## COMMENTS AND ERRATA FOR CHAOTIC DYNAMICS TEXT

Here are corrections and comments as they stand so far. I wish to thank Christopher Grant of BYU for pointing out many of the typos and errors. Please let me know at ggoodson@towson.edu, of any other typos/inaccuracies that you or your students find.

## Geoff Goodson

Page 17, Exercises 1.2.8: the term "eventually periodic" is not defined until Definition 2.3.1(ii). Replace the second sentence by: Show that every point in S is either a fixed point or is mapped to a fixed point.

Page 44, Exercises 2.2.4: should read "into the interval"

Page 44, Exercises 2.2.6(a): replace  $1 - 1/\mu < 1/2$  by  $1 - 1/\mu \le 1/2$ 

Page 50, Exercises 2.3.9 This question would be better in a different section with more explanation. If  $F_{\mu}$  is independent of  $\mu$  and has a super-attracting 2-cycle, then this would violate this exercise.

Page 69, Exercise 2.8.3: the third term should read  $\frac{\mu^3}{1+\mu^3}$ 

Page 71, Lemma 3.1.3 requires that I be a compact interval (I = [a, b] for some  $a, b \in \mathbb{R}$ , a < b, as is mentioned in Theorem 1.2.9).

Page 72, line 2: The two characterizations of  $\alpha$  are not equivalent. Replace I with  $I \cap [p, q]$ .

Page 77, Exercise 3.2.1: The problem is not that f is discontinuous (since it is continuous on its domain), but that its domain is not connected.

Page 78, Exercise 3.2.11: using Sharkovsky's Theorem is not necessary as this can be shown directly.

Page 78, Exercise 3.2.12: should read "Let  $f : [a, b] \to [a, b]$  be a ...". (Otherwise [a, b] = [0, 1], f(x) = 1 - 2x and c = 0 is a counterexample).

Page 82, Example 4.1.5: should also mention that [0, 1] is an open ball.

Page 85, Theorem 4.2.6: The proof of (iii)  $\Rightarrow$  (i) needs some adjustment since not every point of X need be a limit point of A (for example if  $x \in A$  is an isolated point).

Page 94, Exercise 4.3.9: Should read "Let X be a metric space that is not the union of two non-empty open sets".

Page 99, Exercise 4.4.5: Add " $f(x) \neq x$ "

Page 127, line 1: first line should read "Suppose that  $s_j \neq t_j$  for some ..."

Page 130, the parenthetical remark in the proof of Lemma 6.6.2 belongs in the third sentence, not in the second sentence.

Page 131, line -12, replace "=" by " $\subset$ ".

Page 137, Exercises 7.1.7: line 4 should read "Use the conjugacy  $h:[-1,1]\rightarrow [-1,1],$  "

Page 142, Exercises 7.2.2: should read "show that f is a factor of D"

Page 142, Exercises 7.2.3(b): should read "Show that g is a factor of the map  $F: [0,1] \to [0,1], F(x) = 3x \pmod{1}$ "

Page 142, Exercises 7.2.7: assume that f and g are differentiable.

Page 145, Exercise 7.3.3(e) should read "Deduce that if c < 0, then "

Page 149, line -12: " $f'''(x_0) > 0$ " should read " $f'''(x_0) \ge 0$ ".

Page 150, line 7: "negative" should read "non-positive".

Page 150: In the proof of Corollary 8.2.1, the possibility that the basin of attraction is [0, 1] is neglected.

Page 150, Exercises 8.2.6(c): should read "if and only if p'(x) > 0 for all x"

Page 150, line -2, " $L_{\mu}(x)$ " should read " $SL_{\mu}(x)$ ".

Page 153, Exercises 8.2.9(b): should read "Use Lemma 8.1.1 to show.."

Page 164, Exercises 9.2.5: should read " $\subseteq [\mu - \mu^2/2, \mu/2]$ ."

Page 171, line -9: should read "dim $(M) = \lim_{n \to \infty} \frac{\log 20^n}{\log 3^n} = \dots$ "

Page 178, Theorem 10.5.5 needs the hypothesis that  $X \neq \emptyset$ .

Page 180, line 6: Replace "<" by " $\leq$ ".

Page 180, Proposition 10.5.8, line 2 of the proof: Replace " $z = f(y) \in f(U_{\delta}(A))$ " by " $y \in f(U_{\delta}(A))$ "

Page 181, line 2: Replace > by  $\geq$ .

Page 184, line -2, (start of Section 11.1), replace " $b \in \mathbb{Z}^+$ " with " $b \in \mathbb{Z}^+, b > 1$ ".

Page 186, Example 11.1.3(3), replace " $\cdot \overline{00011}$ " with " $\cdot 0\overline{0011}$ ".

Page 186, line 14 should say " $r \in [0,1] \setminus \mathbb{Q}$ " instead of " $r \notin \mathbb{Q} \cap [0,1]$ ".

Page 197, line -6, "f(x) - x" should read " $f^2(x) - x$ ".

Page 198, line -3, In the proof of Coppel's Theorem,  $q \leq r$  , is not used, so should be deleted.

Page 202, line 5: "f(x) < z" should read " $f(x) \le z$ ".

Page 207, Exercise 12.3.10(a): should read "then f has a fixed point z < b..."

Page 210, line 12: Replace " $\nu \in \mathbb{R}^n$ " by " $\nu \in \mathbb{C}^n$ ".

Page 213, line -3: Should read "of algebraic multiplicity two, with a onedimensional eigenspace (geometric multiplicity one),". Page 215, Exercise 13.1.5: should read "has a single eigenvalue of algebraic multiplicity two, ..."

