Cooperative Communications and Networking

Chapter 5

Cooperative Communications with Single Relay

Outline

- System Model
- SER Analysis for DF Cooperative Communications for M-PSK and M-QAM Systems
  1. Closed-Form SER Analysis
  2. SER Upper Bound and Asymptotically Tight Approximation
  3. Optimum Power Allocation
  4. Some Special Scenarios
- SER Analysis for AF Cooperative Communications
  1. SER Analysis by MGF Approach
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  4. Optimum Power Allocation
- Comparison of DF and AF Protocols
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System Model (1)

- Cooperation strategy over two phases:
  - Phase 1: Each user sends signals to destination – also received by other users
  - Phase 2: Each user helps others by forwarding signals it received in Phase 1
- All users transmit signals through orthogonal channels by using TDMA, FDMA or CDMA
- Consider a two-user cooperation scheme

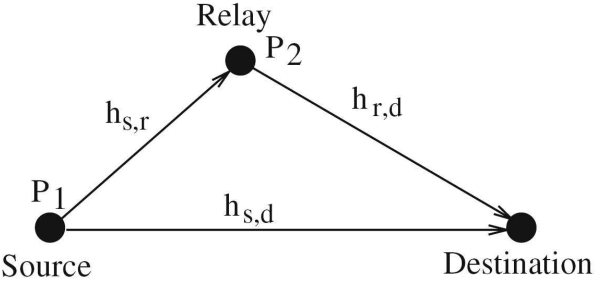


Figure: Simplified cooperation model

System Model (2)

- Phase 1: Received signal $y_{s,d}$ and $y_{s,r}$ at the destination and the relay respectively,

$$\begin{aligned} y_{s,d} &= \sqrt{P_1} h_{s,d} x + \eta_{s,d} \\ y_{s,r} &= \sqrt{P_1} h_{s,r} x + \eta_{s,r} \end{aligned} \tag{1}$$

- Phase 2
  - DF Protocol: If relay decodes correctly, it forwards decoded symbol with power $\tilde{P}_2 = P_2$; otherwise relay remains idle (i.e. $\tilde{P}_2 = 0$)

$$y_{r,d} = \sqrt{\tilde{P}_2} h_{r,d} x + \eta_{r,d} \tag{2}$$

  - **Assumption:** Relay is able to detect whether transmitted symbol is decoded correctly or not *(selective-relaying)*

System Model (3)

- Phase 2
  - AF Protocol: Relay amplifies received signal and forwards it to destination with transmitted power P_2

$$y_{r,d} = \frac{\sqrt{P_2}}{\sqrt{P_1|h_{s,r}|^2 + \mathcal{N}_o}} h_{r,d} y_{s,r} + \eta_{r,d} \tag{3}$$

  - Received signal at the destination can be written as

$$y_{r,d} = \frac{\sqrt{P_1 P_2}}{\sqrt{P_1|h_{s,r}|^2 + \mathcal{N}_o}} h_{r,d} h_{s,r} x + \acute{\eta}_{r,d} \tag{4}$$

  - $\acute{\eta}_{r,d} = \frac{\sqrt{P_2}}{\sqrt{P_1|h_{s,r}^2 + \mathcal{N}_0}} h_{r,d}\eta_{s,r} + \eta_{r,d}$ with variance $\left(\frac{P_2|h_{r,d}|^2}{P_1|h_{s,r}|^2+\mathcal{N}_0} + 1\right)\mathcal{N}_0$
  - Assumptions:
    1. Channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are assumed independent
    2. Channel coefficients are assumed to be known at the receiver
    3. Destination utilizes Maximum Ratio Combining (MRC)
    4. Total transmitted power $P_1 + P_2 = P$

SER Analysis for DF Cooperative Communications

- Closed-form SER analysis for M-PSK and M-QAM systems
  - Combined signal at MRC detector $y = a_1 y_{s,d} + a_2 y_{r,d}$ such that the instantaneous SNR of MRC output is maximized:

$$\gamma = \frac{P_1 |h_{s,d}|^2 + \tilde{P}_2 |h_{r,d}|^2}{\mathcal{N}_0} \tag{5}$$

  - Conditional SER for M-PSK modulation

$$P_{PSK}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{PSK}(\gamma) \triangleq \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}\gamma}{sin^2\theta}\right) d\theta \tag{6}$$

  - Conditional SER for M-QAM modulation

$$P_{QAM}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{QAM}(\gamma) \triangleq 4KQ\left(\sqrt{b_{QAM}\gamma}\right) - 4K^2 Q\left(\sqrt{b_{QAM}\gamma}\right) \tag{7}$$

  - where $b_{PSK} = sin^2(\pi/M)$, $K = 1 - \frac{1}{\sqrt{M}}$ and $b_{QAM} = 3/(M-1)$

SER Analysis for DF Cooperative Communications

- SER Analysis for M-PSK modulation

$$P_{PSK}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{PSK}(\gamma)|_{\tilde{P}_2=0}\Psi_{PSK}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)+\Psi_{PSK}(\gamma)|_{\tilde{P}_2=P_2}\left[1-\Psi_{PSK}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)\right] \quad (8)$$

