

Chapter 13

à Question 1

(i)

Since

$$s - \bar{s} = \frac{-1}{uv} (p - \bar{p})$$

then

$$\begin{aligned} s &= \bar{s} - \frac{1}{uv} [\bar{p} + (p_0 - \bar{p}) e^{-\lambda t} - \bar{p}] \\ &= \bar{s} - \frac{(p_0 - \bar{p})}{uv} e^{-\lambda t} \end{aligned}$$

But

$$s_0 - \bar{s} = \frac{-1}{uv} (p_0 - \bar{p})$$

Therefore

$$s = \bar{s} + (s_0 - \bar{s}) e^{-\lambda t}$$

(ii)

```
In[1]:= Solve[s == 105 -  $\frac{1}{0.1}$  (105 - 5 E-0.011 t - 105), s]
```

```
Out[1]= {{s -> 105. + 50. e-0.011 t}}
```

(iii)

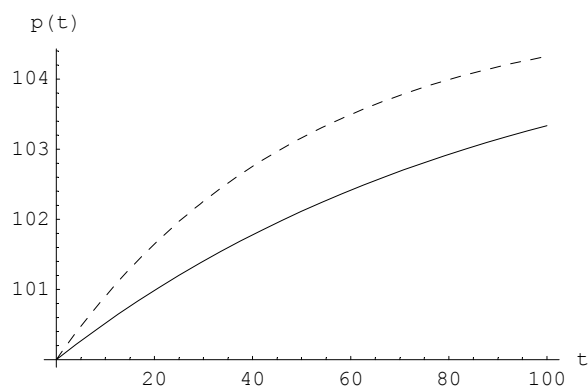
```
In[2]:= pathp1 = 105 - 5 E-0.011 t
```

```
Out[2]= 105 - 5 e-0.011 t
```

```
In[3]:= pathp2 = 105 - 5 E-0.02 t
```

```
Out[3]= 105 - 5 e-0.02 t
```

```
In[4]:= Plot[{pathp1, pathp2}, {t, 0, 100},
  AxesLabel -> {"t", "p(t)"}, PlotStyle -> {{}, {Dashing[{.02}]}}];
```



```
In[5]:= paths1 = 105 + 50 E-0.011 t
```

```
General::spell1 :  
Possible spelling error: new symbol name "paths1" is similar to existing symbol "pathp1".
```

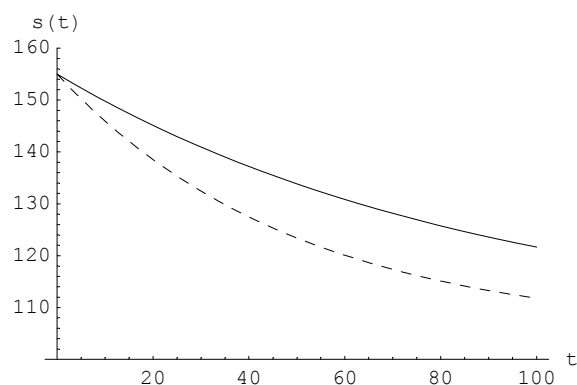
```
Out[5]= 105 + 50 e-0.011 t
```

```
In[6]:= paths2 = 105 + 50 E-0.02 t
```

```
General::spell1 :  
Possible spelling error: new symbol name "paths2" is similar to existing symbol "pathp2".
```

```
Out[6]= 105 + 50 e-0.02 t
```

```
In[7]:= Plot[{paths1, paths2}, {t, 0, 100}, AxesLabel -> {"t", "s(t)"},  
PlotStyle -> {{}, {Dashing[{.02}]}}, PlotRange -> {100, 160}];
```



à Question 2

GM line is $p = s$ and AM line is $p = 110 - 0.1 s$.

```
In[8]:= Solve[{p == s, p == 110 - 0.1 s}, {s, p}]
```

```
Out[8]= {{s -> 100., p -> 100.}}
```

```
In[9]:= am = (m - k y + u rstar + u v sbar - u v s)
```

```
Out[9]= m + rstar u - s u v + sbar u v - k y
```

```
In[10]:= am0 =
```

```
am /. {m -> 105, k -> 0.5, y -> 20, u -> 0.5, rstar -> 10, v -> 0.2, sbar -> 100}
```

```
Out[10]= 110. - 0.1 s
```

```
In[11]:= am1 =
```

```
am /. {m -> 105, k -> 0.5, y -> 20, u -> 0.5, rstar -> 12, v -> 0.2, sbar -> 100}
```

```
Out[11]= 111. - 0.1 s
```

```
In[12]:= Solve[{p == s, p == 111 - 0.1 s}, {s, p}]
```

