

# 4

## Single-interface lossless modes in $\epsilon_r'$ - $\mu_r'$ parameter space

### Tables

Item	Topic	Chapter 3	Chapter 4	Chapter 5	Chapter 6	Chapter 7
1	Interfaces	1	1	2	1	2
2	Types	ENG, DNG	DPS, ENG, DNG, MNG	DPS, ENG, DNG, MNG	ENG	ENG
3	$\epsilon_r, \mu_r$	complex	real	real	complex	complex
4	Dispersion	yes	no	no	no	no
5	Free $\beta$	yes	yes	yes	yes	yes
6	Loaded $\beta$	yes	no	no	yes	yes
7	Configuration	O, K	free	free	O, K	G
8	$\mathcal{R}$	yes	no	no	yes	yes
9	G-H	yes	no	no	no	no
10	$E$ and $H$	no	yes	yes	yes	yes
11	$s_z$	no	yes	yes	yes	yes
12	$s_x$	no	no	no	yes	yes
13	$\eta$	yes	no	no	yes	yes
14	$v_{ph}$ and $v_{group}$	yes	no	no	no	no
15	Charge density	no	no	no	yes	yes

Table 2.10. List of topics investigated in chapters 3-7.

Item	Topic	Chapter 4
1	Interfaces	1
2	Types	DPS, ENG, DNG, MNG
3	$\epsilon_r, \mu_r$	real
4	Dispersion	no
5	Free $\beta$	yes
6	Loaded $\beta$	no
7	Configuration	free
8	$\mathcal{R}$	no
9	G-H	no
10	$E$ and $H$	yes
11	$s_z$	yes
12	$s_x$	no
13	$\eta$	no
14	$v_{\text{ph}}$ and $v_{\text{group}}$	no
15	Charge density	no

Table 4.1. List of topics investigated in this chapter.

Medium	$\epsilon_i$	$\mu_i$	$n_i$	$r_i$	$\phi_i$ (deg)
Cover	$c$	2.25	2.25	2.25	3.18198
Substrate	$s$	3.0625	3.0625	3.0625	4.33103

Table 4.2. The parameters  $\epsilon_i$ ,  $\mu_i$  and  $n_i$ , and the magnitude of the vector,  $r_i$ , and its angle,  $\phi_i$ , for the DPS-type substrate and DPS-type cover. These parameters are used to generate all the media types discussed in this chapter.

Medium	DPS	ENG	DNG	MNG
Cover	$r_c, \phi_c$	$\pi - \phi_c$	$\pi + \phi_c$	$2\pi - \phi_c$
Substrate	$r_s, \phi_s$	$\pi - \phi_s$	$\pi + \phi_s$	$2\pi - \phi_s$

Table 4.3. The sets  $(r_s, \phi_s)$  and  $(r_c, \phi_c)$  used to generate all the media types discussed in this chapter.

$\beta_{\text{TE}}$	$\text{DPS}_{\text{sub.}}$	$\text{ENG}_{\text{sub.}}$	$\text{DNG}_{\text{sub.}}$	$\text{MNG}_{\text{sub.}}$
$\text{DPS}_{\text{cover}}$	0	0	0	4.69043
$\text{ENG}_{\text{cover}}$	0	0	4.69043	0.
$\text{DNG}_{\text{cover}}$	0	4.69043	0	0
$\text{MNG}_{\text{cover}}$	4.69043	0	0	0

Table 4.4. The solutions of  $\beta_{\text{TE}}$  for the 16 media pairs. The valid solutions are those whose magnitude is larger than the refractive indices given in Table 4.2.

Types	DPS/MNG	DNG/ENG
$\delta_{TE}$	4.11553	4.11553
$\gamma_{TE}$	5.6017	5.6017

Table 4.5. The decay constants,  $\delta_{TE}$  and  $\gamma_{TE}$ , for the DPS/MNG-type and DNG/ENG-type structures.

