3.4 Noise in Circuits with Feedback

Outline

- Motivation
- Technique to calculate the equivalent input noise sources
- Shunt-shunt feedback
- Series-series feedback
- Series-shunt feedback
- Shunt-series feedback
- Conclusions

Motivation

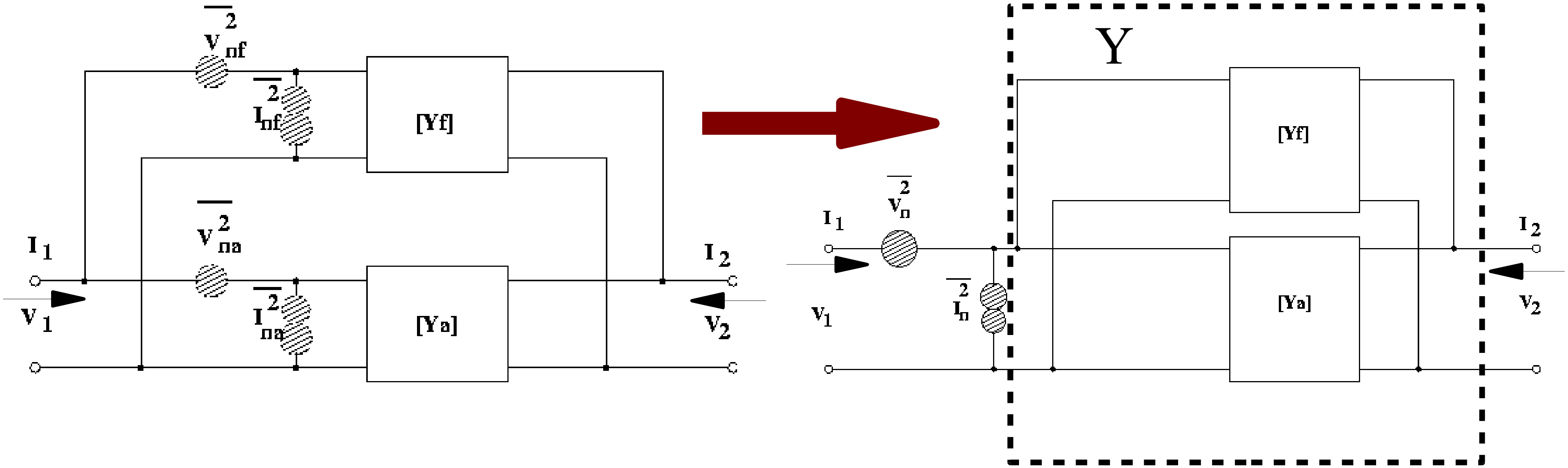
Why study the impact of feedback on noise parameters?

- To understand the impact of negative feedback on:
  - Minimum noise figure
  - Noise impedance
  - Noise matching bandwidth
  - Sensitivity to noise impedance mismatch
- To select the best feedback topology for noise matching
- To select the appropriate feedback topology that minimizes circuit noise in LNAs, mixers, VCOs
- To understand the fundamental trade-offs between
  - noise and DC power
  - Noise and linearity

Technique to derive the input noise sources

- Short circuit the inputs and outputs of the original noisy two-port and of the final noiseless circuit,
    - Calculate the short circuit current at the output in both cases and force them to be equal. You will get the expression for v_n
- Open circuit the inputs and outputs of the original noisy two-port and of the final noiseless circuit
    - Calculate the open circuit voltage at the output in both cases and force them to be equal. You will get the expression for i_n