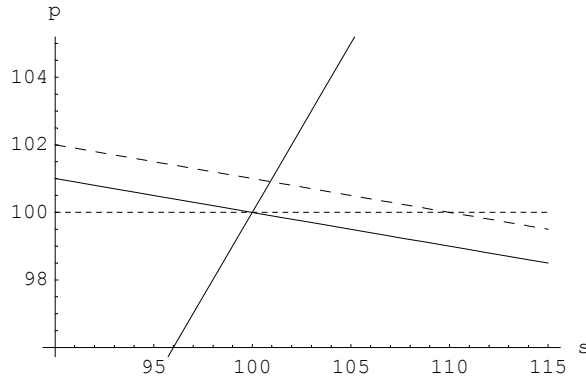
```
Out[12]= {{s -> 100.909, p -> 100.909}}
```

The value on the new AM line at the original price of $p=100$ is

```
In[13]:= Solve[100 == 111 - 0.1 s, s]
```

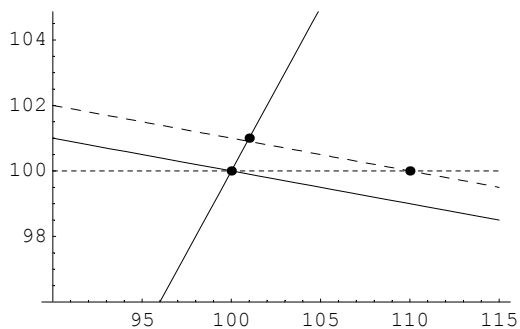
```
Out[13]= {{s -> 110.}}
```

```
In[14]:= lines2 = Plot[{s, am0, am1, 100}, {s, 90, 115}, AxesLabel -> {"s", "p"},
  PlotStyle -> {{}, {}, {Dashing[{.02}]}, {Dashing[{.01}]}}];
```



```
In[15]:= points2 = Graphics[
  {PointSize[.02],
   Point[{100, 100}], Point[{101, 101}], Point[{110, 100}]
  }];
```

```
In[16]:= Show[points2, lines2, Axes -> True,
  PlotLabel -> "Rise in foreign interest rate \n"];
Rise in foreign interest rate
```



System begins with initial equilibrium $(s, p) = (100, 100)$ then moves horizontally across to new AM curve, with value $(s, p) = (110, 100)$. As prices begin to adjust, the system moves up the new AM curve to the new equilibrium at $(s, p) = (101, 101)$.

à Question 3

(i)

Since $\bar{p} = m - k y - u r^*$ and

$$\begin{aligned} p &= m - k y - u(r^* + s^e) \\ &= m - k y - u r^* - u v(\tilde{s} - s) \\ &= m - k y - u r^* - u v(s - \tilde{s}) \end{aligned}$$

Subtracting $\bar{p}$ from p we obtain

$$p - \bar{p} = u v(s - \tilde{s})$$

Or

$$s - \tilde{s} = \frac{1}{uv} (p - \bar{p})$$

(ii)

We have

$$\begin{aligned} \dot{p} &= a(e - y) = a[cy - dr + g + h(s - p) - y] \\ &= a[-(1 - c)y + g + hs - hp - dr] \end{aligned}$$

`In[17]:= pdot = a (- (1 - c) y + g + h s - h p - d r)`

`Out[17]= a (g - h p - d r + h s + (-1 + c) y)`

But

$$m = ky - ur + p$$

`In[18]:= Simplify[Solve[m == k y - u r + p, r]]`

`Out[18]= {{r ->  $\frac{-m + p + k y}{u}$ }}`

`In[19]:= newpdot = pdot /. {r ->  $\frac{-m + p + k y}{u}$ }`

`Out[19]= a (g - h p + h s + (-1 + c) y -  $\frac{d (-m + p + k y)}{u}$ )`

`In[20]:= pdot0 = a (- (1 - c) y + g + h s - h p - d r) /. {p ->  $\bar{p}$ , s ->  $\bar{s}$ , r ->  $\frac{-m + \bar{p} + k y}{u}$ }`

`Out[20]= a (g + (-1 + c) y - h  $\bar{p}$  -  $\frac{d (-m + k y + \bar{p})}{u}$  + h  $\bar{s}$ )`

`In[21]:= finalpdot = Simplify[newpdot - pdot0]`

`Out[21]=  $\frac{a (-d p - h p u + h s u + (d + h u) \bar{p} - h u \bar{s})}{u}$`

Which can be written

$$a h(s - \bar{s}) - a h(p - \bar{p}) - \frac{ad}{u} (p - \bar{p})$$

But from part (i) we know

$$s - \tilde{s} = \frac{1}{uv} (p - \bar{p})$$

Hence,

$$\begin{aligned} \dot{p} &= \frac{ah}{uv} (p - \bar{p}) - a h(p - \bar{p}) - \frac{ad}{u} (p - \bar{p}) \\ &= -a(h + \frac{h}{uv} + \frac{d}{u})(p - \bar{p}) \end{aligned}$$

Let $\lambda = a(h + \frac{h}{uv} + \frac{d}{u})$, then

`In[22]:= DSolve[{p'[t] == -λ (p[t] -  $\bar{p}$ ), p[0] == p0}, p[t], t]`

`Out[22]= {{p[t] ->  $e^{-t\lambda} (p0 - \bar{p} + e^{t\lambda} \bar{p})$ }}`

`In[23]:= Expand[E-tλ (p0 -  $\bar{p}$  + Etλ  $\bar{p}$ )]`

`Out[23]=  $e^{-t\lambda} p0 + \bar{p} - e^{-t\lambda} \bar{p}$`

Which can be expressed $\bar{p} + (p0 - \bar{p}) E^{-\lambda t}$. So the adjustment coefficient in this model is,

$$\lambda = a(h + \frac{h}{uv} + \frac{d}{u})$$

à Question 4

(i)