- After averaging the conditional SER over Rayleigh fading channels $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$

$$\begin{aligned} P_{PSK} &= F_1\left(1+\frac{b_{PSK}P_1\delta_{s,d}^2}{\mathcal{N}_0 sin^2\theta}\right)F_1\left(1+\frac{b_{PSK}P_1\delta_{s,r}^2}{\mathcal{N}_0 sin^2\theta}\right) \\ &+ F_1\left(\left(1+\frac{b_{PSK}P_1\delta_{s,d}^2}{\mathcal{N}_0 sin^2\theta}\right)\left(1+\frac{b_{PSK}P_2\delta_{s,r}^2}{\mathcal{N}_0 sin^2\theta}\right)\right)\left[1-F_1\left(1+\frac{b_{PSK}P_1\delta_{s,r}^2}{\mathcal{N}_0 sin^2\theta}\right)\right] \end{aligned} \quad (9)$$

- where $F_1(x(\theta)) = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\frac{1}{x(\theta)}d\theta$

SER Analysis for DF Cooperative Communications

- SER Analysis for M-QAM modulation

$$P_{QAM}^{h_{s,d},h_{s,r},h_{r,d}} = \Psi_{QAM}(\gamma)|_{\tilde{P}_2=0}\Psi_{QAM}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)+\Psi_{QAM}(\gamma)|_{\tilde{P}_2=P_2}\left[1-\Psi_{QAM}\left(\frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}\right)\right] \tag{10}$$

- After averaging the conditional SER over Rayleigh fading channels $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$

$$\begin{aligned} P_{QAM} = & F_2\left(1+\frac{b_{QAM}P_1\delta_{s,d}^2}{2\mathcal{N}_0 sin^2\theta}\right)F_2\left(1+\frac{b_{QAM}P_1\delta_{s,r}^2}{2\mathcal{N}_0 sin^2\theta}\right) \\ & + F_2\left(\left(1+\frac{b_{QAM}P_1\delta_{s,d}^2}{2\mathcal{N}_0 sin^2\theta}\right)\left(1+\frac{b_{QAM}P_2\delta_{s,r}^2}{2\mathcal{N}_0 sin^2\theta}\right)\right)\left[1-F_2\left(1+\frac{b_{QAM}P_1\delta_{s,r}^2}{\mathcal{N}_0 sin^2\theta}\right)\right] \end{aligned} \tag{11}$$

- where $F_2(x(\theta)) = \frac{4K}{\pi}\int_0^{\pi/2}\frac{1}{x(\theta)}d\theta + \frac{4K^2}{\pi}\int_0^{\pi/4}\frac{1}{x(\theta)}d\theta$

SER Analysis for DF Cooperative Communications

- SER Upper Bound and Asymptotically Tight Approximation
- **Theorem 1:** SER of DF cooperation systems with M-PSK or M-QAM modulation can be upper-bounded as

$$P_s \leq \frac{(M-1)\mathcal{N}_0^2}{M^2} \cdot \frac{MbP_1\delta_{s,r}^2 + (M-1)bP_2\delta_{r,d}^2 + (2M-1)\mathcal{N}_0}{(\mathcal{N}_0 + bP_1\delta_{s,d}^2)(\mathcal{N}_0 + bP_1\delta_{s,r}^2)(\mathcal{N}_0 + bP_2\delta_{r,d}^2)} \quad (12)$$

- If all channel links $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are available (i.e. $\delta_{s,d}^2 \neq 0$, $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$) for sufficiently high SNR, the SER can be tightly approximated as

$$P_s \approx \frac{\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1\delta_{s,d}^2}\left(\frac{A^2}{P_1\delta_{s,r}^2} + \frac{B}{P_2\delta_{r,d}^2}\right) \quad (13)$$

- For M-PSK signals, $A = \frac{M-1}{2M} + \frac{sin\frac{2\pi}{M}}{4\pi}$ and $B = \frac{3(M-1)}{8M} + \frac{sin\frac{2\pi}{M}}{4\pi} - \frac{sin\frac{4\pi}{M}}{32\pi}$
- For M-QAM signals, $A = \frac{M-1}{2M} + \frac{K^2}{\pi}$ and $B = \frac{3(M-1)}{8M} + \frac{k^2}{\pi} - \frac{sin\frac{4\pi}{M}}{32\pi}$
- $b = b_{PSK}$ for M-PSK signals and $b = b_{QAM}/2$ for M-QAM signals

SER Analysis for DF Cooperative Communications

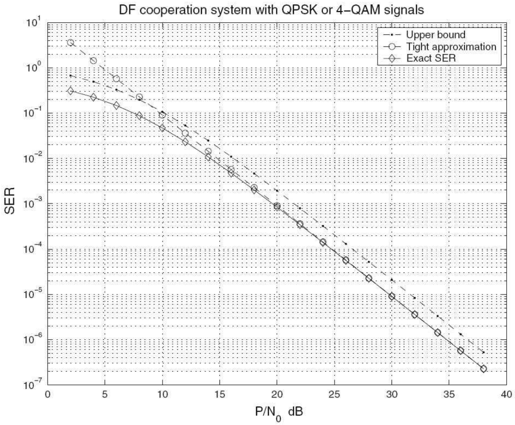


Figure: Comparison of exact SER, the upper bound and the asymptotically tight approximation with QPSK or 4-QAM signals ($\delta^2_{s,d} = \delta^2_{s,r} = \delta^2_{r,d} = 1$), $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P/2$.

