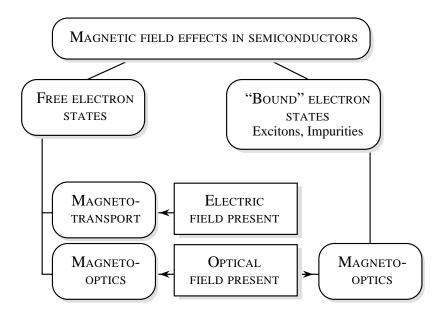
Chapter

11

SEMICONDUCTOR IN MAGNETIC FIELDS



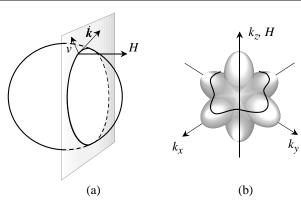
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### CYCLOTRON RESONANCE

At low magnetic field the electron bandstructure is unaffected

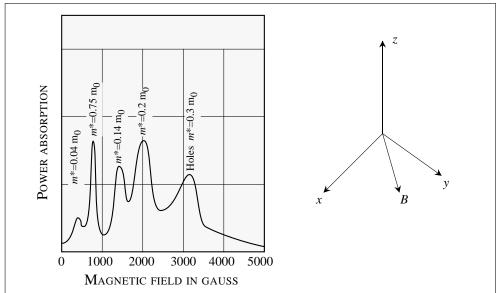
→ electron orbits the constant energy surface with the cyclotron frequency

$$\omega_c = \frac{eB}{m^*}$$



(a) Schematic showing the orbit of an electron in a magnetic field. The electron moves on a constant energy surface in a plane perpendicular to the magnetic field.

(b) Orbit of an electron (or hole) on a constant energy surface that is anisotropic.



Results of cyclotron resonance absorption in germanium at 4 K, at 24 GHz. The static magnetic field is in a (110) plane at  $60^{\circ}$  from a (100) axis. Both electrons and holes are produced by illumination. The peaks correspond to the cyclotron frequency coinciding with 24 GHz. (Adapted from G. Dresselhaus, A.F. Kip, and C. Kittel, *Physical Review*, Vol. 98, 368 (1955).)

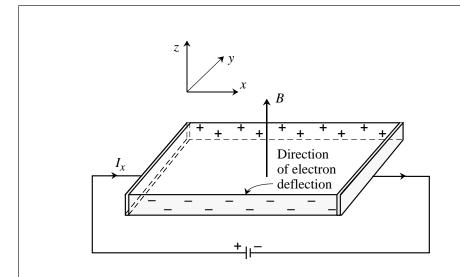
### SEMICLASSICAL THEORY OF MAGNETOTRANSPORT

In Hall effect it is possible to measure: Hall coefficient: 
$$R_H = \frac{F_y}{J_x B} = -\frac{1}{ne}$$
  $n$ -type 
$$= \frac{1}{pe} p$$
-type

Magneto-resistance:  $\rho_H = \frac{F_x}{J_x}$ 

Hall mobility: 
$$=\frac{R_H}{\rho_H} = \mu_H = r_H \cdot \mu$$

 $r_H$  = Hall factor



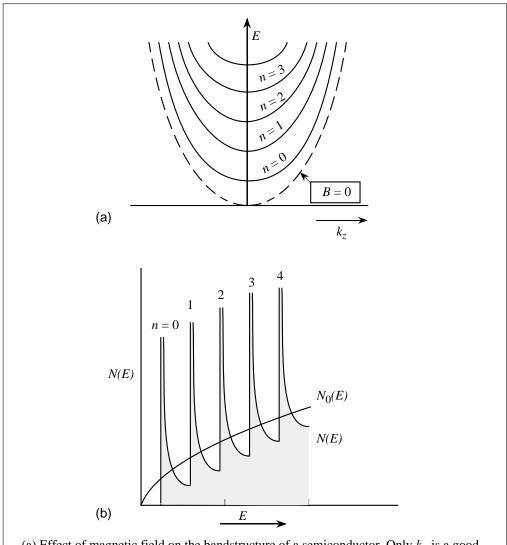
A rectangular Hall sample of an n-type semiconductor.

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### ELECTRONS IN STRONG MAGENTIC FIELDS: LANDAU LEVELS

At high magnetic fields the bandstructure of electrons is altered

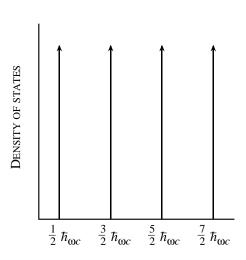
3 Dimensions: 
$$E = \left(n + \frac{1}{2}\right) \hbar \omega_c + \frac{\hbar^2}{2m^*} k_z^2$$



- (a) Effect of magnetic field on the bandstructure of a semiconductor. Only  $k_z$  is a good quantum number. The magnetic field produces quantization in the x-y plane leading to Landau levels. The dashed curve is for zero field.
- (b) Density of staes in a 3D system in the presence of a magnetic field. The density of states develops singularities due to quantization in the plane perpendicular to the magnetic field. Also shown is the zero field density of states for comparison.