Shunt-Shunt Feedback



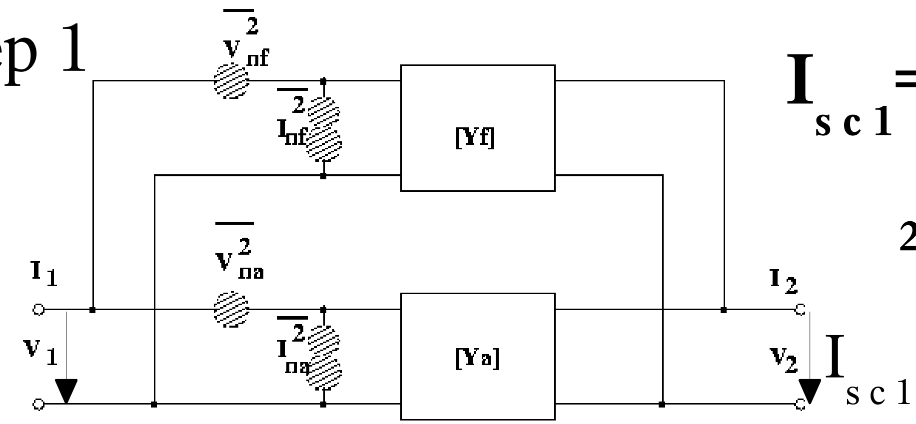
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11f} & Y_{12f} \\ Y_{21f} & Y_{22f} \end{bmatrix} + \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \quad \text{and} \quad [C_y] = [C_{ya}] + [C_{yf}]$$

$$v_n = f(v_{nf}, i_{nf}, v_{na}, i_{na})$$

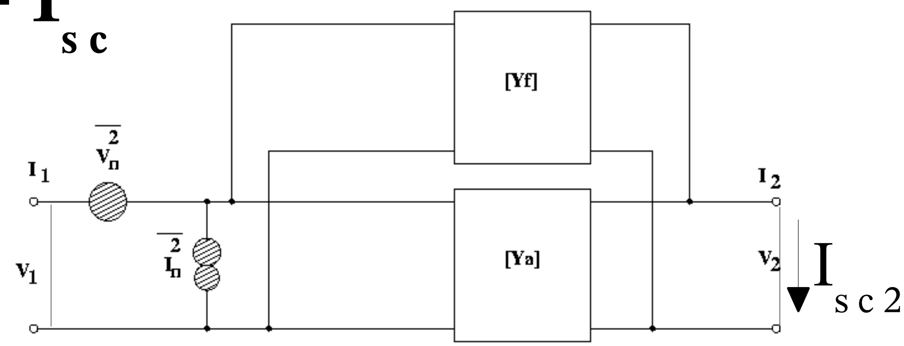
$$i_n = g(v_{nf}, i_{nf}, v_{na}, i_{na})$$