We suppress the t subscript throughout. To derive the GM line let $p_{t+1} - p_t = 0$.

```
In[24]:= e = 0.01 (s - p) + 0.8 (20) - 0.1 r + 5
Out[24]= 21. - 0.1 r + 0.01 (-p + s)

In[25]:= Simplify[Solve[0 == 0.2 (e - 20), p]]
Out[25]= {{p -> 100. - 10. r + 1. s}}
```

We need to eliminate r using equilibrium in the money market.

```
In[26]:= Simplify[Solve[105 == p + 0.5 (20) - 0.25 r, r]]
Out[26]= {{r -> -380. + 4. p}}
```

Furthermore, in equilibrium $r = r^* = 10$, so we can solve for equilibrium p from

```
In[27]:= Solve[105 == p + 0.5 (20) - 0.25 (10), p]
Out[27]= {{p -> 97.5}}
```

Then the GM line is

```
In[28]:= Simplify[NSolve[p == 100 - 10 (-380 + 4 p) + s, p]]
Out[28]= {}
```

Given equilibrium $p = 97.5$, then equilibrium s is given by

```
In[29]:= Solve[97.5 == 95.122 + 0.0243902 s, s]
Out[29]= {{s -> 97.4982}}
```

Which is, to one decimal place the same value.

We can now derive the equation for the AM line as follows.

```
In[30]:= Simplify[Solve[105 == p + 0.5 (20) - 0.25 (10 + 0.25 (97.5 - s)), p]]
Out[30]= {{p -> 103.594 - 0.0625 s}}
```

To summarise:

$$\text{GM line: } p_t = 95.122 + 0.0243902 s_t$$

$$\text{AM line: } p_t = 103.594 - 0.0625 s_t$$

To check these equations

```
In[31]:= Solve[{p == 95.122 + 0.0243902 s, p == 103.594 - 0.0625 s}, {s, p}]
Out[31]= {{s -> 97.5024, p -> 97.5001}}
```

(ii)

We have already established for this model that

$$p_{t+1} - \bar{p} = -a\left(h + \frac{h}{u v} + \frac{d}{u}\right)(p_t - \bar{p})$$

$$p_{t+1} = \bar{p} - \lambda(p_t - \bar{p})$$

with $\bar{p} = 97.5$.

$$\text{In}[32] := \lambda = a \left(h + \frac{h}{u v} + \frac{d}{u} \right)$$

$$\text{Out}[32] = a \left(h + \frac{d}{u} + \frac{h}{u v} \right)$$

$$\text{In}[33] := \text{value}\lambda = \lambda /. \{a \rightarrow 0.2, h \rightarrow 0.01, u \rightarrow 0.25, v \rightarrow 0.25, d \rightarrow 0.1\}$$

$$\text{Out}[33] = 0.114$$

$$\text{In}[34] := \text{Solve}\left[s0 - 97.5 == -\left(\frac{1}{0.25 \cdot 0.25}\right)(100 - 97.5), s0\right]$$

$$\text{Out}[34] = \{\{s0 \rightarrow 57.5\}\}$$

$$\text{In}[35] := \text{pathp} = 97.5 + (100 - 97.5) E^{-0.114 t}$$

$$\text{Out}[35] = 97.5 + 2.5 e^{-0.114 t}$$

$$\text{In}[36] := \text{paths} = \text{Simplify}[97.5 + (57.5 - 97.5) E^{-0.114 t}]$$

General::spell1 :

Possible spelling error: new symbol name "paths" is similar to existing symbol "pathp".

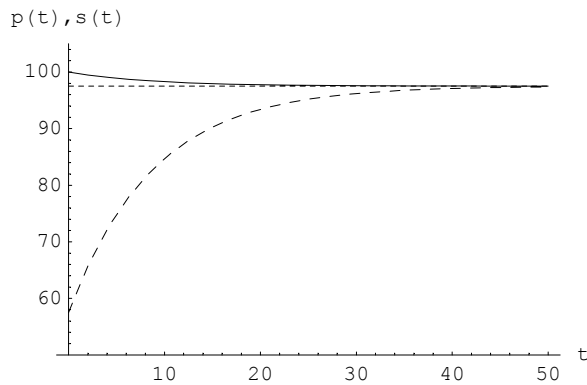
$$\text{Out}[36] = 97.5 - 40. e^{-0.114 t}$$

```
In[37]:= Table[{t, pathp, paths}, {t, 0, 50}] // TableForm
```

```
Out[37]//TableForm=
```