$\beta_{TM}$	DPS <sub>sub.</sub>	ENG <sub>sub.</sub>	DNG <sub>sub.</sub>	MNG <sub>sub.</sub>
DPS <sub>cover</sub>	0	4.69043	0	0
ENG <sub>cover</sub>	4.69043	0	0	0
DNG <sub>cover</sub>	0.	0	0	4.69043
MNG <sub>cover</sub>	0	0	4.69043	0

Table 4.6. The solutions of  $\beta_{TM}$  for the 16 media pairs. The valid solutions, whose magnitude is larger than the refractive indices presented in Table 4.2.

Types	DPS/ENG	DNG/MNG
$\delta_{TM}$	4.11553	4.11553
$\gamma_{TM}$	5.6017	5.6017

Table 4.7. The decay constants,  $\delta_{TM}$  and  $\gamma_{TM}$ , for the DPS/ENG-type and DNG/MNG-type structures.

Mode	Types	a	b
TE	DPS/MNG	201 534.	
TE	DNG/ENG	201 534.	
TM	DPS/ENG		534.956
TM	DNG/MNG		534.956

Table 4.8. The normalization constants of the fields for the TE mode,  $a$ , using Eq. (2.189) and for the TM mode,  $b$ , using Eq. (2.195).

## Figures

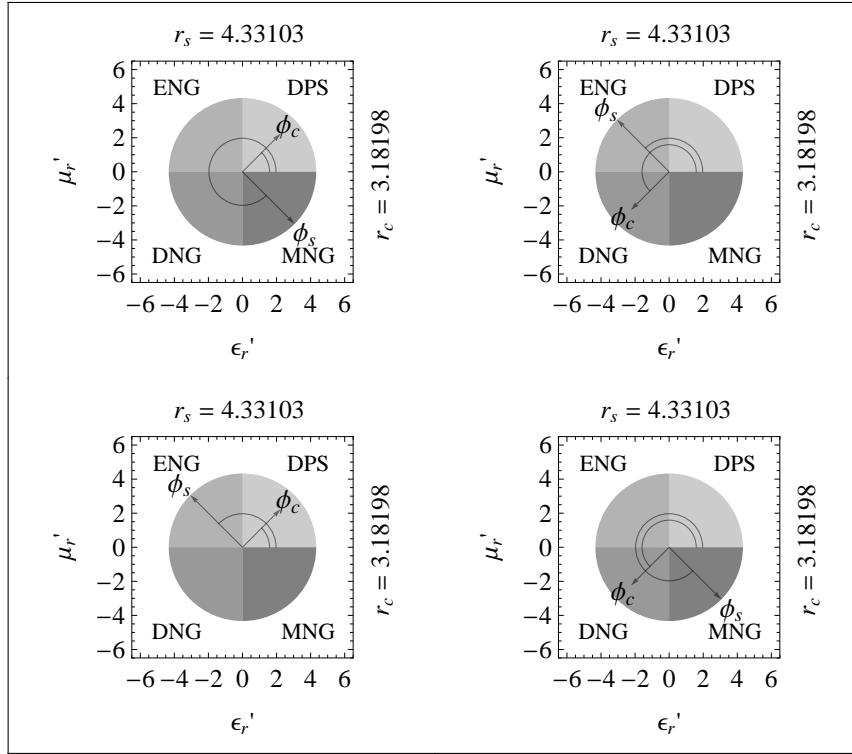


Fig. 4.3. The four media-type pairs that support a single-interface mode, presented in an  $\epsilon_r'$ - $\mu_r'$  parameter space. These pairs consist of the following structures: (top left) DPS/MNG-type, (top right) DNG/ENG-type, (bottom left) DPS/ENG-type and (bottom right) DNG/MNG-type.

