2 Biomass

Here we introduce the case where the resource itself reproduces, occupying Quadrants 3 and 4 simultaneously. We begin with a single resource stock: biomass *B*. It is necessary to account for its growth rate and for the harvest. Steady states are possible when these two are in balance, Quadrant 3; but the descent into Quadrant 4, "resource mining," occurs when harvest continuously exceeds growth, leading to extinction: no *S*, therefore no *Q*. This would be the analog of resource exhaustion for the sterile resource. It is irreversible.

These resources are local in that they live within a regional ecosystem with finite carrying capacity. The market for the harvest, however, is presumed exogenous, dependent on many things other than this particular resource. The application to fishery management is used throughout to fix ideas.

The base case here uses the logistic function for the growth rate. As the resource (e.g., fish) is fugitive, harvesting requires effort (fishers) as well as fish availablility. The formulation needs to add fishing effort *E* at a fundamental level. The economic interaction of growth and harvesting is commonly referred to as "bioeconomic." From a sustainability perspective, there needs to be attention to (a) avoiding the "mining" phenomenon associated with Quadrant 4 extinction; (b) regulating the effort directed at the resource harvesting; and (c) respecting the conditions required for maintenance of the reproducing stock (the ecological carrying capacity).

2.1 GROWTH AND HARVESTING

In the case of sterile resources, we have only one consideration: the rate of its exhaustion and the time frame of complete exhaustion. Exhaustion can be "physical" exhaustion, as in the case of costless production. More realistically, "economic" exhaustion would indicate that the resource can no longer be produced economically – the cost of production exceeds its value. "Political" exhaustion occurs when the resource cannot be produced legally.

By contrast, living systems present the possibility of sustained resource usage, indefinitely. We will therefore be concerned with the possibility of steady states, reflecting a balance between nature and the economy, and the stability of those states. We will start with a simple situation where the living system is characterized by a single descriptor, the biomass B.

The basic natural dynamic for this system is

$$\frac{dB}{dt} = G - H \tag{2.1}$$

with

- $B(t)$ = amount of biomass
- $G(t)$ = biological growth rate
- $H(t)$ = harvesting rate

To close this system, we need relations for both growth and harvesting rates.

2.1.1 Growth

A candidate growth rate function is the logistic function

$$G(B) = gB\left(1 - \frac{B}{K}\right) \tag{2.2}$$

with K the carrying capacity of the system. This growth rate depends on B only. There is positive growth at all positive levels of B, up to the carrying capacity, beyond which G is always negative.

At low B, $G \simeq gB$ and in the absence of harvesting, $dB/dt = gB$, and we expect exponential growth, $B = B_0 \exp(gt)$. This would be a frontier situation, characteristic of, for example, an invasive species in its early history.

At intermediate B, G increases with B, reaching a peak at $B^* = K/2$. Higher B leads to decline in G. The *proportional* rate of growth G/B declines monotonically with B:

$$\frac{G}{B} = g\left(1 - \frac{B}{K}\right) \tag{2.3}$$

This might occur due to crowding, habitat restrictions, food limitation, or attraction of predators.

As $B \to K$, growth shuts off. The simple substitution $\epsilon = K - B$ gives, in the absence of harvesting and near carrying capacity, $d\epsilon/dt = -g\epsilon$. At carrying capacity, B is stable. Departures from K will decay exponentially as $\exp(-gt)$.

Logistic Growth

Figure 2.1 illustrates the logistic growth of B over time and the attendant G history, beginning at very low B and approaching K in the absence of harvesting. The steepest growth occurs midway in the trajectory, at intermediate B.

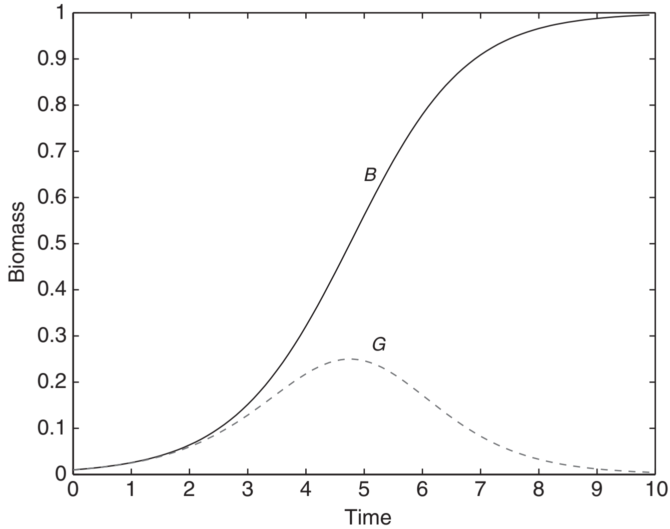

Figure 2.1. Time evolution of B and growth rate G in the absence of harvesting. Logistic growth $G = B(1 - B)$.

The governing equation is

$$\frac{dB}{dt} = gB\left(1 - \frac{B}{K}\right) \tag{2.4}$$

and its solution is

$$B(t) = \frac{K}{1 + Ae^{-gt}} \tag{2.5}$$

$$A = \frac{K - B_0}{B_0} \tag{2.6}$$

where B_0 is the initial condition at time $t = 0$. If initial conditions are small, then A is big. Peak growth occurs at $B = K/2$, at time T_p:

$$Ae^{-gT_p} = 1 \tag{2.7}$$

and therefore

$$gT_p = \ln(A) = \ln\left(\frac{K}{B_0} - 1\right) \tag{2.8}$$

In the limit of small B_0/K,

$$gT_p \simeq \ln\left(\frac{K}{B_0}\right) \tag{2.9}$$

As an example, suppose an invasive species with $g = 0.1/\text{yr}$ is observed at 1% of carrying capacity. The waiting time to peak G would be $T_p = 46$ years.

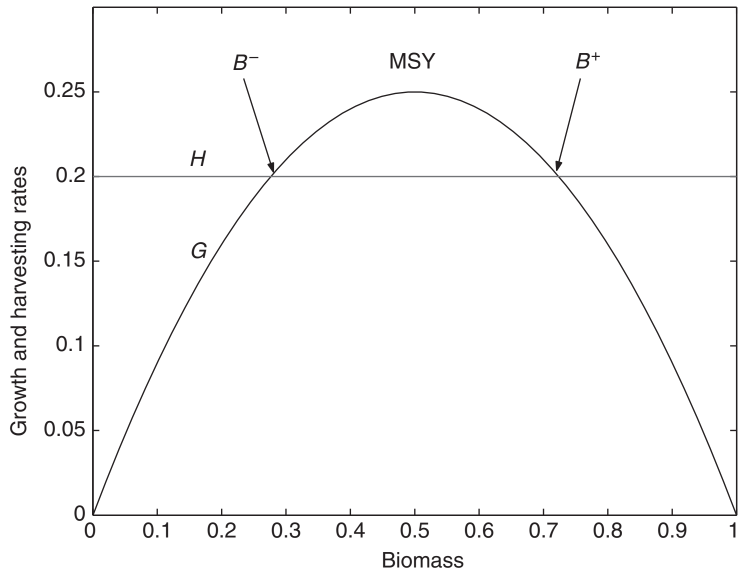

Figure 2.2. Growth and harvesting rates. The case of constant harvest is shown, with logistic growth function $G = B(1 - B)$. There are two intersections where $G = H$; the one on the left, B^-, is unstable.

Steady State

For steady state, we need $G = H$. The highest harvest possible occurs at $B = B^*$, with $H^* = G^* = gK/4$. This is the maximum sustainable yield (MSY). For any fixed value of harvest $H < H^*$, there are two equilibria, B^+ and B^-, symmetrically situated about B^* (Figure 2.2). If harvesting were arranged to be constant, then the right-most of this pair, B^+, would be stable to small perturbations in B; while the left-most B^- would be unstable. Negative perturbations about B^- would result in extinction; positive perturbations would result in growth toward the stable equilibrium at B^+. Harvesting in excess of H^* also results in extinction, as no growth could ever keep up. Thus, we have two recipes for extinction: Operate at low biomass, $B < B^-$, *or* harvest above MSY, $H > H^*$. And there are two conditions for a stable, sustainable harvest: Harvest below MSY, *and* avoid the possibility of large negative disturbances to B, such that B falls below B^-.

2.1.2 Harvest

In considering the harvest rate, we need the concept of harvesters' effort: the number of jobs, machines, etc. involved in active harvesting and their relative employment, activity, or utilization (e.g., number of days per year spent harvesting). We will lump all these factors into a single effort variable E.