0	100.	57.5
1	99.7306	61.8097
2	99.4903	65.655
3	99.2759	69.0861
4	99.0845	72.1474
5	98.9138	74.879
6	98.7615	77.3162
7	98.6256	79.4909
8	98.5043	81.4312
9	98.3961	83.1625
10	98.2995	84.7072
11	98.2134	86.0856
12	98.1365	87.3154
13	98.068	88.4127
14	98.0068	89.3918
15	97.9522	90.2654
16	97.9034	91.0448
17	97.86	91.7403
18	97.8212	92.3609
19	97.7866	92.9146
20	97.7557	93.4086
21	97.7282	93.8494
22	97.7036	94.2428
23	97.6816	94.5937
24	97.6621	94.9068
25	97.6446	95.1862
26	97.629	95.4355
27	97.6151	95.6579
28	97.6027	95.8564
29	97.5917	96.0335
30	97.5818	96.1915
31	97.573	96.3325
32	97.5651	96.4583
33	97.5581	96.5705
34	97.5518	96.6707
35	97.5462	96.76
36	97.5413	96.8397
37	97.5368	96.9109
38	97.5329	96.9744
39	97.5293	97.031
40	97.5262	97.0815
41	97.5233	97.1266
42	97.5208	97.1668
43	97.5186	97.2027
44	97.5166	97.2348
45	97.5148	97.2633
46	97.5132	97.2888
47	97.5118	97.3116
48	97.5105	97.3319
49	97.5094	97.35
50	97.5084	97.3662

```
In[38]:= Plot[{pathp, paths, 97.5}, {t, 0, 50}, PlotRange -> {50, 105},
  PlotStyle -> {{}, {Dashing[{.02}]}, {Dashing[{.01}]}},
  AxesLabel -> {"t", "p(t), s(t)"}];
```



(iii)

The expenditure function in terms of s, p and r remains unaffected. However, the value of r in terms of p now differs.

```
In[39]:= Simplify[Solve[110 == p + 0.5 (20) - 0.25 r, r]]
```

```
Out[39]= {{r -> -400. + 4. p}}
```

Equilibrium price is

```
In[40]:= Solve[110 == p + 0.5 (20) - 0.25 (10), p]
```

```
Out[40]= {{p -> 102.5}}
```

The GM line is then,

```
In[41]:= Simplify[NSolve[p == 100 - 10 (-400 + 4 p) + s, p]]
```

```
Out[41]= {}
```

```
In[42]:= Solve[102.5 == 100 + 0.0243902 s, s]
```

```
Out[42]= {{s -> 102.5}}
```

The AM line is then,

```
In[43]:= Simplify[Solve[110 == p + 0.5 (20) - 0.25 (10 + 0.25 (102.5 - s)), p]]
```

```
Out[43]= {{p -> 108.906 - 0.0625 s}}
```

```
In[44]:= Solve[{p == 100 + 0.0243902 s, p == 108.906 - 0.0625 s}, {s, p}]
```

```
Out[44]= {{s -> 102.497, p -> 102.5}}
```

Hence,

$$dm_t = ds_t = dp_t = 5$$

à Question 5

(i)

The goods market is solved as before.

```
In[45]:= Clear[e]

In[46]:= e = 0.8 (20) + 4 + 0.01 (s - p)

Out[46]= 20. + 0.01 (-p + s)

In[47]:= Simplify[Solve[0 == 0.1 (e - 20), p]]

Out[47]= {{p → 0. + 1. s}}
```

Which is as it should be, a 45^0 line through the origin. The major difference in this model lies in the asset market, the AM line.

In equilibrium $s = \bar{s}$, so $\dot{s}^e = 0$, $bp = 0$ and $r = r^*$.

```
In[48]:= Clear[pbar]

General::spell1 :
Possible spelling error: new symbol name "pbar" is similar to existing symbol "sbar".

In[49]:= Solve[105 == pbar + 0.5 (20) - 0.5 (10), pbar]

Out[49]= {{pbar → 100.}}
```

```
In[50]:= Simplify[Solve[0 == 0.01 (s - p) + 0.0045 (r - 10 - 0.2 (100 - s)), r]]

Out[50]= {{r → 30. + 2.22222 p - 2.42222 s}}
```

```
In[51]:= Simplify[Solve[105 == p + 0.5 (20) - 0.5 (30 + 2.22222 p - 2.42222 s), p]]

Out[51]= {{p → -990.01 + 10.9001 s}}
```

To check

```
In[52]:= Solve[{p == s, p == -990.01 + 10.9001 s}, {s, p}]

Out[52]= {{s → 100., p → 100.}}
```

(ii) Rise in m_s from 105 to 110

The goods line remains as the 45^0 line through the origin. But,

```
In[53]:= Solve[110 == pbar + 0.5 (20) - 0.5 (10), pbar]

Out[53]= {{pbar → 105.}}
```

```
In[54]:= Simplify[Solve[0 == 0.01 (s - p) + 0.0045 (r - 10 - 0.2 (105 - s)), r]]

Out[54]= {{r → 31. + 2.22222 p - 2.42222 s}}
```

```
In[55]:= Simplify[Solve[110 == p + 0.5 (20) - 0.5 (31 + 2.22222 p - 2.42222 s), p]]
```

```
Out[55]= {{p → -1039.51 + 10.9001 s}}
```

```
In[56]:= Solve[{p == s, p == -1039.50 + 10.9001 s}, {s, p}]
```

```
Out[56]= {{s → 104.999, p → 104.999}}
```

Point C arises on the new AM line at a price of 100.

```
In[57]:= Solve[100 == -1039.51 + 10.90001 s, s]
```

```
Out[57]= {{s → 104.542}}
```

Hence, coordinates of point C are $(s, p) = (104.542, 100)$.