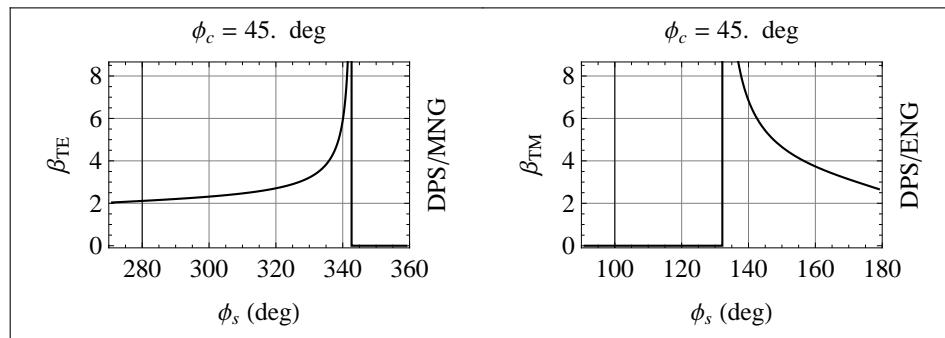


Fig. 4.4. The solutions of single-interface (left) TE and (right) TM modes,  $\beta_{TE}$  and  $\beta_{TM}$ , respectively, in terms of the angle  $\phi_s$ . Here, the cover is DPS-type while the substrate, characterized by  $\phi_s$ , is (left) MNG-type and (right) ENG-type.

