For the waveguide to support the TE_0 mode but not the TE_1 mode,

$$M_{TE} = 1 \quad \Rightarrow \quad 0 < \frac{V}{\pi} - \frac{1}{\pi}\tan^{-1}\sqrt{a_E} \le 1 \quad \Rightarrow \quad \tan^{-1}\sqrt{a_E} < V \le \pi + \tan^{-1}\sqrt{a_E}$$
$$\Rightarrow \quad 1.168 < V \le 4.310 \quad \Rightarrow \quad \frac{1.168}{2.884}\ \mu m < d \le \frac{4.310}{2.884}\ \mu m$$
$$\Rightarrow \quad 405\ nm < d \le 1.494\ \mu m.$$

For the waveguide to support the TM_0 mode but not the TM_1 mode,

$$M_{TM} = 1 \quad \Rightarrow \quad 0 < \frac{V}{\pi} - \frac{1}{\pi}\tan^{-1}\sqrt{a_M} \le 1 \quad \Rightarrow \quad \tan^{-1}\sqrt{a_M} < V \le \pi + \tan^{-1}\sqrt{a_M}$$
$$\Rightarrow \quad 1.393 < V \le 4.535 \quad \Rightarrow \quad \frac{1.393}{2.884}\ \mu m < d \le \frac{4.535}{2.884}\ \mu m$$
$$\Rightarrow \quad 483\ nm < d \le 1.572\ \mu m.$$

For the waveguide to support the TE_0 mode but not the TM_0 mode,

$$M_{TE} = 1 \text{ and } M_{TM} = 0 \quad \Rightarrow \tan^{-1}\sqrt{a_E} < V < \tan^{-1}\sqrt{a_M} \quad \Rightarrow \quad 405\ nm < d \le 483\ nm.$$

3.5.2 Symmetric Slab Waveguides

For a symmetric slab waveguide, $n_3 = n_2$, $a_E = a_M = 0$, and $\gamma_3 = \gamma_2$. Then, it can be seen from (3.136) and (3.141) that for both TE and TM modes, $\tan 2\psi = 0$ so that

$$\psi = \frac{m\pi}{2}, \qquad m = 0, 1, 2, \ldots. \tag{3.154}$$

Therefore, the mode field patterns of a symmetric waveguide given by (3.134) and (3.139) are either even functions of x, varying in space as $\cos h_1 x$ in the core region $-d/2 < x < d/2$, for even values of m, or odd functions of x, varying in space as $\sin h_1 x$ in the core region $-d/2 < x < d/2$, for odd values of m. This characteristic is expected because the mode field pattern in a symmetric structure is either symmetric or antisymmetric. Figure 3.21 shows the field patterns and the corresponding intensity distributions of the first few guided modes of a symmetric slab waveguide.

By using the identity $\tan 2\theta = 2\tan\theta/(1 - \tan^2\theta) = 2\cot\theta/(\cot^2\theta - 1)$ while equating γ_3 to γ_2, the eigenvalue equation in (3.135) for guided TE modes can be transformed to two equations:

$$\tan\frac{h_1 d}{2} = \frac{\gamma_2}{h_1}, \quad \text{for even modes;} \qquad -\cot\frac{h_1 d}{2} = \frac{\gamma_2}{h_1}, \quad \text{for odd modes.} \tag{3.155}$$

These two equations can be combined in one eigenvalue equation for all guided TE modes:

$$\tan\left(\frac{h_1 d}{2} - \frac{m\pi}{2}\right) = \frac{\gamma_2}{h_1} = \frac{\sqrt{V^2 - h_1^2 d^2}}{h_1 d}, \qquad m = 0, 1, 2, \ldots, \tag{3.156}$$