(iii)

In the new equilibrium $p = s = 105$ and hence $dp = ds = 5 = dm$.

à Question 6

(i) Model 13.4

```
In[58]:= solr = Solve[0 == h (s - p) + b (r - rstar - v (sbar - s)), r]
```

```
Out[58]= {{r → -\frac{-hp - brstar + hs + bsv - bsbar v}{b}}}
```

```
In[59]:= valuer = solr[[1, 1, 2]]
```

```
Out[59]= -\frac{-hp - brstar + hs + bsv - bsbar v}{b}
```

```
In[60]:= valuep = m - k y + u valuer
```

```
General::spell11 :
```

```
Possible spelling error: new symbol name "valuep" is similar to existing symbol "valuer".
```

```
Out[60]= m - \frac{u (-hp - brstar + hs + bsv - bsbar v)}{b} - k y
```

```
In[61]:= valuepbar = m - \frac{u (-hpbar - brstar + h sbar + b sbar v - b sbar v)}{b} - k y
```

```
Out[61]= m - \frac{(-hpbar - brstar + h sbar) u}{b} - k y
```

```
In[62]:= Simplify[valuep - valuepbar]
```

```
Out[62]= \frac{u (h (p - pbar - s + sbar) + b (-s + sbar) v)}{b}
```

Which can be written,

$$-(u v + \frac{uh}{b})(s - \bar{s}) + \frac{uh}{b}(p - \bar{p})$$

Hence,

$$\begin{aligned}(p - \bar{p}) &= -(u v + \frac{u h}{b})(s - \bar{s}) + \frac{u h}{b}(p - \bar{p}) \\ (1 - \frac{u h}{b})(p - \bar{p}) &= -(u v + \frac{u h}{b})(s - \bar{s}) \\ (s - \bar{s}) &= -\frac{(1 - \frac{u h}{b})}{(u v + \frac{u h}{b})}(p - \bar{p})\end{aligned}$$

Since in equilibrium $r = r^* + v(\bar{s} - s) = r^* - v(s - \bar{s})$, then

```
In[63]:= p_dot = a (c y - d (rstar - v (s - sbar)) + g + h (s - p) - y)
Out[63]= a (g + h (-p + s) - d (rstar - (s - sbar) v) - y + c y)

In[64]:= p_dot0 = a (g + h (-pbar + sbar) - d (rstar - (sbar - sbar) v) - y + c y)
Out[64]= a (g - d rstar + h (-pbar + sbar) - y + c y)

In[65]:= Simplify[p_dot - p_dot0]
Out[65]= a (h (-p + pbar + s - sbar) + d (s - sbar) v)
```

Which can be written,

$$a(d v(s - \bar{s}) + h(s - \bar{s}) - h(p - \bar{p}))$$

and substituting the value for $(s - \bar{s})$, we get for $\dot{p}$

$$\begin{aligned}\dot{p} &= a \left\{ -(h + d v) \left[\frac{1 - (u h/b)}{u v + (u h/b)} \right] (p - \bar{p}) - h(p - \bar{p}) \right\} \\ &= -a \left\{ (h + d v) \left[\frac{1 - (u h/b)}{u v + (u h/b)} \right] + h \right\} (p - \bar{p})\end{aligned}$$

and for λ we have,

$$\lambda = a \left\{ \frac{(h + d v)[1 - (u h/b)]}{u v + (u h/b)} + h \right\}$$

```
In[66]:= newλ = a ( (h + d v) (1 - (u h / b)) / (u v + (u h / b)) + h )
Out[66]= a ( h + (1 - (h u / b)) (h + d v) / (h u / b + u v) )
```

(ii) Limit for $d \rightarrow \infty$

$$a \left(h + \frac{(1 - \frac{h u}{b}) (h + d v)}{\frac{h u}{b} + u v} \right)$$

```
In[67]:= Limit[newλ, d -> 0]
```

$$\text{Out[67]} = \frac{a b h (1 + u v)}{u (h + b v)}$$

which is the same as

$$a h \left\{ \frac{1 - (u h/b)}{u v + (u h/b)} + 1 \right\}$$

(iii) Limit for $b \rightarrow \infty$

```
In[68]:= Limit[newλ, b -> ∞]
```

$$\text{Out[68]} = \frac{a (h + d v + h u v)}{u v}$$

which is the same as

$$a \left\{ \frac{h}{uv} + \frac{d}{u} + h \right\}$$

(iv) Limit for $d \rightarrow 0$ and $b \rightarrow \infty$

```
In[69]:= Limit[Limit[newλ, d -> 0], b -> ∞]
```

```
Out[69]= a h (1 + 1/(u v))
```

which is the same as

$$a h \left(\frac{1}{uv} + 1 \right)$$

à Question 7

(i)

