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where  $\hat{K}$  describes the propagation direction and  $K = 2\pi/\Lambda = \Omega/v_a$  is the propagation constant with  $v_a$  being the acoustic velocity. A *standing* plane acoustic wave is a combination of two contrapropagating traveling waves of equal amplitude, wavelength, and frequency:

$$\mathbf{u}(\mathbf{r},t) = \mathcal{U}\cos\left(\mathbf{K}\cdot\mathbf{r}\right)\cos\Omega t. \tag{2.80}$$

An acoustic wave polarized in the direction of **K** is known as a *longitudinal wave*, while one with a polarization perpendicular to K is called a *transverse wave*. For any given direction of propagation in a medium, there are *three* orthogonal acoustic normal modes of polarization: one longitudinal or quasi-longitudinal mode, and two transverse or quasi-transverse modes.

The mechanical strains associated with deformation are described by a symmetric strain *tensor*,  $\mathbf{S} = [S_{ij}]$ , defined by

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{2.81}$$

where the indices i, j = x, y, z. The three tensor elements  $S_{xx}, S_{yy}$ , and  $S_{zz}$  are *tensile strains*, while the other elements  $S_{yz} = S_{zy}$ ,  $S_{zx} = S_{xz}$ , and  $S_{xy} = S_{yx}$  are *shear strains*. In addition, there is an antisymmetric *rotation tensor*,  $\mathbf{R} = |R_{ii}|$ , defined by

$$R_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right).$$
(2.82)

Clearly,  $R_{xx} = R_{yy} = R_{zz} = 0$ , while  $R_{yz} = -R_{zy}$ ,  $R_{zx} = -R_{xz}$ , and  $R_{xy} = -R_{yx}$ . For elastic deformation caused by an acoustic wave, all of the strain and rotation tensor elements are space- and time-dependent quantities.

Mechanical strain in a medium causes changes in the optical property of the medium due to the *photoelastic effect*. The basis of acousto-optic interaction is the *dynamic photoelastic effect* in which the periodic time-dependent mechanical strain and rotation caused by an acoustic wave induce periodic time-dependent variations in the optical properties of the medium. The photoelastic effect is traditionally defined in terms of changes in the elements of the relative impermeability tensor:

$$\eta_{ij}(\mathbf{S}, \mathbf{R}) = \eta_{ij} + \Delta \eta_{ij}(\mathbf{S}, \mathbf{R}) = \eta_{ij} + \sum_{k,l} \left( p_{ijkl} S_{kl} + p'_{ijkl} R_{kl} \right),$$
(2.83)

where  $p_{iikl}$  are dimensionless *elasto-optic coefficients*, also called *strain-optic coefficients* or photoelastic coefficients, and  $p'_{ijkl}$  are dimensionless rotation-optic coefficients. Both are fourthorder tensors. Because  $\eta_{ij} = \eta_{ji}$  and  $S_{kl} = S_{lk}$ , the  $[p_{ijkl}]$  tensor is symmetric in *i* and *j* and in *k* and l. Because  $\eta_{ij} = \eta_{ji}$  and  $R_{kl} = -R_{lk}$ , the  $[p'_{ijkl}]$  tensor is symmetric in i and j but is antisymmetric in k and l.

The photoelastic effect exists in all matter, including centrosymmetric crystals and isotropic materials, because the  $[p_{ijkl}]$  tensor never vanishes in any material though the  $[p'_{ijkl}]$  tensor vanishes in isotropic materials and cubic crystals. Acousto-optic interactions are not precluded by any symmetry property of a material. The tensor form of  $[p_{ijkl}]$  for a crystal is determined by the point group of the crystal. The  $[p'_{ijkl}]$  tensor elements of a crystal are determined by the birefringence of the crystal. If the indices i, j, k, are l referenced to the principal axes of a crystal, we have