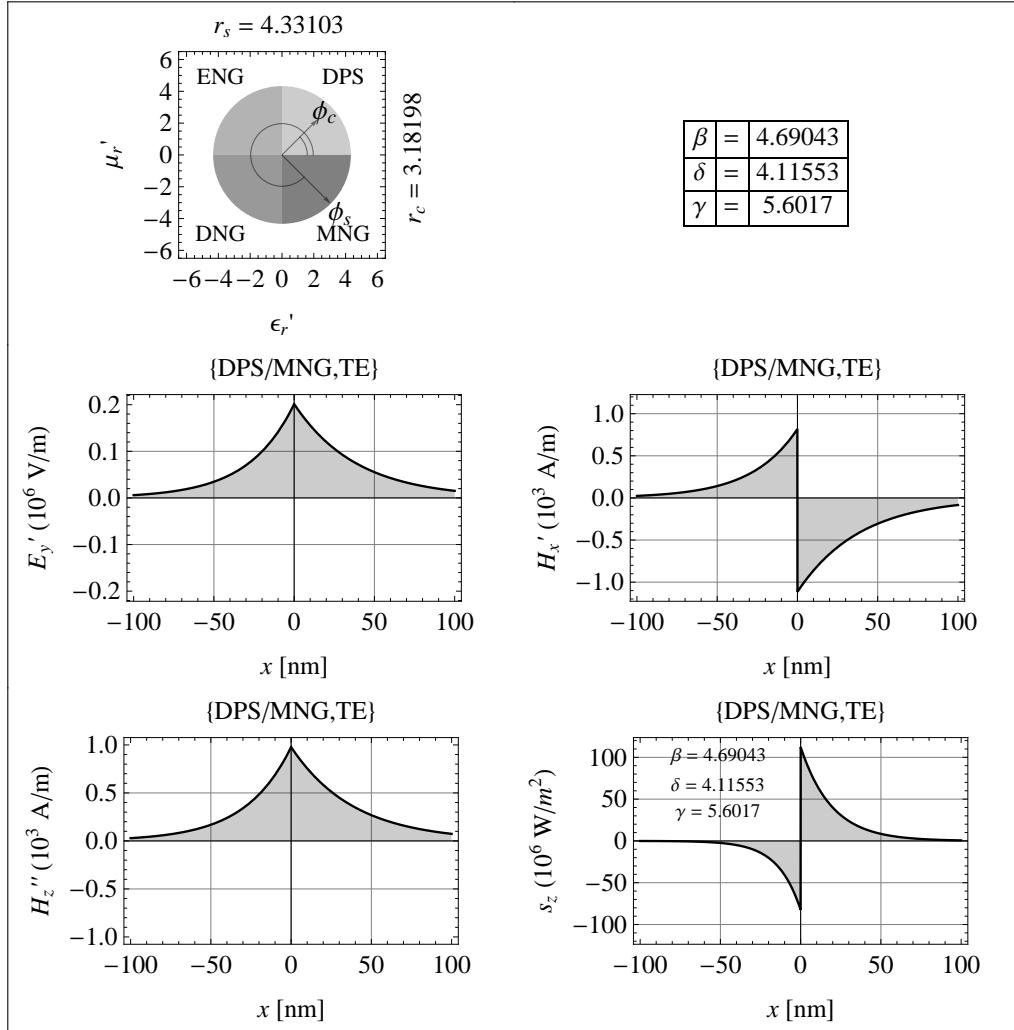


Fig. 4.6. The complete solution of the TE mode belonging to the DPS/MNG-type structure: (top left) the presentation of the structure in an  $\epsilon_r'$ - $\mu_r'$  parameter space, (top right) the propagation constant,  $\beta$ , and the decay constants in the cover and substrate,  $\delta$  and  $\gamma$ , (center left)  $E_y'$ , (center right)  $H_x'$ , (bottom left),  $H_z''$ , and (bottom right)  $s_z$ . Note that for the substrate  $x < 0$  and for the cover  $x > 0$ .