(a) Solve for equilibrium

```
In[70]:= Solve[105 == p + 0.5 (20) - 0.5 (10), p]
```

```
Out[70]= {{p -> 100.}}
```

(b) GM line

To derive the GM line we first solve for r and then substitute this into the equation for $\dot{p}$ and then let this in turn have the value of 0 and solve for p .

```
In[71]:= Simplify[Solve[m == p + k y - u r, r]]
```

```
Out[71]= {{r -> (-m + p + k y)/u}}
```

```
In[72]:= Simplify[Solve[0 == -(1 - c) y - d ( (-m + p + k y)/u ) + g + h (s - p), p]]
```

```
Out[72]= {{p -> (u (g + h s + (-1 + c) y) + d (m - k y)) / (d + h u)}}
```

Which can be expressed

$$p = \frac{-[(1-c)+(d k/u)]y}{h+(d/u)} + \frac{g+(d/u)m}{h+(d/u)} + \left(\frac{h}{h+(d/u)} \right) s$$

```
In[73]:= intGM4 = -((1 - c) + (d k / u)) y + g + (d / u) m / (h + (d / u)) / .
```

```
{c -> 0.8, d -> 0.1, k -> 0.5, u -> 0.5, y -> 20, g -> 5, m -> 105, h -> 0.01}
```

```
Out[73]= 95.2381
```

```
In[74]:= slopeGM4 = h / (h + (d / u)) / . {h -> 0.01, d -> 0.1, u -> 0.5}
```

```
Out[74]= 0.047619
```

Hence, the equation for the GM line is

$$p = 95.2381 + 0.047619s$$

(c) AM line

```
In[75]:= Simplify[Solve[0 == h (s - p) + b (r - rstar - v (sbar - s)), r]]
```

```
Out[75]= {{r -> rstar + \frac{h (p - s)}{b} + (-s + sbar) v}}
```

```
In[76]:= Simplify[Solve[m == p + k y - u \left( rstar + \frac{h (p - s)}{b} + (-s + sbar) v \right), p]]
```

```
Out[76]= {{p -> \frac{-h s u + b (m + rstar u - s u v + sbar u v - k y)}{b - h u}}}
```

Which can be expressed

$$p = \frac{m - k y + u r^*}{1 - (u h / b)} + \frac{u v s}{1 - (u h / b)} - \frac{(u v + (u h / b))}{1 - (u h / b)}$$

```
In[77]:= intAM4 = \frac{m - k y + u rstar + u v sbar}{1 - (u h / b)} /. {m -> 105, k -> 0.5, y -> 20,
rstar -> 10, u -> 0.5, v -> 0.2, sbar -> 100, b -> 0.004, h -> 0.01}
```

General::spell1 :

Possible spelling error: new symbol name "intAM4" is similar to existing symbol "intGM4".

```
Out[77]= -440.
```

```
In[78]:= slopeAM4 = \frac{-(u v + (u h / b))}{1 - (u h / b)} /. {u -> 0.5, v -> 0.2, h -> 0.01, b -> 0.004}
```

General::spell1 :

Possible spelling error: new symbol name "slopeAM4" is similar to existing symbol "slopeGM4".

```
Out[78]= 5.4
```

Hence, equation for the LM curve is

$$p = -440 + 5.4 s$$

We can check the equilibrium results by solving

```
In[79]:= Solve[{p == 95.2381 + 0.047619 s, p == -440 + 5.4 s}, {s, p}]
```

```
Out[79]= {{s -> 100., p -> 100.}}
```

(ii) Rise in money supply from 105 to 110

```
In[80]:= Solve[110 == p + 0.5 (20) - 0.5 (10), p]
```

```
Out[80]= {{p -> 105.}}
```

Only the intercept on the GM line and the AM lines are affected.

```
In[81]:= intGM42 = \frac{-((1 - c) + (d k / u)) y + g + (d / u) m}{h + (d / u)} /.
{c -> 0.8, d -> 0.1, k -> 0.5, u -> 0.5, y -> 20, g -> 5, m -> 110, h -> 0.01}
```

```
Out[81]= 100.
```

```
In[82]:= intAM42 = 
$$\frac{m - k y + u rstar + u v sbar}{1 - (u h / b)}$$
 /. {m -> 110, k -> 0.5, y -> 20,
      rstar -> 10, u -> 0.5, v -> 0.2, sbar -> 105, b -> 0.004, h -> 0.01}

General::spell1 :
  Possible spelling error: new symbol name "intAM42" is similar to existing symbol "intGM42".

Out[82]= -462.
```

Hence, the two new equations are

$$p = 100 + 0.047619s$$

$$p = -462 + 5.4s$$

The new equilibrium is

```
In[83]:= Solve[{p == 100 + 0.047619 s, p == -462 + 5.4 s}, {s, p}]

Out[83]= {{s -> 105., p -> 105.}}
```

Point C on the new AM line is at a price of $p = 100$.