The harvest depends on the effort and the biomass. A simple relation is

$$H(B, E) = hEB \tag{2.10}$$

wherein h represents the harvesting technology. Increases in effort, biomass, or technology increase the harvest, linearly in this case. Absence of any of these factors guarantees zero harvest.

With this closure, we can describe the steady state in which $H = G$:

$$hEB = gB\left(1 - \frac{B}{K}\right) \tag{2.11}$$

and thus,

$$E = \frac{g}{h}\left(1 - \frac{B}{K}\right) \tag{2.12}$$

or equivalently,

$$\frac{hE}{g} + \frac{B}{K} = 1 \tag{2.13}$$

So for this fishery in steady state, E is linear in B. We may operate at any combination of B and E (fish and fishers) on this line; at one extreme, $(E, B) = (0, K)$, and we have the natural carrying capacity with no effort and no harvest; while at the other extreme, $(E, B) = \left(\frac{g}{h}, 0\right)$, we have a system at infinitesimal biomass, essentially no harvest, and much effort devoted to keeping it there. In the middle, we have the MSY point $(E, B) = \left(\frac{g}{2h}, K/2\right)$ with $H = H^*$. If effort were able to be controlled, we could choose among these equilibria or any other (E, B) pairs along the line represented by Equation 2.12.

Figures 2.3 and 2.4 illustrate logistic and depensatory growth, with harvesting as in Equation 2.10.

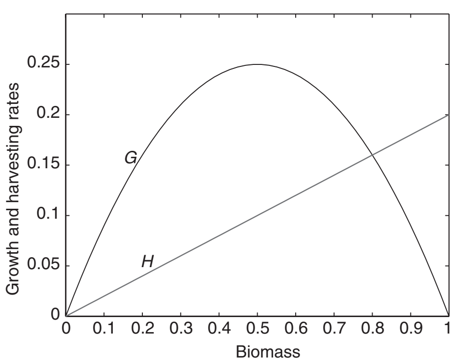

Figure 2.3. Gowth and harvesting rates with $H = hEB$, for a representative value $hE = .2$. The logistic growth function is compensatory at low B; the unstable intersection is absent.

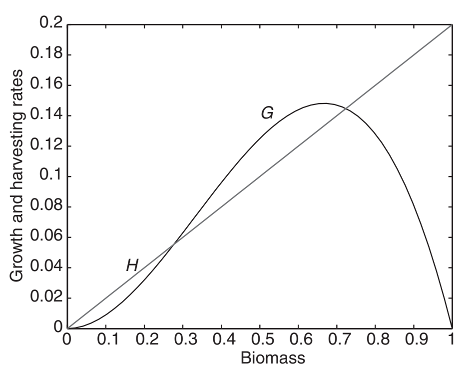

Figure 2.4. Growth and harvesting rates. $H = hEB$ as in Figure 2.3, but $G = B^2(1 - B)$. This growth curve is depensatory at low B, with an unstable descent to extinction in that low B range.

2.1.3 Rent

Next, introduce rent as the net profit resulting from the harvest. We will take the sales price for the harvest, p, to be constant and the wage or cost of effort, c, also as constant.[1] So, rent R is given by

$$\pi = pH - cE \tag{2.14}$$

In the steady state, $H = G$ and E is given by Equation 2.12, so

$$\pi(B) = pgB\left(1 - \frac{B}{K}\right) - c\frac{g}{h}\left(1 - \frac{B}{K}\right) \tag{2.15}$$

Now we can characterize three interesting steady-state points above by the four descriptors B, H, E, and R (fish, food, effort, and money).

Point 0: (resource extinction)

$$\begin{aligned} B &= 0 \\ H &= 0 \\ E &= g/h \\ \pi &= -cg/h \end{aligned} \tag{2.16}$$

Point MSY: (maximum sustainable yield)

$$\begin{aligned} B &= K/2 \\ H &= gK/4 = H^* \\ E &= g/2h \\ \pi &= pgK/4 - cg/2h \end{aligned} \tag{2.17}$$

Point K: (carrying capacity)

$$\begin{aligned} B &= K \\ H &= 0 \\ E &= 0 \\ \pi &= 0 \end{aligned} \tag{2.18}$$

It is interesting to notice that the MSY point may produce either positive or negative rent; it is not characterized by an economic criterion, but rather by a biological one. Point K is also an exclusively biological one, with no harvesting effort and no rent.

[1] Here we assume large external markets in food (p) and effort (c). This resource is on the margin of a large economy, which it does not affect; p and c are constants.

Point 0, however, requires maximal effort to maintain the level $B = 0$. Many other equilibria are possible, between points $B = 0$ and $B = K$, with an implied steady-state trade-off between E and B.

Aside: It is interesting to reexpress rent in terms of B and H. When $H = hEB$, we always have

$$E = \frac{H}{hB} \tag{2.19}$$

and therefore,

$$\pi = pH - cE \equiv \left(p - \frac{c}{hB}\right) H \tag{2.20}$$

(This does not assume a steady harvest.) Immediately, we have $\pi = \pi(H, B)$, and the "cost of harvest" is c/hB – that is, dependent on B. This is a "stock effect." We also identify $B = c/hp$ as the condition of zero rent; this parameter B_0 is used extensively below:

$$B_0 \equiv \frac{c}{hp} \tag{2.21}$$

The general rent formula is as above, $\pi = \pi(H, B)$. In the steady state, with $G = H$,

$$\pi_s = \pi(G, B) \tag{2.22}$$

$$= \left(p - \frac{c}{hB}\right) G \tag{2.23}$$

$$\frac{d\pi_s}{dB} = \left(p - \frac{c}{hB}\right) \frac{\partial G}{\partial B} + \left(\frac{c}{hB^2}\right) G \tag{2.24}$$

This formulation of π is a useful alternative to that formulated explicitly in terms of effort, $\pi = pH - cE$.

Which equilibrium is likely to occur? That depends on the conditions under which the fishery is operated and the criterion on which that is based.

2.2 ECONOMIC DECISION RULES

Here we will assume that harvesting can be made profitable over some range of B, as in Figure 2.5. The opposite is easy to envision – for example, when c is very large and rent is always negative.[2] Under these conditions, the resource is *economically extinct*; economic actors would abandon all harvesting, and the steady state would be at the carrying capacity, point K, with no economic effort, harvest, or rent. A steady harvest does not pay.

Assuming harvesting is profitable over some range, we have two interesting steady states.

[2] This occurs in this fishery when $c/ph > K$, that is, $B_0 > K$.

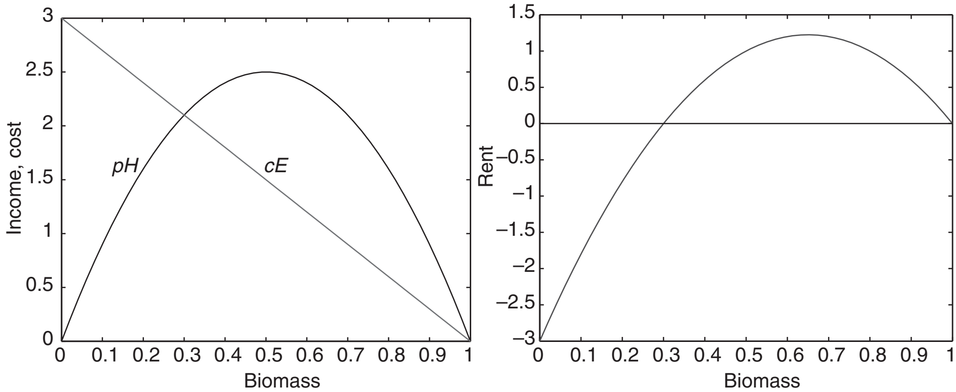

Figure 2.5. Income pH and expenses cE (*left*) and rent π (*right*), as a function of B, in steady state for logistic growth $G = gB(1 - B/K)$ and $H = hEB$. Parameters: $(g, h, K) = 1$; $p = 10$; $c = 3$.