SER Analysis for DF Cooperative Communications

- Optimum Power Allocation
  - Based on the SER asymptotic approximation at high SNR
  - Optimize SER performance with constraint $P = P_1 + P_2$

$$G(P_1, P_2) = \frac{1}{P_1 \delta_{s,d}^2} \left(\frac{A^2}{P_1 \delta_{s,r}^2} + \frac{B}{P_2 \delta_{r,d}^2} \right) \quad (14)$$

  - **Theorem 2:** In DF cooperation systems with M-PSK or M-QAM modulation, if all channel links are available (i.e. $\delta_{s,d}^2 \neq 0$, $\delta_{s,r}^2 \neq 0$ and $\delta_{r,d}^2 \neq 0$), then for sufficiently high SNR, the optimum power allocation is

$$\begin{aligned} P_1 &= \frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}} P, \\ P_2 &= \frac{2\delta_{s,r}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}} P \end{aligned} \quad (15)$$

  - Asymptotic optimum power allocation does not depend on channel link between source and destination
  - Depends on channel link between source and relay and between relay and destination

SER Analysis for DF Cooperative Communications

- Comments on asymptotic power allocation:
  - Notice that:

$$\frac{1}{2} < \frac{P_1}{P} < 1 \quad \textit{and} \quad 0 < \frac{P_2}{P} < \frac{1}{2} \tag{16}$$

  - More power should be assigned to the source and less power to the relay
  - If $\delta_{s,r}^2 << \delta_{r,d}^2$, then $P_1 \to P$ and $P_2 \to 0$
  - If $\delta_{s,r}^2 >> \delta_{r,d}^2$, then both P_1 and P_2 approach $P/2$
  - If $\delta_{s,r}^2 = \delta_{r,d}^2$, then

$$\begin{aligned} P_1 &= \frac{1+\sqrt{1+8(A^2/B)}}{3+\sqrt{1+8(A^2/B)}} P, \\ P_2 &= \frac{2}{3+\sqrt{1+8(A^2/B)}} P \end{aligned} \tag{17}$$

  - Examples:
    1. BPSK: $P_1 = 0.5931P$ and $P_2 = 0.4069P$
    2. QPSK: $P_1 = 0.6270P$ and $P_2 = 0.3730P$
    3. 16-QAM: $P_1 = 0.6495P$ and $P_2 = 0.3505P$
  - Larger constellation size, more power at the source

SER Analysis for DF Cooperative Communications

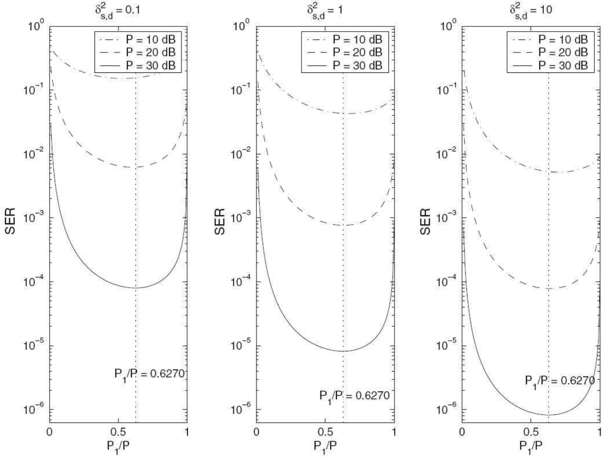


Figure: QPSK SER of DF cooperation systems with $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$: (a) $\delta^2_{s,d}$; (b) $\delta^2_{s,d} = 1$; and (c) $\delta^2_{s,d} = 10$

SER Analysis for DF Cooperative Communications

- Some Special Scenarios
  1. Case 1: If the channel link between relay and destination is not available (i.e. $\delta_{r,d} = 0$), the optimum power allocation is $P_1 = P$ and $P_2 = 0$ (that is, use direct transmission from source to destination)
  2. Case 2: If the channel link between source and relay is not available (i.e. $\delta_{s,r} = 0$), the optimum power allocation is also $P_1 = P$ and $P_2 = 0$
  3. Case 3: If the channel link between source and destination is not available (i.e. $\delta_{s,d} = 0$), then:

$$\begin{aligned} P_1 &= \tfrac{\delta_{r,d}}{\delta_{s,r}+\delta_{r,d}} P, \\ P_2 &= \tfrac{\delta_{s,r}}{\delta_{s,r}+\delta_{r,d}} P \end{aligned} \tag{18}$$