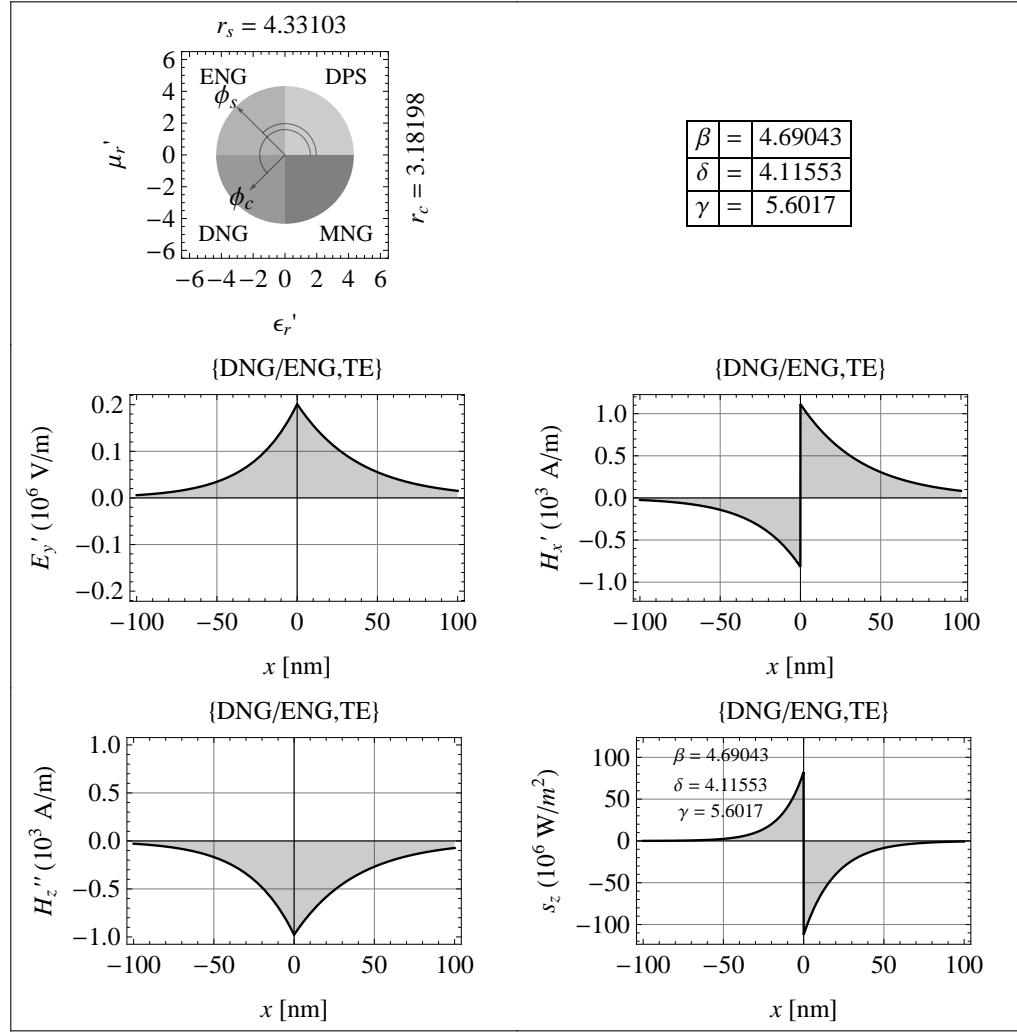


Fig. 4.7. The complete solution of the TE mode belonging to the DNG/ENG-type structure: (top left) the presentation of the structure in an  $\epsilon_r'$ - $\mu_r'$  parameter space, (top right) the propagation constant,  $\beta$ , and the decay constants in the cover and substrate,  $\delta$  and  $\gamma$ , (center left)  $E_y'$ , (center right)  $H_x'$ , (bottom left),  $H_z''$ , and (bottom right)  $s_z$ .