2.2.1 Free-Access Equilibrium

In a *laissez-faire* system, with free access to the fishery, we hypothesize that new effort (fishers) will enter the business as long as the value of the harvest exceeds the cost of its production – essentially, as long as $\pi > 0$. (Recall that "cost" includes the cost of all inputs to production, *including reasonable/necessary return on investment* in equipment; positive rent implies a windfall situation where all of these costs are paid, including all returns on captital invested or borrowed; and there is still income left over.) The condition of vanishing steady rent (Equation 2.15) characterizes this equilibrium:

$$\pi(B) = pgB\left(1 - \frac{B}{K}\right) - c\frac{g}{h}\left(1 - \frac{B}{K}\right) = 0 \tag{2.25}$$

Setting $\pi(B) = 0$ gives the equilibrium at $B = \frac{c}{ph} \equiv B_0$; we identify this equilibrium value as B_0 for convenience. This is the "open-access," or "free-entry," point. Because positive rent is possible at lower effort, this point is sometimes characterized as the "rent dissipation" point, arrived at by increasing E freely until π is driven to zero. Key quantities at this point are

Point R_0: (free entry)

$$\begin{aligned} B &= B_0 = \frac{c}{ph} \\ H &= gB_0\left(1 - \frac{B_0}{K}\right) \\ E &= E_0 \equiv \frac{g}{h}\left(1 - \frac{B_0}{K}\right) \\ \pi &= 0 \end{aligned} \tag{2.26}$$

The point R_0 is stable under these conditions. Effort may not increase without encountering negative rent, and vice versa.

There is another point where rent vanishes, at $B = K$. It is economically unstable under open access; $E = 0$ there, but positive rent at $E > 0$ would encourage entry of effort, such that the stable equilibrium R_0 is approached, with $B < K$ and $E > 0$.[3]

2.2.2 Controlled-Access Equilibrium

In a *monopoly-operated* system, effort may be controlled. It is visually apparent that more rent may be earned by reducing effort relative to the open-system equilibrium (B_0). The controlled-access equilibrium is defined by the condition of rent maximation. Taking the derivative of Equation 2.15 with respect to B gives us the extremum condition:

$$\frac{d\pi(B)}{dB} = pg - 2\frac{pg}{K}B + \frac{cg}{hK} = 0 \tag{2.27}$$

Solving for B gives the equilibrium:

Point R^*: (maximum rent, controlled access)

$$\begin{aligned} B &= \frac{K + B_0}{2} \\ H &= gK\left(1 - [B_0/K]^2\right)/4 \\ E &= E_0/2 = \frac{1}{2}\frac{g}{h}\left(1 - \frac{B_0}{K}\right) \\ \pi &= \pi_{\text{MSY}} + (pg/4K)B_0^2 \end{aligned} \tag{2.28}$$

This point requires monopoly control of effort.[4] Compared with the open-access case, we have less effort, more (positive) rent, and higher B. A standard characterization is that there is less work being done, more money made, and a higher biomass. The effect on harvest H is ambiguous and depends on the parameters.

Table 2.1 summarizes the five steady states discussed so far.

Table 2.1. Steady-state operating options for $H = hEB$; $G = gB(1 - B/K)$. $B_0 \equiv c/ph$ is the open-access (free-entry) equilibrium. When $B_0 > K$, the resource is economically extinct.

		B	H	E	π
0	Extinction	0.	0.	g/h	$-cg/h$
MSY	Maximum harvest	$K/2$	$gK/4 = H^*$	$g/2h$	$pgK/4 - cg/2h$
K	Carrying capacity	K	0.	0.	0.
R_0	Free entry	$B_0 \equiv c/ph$	$gB_0(1 - B_0/K)$	$E_0 \equiv g(1 - B_0/K)/h$	0.
R^*	Maximum rent	$(K + B_0)/2$	$gK\left(1 - [B_0/K]^2\right)/4$	$E_0/2$	$\pi_{MSY} + (pg/4K)B_0^2$

[3] As above, this assumes $B_0 \equiv c/ph < K$; otherwise, the resource is economically extinct.

[4] Notice that for the costless case, point R^* is the same as point MSY.

Control of harvesting is often conceived as one or more measures aimed at effort control, harvest control, technology control, market control, or political control. All are normally achieved by a permitting/inspection process. There are several generic forms:

- Effort at harvesting
  - Days at sea
  - Vessel licensing
- Harvest control
  - Count landed harvest
  - Inspection at sea
  - Quotas
- Technology control
  - Information technology
  - Vesssel size, speed
  - Method of capture/storage
- Market control
  - Tax the product
  - Tax the landed harvest
  - Tax the effort
- Political control
  - Penalties for violation: civil, criminal, economic

2.3 EFFORT DYNAMICS

It is easy to extend a formal description of effort adjustment. The discussion above describes this in terms of rent: When $\pi > 0$, effort in the open-access system increases; and when $\pi < 0$, effort diminishes. We formalize this with a first-order rate $\nu\pi$. Together with the system described so far, we have the dynamic system

$$\frac{dB}{dt} = G - H \tag{2.29}$$

$$H = hEB \tag{2.30}$$

$$G = gB\left(1 - \frac{B}{K}\right) \tag{2.31}$$

$$\pi = pH - cE \tag{2.32}$$

$$\frac{dE}{dt} = \nu\pi \tag{2.33}$$

There are two dynamic state variables (B, E) and three constitutive relations defining the auxiliary variables (H, G, π). Fixed parameters include ν, h, g, K, p, and c. As a second-order nonlinear system, we have the potential for extravagant behavior, and the parameter ν remains to be explored.

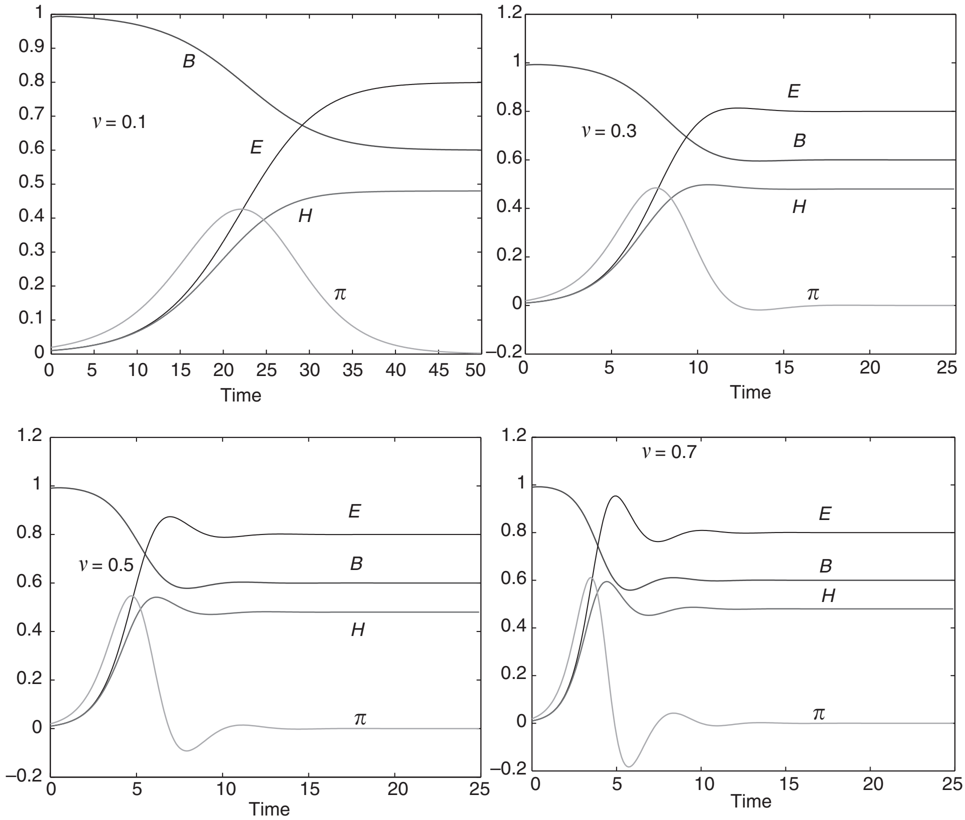

Figure 2.6. Dynamics of the free-access solution as a function of $\boldsymbol{\nu}$, as indicated. Logistic growth $G = gB(1 - B/K)$ and $H = hEB$. Parameters: $(g, h, K) = (2, 1, 1)$; $(p, c) = (5, 3)$. Euler (explicit forward) integration, $\Delta t = 0.1$. Equilibrium values are $(B, E, H, \boldsymbol{\pi}) = (0.6, 0.8, 0.48, 0.0)$.

The program **fish1_4.m** (**Fish1.4.xls**) illustrates this; results appear in Figure 2.6. Equilibrium values are $B = 0.6$, $E = 0.8$, $H = 0.48$, and $\pi = 0.0$. There is an orderly approach to equilibrium at low ν; during this approach, rent rises, peaks, and returns toward zero as the equilibrium is approached. Exploitation during this period is lucrative. Increasing ν to 0.3 speeds up the process but otherwise adds little to the dynamic. $\nu = 0.5$ and 0.7 continue this trend; in addition, an overshoot is introduced, following which rent becomes negative, E reduces (an employment layoff), and H recovers, all in an orderly approach to the same equilibrium.