     In this case, the system reduces to a two-hop communication scenario

SER Analysis for AF Cooperative Communications

- SER Analysis by MGF Approach
  - Relay not only amplifies received signal but also noise
  - Noise at $\grave{\eta}_{r,d}$ destination in Phase 2 is zero-mean complex Gaussian random variable with variance $\left(\frac{P_2|h_{r,d}|^2}{P_1|h_{s,r}|^2+\mathcal{N}_0}+1\right)\mathcal{N}_0$
  - With knowledge of channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$, the output of MRC detector is $y = a_1 y_{s,d} + a_2 y_{r,d}$, where

$$a_1 = \frac{\sqrt{P_1}h_{s,d}^*}{\mathcal{N}_0} \quad and \quad a_2 = \frac{\sqrt{\frac{P_1 P_2}{P_1|h_{s,r}|^2}+\mathcal{N}_0}\,h_{s,r}^* h_{r,d}^*}{\left(\frac{P_2|h_{r,d}|^2}{P_1|h_{s,r}|^2+\mathcal{N}_0}+1\right)\mathcal{N}_0} \tag{19}$$

  - Instantaneous SNR at MRC output is $\gamma = \gamma_1 + \gamma_2$, where $\gamma_1 = P_1|h_{s,d}|^2/\mathcal{N}_0$ and

$$\gamma_2 = \frac{1}{\mathcal{N}_0}\frac{P_1 P_2|h_{s,r}|^2|h_{r,d}|^2}{P_1|h_{s,r}|^2 + P_2|h_{r,d}|^2 + \mathcal{N}_0} \tag{20}$$

SER Analysis for AF Cooperative Communications

- SER Analysis by MGF Approach
  - The instantaneous SNR γ_2 can be tightly upper bounded

$$\tilde{\gamma}_2 = \frac{1}{\mathcal{N}_0} \frac{P_1 P_2 |h_{s,r}|^2 |h_{r,d}|^2}{P_1 |h_{s,r}|^2 + P_2 |h_{r,d}|^2} \tag{21}$$

  - Equation (21) represents the harmonic mean of two exponential random variables $X_1 = P_1 |h_{s,r}|^2 / \mathcal{N}_0$ and $X_2 = P_2 |h_{r,d}|^2 / \mathcal{N}_0$
  - Conditional SER for M-PSK modulation

$$P_{PSK}^{h_{s,d}, h_{s,r}, h_{r,d}} \approx \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}(\gamma_1 + \tilde{\gamma}_2)}{sin^2\theta} \right) d\theta \tag{22}$$

  - Conditional SER for M-QAM modulation

$$P_{QAM}^{h_{s,d}, h_{s,r}, h_{r,d}} \approx 4KQ\left(\sqrt{b_{QAM}(\gamma_1 + \tilde{\gamma}_2)} \right) - 4K^2 Q\left(\sqrt{b_{QAM}(\gamma_1 + \tilde{\gamma}_2)} \right) \tag{23}$$

  - where $b_{PSK} = sin^2(\pi/M)$, $K = 1 - \frac{1}{\sqrt{M}}$ and $b_{QAM} = 3/(M-1)$

SER Analysis for AF Cooperative Communications

- SER Analysis by MGF Approach
  - Let the MGF of a random variable Z be defined as

$$\mathcal{M}_Z(s) = \int_{\infty}^{\infty} \exp(-sz) p_Z(z) dz \tag{24}$$

  - By averaging over the Rayleigh fading channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$

$$P_{PSK} \approx \frac{1}{\pi} \int_0^{(M-1)\pi/M} \mathcal{M}_{\gamma_1}\left(\frac{b_{PSK}}{sin^2\theta}\right) \mathcal{M}_{\tilde{\gamma}_2}\left(\frac{b_{PSK}}{sin^2\theta}\right) d\theta \tag{25}$$

$$P_{QAM} \approx \left[\frac{4K}{\pi} \int_0^{\pi/2} - \frac{4K^2}{\pi} \int_0^{\pi/4}\right] \mathcal{M}_{\gamma_1}\left(\frac{b_{QAM}}{2sin^2\theta}\right) \mathcal{M}_{\tilde{\gamma}_2}\left(\frac{b_{QAM}}{2sin^2\theta}\right) \tag{26}$$

  - Since $\gamma_1 = \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0}$ has an exponential distribution with parameter $\frac{\mathcal{N}_0}{P_1\delta_{s,d}^2}$, hence $\mathcal{M}_{\gamma_1}(s) = \frac{1}{1+\frac{sP_1\delta_{s,d}^2}{\mathcal{N}_0}}$

SER Analysis for AF Cooperative Communications

- SER Analysis by MGF Approach
  - The MGF of γ_2 is given by

$$\mathcal{M}_{\tilde{\gamma}_2}(s) = \frac{16\beta_1\beta_2}{3(\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s)^2}\times \left[\frac{4(\beta_1+\beta_2)}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s} + {}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right) {}_2F_1\left(2,\frac{1}{2};\frac{5}{2};\frac{\beta_1+\beta_2-2\sqrt{\beta_1\beta_2}+s}{\beta_1+\beta_2+2\sqrt{\beta_1\beta_2}+s}\right)\right], \quad (27)$$

  where $\beta_1 = \mathcal{N}_0/(P_1\delta_{s,r}^2)$, $\beta_2 = \mathcal{N}_0/(P_2\delta_{r,d}^2)$, and ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function

