# Focal mechanisms inversion manual.

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## Introduction

This is a set of codes for estimating principal stress directions and principal stress ratio from focal mechanisms. This inversion is based on the approximate method proposed by Gephart and Forsyth (1984) with an additional step to determine the best model with a generalized least squares method (Julien and Cornet, 1987). An initial step is added to check whether the focal mechanisms verify the hypothesis of independence (Maury et al, 2014). This hypothesis is expressed in terms of a minimum distance from previous events. This minimum distance depends on the magnitude of the previous events.

## Approximate method

The problem is considered from an angular point of view. We are looking for the minimum angle that brings into coincidence the fault plane geometry with a geometry that satisfies the hypothesis H1 (faults slip in the resolved shear stress direction).The question is how to define this minimum angle. Defining it as the angle between the observed and predicted slip in the fault plane is a possibility. The normal, n, is, in this case, the axis of rotation. However any other rotation axis bringing into coincidence the observed and predicted slip may also be appropriate. The approximate solution considers only three rotation axes for the minimum angle n, n^s and s, where s is the slip vector. The rotation angle can then be expressed with the direct cosines $\beta_{ij}$ (angle cosines between the local coordinates axis (n, n^s, s) and the principal stress axis. Gephart and Forsyth show that these three axes sample properly the solution space. To avoid a choice among the two nodal planes, the inversion is performed for both planes and the plane selected as fault plane is that which results with the smallest misfit.
An L1-norm is used for evaluating the misfit. If a focal mechanism is not properly constrained it may happen that the wrong nodal plane is selected as fault plane. L1-norm places less weight on erratic data than does the L2-norm. The misfit is defined as the smallest rotation about the three axes n, n^s and s that brings one nodal plane, its slip direction and sense, into an orientation consistent with the stress model. This definition takes into account the error in the orientation of the nodal planes.

## Least squares step

We assume the parameters, strike, dip and rake of the K focal mechanisms, are independent and obey a Gaussian law. The a priori stress tensor of this inversion is the best stress tensor determined in the approximate method. Let $C_0$ be the corresponding a priori covariance matrix i.e. the a priori covariance matrix of the parameters plus the stress tensor reduced components. Since the variables are independent $C_0$ is diagonal.
Let x0 be the vector whose components are the measured parameters and the four unknowns.
Among the L-dimension space considered there exists a set S of points, x, which satisfy:
$f(x)=s_j\underline{T}n_j-[(\underline{T}n_j)**2-(n_j\underline{T}n_j)**2]**1/2=0$

where T is the tensor of the 3 Euler angles ($\psi,\varphi,\theta$) and the relative magnitude R.
This equation can be rewritten as:

$f(x)=0$
$(x-x0)^T C_0^{-1}(x-x0)$

Which is equivalent to:

$x=x0+C_0.F^T.(F..C_0.F^T)^{-1}[F.(x-x0)-f(x)]$

where F is the matrix of the partial derivatives of f(x) taken at point x.
This equation is solved by an iterative algorithm using the fixed point method:

$x_{k+1}=x0+C_0.F^T{}_k.(F_k..C_0.F^T{}_k)^{-1}[F_k.(x_k-x0)-f(x_k)]$

## Independence of focal mechanisms

With respect to the hypothesis of independent focal mechanisms we identify zones of exclusion for all earthquakes, so as to make sure that all focal mechanisms under consideration for the inversion are indeed sampling the natural unperturbed stress field.
The zone of influence of an earthquake is very roughly approximated by a sphere characterized by its radius Rad. Rad is estimated from the size for the earthquake source. We have adopted Pearson [1982] relation between rupture radius and seismic moment:

$$\log(r)=0.33*\log(Mo)-4.18$$

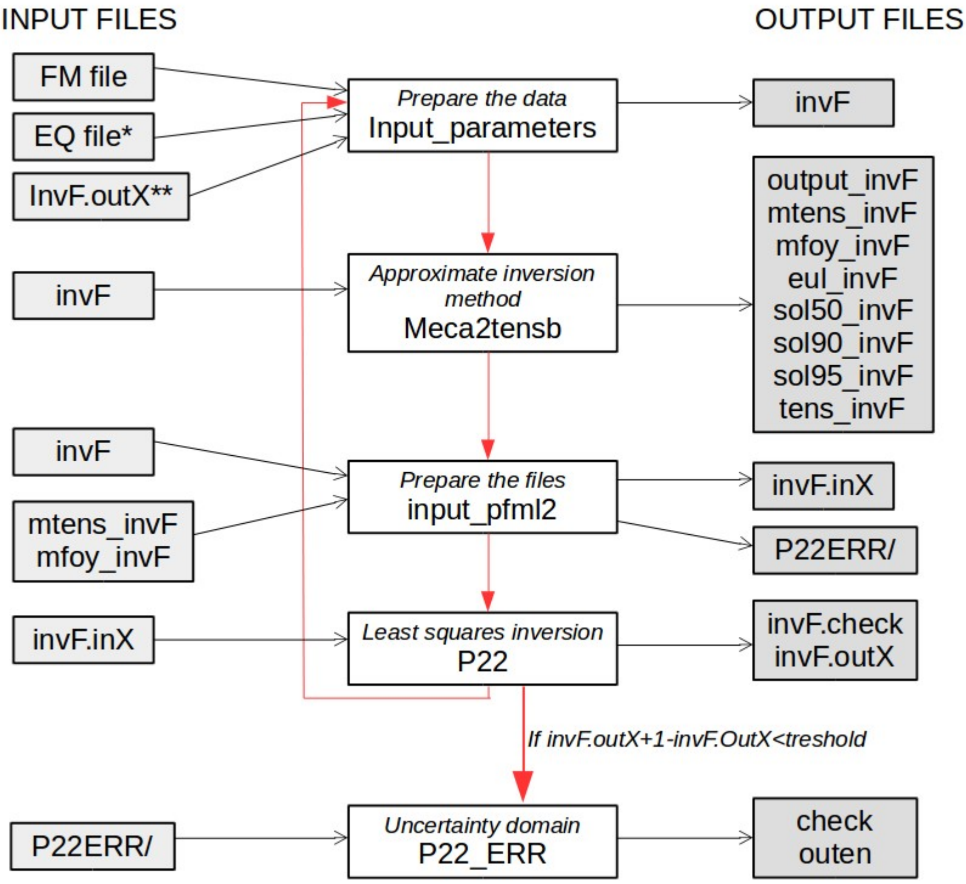
Where Mo is expressed in dyne.cm and r is expressed in meters.
The rupture radius is then connected to the moment magnitude:

$$\log(r)=0.5*Mw+1.12$$

Then, for each event the distance to all previous events is determined and the minimum distance to a previous event, Rmin, is calculated.
Considering event $E_i$, if any previous event $E_j$ ($1\leq j\leq i-1$) has occurred within volume V of radius Rad=rfac*r (Rmin<Rad) we exclude Ei from our dataset.

## Flow chart

The stress tensor is determined using the approximate method in the first step. In addition a weighting factor is added to the misfit computation to take into account the uncertainty on the nodal planes determination. The first step finds which nodal plane yields the minimum misfit and this plane is used as fault plane in the second step. Moreover before performing the second step all focal mechanisms that yield a rotation greater than the uncertainty on their fault plane determination are rejected. The second step does not use the exact method but a least squares inversion. In a last step we check if the nodal planes chosen as fault planes are those with the smallest misfit for the stress field determined. If it is not, a new stress determination is performed. The a priori stress tensor is the one determined by the least squares inversion. This iterative process converges in four to five steps. The use of a L2-norm is justified by the selection of the fault planes in the first step prior to the second inversion.
A graphic method is used to determine the general uncertainty domain. For each focal mechanism, the three parameters (strike, dip, and rake) are assumed independent and a density probability function is calculated for each parameter. Then 1000 random realizations of each Gaussian are generated and with each dataset an inversion is performed. We can determine the 90% confidence limit by keeping the 90% of the solutions nearest to the final one by using Etchecopar et al. definition of the distance between two stress tensors.




*Needed only if the distance criterion is used
**Needed after the first step

## Codes

An explanation of the usage of each program is written next. Some information can also be found at the beginning of each program.