Figure 2.7 presents the high ν extension of this case. The equilibrium solutions are unchanged, yet all hope for monotone solutions is gone in these rapid-response-to-rent scenarios. The overshoot trend identified above at low ν is here amplified, and (ultimately) wild oscillations are evident in B, E, π (i.e., economy, employment, and ecology). A complex periodicity of about two years is apparent. Amplitude increases with ν; the damping rate decreases with ν. High ν clearly is producing boom-or-bust cycling in this system.

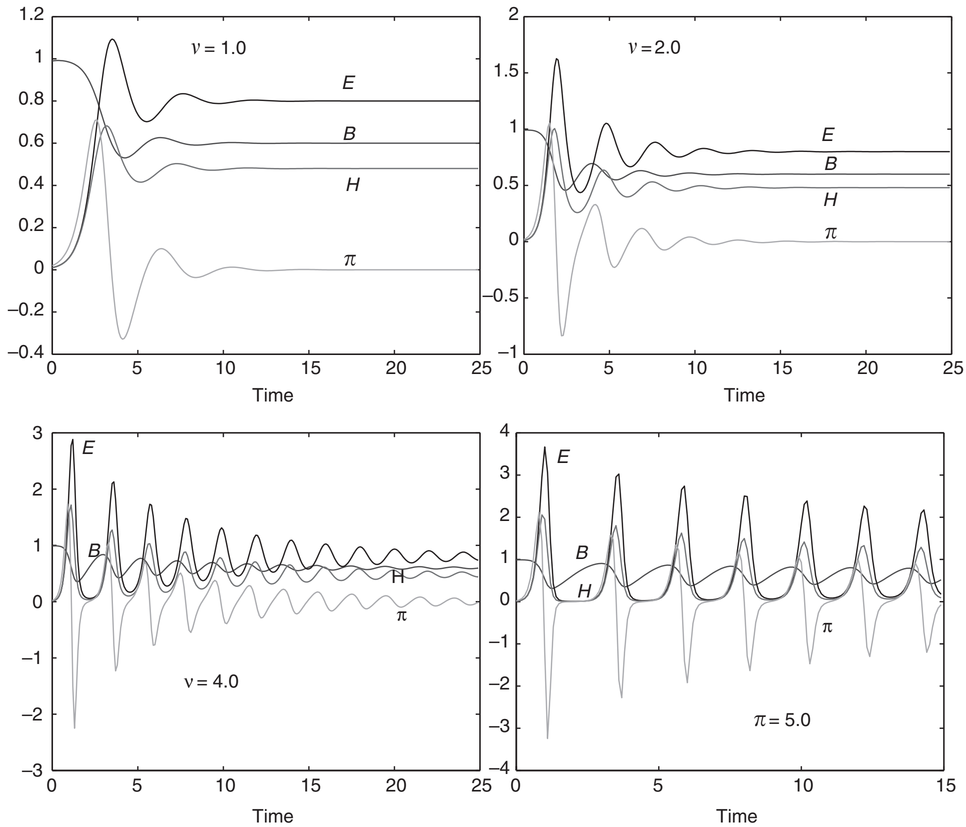

Figure 2.7. Continuation of Figure 2.6, for larger values of ν.

Program **Fish1.4-RAND.xls** adds independent stochastic disturbances to this dynamic, through G and dE/dt. These are autocorrelated as in the Appendix. Program **Fish1.4-Cubic.xls** uses cubic growth function, with depensation exaggerated by the incorporation of a minimum viable biomass $\underline{B}$:

$$G(B) = gB(B - \underline{B})(1 - B/K) \tag{2.34}$$

Either depensatory growth, or the entry of stochastic disturbances, can exaggerate the negative effects of rapid effort dynamics (high ν).

2.4 INTERTEMPORAL DECISIONS: THE INFLUENCE OF *r*

In previous analyses, we ignored the effect of r, the growth rate of money invested in a productive economy. We have concentrated solely on the sustainable harvest, correct for $r = 0$. Here we will reinstate the effect of $r > 0$.

We imagine a situation where a harvester discovers a resource in the unexploited state $B = K$. Faced with the option to establish a steady-state harvest, B must be reduced to some level $B < K$. That initial harvest, assumed instantaneous for the moment, would be profitable and the proceeds invested at interest rate r.

2.4.1 Costless Harvesting

The sale of the initial harvest is $p(K - B)$, which is invested at interest rate r. The harvest at B is then sustained, and its sale value is $pG(B)$. Annual income π is thus the sum of the sale of the sustainable harvest plus the investment earnings from the initial harvest:

$$\pi = pG(B) + rp(K - B) \tag{2.35}$$

Its maximum is found by differentiating

$$\frac{d\pi}{dB} = p\frac{dG}{dB} - rp = 0 \tag{2.36}$$

and the optimal point is at

$$\frac{dG}{dB} = r \tag{2.37}$$

For the logistic growth curve

$$G(B) = gB\left(1 - \frac{B}{K}\right) \tag{2.38}$$

we have

$$\frac{dG}{dB} = g\left(1 - \frac{2B}{K}\right) \tag{2.39}$$

Figure 2.8 illustrates this balance. The steady B will always be below the MSY point, as dG/dB is negative above that. The maximum growth rate is g; when $r > g$, extinction is the "rational" solution. Otherwise, the sustainable solution is between extinction and the MSY point.

It is clear that the scenario given initially can be relaxed; starting from *any* initial B, we arrive by the same reasoning at the desired equilibrium: balancing the annual yield of the initial harvest against the annual yeild of steady harvesting.

The consequence of this is striking: Such a "rational economic actor" with guaranteed and exclusive access to the resource would always find the steady equilibrium B and H on the rising limb of the growth curve, *below* the MSY point.[5] For slow-growing resources, where $dG/dB < r$ irrespective of abundance B, such an approach would result in extinction; the resource growth cannot equal financial investments. Money grows faster than the resource under all conditions!

[5] Under costless harvesting, point MSY is also the maximum rent equilibrium R^* identified above.

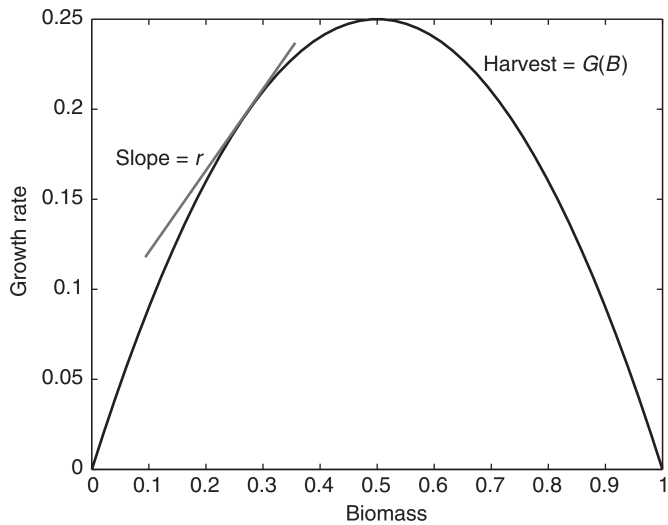

Figure 2.8. Growth curve indicating the intertemporal equilibrium where $dG/dB = r$, for costless harvesting. The equilibrium is at $B \simeq .28$ in this illustration.

It would appear from this analysis that a strictly financial criterion for a "renewable" resource is $dG/dB > r$; resources with slower growth would be exploited as if they were sterile and exhaustible under free access.

Notice that here we have no explanation for r, unlike the biology that creates G. A fuller examination of this balance would need to develop the relationship between the growth rate of money invested and the existence and growth of natural resources.

Notice, too, that we have now developed two different criteria for *economic extinction*:

- When there is B but it is too costly to harvest, then $B \to K$.
- When G is too slow, it is attractive to harvest all of B and invest it in the economy at growth rate r, *presumed sustainable*.

These are dramatically different situations!