- Simple MGF Expression for the Harmonic Mean
- **Theorem 3:** Suppose that X_1 and X_2 are two independent random variables with pdf $p_{X_1}(x)$ and $p_{X_2}(x)$ defined for all $x \geq 0$. Then the pdf of $Z = \frac{X_1X_2}{X_1+X_2}$ is given by

$$p_Z(z) = \left(z\int_0^1 \frac{1}{t^2(1-t)^2}p_{X_1}\left(\frac{z}{1-t}\right)p_{X_2}\left(\frac{z}{t}\right)\right)\cdot U(z) \qquad (28)$$

in which $U(z) = 1$ for $z \geq 0$ and $U(z) = 0$ for $z < 0$

SER Analysis for AF Cooperative Communications

- Simple MGF Expression for the Harmonic Mean
  - **Theorem 4:** Let X_1 and X_2 be two independent exponential random variables with parameters β_1 and β_2, respectively. Then, the MGF of $Z = \frac{X_1 X_2}{X_1+X_2}$ is given by

$$\mathcal{M}_Z(s) = \frac{(\beta_1-\beta_2)^2+(\beta_1+\beta_2)s}{\Delta^2} + \frac{2\beta_1\beta_2 s}{\Delta^3}\ln\frac{(\beta_1+\beta_2+s+\Delta)^2}{4\beta_1\beta_2} \quad (29)$$

    for any $s > 0$, in which $\Delta = \sqrt{(\beta_1-\beta_2)^2+2(\beta_1+\beta_2)s+s^2}$. Furthermore, if β_1 and β_2 go to zero, then the MGF of Z can be approximated as $\mathcal{M}(s) \approx \frac{\beta_1+\beta_2}{s}$

- Closed-Form SER Expressions and Asymptotically Tight Approximations
  - SER formulation for M-PSK signals can be approximated as

$$P_{PSK} \approx \frac{B}{b_{PSK}^2}\beta_0(\beta_1+\beta_2), \quad (30)$$

    where $B = \frac{1}{\pi}\int_0^{(M-1)\pi/M} sin^4\theta d\theta = \frac{3(M-1)}{8M} + \frac{sin\frac{2\pi}{M}}{4\pi} - \frac{sin\frac{4\pi}{M}}{32\pi}$

SER Analysis for AF Cooperative Communications

- Closed-Form SER Expressions and Asymptotically Tight Approximations
  - SER formulation for M-QAM signals can be approximated as

$$P_{QAM} \approx \frac{4B}{b_{QAM}^2}\beta_0(\beta_1 + \beta_2), \tag{31}$$

where $B = \left[\frac{4K}{\pi}\int_0^{\pi/2} - \frac{4K^2}{\pi}\int_0^{\pi/4}\right] sin^4\theta d\theta = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}$

  - **Theorem 5:** At sufficiently high SNR, the SNR of the AF cooperation systems with M-PSK or M-QAM modulation can be approximated as

$$P_s \approx \frac{B\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1\delta_{s,d}^2}\left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right) \tag{32}$$

  - For M-PSK signals, $b = b_{PSK}$ and $B = \frac{3(M-1)}{8M} + \frac{sin\frac{2\pi}{M}}{4\pi} - \frac{sin\frac{4\pi}{M}}{32\pi}$
  - For M-QAM signals, $b = b_{QAM}/2$ and $B = \frac{3(M-1)}{8M} + \frac{K^2}{\pi}$

SER Analysis for AF Cooperative Communications

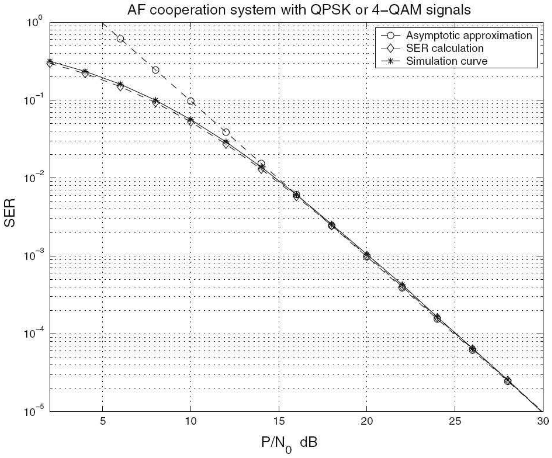


Figure: Comparison of the SER approximations for AF cooperation system with QPSK or 8-QAM - $\delta_{s,d}^2 = \delta_{s,r}^2 = \delta_{r,d}^2 = 1$, $\mathcal{N}_0 = 1$, and $P_1/P = 2/3$ and $P_2/P = 1/3$