# LANDAU LEVELS IN 2-DIMENSIONS

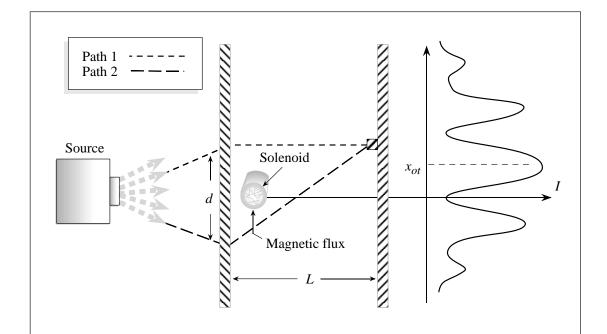
A singular density of states is produced: 
$$E_{nl} = E_n + \left(l + \frac{l}{2}\right)\hbar \; \frac{eB_2}{m_{//}}$$



Singular density of states in an ideal 2D system in the presence of a magnetic field.

## AHARONOV-BOHM EFFECT: QUANTUM INTERFERENCE

Current 
$$\propto \cos \frac{e\phi}{\hbar}$$
;  $\Phi = B \cdot A$ 



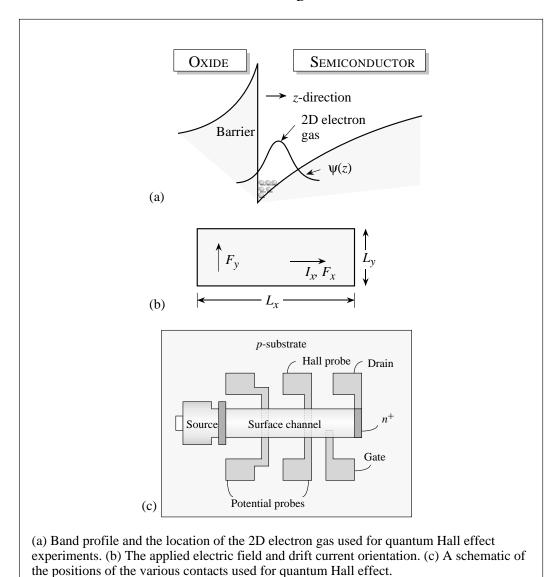
A magnetic field can influence the motion of electrons even though it exists only in regions where there is an arbitrarily small probability of finding the electrons. The interference pattern of the electrons can be shifted by altering the magnetic field.

## QUANTUM HALL EFFECT

Hall effect measurement at low temperature in 2D systems where scattering time is very long, compared to 1/cyclotron frequency

$$\Rightarrow \omega_c \tau >> 1$$

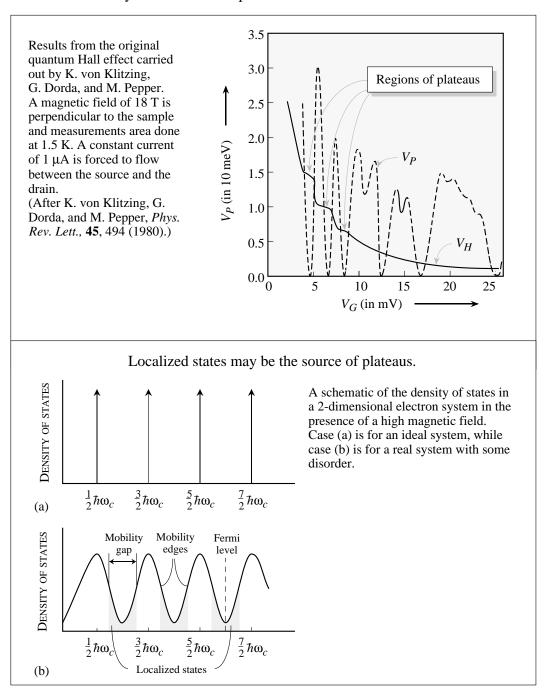
$$\sigma_{xx} \rightarrow 0; \sigma_{xy} = -\frac{ne}{B}$$



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## QUANTUM HALL EFFECT

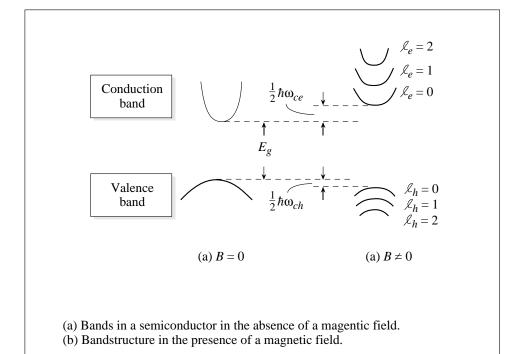
Hall resistivity is found to have plateaus.



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### MAGNETO-OPTIC STUDIES

A magnetic field causes splitting of the electron and hole bands into Landau levels. Optical transitions can reveal bandstructure properties.



Transition energies:

$$E_{mo} = E_g + \frac{\hbar^2 kz^2}{2\mu} + (\rho + 1/2)\hbar\omega_{ceh}$$

$$\omega_{ceh} = \frac{eB}{\mu}$$

$$\mu = \text{e-h reduced mass}$$

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