2.4.2 Costly Case

The same exercise with costly harvesting can be done. Consider operating at the harvest rate $H = hBE$, with sustainable annual rent $\pi_s = \pi_s(B)$, as in Figure 2.5. We contemplate an *instantaneous surge in effort* ΔE for a small period of time Δt (Figure 2.9); with a related instantaneous surge in harvest ΔH over the same small Δt:

$$\Delta H = hB\Delta E \tag{2.40}$$

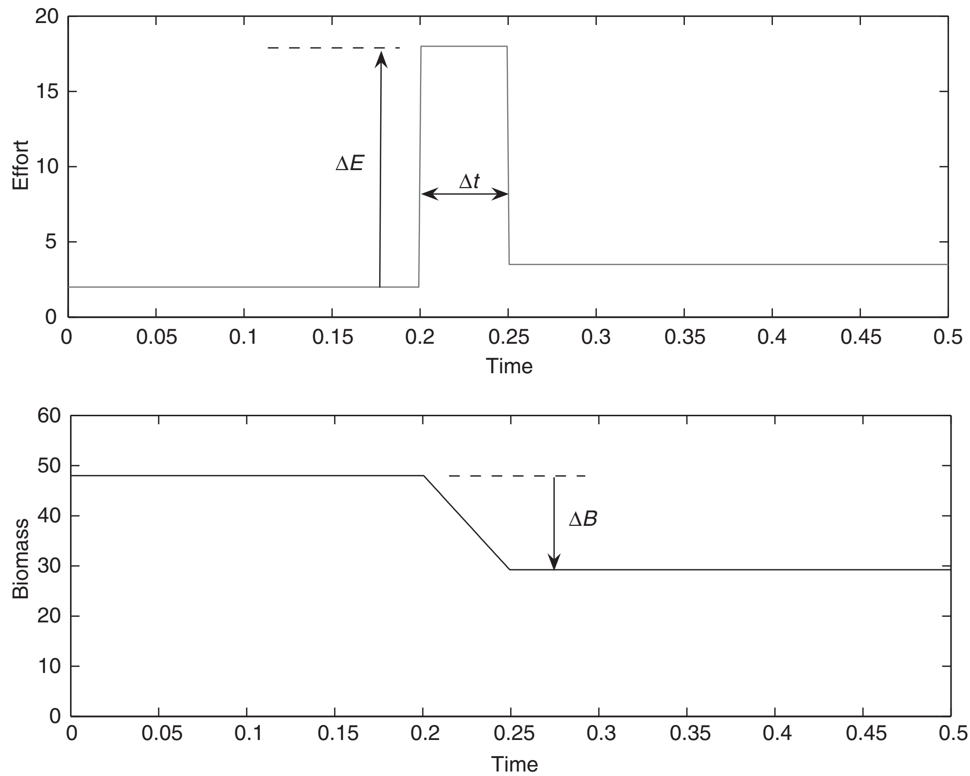

Figure 2.9. Illustration of an instantaneous surge in E and the resulting change in B. Both levels of B are to be harvested sustainably.

The instant, one-time rent quantum would be

$$\Delta\pi_i = (p\Delta H - c\Delta E)\Delta t \tag{2.41}$$

$$= \left(p - \frac{c}{hB}\right)\Delta H \Delta t \tag{2.42}$$

As a result of this change, all future harvests will be based on a decreased B:

$$\Delta B = -\Delta H \Delta t \tag{2.43}$$

and the sustainable rent $\pi_s(B)$ would change by the amount

$$\Delta\pi_s = \frac{d\pi_s}{dB}\Delta B \tag{2.44}$$

The two effects from the instantaneous change are: $\Delta\pi_i$, the instant, one-time rent quantum in the bank (due to the instantaneous ΔE and ΔH); and the changed sustainable rent $\Delta\pi_s$ (due to the permanent change in ΔB). On an annual basis, the net change is

$$\Delta\pi = \Delta\pi_s + r\Delta\pi_i \tag{2.45}$$

$$= \left[\frac{d\pi_s}{dB} + r\left(-p + \frac{c}{hB}\right)\right]\Delta B \tag{2.46}$$

$$= \left[\frac{d\pi_s}{dB} - rp\left(1 - \frac{B_0}{B}\right)\right]\Delta B \tag{2.47}$$

(Earlier we introduced the open-access equilibrium $B_0 \equiv c/ph$.) Indifference about this trade-off must characterize the equilibrium point; $\Delta\pi = 0$:

$$\frac{d\pi_s}{dB} = rp\left(1 - \frac{B_0}{B}\right) \tag{2.48}$$

When costs are zero, we recover the costless case above (Equation 2.37), with $\pi_s = pG$. This result generalizes that. Clearly, the controlled access point has

$$\frac{d\pi_s}{dB} = 0 \tag{2.49}$$

and assuming profitable harvesting, then here we have

$$\frac{d\pi_s}{dB} > 0 \tag{2.50}$$

and we can see that we are moving to lower B in Figure 2.5. The intertemporal consideration has required sustainable operation at a lower B than we had before: It is profitable to cut back on B once, invest the proceeds in the economy, and harvest the interest plus the remaining $G(B)$ sustainably thereafter. When $r = 0$, we recover the previous equilibrium R^* (the rent-maximizing controlled access point).

Base Case: Logistic Growth

For the base case

$$G = gB\left(1 - \frac{B}{K}\right) \tag{2.51}$$

$$H = hEB \tag{2.52}$$

we have from Equations 2.15 and 2.27

$$\pi(B) = pgB\left(1 - \frac{B}{K}\right) - c\frac{g}{h}\left(1 - \frac{B}{K}\right) \tag{2.53}$$

$$\frac{d\pi(B)}{dB} = pg - 2\frac{pg}{K}B + \frac{cg}{hK} \tag{2.54}$$

Assembling Equation 2.48, we obtain

$$1 - 2\frac{B}{K} + \frac{B_0}{K} = \frac{r}{g}\left(1 - \frac{B_0}{B}\right) \tag{2.55}$$

A little algebra gives the quadratic equation

$$\beta^2 - \beta\frac{(1+\beta_0-\rho)}{2} - \frac{\rho\beta_0}{2} = 0 \tag{2.56}$$

where we have introduced the normalized variables

$$\beta \equiv B/K \tag{2.57}$$

$$\rho \equiv r/g \tag{2.58}$$

$$\beta_0 \equiv B_0/K \equiv c/phK \tag{2.59}$$

The roots of this are[6]

$$\beta = \left[\frac{1+\beta_0-\rho}{4}\right] \pm \sqrt{\left[\frac{1+\beta_0-\rho}{4}\right]^2 + \frac{\rho\beta_0}{2}} \tag{2.60}$$

This result reproduces earlier results:

- In the costless case ($\beta_0 = 0$), $\beta = \frac{1-\rho}{2}$
- In the no-interest case ($\rho = 0$), $\beta = \frac{1+\beta_0}{2}$

In the interesting intermediate cases, $\beta_0 < \beta < \frac{1+\beta_0}{2}$. In the limit of vanishing ρ, we get the closed-access equilibrium case identified above. As ρ increases, β is reduced

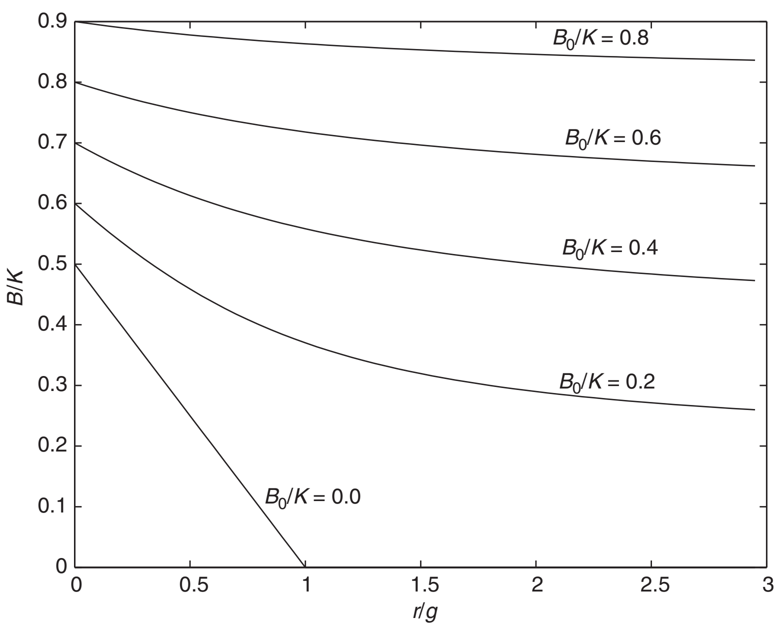

Figure 2.10. Equilibrium $\beta \equiv B/K$ versus $\rho \equiv r/g$ from Equation 2.60. This is the intemporal, rent-maximizing equilibrium for the base case (logistic) fishery. $B_0 \equiv c/ph$ is the open-access equilibrium.