SER Analysis for AF Cooperative Communications

- Optimum Power Allocation
  - For fixed total power $P = P_1 + P_2$, minimize

$$G(P_1, P_2) = \frac{1}{P_1 \delta_{s,d}^2} \left(\frac{1}{P_1 \delta_{s,r}^2} + \frac{1}{P_2 \delta_{r,d}^2} \right) \quad (33)$$

  - **Theorem 6:** For sufficiently high SNR, the optimum power allocation for the AF cooperation systems with either M-PSK or M-QAM modulations is given by

$$P_1 = \frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}} P \quad \textit{and} \quad P_2 = \frac{2\delta_{s,r}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}} P \quad (34)$$

  - Optimum power allocation for AF cooperation systems is not modulation-dependent (due to the fact that relay amplifies-and-forwards received signals despite its modulation)
  - As in DF cooperation systems, optimum power allocation does not depend on the channel link between source and destination

SER Analysis for DF and AF Protocols

- SER performance of DF systems can be approximated as

$$P_s \approx \frac{\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1 \delta_{s,d}^2} \left(\frac{A^2}{P_1 \delta_{s,r}^2} + \frac{B}{P_2 \delta_{r,d}^2} \right) \Rightarrow P_s \approx \Delta_{DF}^{-2} \left(\frac{P}{\mathcal{N}_0} \right)^{-2}, \quad (35)$$

where $\Delta_{DF} = \frac{2\sqrt{2} b \delta_{s,d} \delta_{s,r} \delta_{r,d}}{\sqrt{B}} \frac{\left(\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2} \right)^{1/2}}{\left(3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/N)\delta_{r,d}^2} \right)^{3/2}}$

- SER performance for AF systems can be approximated as

$$P_s \approx \frac{B\mathcal{N}_0^2}{b^2} \cdot \frac{1}{P_1 \delta_{s,d}^2} \left(\frac{1}{P_1 \delta_{s,r}^2} + \frac{1}{P_2 \delta_{r,d}^2} \right) \Rightarrow P_s \approx \Delta_{AF}^{-2} \left(\frac{P}{\mathcal{N}_0} \right)^{-2}, \quad (36)$$

where $\Delta_{AF} = \frac{2\sqrt{2} b \delta_{s,d} \delta_{s,r} \delta_{r,d}}{\sqrt{B}} \frac{\left(\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2} \right)^{1/2}}{\left(3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2} \right)^{3/2}}$

Comparison DF and AF Protocols

- Define cooperation gain ratio $\lambda = \Delta_{DF}/\Delta_{AF}$ which is given by

$$\lambda = \left(\frac{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}}{\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}} \right)^{1/2} \left(\frac{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8(A^2/B)\delta_{r,d}^2}} \right)^{3/2} \tag{37}$$

Cases	Cooperation Gain Ratio λ (M Large)
Case 1: $\delta_{s,r}^2 << \delta_{r,d}^2$	M-PSK: $\lambda \approx 1.2247 > 1$ and M-QAM: $\lambda \approx 1.0175 > 1$
Case 2: $\delta_{s,r}^2 >> \delta_{r,d}^2$	Almost the same for M-PSK and M-QAM
Case 3: $\delta_{s,r}^2 = \delta_{r,d}^2$	M-PSK: $\lambda \approx 1.0635 > 1$ and M-QAM: $\lambda \approx 1.0058 > 1$

- Comments:

  1. Case 1: Cooperation gain for DF is always larger than AF (but more significant for M-PSK than M-QAM)
  2. Case 2: It is preferred to use AF to reduce complexity (since cooperation gain is almost the same)
  3. Case 3: For large modulation size, gain of DF compared to AF is negligible

Comparison of DF and AF Protocols

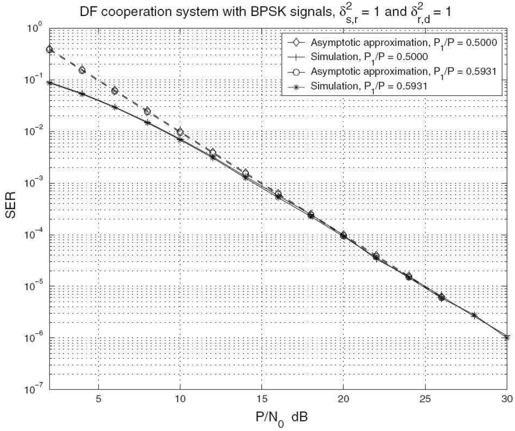


Figure: Performance of DF cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = \delta^2_{r,d} = 1$ and $\mathcal{N}_0 = 1$

Comparison of DF and AF Protocols

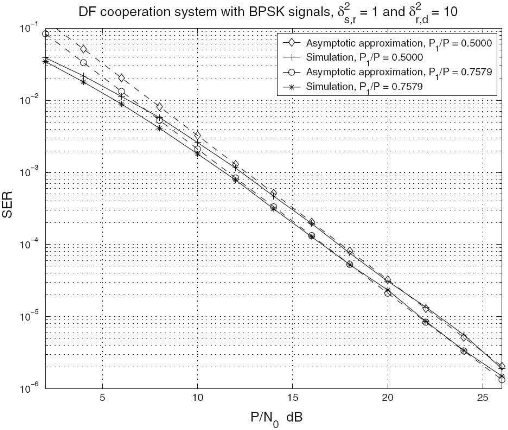


Figure: Performance of DF cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = 1$, $\delta^2_{r,d} = 10$ and $\mathcal{N}_0 = 1$