Calculating the input noise sources

Step 1

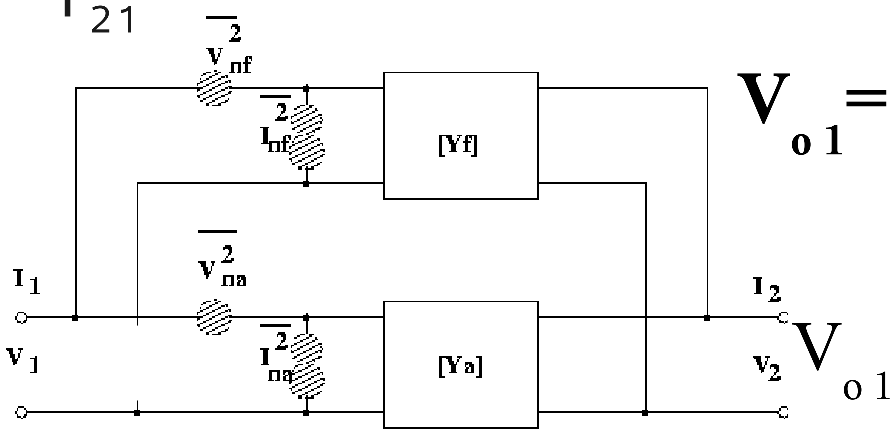


$I_{sc1} = I_{sc}$ 2

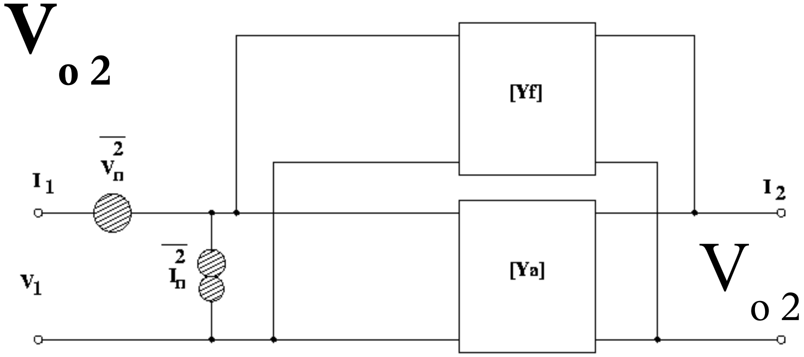


$$v_n = \frac{Y_{21f} v_{nf} + Y_{21a} v_{na}}{Y_{21}}$$

Step 2



$V_{o1} = V_{o2}$



$$i_n = i_{nf} + i_{na} + \frac{Y_{11a} Y_{21f} - Y_{21a} Y_{11f}}{Y_{21}} v_{nf} + \frac{Y_{11f} Y_{21a} - Y_{21f} Y_{11a}}{Y_{21}} v_{na}$$

Unilateral amplifier approximation

$$Y_{21} \approx Y_{21a}; \quad Y_{12} \approx Y_{12f}$$

$$v_n \approx v_{na}$$

$$i_n = i_{nf} + i_{na} + Y_{11f}\left(v_{na} - v_{nf}\right)$$

$$i_n = i_u + Y_{cor} v_n$$

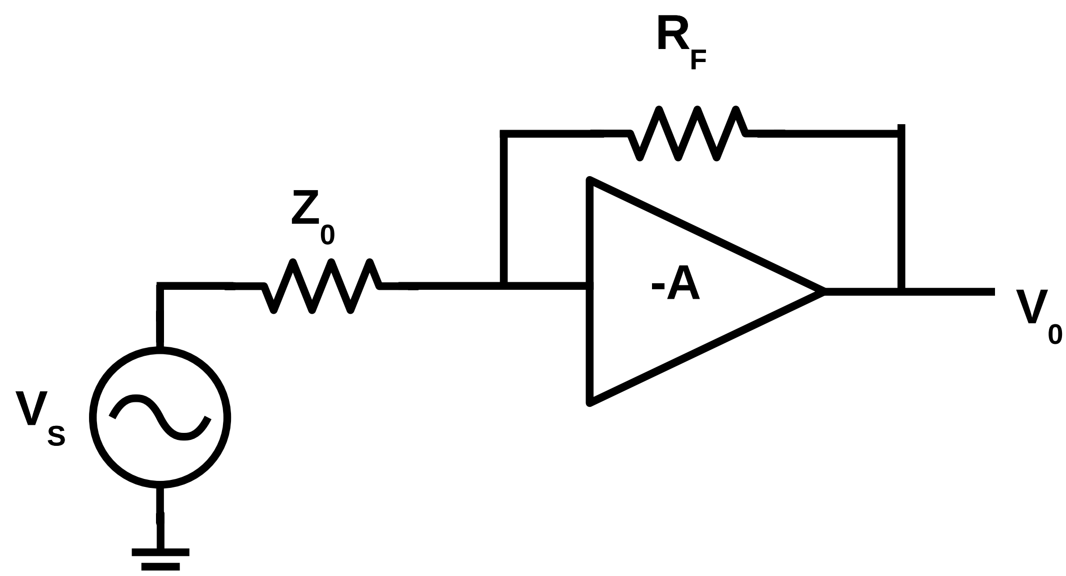
$$R_n \approx R_{na}$$

$$G_u = G_{uf} + G_{ua} + \left|Y_{corf} - Y_{11f}\right|^2 R_{nf}$$

$$Y_{cor} = Y_{cora} + Y_{11f}$$

Noise in Circuits with Shunt-Shunt Feedback

- Noise voltage is that of main amplifier
- Noise currents add
- Z_{in} and Z_{sopt} both decrease
- F_{MIN} increases if feedback has resistive elements
- Use shunt feedback only when amplifier noise impedance is higher than source impedance