## Name: Input_parameters.m

For use with    Octave (or Matlab)

Octave script to create an input file for the focal mechanism inversion from FM file. An internal program can optionally be called (recommended) to verify the independence of the FM used. This additional program is necessary when using earthquakes from seismic crisis that can be linked to each other. If this step is taken an additional file of earthquakes location is required.
The relation between rupture radius and seismic moment can be modified in exclusion.m (l.33) if a relation other than Pearson's is needed.

Input

FM file: location, magnitude and angle for the first nodal plane
example

| |
|---|
| 1980 7 15 12 17 21.3 47.6727  7.4825 13.57 4.7 4.4 130.0  81.0 171.0 32 24 |

(1980) Year
(7) Month

(15) Day
(12) Hour
(17) Min
(21.3) sec
(47.6727) Lat
(7.4825) Lon
(13.57) Depth
(4.7) ML (not used 0 is ok)
(4.4) Mw
(130.0) Strike
(81.0) Dip
(171.0) Rake
(32) Uncertainty on first nodal plane
(24) Uncertainty on second nodal plane

Earthquake file (optional): location and magnitude
example

```
year month day hour min sec lat lon dep ml mw MF
80 7 15 12 17 21.27 47.67  7.48 13.57 4.7 4.4 F
```

Line 1:
free format, header
Line 2-N:
(80) year
(7) month
(15) day
(12) hour
(17) min
(21.27) sec
(47.67) lat
(7.48) lon
(13.57) depth
(4.7) ml
(4.4) mw
(F) indicator whether a focal mechanism exist (F) or not (N)

Output

File named by the user (named here invF) that contains the information necessary to invert the focal mechanisms.

## Name: Meca2tensb.f

For use with fortran

Fortran code to calculate the principal stress directions and stress ratio from focal mechanisms. This is an approximate method because it only looks at rotation around (n, n^s and s) for bringing the slip vector into coincidence with the resolved shear stress.
The advantage of this method is that it does not require the choice of a fault plane nor adding a supplementary hypothesis to avoid this problem.

Input

None (only the output of input_parameters is needed).
Output

output_invF: description of the results of the inversion step by step
mtens_invF: best stress tensor, with the first line corresponding to the stress tensor with the lowest global misfit, and the following lines corresponding to the stress tensors with the lowest reduced misfit explaining a decreasing number of FM (see output_invF to obtain the number of FM explained by each solution)
mfoy_invF: give the initial FM, the recalculated ones the minimum rotation for each nodal plane and the nodal plane chosen as fault plane
eul_invF: euler angles for the 90% confidence limits area
sol50_invF (resp sol95): stress tensors within the 50% confidence limits
sol90_invF (resp sol95): stress tensors within the 90% confidence limits  but not the 50% conf. lim.(resp 95% but not the 90% conf.lim.)
tens_invF: planes not consistent with the best stress tensors explaining the highest number of MF

## Name: input_pfml2.m

For use with    Octave or Matlab

Matlab script to prepare the input file of the least squares inversion from the output of the approximate method. Exclusion of FM with a rotation greater than their uncertainty, generation of 1000 random FM from the input file to use to determine the uncertainty domain.

Input

None (only the output of Meca2tensb is needed).
A folder where the random realizations will be stocked must be created before using this script.

Ouput

invF.inX (X= number of the inversion)
P22ERR/errXXX input files for P22_ERR

## Name: P22.FOR

For use with    fortran

Fortran code to determine the best principal stress directions and stress ratio with a least squares inversion. The solution of Meca2tensb is used as an initial model.

Input

None (only the output of input_pfml2 is needed).

Output

invF.check: description of the results of the inversion step by step a posteriori covariance matrix and recalculed FM
invF.outX: best stress tensor (3 euler angles + R)

## Name: P22_ERR.FOR

For use with fortran

Fortran code to determine the best principal stress directions and stress ratio with a least squares inversion for the random realizations picked in input_pfml2.m. The solution of Meca2tensb is used as an initial model.

Input

None (only the output of input_pfml2 is needed).

Output

output best stress tensor (3 euler angles + R) for each random realization.
check: description of the results of the inversion step by step a posteriori covariance matrix and recalculated FM

## Name: plot_pfml2.m

For sue with Octave (or Matlab)

Octave script to plot the results of P22.FOR and P22_ERRFOR

For references, see main volume.

**Examples of imput files** are given by files loctot and locmec and are discussed in Maury et al., 2013 (see complete reference in book).