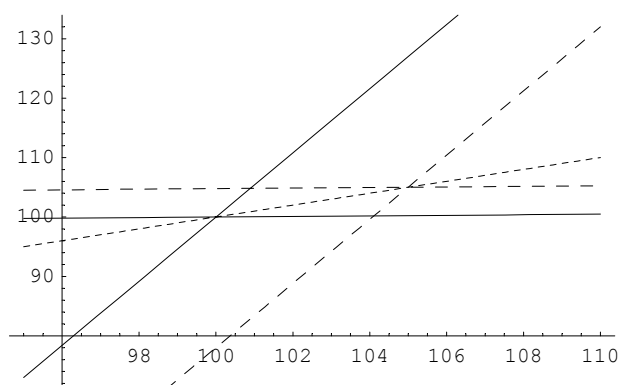
```
In[84]:= Solve[100 == -462 + 5.4 s, s]

Out[84]= {{s -> 104.074}}
```

Hence, the coordinates of point C are $(s, p) = (104.074, 100)$.

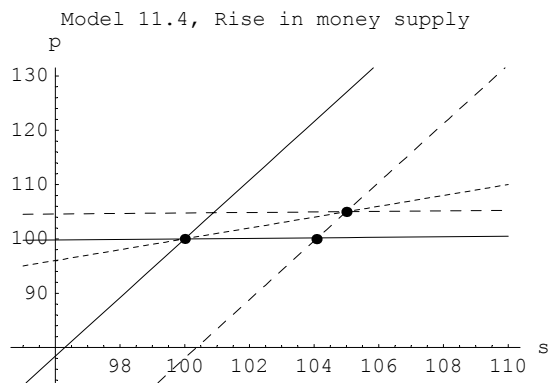
Also, we have that $dp = ds = 5 = dm$.

```
In[85]:= lines7 = Plot[{s, 95.2381 + 0.047619 s, -440 + 5.4 s,
      100 + 0.047619 s, -462 + 5.4 s}, {s, 95, 110}, PlotStyle ->
      {{Dashing[{.01]}], {}, {}, {Dashing[{.02]}], {Dashing[{.02]}]}}];
```



```
In[86]:= points7 = Graphics[{PointSize[.02],
      Point[{100, 100}], Point[{105, 105}], Point[{104.074, 100}]}];
```

```
In[87]:= Show[points7, lines7, Axes -> True, AxesLabel -> {"s", "p"},
  PlotLabel -> "Model 11.4, Rise in money supply\n"];
```



à Question 8

(i) Equilibrium

```
In[88]:= Clear[pdot, sdot]
```

```
General::spell1 :
Possible spelling error: new symbol name "sdot" is similar to existing symbol "pdot".
```

```
In[89]:= pbar = 105 - 0.5 (20) + 0.5 (18)
```

```
Out[89]= 104.
```

Hence, $\bar{p} = \bar{s} = 104$

(ii) Characteristic roots

```
In[90]:= Simplify[pdot = 0.01 (0.1 (s - p) + 0.8 (20) + 4 - y)]
```

```
Out[90]= 0.2 - 0.001 p + 0.001 s - 0.01 y
```

```
In[91]:= newpdot = -0.001 (p - p̄) + 0.001 (s - s̄)
```

```
Out[91]= -0.001 (p - p̄) + 0.001 (s - s̄)
```

```
In[92]:= sdot = (1 / 0.5) (s - s̄)
```

```
Out[92]= 2. (s - s̄)
```

```
In[93]:= matrixA =  $\begin{pmatrix} -0.001 & 0.001 \\ 2 & 0 \end{pmatrix}$ 
```

```
Out[93]= {{-0.001, 0.001}, {2, 0}}
```

```
In[94]:= Eigenvalues[matrixA]
```

```
Out[94]= {-0.0452242, 0.0442242}
```

(iii) Saddle-path equations

(a)

```

In[95]:= Eigenvectors[matrixA]
Out[95]= {{-0.0226063, 0.999744}, {-0.0221067, -0.999756}}

In[96]:= matrixA - 0.0442242 IdentityMatrix[2]
Out[96]= {{-0.0452242, 0.001}, {2, -0.0442242}}

In[97]:= Simplify[Solve[-0.0452242 (p - 104) + 0.001 (s - 104) == 0, p]]
Out[97]= {{p -> 101.7 + 0.0221121 s}}

```

(b)

```

In[98]:= matrixA + 0.0452242 IdentityMatrix[2]
Out[98]= {{0.0442242, 0.001}, {2, 0.0452242}}

In[99]:= Simplify[Solve[0.0442242 (p - 104) + 0.001 (s - 104) == 0, p]]
Out[99]= {{p -> 106.352 - 0.0226121 s}}

```

(iv) Rise in the money supply to 110

The change in the money supply has no affect on the dynamics of the system, i.e., the matrix of the system remains unaffected. However, the equilibrium value changes to

```

In[100]:= newpbar = 110 - 0.5 (20) + 0.5 (18)
Out[100]= 109.