Comparison of DF and AF Protocols

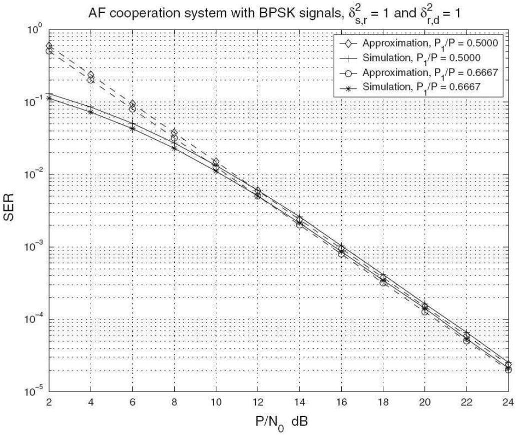


Figure: Performance of AF cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = \delta^2_{r,d} = 1$ and $\mathcal{N}_0 = 1$

Comparison of DF and AF Protocols

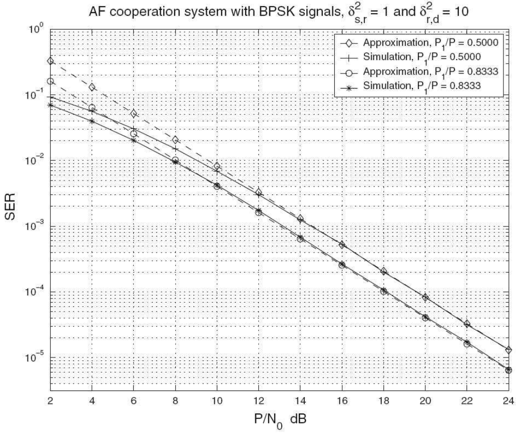


Figure: Performance of AF cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = 1$, $\delta^2_{r,d} = 10$ and $\mathcal{N}_0 = 1$

Comparison of DF and AF Protocols

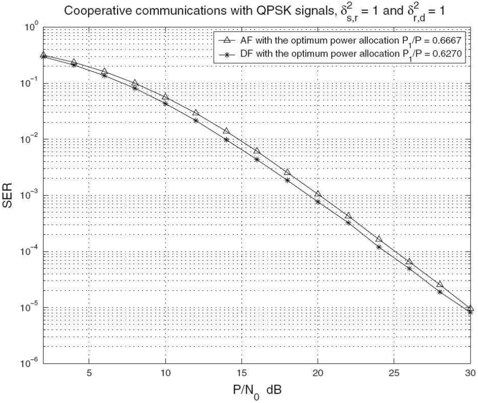


Figure: Performance comparison of the cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = \delta^2_{r,d} = 1$ and $\mathcal{N}_0 = 1$

Comparison of DF and AF Protocols

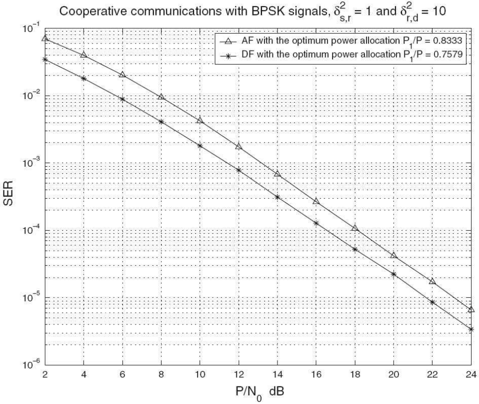


Figure: Performance comparison of the cooperation systems with BPSK: optimum power versus equal power allocation - $\delta^2_{s,r} = 1$, $\delta^2_{r,d} = 10$ and $\mathcal{N}_0 = 1$

Trans-Modulation in Wireless Relay Networks (1)

- Trans-modulation design for Decode-and-Forward relay networks
  - Re-mapping of constellation points at relay nodes to minimize symbol error rate (SER) - increases Euclidean distance between different transmitted symbols
  - Repetition coding vs. constellation re-assignment
- DF relay node decides whether received signal decoded correctly before re-transmission to destination

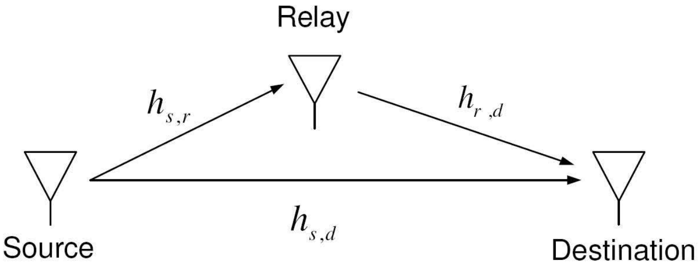


Figure: Simplified system model for the single-relay DF

Trans-Modulation in Wireless Relay Networks (2)