Application example #1: Parallel Connection of N Identical 2-Ports

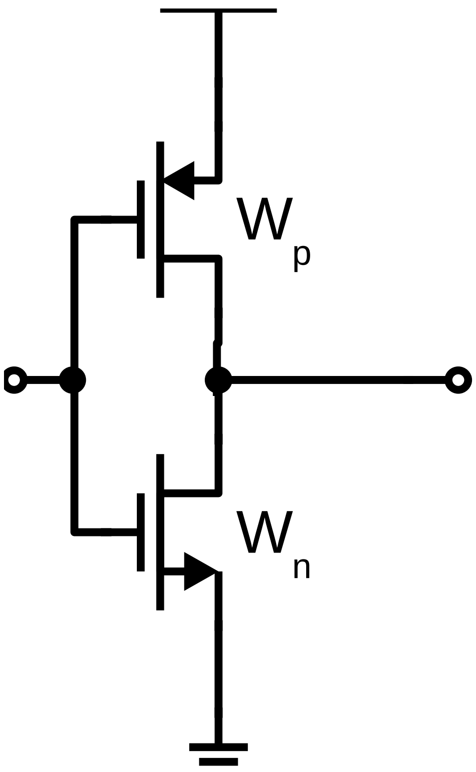
$$C_y = \sum_{i=1}^{i=N} C_{iy} = N \times C_{iy} \ \ and \ \ Y = \sum_{i=1}^{i=N} Y_i = N \times Yi \qquad R_n = \frac{C_{y22}}{|Y_{21}|^2} = \frac{N C_{iy22}}{N^2 |Y_{i21}{}^2|} = \frac{R_{ni}}{N}$$

$$Y_{cor} = Y_{11} - \frac{C_{y12}}{C_{y22}} Y_{21} = N Y_{i11} - \frac{N Ci_{y12}}{N C_{iy22}} N Y_{i21} = N Y_{icor}$$

$$G_u = C_{y11} - R_n |Y_{11} - Y_{cor}|^2 = N C_{iy11} - \frac{R_{ni}}{N} |N Y_{i11} - N Y_{icor}|^2 = N G_{ui}$$

- Ci_y, Yi, R_{ni}, F_{MINi} etc. represent the params of 2-port *i*.
- C_y, Y, R_n, F_{MIN}, G_u represent the params of the final 2-port
- $R_n = R_{ni}/N$, $Y_{spt} = N\times Y_{sopti}$, $F_{MIN} = F_{MINi}$
- By connecting transistors/fingers in parallel the F_{MIN} is preserved while Z_{SOPT} and sensitivity to source mismatch are reduced

Useful case #2: CMOS Inverter



$$W_p = kW_n,$$

$$g'_{m\,p} = g'_{m\,n}/k,\ C'_{gsp} = C'_{gsn};\ C'_{gdp} = C'_{gdn},\ C'_{dbp}$$

$$= C'_{dbn};\ g_{mn}=g_{mp};\ I_{Dn}=I_{Dp};\ R_{gn}=R_{gp},\ kg_{op}=g_{on}\ .$$

$$F_{MIN,CMOS}-1=\frac{1+k}{2}\left[F_{MIN,n-MOS}-1\right]$$

$$\frac{1}{2\pi f_{T,CMOS}}=\frac{1+k}{2}\,\frac{C_{gs,nMOS}+C_{gd,nMOS}}{g_{m,nMOS}}+\frac{1+k}{2}C_{gd,nMOS}\left(R_{s,nMOS}+R_{d,nMOS}\right);\ f_{T,CMOS}=\frac{2}{1+k}f_{T,nMOS}$$

$$1+k$$

$$\frac{g_m}{I_{DS}}=\frac{2\,g_{m,nMOS}}{I_{DS}};\ \frac{R_n}{I_{DS}}=\frac{R_{n,nMOS}}{2\,I_{DS}};\ \frac{R_{sopt}}{I_{DS}}=\frac{R_{sopt,nMOS}}{¿\,I_{DS}};\ X_{sopt}=\frac{X_{sopt,nMOS}}{1+k};\ X_{IN}=\frac{X_{IN,nMOS}}{1+k}$$