[6] The positive option is correct here; the negative option is useless, returning $\beta < 0$.

toward its open-access counterpart β_0. Programs **Fish1.6.xls** and **Fish1_6.m** simulate this. Figure 2.10 illustrates this.

2.4.3 Costly Case: A General Expression

An alternative, and equivalent, view of the intertemporal case is useful and common. Here we note that with $H = hEB$, we always have

$$E = \frac{H}{hB} \tag{2.61}$$

and thus the sustainable rent is

$$\pi = pH - cE \tag{2.62}$$

$$= \left[p - \frac{c}{hB}\right] H \tag{2.63}$$

$$= \pi(H, B) \tag{2.64}$$

The cost term c/hB has a dependence on the resource "stock" B as well as the harvest H. In this formulation, the cost of harvesting increases as the biomass is reduced.[7] The partial derivatives are

$$\frac{\partial \pi}{\partial B} = \frac{cH}{hB^2} > 0 \tag{2.65}$$

$$\frac{\partial \pi}{\partial H} = p - \frac{c}{hB} \tag{2.66}$$

More general harvesting rate functions $H(E, B)$ are possible and the form

$$\pi = \pi(H, B) \tag{2.67}$$

is quite general. In these general terms, the intertemporal trade-off is as before:

$$r\frac{\partial \pi}{\partial H}\Delta H \Delta t + \Delta\pi = 0 \tag{2.68}$$

with the first term coming from the one-time harvest and the second term coming from the reduced sustainable harvest. Expanding the second term, we have both the harvest and stock effects:

$$\Delta\pi = \frac{\partial \pi}{\partial B}\Delta B + \frac{\partial \pi}{\partial H}\frac{dH}{dB}\Delta B \tag{2.69}$$

$$= \frac{\partial \pi}{\partial B}\Delta B + \frac{\partial \pi}{\partial H}\frac{dG}{dB}\Delta B \tag{2.70}$$

(We assume the steady balance $H = G$.) Adding the fact $\Delta B = -\Delta H \Delta t$, the result is

$$r\frac{\partial \pi}{\partial H} = \frac{\partial \pi}{\partial B} + \frac{\partial \pi}{\partial H}\frac{dG}{dB} \tag{2.71}$$

[7] The "stock effect" on cost of production was seen in the nonrenewable case earlier, $C(S)$ in Chapter 1.

and a little rearrangement gives

$$\frac{dG}{dB} = r - \left[\frac{\partial\pi/\partial B}{\partial\pi/\partial H}\right] \tag{2.72}$$

This relation is quite general; it does not depend on the specific closures for $G(B)$ and $H(E, B)$ introduced above. It recovers the simple costless case, Equation 2.37, when c and therefore $\partial\pi/\partial B$ vanishes. Otherwise, $\partial\pi/\partial B > 0$ and the equilibrium moves toward smaller dG/dB, higher B.

When $r = 0$, this point recovers the equilibrium R^* (the peak sustainable rent for controlled access), as there is no value in financial investment and the trade-off is reduced to $d\pi/dB = 0$.

Equation 2.72 is referred to by several authors as the "fundamental equation of renewable resources" (e.g., Conrad (1999), Equations 1.16 and 3.5).

From the three demonstrations here, we can see that the general case puts the optimal intertemporal trade-off in between the simple closed-access optimum R^* (valid when $r = 0$) and the costless limit $dG/dB = r$.

2.5 TECHNOLOGY

The technology of harvesting has been assumed constant up to now. It clearly figures in our formulation of harvest rate, via the parameter h:

$$H = hEB \tag{2.73}$$

Advances in technology plainly amplify effort and make possible harvesting at lower biomass or effort or both. On what do h and dh/dt depend? A realistic treatment of technology is needed for a full theory of natural resource dynamics; here we simply speculate on the form it might take.

We denote by $I(t)$ the innovation rate:

$$\frac{dh}{dt} = I \tag{2.74}$$

Necessity is the mother of invention; and in a crude way, the purpose of entrepreneurship is to create rent where currently there is none. So we suppose that π drives innovation. That might take a form such as

$$I = I_0 e^{-\beta\pi} \tag{2.75}$$

At low rent, there is incentive to innovate; but as rent rises, there is less urgency. (Recall this is a rate of innovation.) Naturally, negative innovation and/or negative rent is not meaningful here, so we limit ourselves to the first quadrant of this plot (Figure 2.11). Implied is that $I = 0$ for $\pi \leq 0$.

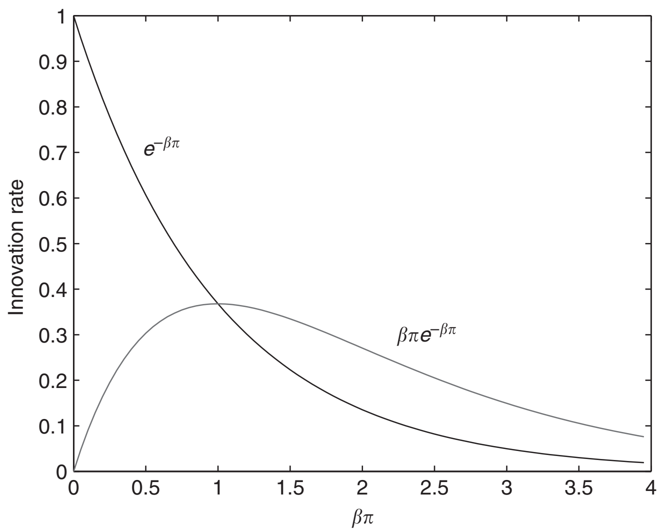

Figure 2.11. Candidate innovation rate functions. $I_0 = 1$; $\boldsymbol{\beta} = 1$.

We can do better for low π. The abrupt termination there would be better modeled with a gentler approach to the origin. A candidate with the correct shape is

$$I = I_0 \beta \pi e^{-\beta \pi} \tag{2.76}$$

This form is also plotted in Figure 2.11. It contains the idea that if π is small, there is little reinvestment in harvesting technology. As rising π attracts innovation investment, whether from within the harvesting industry or from without. But it is self-extinguishing at very high π as in the previous form.

Finally, there is the possibility of technical saturation: a ceiling $\overline{h}$ that limits I:

$$I = I_0 \beta \pi e^{-\beta \pi} \left(\frac{\overline{h} - h}{\overline{h}} \right) \tag{2.77}$$

or a similar effect without the absolute ceiling:

$$I = I_0 \beta \pi e^{-\beta \pi} \left(\frac{\overline{h}}{h} \right) \tag{2.78}$$

The ultimate form suggested here has three parameters $(I_0, \beta, \overline{h})$ and two state variables (h, π):

$$\frac{dh}{dt} = I_0 \beta \pi e^{-\beta \pi} \left(\frac{\overline{h} - h}{\overline{h}} \right) \tag{2.79}$$

Program **Fish1.5.xls** simulates this. The technology grows during exploitation. Technology growth amplifies human effort, frees it to work on other things, and compensates for scarcity. But ultimately, we must confront its effect on the growing resource B. At constant E, for example, we find that increasing h eventually causes resource extinction in a free-entry system. Effectively, unbounded h brings us to the costless harvesting limit that, in a free entry system, drives $B \to 0$ (Figure 2.12). Some form of effort and/or technology control is needed on the part of the owner if this scenario is to be avoided.

The dynamics of this system are illustrated in Figure 2.13 for a case studied earlier (Figure 2.6). Clearly, the addition of innovation here has resulted in enhanced cycling of the fishery and a general trajectory toward lower biomass.

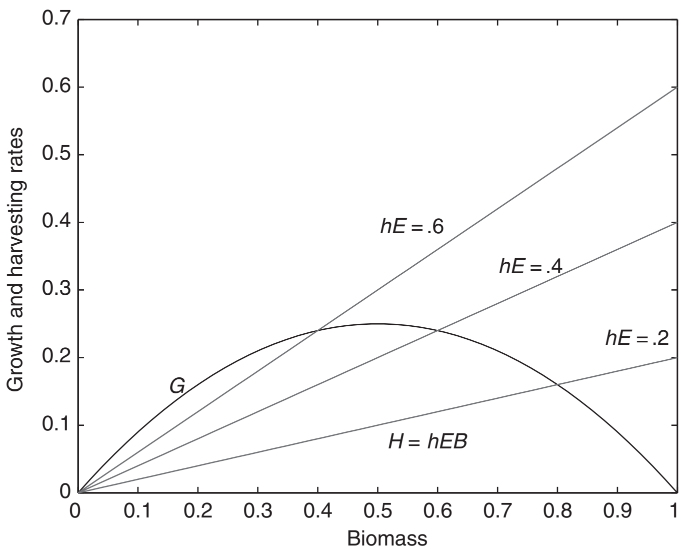

Figure 2.12. Growth ($G = gB[1-B]$) and harvesting ($H = hEB$) rates. Increasing technology h moves the equilibrium toward the left (lower B) unless compensated for by decreasing effort E. Extinction is reached when $hE > g$ for these rates.