Page 215, Exercise 13.1.6: should read "Show A has a single eigenvalue of geometric multiplicity one, ..."

Page 216, Exercise 13.1.7(d): should read "... triangle  $\{(\tau, \Delta) : \Delta < 1, 4\Delta > \tau^2\}$ . ..."

Page 226, line 7: should read "Let  $(x_1, x_2)^t \in \mathbb{T}^2$  ..."

Page 234, Exercise 14.2.5: delete the hint.

Page 234, Section 14.3, line 10: should read "as the closure of the repelling periodic points of f."

Page 238, Exercise 14.3.8: should read "...and the set  $D_r = \{z \in \widehat{\mathbb{C}} : d(z, 0) > r\}$  is open ..."

Page 239, Section 14.4, line 10: should read "so they represent all such polynomials ..."

Page 243, The last paragraph of Theorem 14.4.5: should start: "The continuity of  $F_c$  implies that each of the sets  $F_c^{-n}A$  is closed and they are non-empty (from the Fundamental Theorem of Algebra)." In addition, there are missing A's in the last three lines of the proof.

Page 244, The Julia set on the left is actually for c = -.4 - .3i.

Page 244, line 1 of the proof, delete the "a" before "closed". At the end of line 4 of the proof, add "(other than  $\mathbb{C}$  itself, which is a non-empty open set in  $\mathbb{C}$  having empty boundary)." In the last line of the proof, replace "invariant" by "completely invariant".

Page 246, Theorem 14.4.9(a) should read "If  $0 \notin K(F_c)$ , then the Julia set of  $F_c$  is a type of Cantor set (a totally disconnected set called *fractal dust*)."

Page 254, Section 14.5, line 10: should read "... $z^2 - z + c = 0$  and  $|f'_c(z_1)| < 1$ ...". There is a missing closed parenthesis on line -10, and one of the repeated factors there should have a minus in place of a plus.

Page 255, Exercise 14.5.1(c): should read "(Hint: Modify the proof of Proposition 14.4.4) ..."

Page 262: Remove Exercise 14.6.2(c) as it appears earlier in the text (see Exercises 14.3.3), and relabel Exercise 14.6.2(d) as Exercise 14.6.2(c).

Page 270, Exercise 14.7.8: line 2 should read: "Prove that if  $z \neq 0$  is purely imaginary...".

Page 270, Exercise 14.7.11: should read: (a) If g = f/f', where f is sufficiently many times differentiable, derive the formulas for Halley's Method, of Section 14.7.12 for  $N_g$  and  $N'_g$ .

(b) If instead  $g = f/(f')^{1/2}$ , prove that  $N'_g(\alpha) = N''_g(\alpha) = 0$  and  $N'''_g(\alpha) = -Sf(\alpha)$ , where Sf is the Schwarzian derivative of f.

Page 276, line -3: should read: " $\mathcal{A}^{\mathbb{N}}$  is the set..."

Page 293, Exercise 16.1.4(c): We should assume that m is even. The question is better read as: "is rotated through  $-\pi/3$  radians and then translated by  $\alpha(2^n)$ , show...". (First rotate keeping  $\alpha(0)$  fixed at the origin, then translate).

Page 310: Exercise 16.4.7: Interchange exercises (b) and (c). (Since  $\gamma$  is only defined in (c)).

Page 312, In Definition 17.1.4 insert "(Limit points of the sequence  $(x_n)$  are simply those of the set  $\{x_n : n \in \mathbb{N}\}$ ).", at the end of the sentence in line 2 (limit points have been defined only for sets up to this point).

Page 326, Exercise 17.4.6(c): Need the added hypothesis that y be periodic, not just in O(x).

Page 337, line -11: The inclusions go the wrong way - replace " $A_0 \subseteq A_1 \subseteq A_2 \subseteq \cdots \subseteq A_{m-1}$ ", by " $A_0 \supseteq A_1 \supseteq A_2 \supseteq \cdots \supseteq A_{m-1}$ ".

Page 345, Lemma 18.3.2: In the first part of the proof, assume that w is not a periodic sequence, as the result is clear in this case. Similarly in the second part, w may be a periodic sequence.

Page 345, lines 6 and 7 should read: "Note that O(u) is a finite set if and only if u is shift periodic or shift eventually periodic".

Page 346. In Example 18.3.7, it may be useful to avoid confusion by changing the subscripts to superscripts.

Page 347, line 2 should read: " $w_n = 111...11$ , a string of 1's having the same length as  $u_n$ ".

Page 349, Exercise 18.3.4(a) should read "Conversely, show that if O(u) is finite, then u is periodic or eventually periodic".

Page 353. In Exercise 18.4.6 there should be a capital F in the right-hand side of the displayed equation. Also in part (b) there are a couple of places where "f" needs to be changed to "F".

Page 357, line 4: delete the "a" before Sturmian.

Page 361, Proposition 19.2.2, line 2: replace " $f_{\alpha}(n)$ " by " $f_{\alpha}(i)$ ".

Page 362, line 4: delete "essentially".

Page 362, line 12. Can use the notation " $\alpha = [0,\overline{1}]$ ", in place of " $\alpha = [0,1,1,1,\ldots]$ ".

Page 363, line -8, replace " $\theta$ " by " $\theta_k$ ".

Page 369, The indexing is off in the proof of Theorem 19.4.1. To fix this, in line 9 of page 369, replace " $E_n$ " by " $E_{n-1}$ ". Do the same in line 1 of Lemma 19.4.3 on

Page 380, lines 3 and 8, replace "open set" by "nonempty open set".