Noise in Circuits with Series-Series Feedback

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11f} & Z_{12f} \\ Z_{21f} & Z_{22f} \end{bmatrix} + \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \quad \text{and} \quad [C_z] = [C_{za}] + [C_{zf}]$$

$$v_n = v_{nf} + v_{na} + \frac{Z_{11a} Z_{21f} - Z_{21a} Z_{11f}}{Z_{21}} i_{nf} + \frac{Z_{11f} Z_{21a} - Z_{21f} Z_{11a}}{Z_{21}} i_{na}$$

$$i_n = \frac{Z_{21f} i_{nf} + Z_{21a} i_{na}}{Z_{21}}$$

Noise in Circuits with Series-Series Feedback

$$Z_{21} \approx Z_{21a}; \quad Z_{12} \approx Z_{12f}$$

$$i_n \approx \frac{Z_{21a}}{Z_{21}} i_{na} \approx i_{na}$$

$$v_n = v_{nf} + v_{na} + Z_{11f}(i_{na} - i_{nf})$$

$$v_n = v_u + Z_{cor} i_n$$

$$G_n \approx \frac{|Z_{21a}|^2}{|Z_{21}|^2} G_{na} \approx G_{na}$$

$$Z_{cor} = Z_{cora} + Z_{11f}$$

$$R_u = R_{uf} + R_{ua} + |Z_{corf} - Z_{11f}|^2 G_{nf}$$

Noise in Series-Series Feedback

- Noise current is that of main amplifier
- Noise voltages add
- Z_{in} and Z_{sopt} both increase!
- F_{MIN} increases if feedback has resistive elements
- Use series feedback only when amplifier noise impedance is smaller than source impedance

Series Connection of N Identical 2-Ports

- $G_n = G_{ni}/N$
- $Z_{sopt} = N \times Z_{sopt\,i}$
- $F_{MIN} = F_{MIN\,i}$
- By connecting 2 identical transistors in series the minimum noise figure is preserved while the optimum source impedance and sensitivity to source mismatch are reduced ....

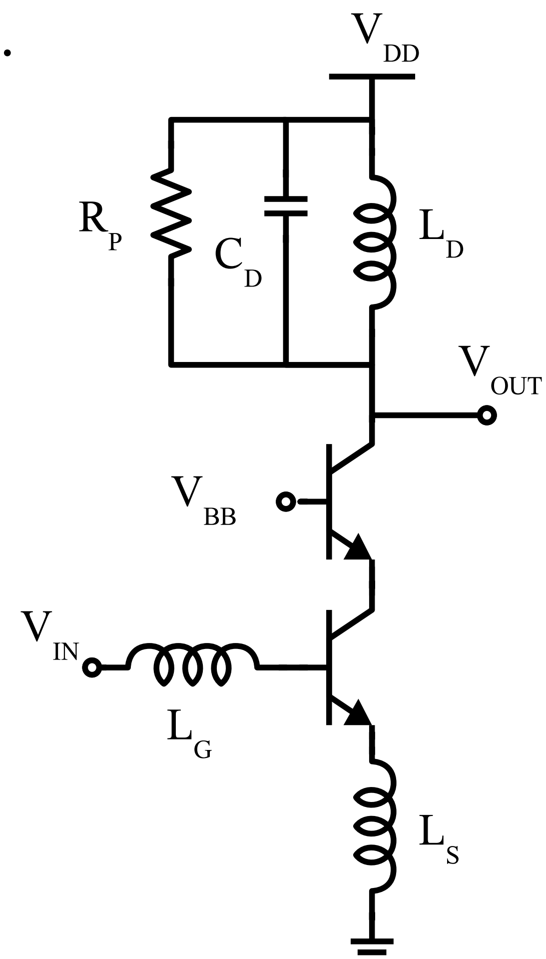
Example: Lossless Series Inductive Feedback