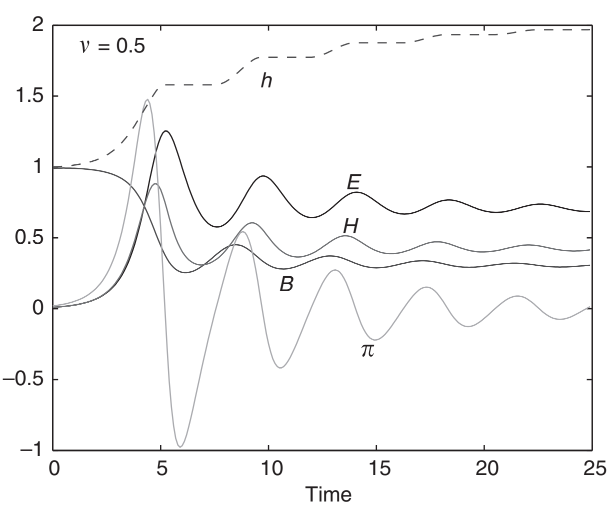

Figure 2.13. Dynamic adjustment as in Figure 2.6 for $\nu = 0.5$, but with innovation parameters $(I_0, \beta, \overline{h}) = (1, 0.5, 5)$ and initial $h = 1$.

Other forms of technological saturation functions $S(h)$ are useful. Above, we used the form

$$S(h) = 1 - \frac{h}{\overline{h}} \tag{2.80}$$

This has the anomaly that exceeding the ceiling $\overline{h}$ results in negative innovation. A better form is common:

$$S(h) = \frac{1}{1 + \frac{h}{\overline{h}}} \tag{2.81}$$

In this form, $\overline{h}$ is called the "half-saturation constant," as $S = .5$ when $h = \overline{h}$. In this form, S is positive for all positive values of h, decreasing monotonically.

2.6 RECAP

This chapter has developed many basic bioeconomic interactions among fish and fishers – more generally, prey and predator or resource and people. The basic bioeconomic model can be recognized as the "Gordon-Scott fishery" [32, 80]. It is used in many expositions, including resource economics generally – for example, Conrad [12], and in fisheries treatments specifically: Clark [9], van den Bergh [90], Grafton et al [34], and Mangel [59]. These works and many other typically go much further into the description of biological populations, resonating well with the subsequent Chapters 3 and 4 herein.

Critical issues reveal the necessity of sustaining the ecosystem; the biological populations hosted; and the economic interactions with people. Achieving this set of outcomes will require a high degree of practicality in diverse cases. A realistic view of local/private incentives, ownership, and the common object is needed, as is a careful assessment of individual objectives and motivations. The simple model used here illumines the multiplicity of criteria that might be relevant: the biomass, the harvest, the rent, the jobs. All have found their their place in practical systems.

Several works address operational issues encountered in fisheries management. Included are Walters and Martell [91], Hillborn and Walters [39], and Clark [10]. As used here, fisheries represent a case study of the more general class of living renewables.

The critical issue of management of common-pool resources is fundamental to the considerations introduced here. The reader is referred to the important contributions of Ostrom et al. [71, 70] and Sandler [79], for a blend of theory and experiment. These are important contributions to natural resource management, touching fundamental social science issues broadly.

2.7 PROGRAMS

The following programs illustrate the ideas in these lectures:

Fish1.4.xls (**fish1_4.m**) simulates the dynamics of B and E under open-access conditions.

Fish1.4-RAND.xls adds independent stochastic disturbances to G and dE/dt. Both time series are autocorrelated.

Fish1.4-Cubic.xls adds depensation to G and a minimum viable biomass parameter $\underline{B}$.

Fish1.5-RAND.xls adds dynamics of technological innovation to **Fish1.4-RAND.xls**.

Fish1.6.xls (**fish1_6.m**) computes the equilibrium for the costly harvesting of logistic growth, with intertemporal considerations of interest rate, as in Section 2.4.2.

2.8 PROBLEMS

1 Logistic growth function:

(a) Express the logistic growth function, Equation 2.2, in terms of the departure from carrying capacity $\epsilon \equiv K - B$.

(b) Show that for small ϵ,

$$\frac{d\epsilon}{dt} = -g\epsilon \tag{2.82}$$

2 Logistic growth, no harvesting. Find the waiting time T_p until peak growth rate occurs for the following:

(a) $g = .08/\text{yr}$, $B_0/K = 5\%$

(b) An endangered species with slow growth, $g = .04/\text{yr}$, and low abundance $B_0/K = 10\%$

(c) An invasive species at low initial abundance $B_0/K = 7\%$, and a doubling time of 20 years

3 For the steady-state fishery:

$$G = gB(1 - B/K) \tag{2.83}$$

$$H = hEB \tag{2.84}$$

$$\pi = pH - cE \tag{2.85}$$

Plot harvest as a function of price p, for the open-access (Rent = 0) and for the closed-access (Rent-maximizing) cases. Use these parameters: $(K, c, g, h) = 1$. Explain your results.

4 Consider a fishery with growth given as $G = g(1 - B/K)$ and all other relations unchanged from the case in Problem 3. (Be careful about G here; it is not hard, just different.)

(a) Solve for B, H, E, and rent under open access.

(b) Repeat for the closed-access case.

(c) Plot H versus p as in Problem 3.

(d) Compare the two management regimes.

5 Simulation exercises with the **FISH1.4** model. This model solves the simple logistic fishery

$$G = gB(1 - B/K) \quad (2.86)$$

$$H = hEB \quad (2.87)$$

$$\pi = pH - cE \quad (2.88)$$

with dynamics

$$\frac{dB}{dt} = G - H \quad (2.89)$$

$$\frac{dE}{dt} = \nu\pi \quad (2.90)$$

The standard parameter setup is

Time	dt	1
Growth	g	2
	K	1
Harvesting	h	1
	ν	0.5
Value	p	5
	c	3

Confirm: With all other parameters fixed, what values of ν lead to (a) orderly (monotonic) approach to equilibrium; (b) oscillatory approach to equilibrium; (c) eternal ocsillations; and (d) instability or extinction.

6 Repeat the simulations from Problem 5, with a stochastic disturbance ϵ affecting the growth function:

$$G = gB(1 - B/K)\,(1 + \epsilon) \quad (2.91)$$

with ϵ a random number with zero mean, variance σ^2. Use the three values of ν found in Problem 5 and values of autocorrelation $\rho = 0$, .50, .75, .90, and .95. Study and report the effect of disturbance size σ in each of the cases.

7 Add depensation to Problem 5 by altering the growth function to be

$$G = g(1 - B/K)(B - 0.25) \quad (2.92)$$

Restudy Problem 5. Are your conclusions about parameter ranges, extinction possibility, etc. altered?

8 Repeat the simulations from Problem 7, with a stochastic disturbance ϵ affecting the growth function:

$$G = g(1 - B/K)(B - 0.25)\,(1 + \epsilon) \quad (2.93)$$

with ϵ a random number with zero mean, variance σ^2. Use the three values of ν found above (Problem 7) and values of autocorrelation $\rho = 0$, .50, .75, .90, and .95. Study and report the effect of disturbance size σ in each case.

9 For the standard fishery:

$$G = gB(1 - B/K) \tag{2.94}$$

$$H = hEB \tag{2.95}$$

$$\pi = pH - cE \tag{2.96}$$

with parameters $g = 1$, $h = 1$, $K = 1$, $p = 5$, $c = 3$. Currently, this fishery is open access and at steady state. There are numerous small fishing businesses. It is decided to change to a controlled-access regime and to maximize rent. The essence of the problem is reducing fishing effort – that is, all fishers will not continue to fish at the same rate.

- **(a)** One proposal is to *sell permits* that allow access to the fishery. One permit would allow .01 unit of fishing effort per year. How many permits need to be sold each year, and what is their price?
- **(b)** Instead, *fish will be taxed* at the dock, with fishers paying a tax, t, per unit of harvest. What value of tax is needed?
- **(c)** Instead, the *government will do the fishing* and issue no permits. Current fishers will be offered government jobs, either fishing or teaching ecology courses in the local high school. Both will be paid the same wage, that of a current fisher. What proportion of the current fishers will become teachers?