- Received signals $y_{s,d}$ and $y_{s,r}$ at the destination and relay nodes

$$\begin{aligned} y_{s,d} &= \sqrt{P_s} h_{s,d} x_s + \eta_{s,d} \\ y_{s,r} &= \sqrt{P_s} h_{s,r} x_s + \eta_{s,r} \end{aligned} \quad (38)$$

- Received signal at destination from the relay after DF (assuming correct decoding)

$$y_{r,d} = \sqrt{P_r} h_{r,d} x_r + n_{r,d} \quad (39)$$

- Pairwise symbol error probability (PSEP) between two possible transmitted symbols at destination

$$\begin{aligned} &\text{Pr}\{\boldsymbol{x}_1 \rightarrow \boldsymbol{x}_2\} = \\ &\quad Pr\{\boldsymbol{x}_1 \rightarrow \boldsymbol{x}_2 | \boldsymbol{x}_1, relay\ decodes\ erroneously\} \times Pr\{relay\ decodes\ erronoeously\} + \\ &\quad Pr\{\boldsymbol{x}_1 \rightarrow \boldsymbol{x}_2 | \boldsymbol{x}_1, relay\ decodes\ correctly\} \times Pr\{relay\ decodes\ correctly\} \end{aligned} \quad (40)$$

- where $\boldsymbol{x}_1 = \left[\sqrt{P_s} x_{s_1} \sqrt{P_r} x_{r_1}\right]$ and $\boldsymbol{x}_2 = \left[\sqrt{P_s} x_{s_2} \sqrt{P_r} x_{r_2}\right]$

Trans-Modulation in Wireless Relay Networks (3)

- Let $PSEP_r = Pr\{\boldsymbol{x}_1 \rightarrow \boldsymbol{x}_2 | \boldsymbol{x}_1, \textit{relay decodes correctly}\}$ which can be shown to be

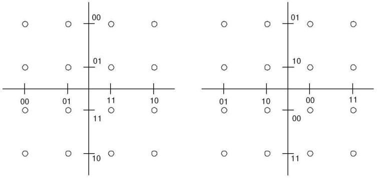
$$PSEP_r = E\left\{Q\left(\sqrt{\frac{1}{2N_o}(P_s|h_{s,d}|^2|x_{s_1} - x_{s_2}|^2 + P_r|h_{r,d}|^2|x_{r_1} - x_{r_2}|^2)}\right)\right\} \tag{41}$$

- An upper bound on $PSEP_r$ can be shown to be

$$PSEP_r \leq \frac{3N_o^2}{\sigma_{s,d}^2 \sigma_{r,d}^2 P_s P_r |x_{s_1} - x_{s_2}|^2 |x_{r_1} - x_{r_2}|^2} \tag{42}$$

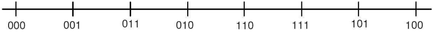
- Constellation reassignment at relay to better separate symbols by maximizing $|x_{s_1} - x_{s_2}|^2 |x_{r_1} - x_{r_2}|^2$
- Exhaustive search over all possible relay constellation assignments is complex and impractical
- Heuristic approach: rearrange rows and then columns to ensure any two adjacent rows (columns) in the source constellation

Trans-Modulation in Wireless Relay Networks (4)

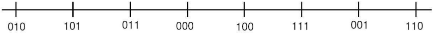


(a) Source constellation (b) Relay constellation

Figure: Trans-modulation for 16-QAM constellation



(a) Source constellation (real axis)



(b) Relay constellation (real axis)

Figure: Trans-modulation for 64-QAM constellation

Trans-Modulation in Wireless Relay Networks (5)

- Two cases:
  1. Relay close to source ($\sigma^2_{s,r} = 10, \sigma^2_{r,d} = 1$)
  2. Relay close to destination ($\sigma^2_{s,r} = 1, \sigma^2_{r,d} = 10$)
- 2 dB gain for 16-QAM and about 3 dB gain for 64-QAM when relay is sloe to source

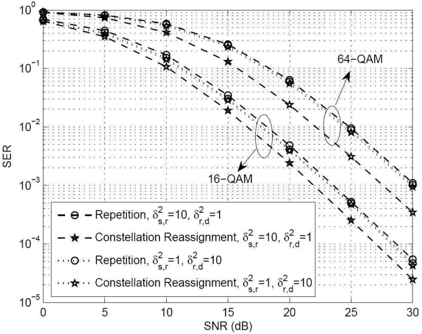


Figure: SER for single-relay DF using 16-QAM and 64-QAM constellations

Conclusions

- For DF cooperation systems:
  - Optimum power allocation does not depend on the direct link between source and destination – only on channel links relay to the relay
  - Optimal power allocation is modulation-dependent (i.e. depends on specific M-PSK or M-QAM modulation)
- For AF cooperation systems:
  - Optimum power allocation is modulation-independent
- In general, the performance of DF cooperation is better than that of its AF counter part; but more complex
- Trans-modulation can significantly improve the performance of DF cooperation systems