- Add loss-less series element L_{sk}
- The series feedback network is formed by L_S and L_G.
- $V_{nf}=i_{nf} = 0$; $Z_{corf}=0$; $Z_{11f} = j\omega L_G + j\omega L_s$, $Z_{21a} = j R_P f_T/f$

$$G_n = \frac{G_{na}}{\left|1+\frac{j\omega L_S}{Z_{21a}}\right|^2} \approx G_{na} \qquad R_u = R_{ua}$$

$$Z_{cor} = Z_{cora} + j\omega(L_s + L_g)$$

- F_{MIN} is slightly improved! M is preserved.
- R_{sopt} is unchanged (slightly increased at higher f)
- X_{sopt} is modified



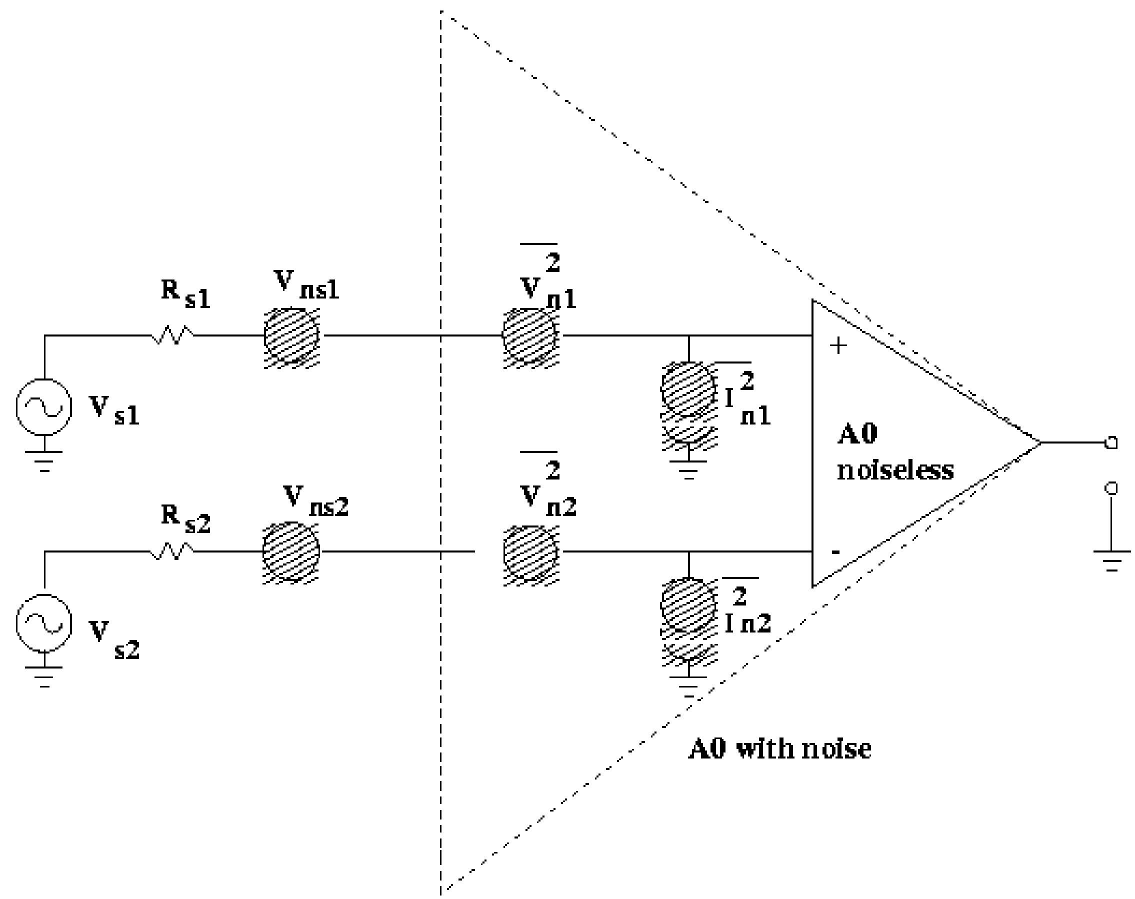
$$Z_{in} = Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22}+Z_L} \approx R_E + R_b + \frac{2\pi R_P f_T L_S + \omega^2 L_S^2}{R_P} \approx R_E + R_b + \omega_T L_S$$