10 Consider the following fishery:

$$G = gB^2(1 - B/K) \tag{2.97}$$

$$H = hEB \tag{2.98}$$

$$\pi = pH - cE \tag{2.99}$$

This is the standard model except that the growth function G is different. (Be careful about this before starting!) Parameters are $g = 2$, $h = 1$, $K = 1$, $p = 5$, $c = 3$.

- **(a)** Sketch the growth function G versus B. Pay attention to the slope and curvature near the origin.
- **(b)** Is there depensation?
- **(c)** What is the maximum sustainable yield? And what is the associated value of B?
- **(d)** Find B, H, and E under controlled-access conditions.

11 Given the usual harvesting and rent relations

$$H = hEB \tag{2.100}$$

$$\pi = pH - cE \tag{2.101}$$

Consider the following growth functions:

(i) $G = \frac{1}{B}$
(ii) $G = \frac{1}{B} - \frac{1}{K}$
(iii) $G = 1 - e^{-B}$
(iv) $G = \sqrt{B}$
(v) $G = Be^{-B}$
(vi) $G = g \sin\left(\frac{B}{K}\Pi\right)$ (Here Π is the geometry constant 3.1416, as distinct from π, which indicates rent.)
(vii) $G = gB/\left(1 + \frac{B}{K}\right)$

For each case, find B, E, H, π for

(a) the open-access equilibrium.
(b) the closed-access equilibrium.
(c) the closed-access intertemporal equilibrium for $r = 0$ and $c = 0$.
(d) the closed-access intertemporal equilibrium for $r > 0$ and $c = 0$.
(e) the closed-access intertemporal equilibrium for $r > 0$ and $c \neq 0$.

Express your answers in terms of the parameters p, c, g, h, K, r, and, where useful, $B_0 \equiv c/ph$.

12 A fishery is characterized by

$$G = gBe^{-B}$$

$$H = hEB$$

Parameters: $(g, h, p, c) = (2, 3, 5, 2)$.

(a) Find the open-access steady state: E, B, H, π.
(b) Find the closed-access steady state: E, B, H, π.
(c) Find the MSY steady state: E, B, H, π.

13 For the standard fishery,

$$G = gB\left(1 - \frac{B}{K}\right)$$

$$H = hEB$$

Parameters: $(g, h, p, c, K) = (1, 1, 4, 1, 1)$. This fishery is now operating in open-access steady state. It is a concern that the level of B is too low, risking extinction. Therefore, it is desired to (i) close the access and keep the *effort constant*; and simultaneously, (ii) *restrict the harvesting technology*.

(a) What is the open-access status quo, (E, B, H, π)?
(b) What value of h is needed to move this fishery to the desired new steady state at MSY? (Note that there is no change in effort.)
(c) After making this change in h, what are the new steady-state values of (E, B, H, π)? Which have changed and in what direction?
(d) The rent will be distributed equally among the fishers. What is the payment per unit of effort?

(e) What is the *effective* compensation per unit of effort?

(f) Comment: The harvest is larger, the technology is cruder, the effort is the same, but the fishers are getting more money. Is this right, and why?

14 Given

$$H = hEB$$

$$\pi = pH - cE$$

and *any* growth function $G = G(B)$. Show that in steady state the competitive (open-access) equilibrium biomass is independent of the form of G:

$$B = \frac{c}{ph}$$

15 A fishery is described by the growth function

$$G(B) = g\left[1 - e^{-\alpha B}\right] \tag{2.102}$$

with constants g and α. Under costless production, what value of interest rate r will result in extinction?

16 A certain marine mammal population has a doubling time of 20 years in the absence of harvesting. The prevailing interest rate is 5% per year. Assuming costless production, is this species endangered?

17 A fishery has

$$G = g\frac{B}{(1 + B/K)} \tag{2.103}$$

$$H = hEB \tag{2.104}$$

with $g = .8$ and $K = 2$. If effort is held fixed at $E = 5$, what value of technology h will result in extinction?

18 Here is a fishery to be operated in steady state:

$$G = g(1 - B/K)B^2 \tag{2.105}$$

$$H = hEB \tag{2.106}$$

$$\pi = pH - cE \tag{2.107}$$

(Notice that the last term in G is squared; this is depensatory.) Parameters: $(g, h, K, p, c) = (2, 1, 1, 5, 3)$.

(a) In open-access mode, steady state, what are (B, E, H, π)?

(b) Keeping E fixed from (a): What is the maximum value of technology h beyond which biological extinction is guaranteed? (Hint: Consider H and G; this is a biological question, not an economic one.)

(c) Ignore (b): We wish to add a tax to the price of fish sold in order to achieve $B = 0.9$. (This money will be used to support public education.) What value of tax is needed, and how much total tax is to be collected?

(d) Ignore (b) and (c): The cost of effort will rise due to a new minimum wage law under study: c becomes $c + d$. What is the relation between jobs E and the extra wage d?

(e) What is the maximum value of d above which we will have economic extinction – that is, unprofitable fishing for all $0 < B < K$?

19 Explorers encountered a new fishery at high abundance a few years ago. After a brief period of costless mining, they are now operating it sustainably at $B = .35K$. Parameters: $p = 5$, $c = 0$, $r = .05$. It is thought that the growth function is

$$G = g\frac{B}{(1 + B/K)} \tag{2.108}$$

(Notice that B can be higher than K here.)

(a) What is the value of g?

(b) What change in interest rate would lead to extinction?

(c) A worldwide depression causes r to shrink to .03. What will happen to B? What change in sustainable harvest will occur? (Continue to assume that $c = 0$.)

20 Same as Problem 19, except that there is new information about the growth function G. It is now thought that

$$G = g\left(1 - \frac{B}{K}\right)B^2 \tag{2.109}$$

(Notice that the last term is squared.)

(a) What is the value of g?

(b) Is this fishery operating in a depensatory regime?

21 A fishery is described by

$$G = B(1 - B) \tag{2.110}$$

$$H = hB\sqrt{E} \tag{2.111}$$

$$\pi = 2H - 3E \tag{2.112}$$

(Notice the $\sqrt{E}$ term.) The parameter h represents harvesting technology.

(a) Find the MSY point and the values of H, E, B, π there.

(b) Find the the values of H, E, B, π for an open-access fishery.

(c) Find the maximum sustainable rent possible and the values of H, E, B there.

(d) Plot steady-state rent versus h for both open and closed-access cases.

22 (See Section 2.4.2.) A fishery is described by

$$G = gB\left(1 - \frac{B}{K}\right) \tag{2.113}$$

$$H = hEB \tag{2.114}$$

$$\pi = pH - cE \tag{2.115}$$

A pioneer has encountered this fishery at carrying capacity. She proposes a one-time large harvest, reducing the biomass to $B1$; investing the proceeds forever at interest rate r; and thereafter, harvesting sustainably under controlled-access conditions. (Note: The cost of harvesting is not zero.) Parameters: $[K, g, h] = [1, .06, 1]$; $[p, c, r] = [5, 3, .10]$.

- **(a)** Compute $B1$.
- **(b)** This fishery has been operating sustainably at $B1$ for several years. Suddenly there is a financial crisis, and r drops permanently to zero. Predict the effect on the fishery under controlled access. Use mathematical reasoning, Also explain the result graphically and in common sense terms.
- **(c)** Ignore (b). Is it possible that technological advances will justify extinction in the controlled-access case? If yes, under what conditions? Are they met here?

23 Consider the fishery as in Problem 4, with $H = hEB$ and $G = g(1 - B/K)$, unchanged; *but* with a minimum viable biomass $\underline{B}$ such that $G = 0$ for $B < \underline{B}$. (In other words, G crashes to zero if B falls below $\underline{B}$.) Assume the rent-maximixing (closed-access) solution from Problem 4 and that this fishery is in steady state.

- **(a)** Sketch the growth function $G(B)$. In what range of B is there depensation?
- **(b)** There is a food shortage; fish prices rise. What value of price will cause the extinction of this fishery?
- **(c)** There is no food shortage, but suddenly there is a stock market boom, and it is suggested that the fishery regulation board harvest some extra fish, sell them, and buy industrial stocks with the cash. The expected annual return on this investment is r. Compute the new optimal level $B1$. (Hint: First express steady-state rent π_s in terms of B; then use Equation 2.48 from the text.)
- **(d)** At what value of r will the fishery become extinct?