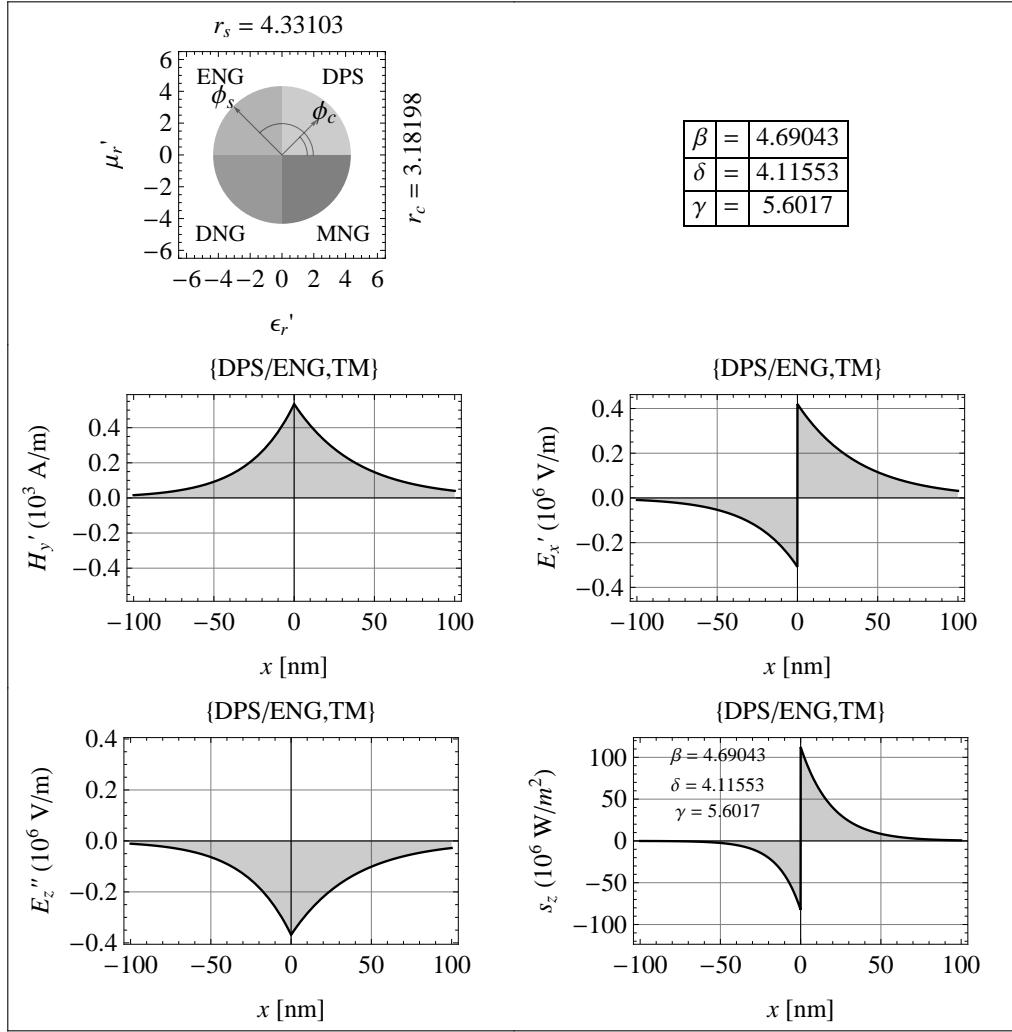


Fig. 4.8. The complete solution of the TM mode belonging to the DPS/ENG-type structure: (top left) the presentation of the structure in an  $\epsilon_r'$ - $\mu_r'$  parameter space, (top right) the propagation constant,  $\beta$ , and the decay constants in the cover and substrate,  $\delta$  and  $\gamma$ , (center left)  $H_y'$ , (center right)  $E_x'$ , (bottom left)  $E_z''$ , and (bottom right)  $s_z$ .