Noise in Series-Series Feedback: Diff Stage

$$v_{o1} = A\left[\left(v_{ns1} + v_{n1} + Z_{s1} i_{n1}\right) - \left(v_{ns2} + v_{n2} + Z_{s2} i_{n2}\right)\right]$$

$$v_{o2} = A\left(v_{ns1} - v_{ns2}\right)$$

$$F = \frac{\overline{v_{o1}^{2}}}{\overline{v_{o2}^{2}}}$$



Noise in Series-Series Feedback: Diff Stage

$$F = 1 + \frac{\overline{v_{u1}^2} + \overline{v_{u2}^2}}{\overline{v_{ns1}^2} + \overline{v_{ns2}^2}} + \frac{\left|Z_{cor1} + Z_{s1}\right|^2 \overline{i_{n1}^2} + \left|Z_{cor2} + Z_{s2}\right|^2 \overline{i_{n2}^2}}{\overline{v_{ns1}^2} + \overline{v_{ns2}^2}}$$

$$F = 1 + \frac{R_{u1} + R_{u2}}{R_{s1} + R_{s2}} + \frac{\left|Z_{cor1} + Z_{s1}\right|^2 G_{n1} + \left|Z_{cor2} + Z_{s2}\right|^2 G_{n2}}{R_{s1} + R_{s2}}$$

$$Z_{s1} = Z_{s2} = \frac{Z_s}{2}; R_{u1} = R_{u2}; G_{n1} = G_{n2}; Z_{cor1} = Z_{cor2}$$

Noise in Series-Series Feedback: Diff Stage

$$F = 1 + \frac{2R_{u1}}{R_s} + \frac{\left|2Z_{cor1} + Z_s\right|^2 \frac{G_{n1}}{2}}{R_s}$$

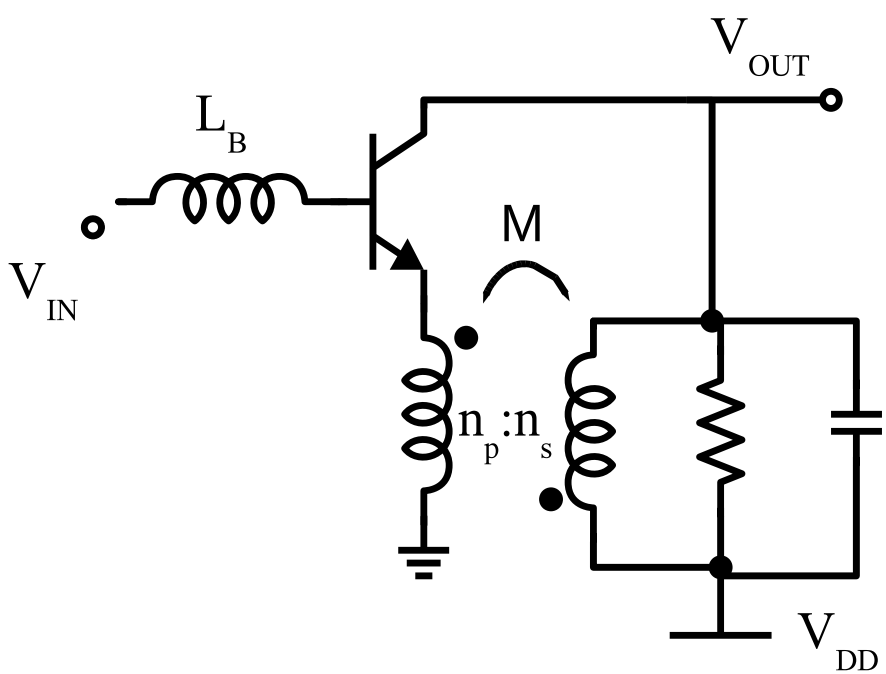
$$R_u = 2R_{u1}; G_n = \frac{G_{n1}}{2}; Z_{cor} = 2Z_{cor}$$

