62
2.3 The laws of electromagnetism	67

2.3.1	The conservation of charge	67
2.3.2	The Lorentz force law	68
2.3.3	The transformation of electric and magnetic fields	73
2.3.4	The Maxwell equations	74
2.3.5	Four-tensors	75
Chapter 3	Geometry and curved spacetime	80
Introduction		80
3.1	Line elements and differential geometry	82
3.1.1	Line elements in a plane	82
3.1.2	Curved surfaces	85
3.2	Metrics and connections	90
3.2.1	Metrics and Riemannian geometry	90
3.2.2	Connections and parallel transport	92
3.3	Geodesics	97
3.3.1	Most direct route between two points	97
3.3.2	Shortest distance between two points	98
3.4	Curvature	100
3.4.1	Curvature of a curve in a plane	101
3.4.2	Gaussian curvature of a two-dimensional surface	102
3.4.3	Curvature in spaces of higher dimensions	104
3.4.4	Curvature of spacetime	106
Chapter 4	General relativity and gravitation	110
Introduction		110
4.1	The founding principles of general relativity	111
4.1.1	The principle of equivalence	112
4.1.2	The principle of general covariance	116
4.1.3	The principle of consistency	124
4.2	The basic ingredients of general relativity	126
4.2.1	The energy–momentum tensor	126
4.2.2	The Einstein tensor	132
4.3	Einstein’s field equations and geodesic motion	133
4.3.1	The Einstein field equations	134
4.3.2	Geodesic motion	136
4.3.3	The Newtonian limit of Einstein’s field equations	138
4.3.4	The cosmological constant	139
Chapter 5	Schwarzschild spacetime	144
Introduction		144
5.1	The metric of Schwarzschild spacetime	145

5.1.1	The Schwarzschild metric	145
5.1.2	Derivation of the Schwarzschild metric	146
5.2	Properties of Schwarzschild spacetime	151
5.2.1	Spherical symmetry	151
5.2.2	Asymptotic flatness	152
5.2.3	Time-independence	152
5.2.4	Singularity	153
5.2.5	Generality	154
5.3	Coordinates and measurements in Schwarzschild spacetime	154
5.3.1	Frames and observers	155
5.3.2	Proper time and gravitational time dilation	156
5.3.3	Proper distance	159
5.4	Geodesic motion in Schwarzschild spacetime	160
5.4.1	The geodesic equations	161
5.4.2	Constants of the motion in Schwarzschild spacetime	162
5.4.3	Orbital motion in Schwarzschild spacetime	166

Chapter 6 Black holes 171

	Introduction	171
6.1	Introducing black holes	171
6.1.1	A black hole and its event horizon	171
6.1.2	A brief history of black holes	172
6.1.3	The classification of black holes	175
6.2	Non-rotating black holes	176
6.2.1	Falling into a non-rotating black hole	177
6.2.2	Observing a fall from far away	179
6.2.3	Tidal effects near a non-rotating black hole	183
6.2.4	The deflection of light near a non-rotating black hole	186
6.2.5	The event horizon and beyond	187
6.3	Rotating black holes	192
6.3.1	The Kerr solution and rotating black holes	192
6.3.2	Motion near a rotating black hole	194
6.4	Quantum physics and black holes	198
6.4.1	Hawking radiation	198
6.4.2	Singularities and quantum physics	200

Chapter 7 Testing general relativity 204

	Introduction	204
7.1	The classic tests of general relativity	204
7.1.1	Precession of the perihelion of Mercury	204

7.1.2	Deflection of light by the Sun	205
7.1.3	Gravitational redshift and gravitational time dilation	206
7.1.4	Time delay of signals passing the Sun	211
7.2	Satellite-based tests	213
7.2.1	Geodesic gyroscope precession	213
7.2.2	Frame dragging	214
7.2.3	The LAGEOS satellites	215
7.2.4	Gravity Probe B	216
7.3	Astronomical observations	217
7.3.1	Black holes	217
7.3.2	Gravitational lensing	223
7.4	Gravitational waves	226
7.4.1	Gravitational waves and the Einstein field equations	226
7.4.2	Methods of detecting gravitational waves	229
7.4.3	Likely sources of gravitational waves	231
Chapter 8	Relativistic cosmology	234
	Introduction	234
8.1	Basic principles and supporting observations	235
8.1.1	The applicability of general relativity	235
8.1.2	The cosmological principle	236
8.1.3	Weyl's postulate	240
8.2	Robertson–Walker spacetime	242
8.2.1	The Robertson–Walker metric	243
8.2.2	Proper distances and velocities in cosmic spacetime	245
8.2.3	The cosmic geometry of space and spacetime	247
8.3	The Friedmann equations and cosmic evolution	251
8.3.1	The energy–momentum tensor of the cosmos	251
8.3.2	The Friedmann equations	254
8.3.3	Three cosmological models with $k = 0$	256
8.3.4	Friedmann–Robertson–Walker models in general	259
8.4	Friedmann–Robertson–Walker models and observations	263
8.4.1	Cosmological redshift and cosmic expansion	263
8.4.2	Density parameters and the age of the Universe	269
8.4.3	Horizons and limits	270
Appendix		277
Solutions		279
Acknowledgements		308
Index		309