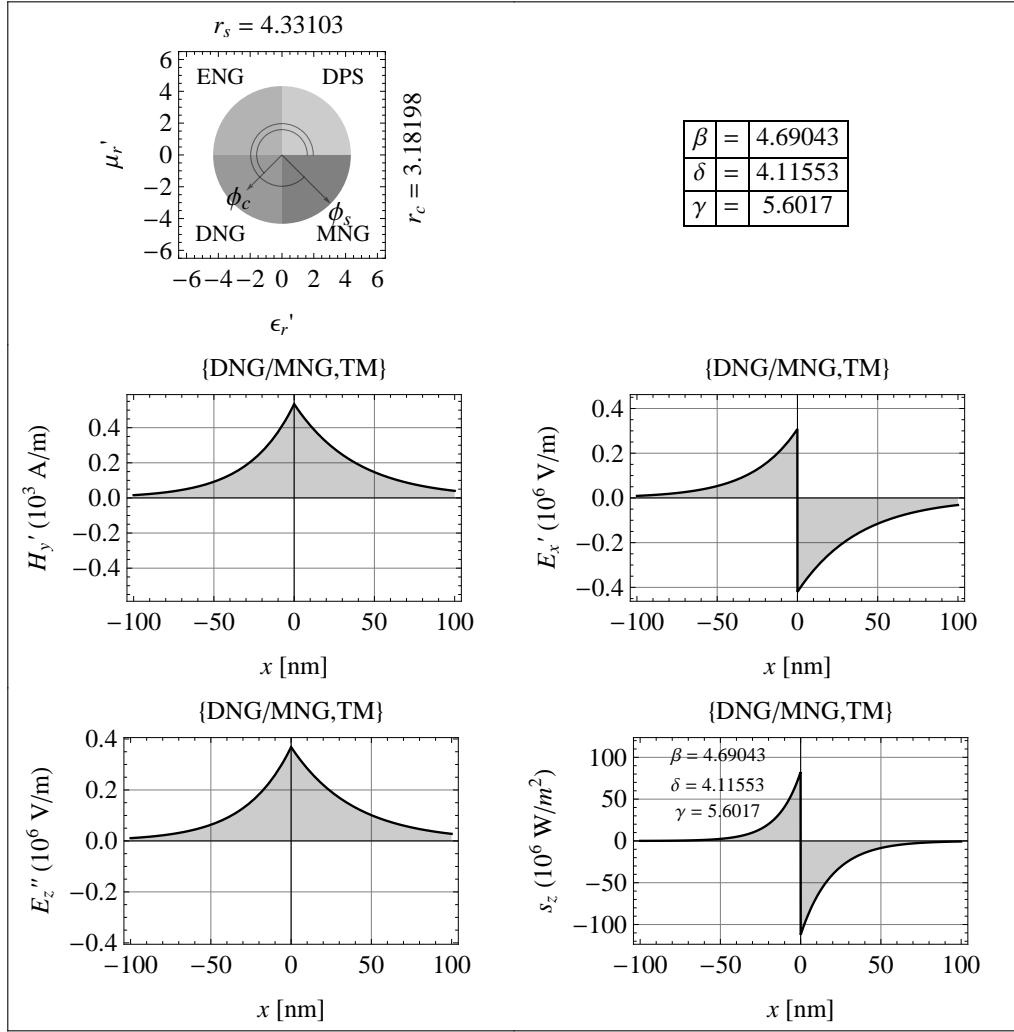


Fig. 4.9. The complete solution of the TM mode belonging to the DNG/MNG-type structure: (top left) the presentation of the structure in an  $\epsilon_r'$ - $\mu_r'$  parameter space, (top right) the propagation constant,  $\beta$ , and the decay constants in the cover and substrate,  $\delta$  and  $\gamma$ , (center left)  $H_y'$ , (center right)  $E_x'$ , (bottom left)  $E_z''$ , and (bottom right)  $s_z$ .

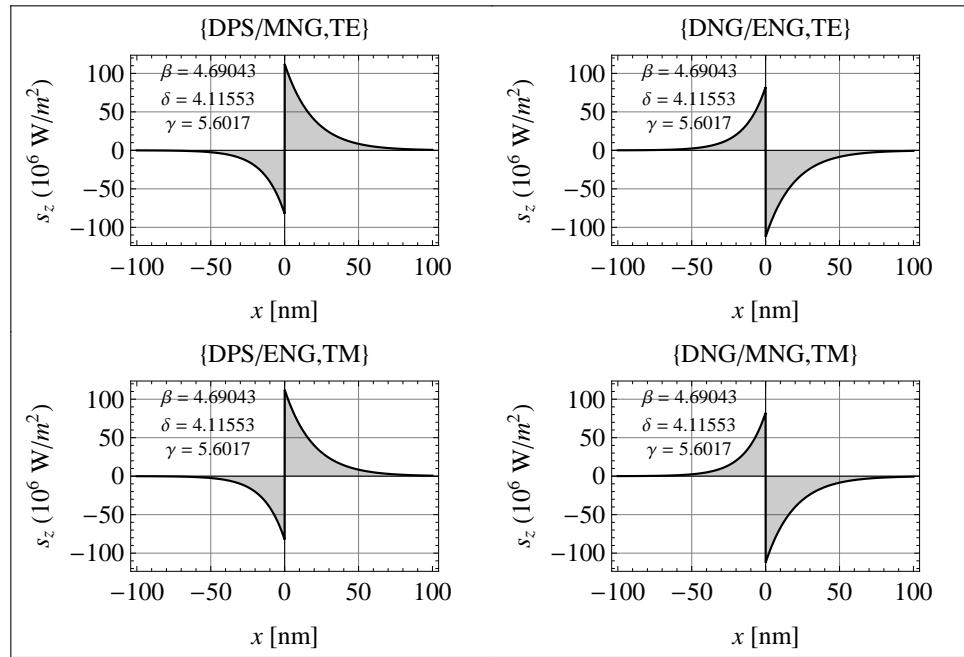


Fig. 4.10. Comparison of the local power flow,  $s_z$ , in the substrate ( $x < 0$ ) and cover ( $x > 0$ ) for the (top left) DPS/MNG-type, (top right) DNG/ENG-type, (bottom left) DPS/ENG-type and (bottom right) DNG/MNG-type. Here, the two figures at the top and at the bottom belong to the TE and TM modes, respectively. Note that in each case  $s_z$  in the cover and substrate point in the opposite direction.

## Exercises

- (1) Table 4.1 includes only a partial list of the topics included in Table 2.2. Try addressing all the rest of the topics using the model metamaterial of this chapter.
- (2) Discuss the meaning of electric and magnetic charge density waves in the DPS/MNG-, DNG/ENG-, DPS/ENG and DNG/MNG-type structures in the context of a metamaterial.

## References

- [1] I. V. Shadrivov, A. A. Sukhorukov and Y. S. Kivshar. Guided modes in negative-refractive-index waveguides. *Phys. Rev. E* 67 (2003) 057602.
- [2] M. J. Adams. *An Introduction to Optical Waveguides* (Wiley Interscience, 1981).