$$X_{soptdif} = 2X_{sopt}; R_{soptdif} = \sqrt{4R_{cor1}^2 + 4\frac{R_{u1}}{G_{n1}}} = 2R_{sop}$$

$$F_{MINdif} = 1 + \left|2R_{sopt} + 2R_{cor1}\right| \frac{G_{n1}}{2} = F_{MIN}$$

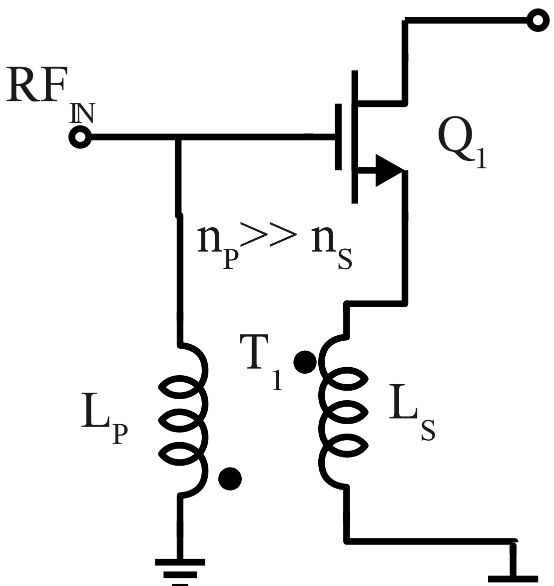
Series-shunt feedback example

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11f} & h_{12f} \\ h_{21f} & h_{22f} \end{bmatrix} + \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} \quad \text{and} \quad [C_h] = [C_{ha}] + [C_{hf}]$$



Shunt-series feedback example

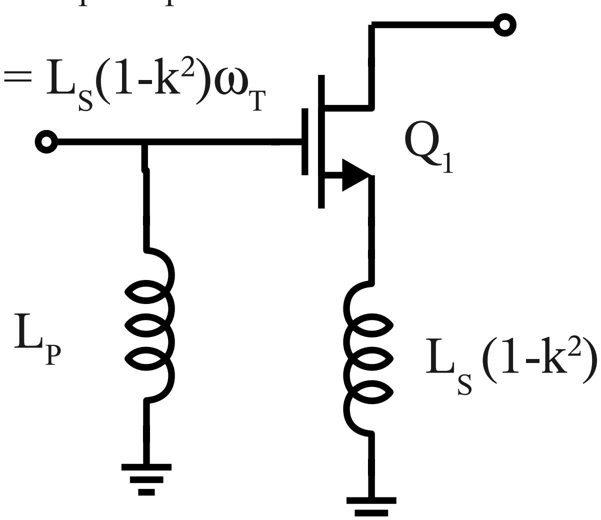
$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} g_{11f} & g_{12f} \\ g_{21f} & g_{22f} \end{bmatrix} + \begin{bmatrix} g_{11a} & g_{12a} \\ g_{21a} & g_{22a} \end{bmatrix} \quad \text{and} \quad [C_g] = [C_{ga}] + [C_{gf}]$$



$\beta = g_{12} = M / L_P$

$a_I = f_T/f$

$Z_{IN} = L_S(1-k^2)\omega_T$



Summary

- Feedback used for *noise* and signal impedance matching
- Fundamental properties of negative feedback with respect to noise:
  - Tor reduce noise impedance matching sensitivity minimize R_n/G_n
  - Noise impedance matching over broad band: want B_{sopt}/G_{sopt} or X_{sopt}/R_{sopt} minimized i.e. low-Q noise impedance
- Lossy feedback helps matching at the expense of increased *NF*
- Lossless feedback does not affect the minimum noise figure
- BUT: Lossless feedback does not affect R_{sopt}