```

Hence, $\bar{p} = \bar{s} = 109$

```

In[101]:= Simplify[Solve[-0.0452242 (p - 109) + 0.001 (s - 109) == 0, p]]
Out[101]= {{p -> 106.59 + 0.0221121 s}}

In[102]:= Simplify[Solve[0.0442242 (p - 109) + 0.001 (s - 109) == 0, p]]
Out[102]= {{p -> 111.465 - 0.0226121 s}}

```

à Question 9

(i) Initial situation

```

In[103]:= Clear[pdot, sdot, pbar]
In[104]:= pbar = 105 - 0.5 (20) + 0.5 (10)
Out[104]= 100.

```

Hence, $\bar{p} = \bar{s} = 100$.

(ii) Saddlepaths


```

In[105]:= pdot = -0.1 (0.01) (p - p̄) + 0.1 (0.01) (s - s̄)
Out[105]= -0.001 (p - p̄) + 0.001 (s - s̄)

In[106]:= sdot = (0.8 / 0.5) (p - p̄) + (0.2 / 0.5) (s - s̄)
Out[106]= 1.6 (p - p̄) + 0.4 (s - s̄)

In[107]:= matrixA7 =  $\begin{pmatrix} -0.001 & 0.001 \\ 1.6 & 0.4 \end{pmatrix}$ 
Out[107]= {{-0.001, 0.001}, {1.6, 0.4}}

In[108]:= Eigenvalues[matrixA7]
Out[108]= {0.403951, -0.00495109}

In[109]:= Eigenvectors[matrixA7]
Out[109]= {{-0.00246943, -0.999997}, {-0.245358, 0.969433}}

In[110]:= matrixA7 - 0.403951 IdentityMatrix[2]
Out[110]= {{-0.404951, 0.001}, {1.6, -0.003951}}

In[111]:= Simplify[Solve[-0.404951 (p - 100) + 0.001 (s - 100) == 0, p]]
Out[111]= {{p → 99.7531 + 0.00246943 s}}

In[112]:= matrixA7 + 0.00495109 IdentityMatrix[2]
Out[112]= {{0.00395109, 0.001}, {1.6, 0.404951}}

In[113]:= Simplify[Solve[0.00395109 (p - 100) + 0.001 (s - 100) == 0, p]]
Out[113]= {{p → 125.309 - 0.253095 s}}

```

The saddlepaths are, then,

$$p = 99.7531 + 0.00246943 s$$

$$p = 125.309 - 0.253095 s$$

(iii) Resource discovery

```

In[114]:= Solve[{p == s +  $\frac{4 - (1 - 0.8) (20) + 2 (0.5)}{0.01}$ ,
                p ==  $\left(\frac{105 - 0.5 (20) - 0.3 + 0.5 (10)}{0.8}\right) - \left(\frac{0.2}{0.8}\right) s$ }, {s, p}]
Out[114]= {{s → 19.7, p → 119.7}}

```

The matrix of the system remains unaffected, and so we have the same eigenvalues.

```

In[115]:= matrixA7 - 0.403951 IdentityMatrix[2]
Out[115]= {{-0.404951, 0.001}, {1.6, -0.003951}}

In[116]:= Simplify[Solve[-0.404951 (p - 119.7) + 0.001 (s - 19.7) == 0, p]]
Out[116]= {{p → 119.651 + 0.00246943 s}}

```

```
In[117]:= matrixA7 + 0.00495109 IdentityMatrix[2]
Out[117]= {{0.00395109, 0.001}, {1.6, 0.404951}}

In[118]:= Simplify[Solve[0.00395109 (p - 119.7) + 0.001 (s - 19.7) == 0, p]]
Out[118]= {{p -> 124.686 - 0.253095 s}}
```

Hence, the saddle path equations are

$$p = 119.651 + 0.00246943 s$$

$$p = 124.686 - 0.253095 s$$

à Question 11

■ (i)

```
In[119]:= Clear[s, sdot]
In[120]:= Simplify[Solve[106 - s - 1 == 0.5 (20) - 0.5 (10 + sdot), sdot]]
Out[120]= {{s -> -200. + 2. s}}
```

Hence the differential equation is

$$ds/dt = -200 + 2s$$

■ (ii)

```
In[121]:= Simplify[Solve[0 == -200 + 2 s, s]]
Out[121]= {{s -> 100}}
```

à Question 12

```
In[122]:= Clear[p, s, sdot, pstar, rstar, λ]

General::spell1 :
Possible spelling error: new symbol name "pstar" is similar to existing symbol "rstar".

In[123]:= m - s - pstar - k y + u (rstar + sdot)
Out[123]= m - pstar - s + (rstar + sdot) u - k y

In[124]:= m - s - pstar - k y + u (rstar + sdot) /. sdot -> λ - pstar
Out[124]= m - pstar - s - k y + u (-pstar + rstar + λ)
```

Hence,

$$sbar = m - pstar - k y + u(rstar + λ - pstar)$$