X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 10. X-ray& EUV Optics Homework Problems

10.1 (a) Calculate and plot reflectivity versus photon energy for iridium (Z = 77) at a 0.45° glancing angle (7.8 mrad, 27 arc min). Use the website, www.cxro.LBL/optical_constants . Plot your results as reflectivity on a linear vertical scale vs. photon energy on a semilog horizontal scale. Also calculate and plot on the same graph reflectivity for a gold (Z=79) mirror at 2°, and nickel (Z=28) at 1.7°. Label the axes and the three curves. Compare your results with Figure 10.2 on page 450. (b) How might these mirrors be used in an x-ray telescope?

10.2 The Chandra X-ray Observatory, an orbiting x-ray telescope launched July 1999, utilizes four nested pairs of iridium coated Wolter Type I optics (parabola + hyperbola) to provide sub-arcsecond imaging covering the spectral region from 0.08 to 10 keV, and gratings to provide fine spectra. In preparation for selection of the mirror coatings several candidate materials were considered. Reflectivity calculations for iridium mirrors were compared with measured values. (a) Calculate the reflectivity of an iridium x-ray mirror for 6.4 and 8.1 keV x-rays, for incidence angles from 10 to 55 arcminutes, and compare your results with those presented in the 2014 RSI paper by D.A. Schwarz, his Figure 3. (b) Discuss the impact of *Chandra* on the understanding of astrophysical objects such as supernova, astrophysical jets, galaxy clusters, massive black holes, and dark matter. See the papers D.A. Schwartz,"The Chandra X-ray Observatory", *Rev.Sci.Instr.* **85**, 061101(2014); M.C. Weisskopf, B. Brinkman, C. Canizares, G. Garmire, S. Murray and L.P. van Speybroeck, "An Overview of the Performance and Scientific Results from the Chandra X-ray Observatory", *Pub.Astron.Soc.Pacific*, **114**(791), 1 (January 2002); and M. Santos-Lleno, N. Schartel, H. Tananbaum, W. Tucker and M. Wisskopf, "The First Decade of Science with Chandra and XMM-Newton", *Nature* **462**, 997 (24 December 2009).

10.3 (a) Describe some general considerations relevant to the choice of materials for a multilayer interference coating for use at a given photon energy. (b) Comment on the choice of angle of incidence, required *d*-spacing, choice of the low-Z "spacer" material, choice of the high-Z "scattering" material, optimum choice of Γ, required number of bi-layer pairs (N), and specification on substrate roughness. Be sure to consider materials compatibility, interface roughness, and chemical stability at the exposed surface.

10.4 You are designing an EUV telescope to observe the solar corona in the spectral vicinity of the Fe^{+10} (ten times ionized iron) emission line at 18.04 nm.
(a) What multilayer coating might you use for a normal incidence optic?
(b) Give the materials, d-spacing, Γ, σ, and N.
(c) What reflectivity do you anticipate? For information on spectral emission lines of highly ionized atoms of interest for x-ray astronomy. See the paper by G.A. Doschek and R.D. Cowan, "A Solar Spectral Line List Between 10 and 200 Å Modified for Application to High Spectral Resolution X-Ray Astronomy," *Astro. Phys. J. Suppl.* ***56***, 67 (1984).

10.5 (a) Plot the reflectivity of a Mo/Si coating for wavelengths from 12 to 15 nm, at a near normal incidence angle of 85°, assuming d = 6.89 nm, Γ = 0.44, and N = 40. Show results for a perfectly smooth, infinitely sharp interface (σ = 0), and for interfaces characterized by σ = 0.5 nm rms. (b) What does this indicate regarding surface finish specifications for high mirror reflectivity? (c) What throughput would be expected for a nine-mirror optical system with σ = 0.5 nm rms? Feel free to use the multilayer sub-routine at the website www.cxro.LBL.gov/optical_constants.

10.6 (a) Repeat problem 10.5 for a Mo/Be multilayer coating with d = 5.7 nm, Γ = 0.40, N = 40, and σ = 0.1 nm rms, for wavelengths from 11 to 15 nm.
(b) Compare the peak reflectivity and relative spectral bandpass to that of Mo/Si at $\sigma = 0.1$ nm rms.
(c) How does this change if one employs 70 bi-layer pairs of Mo/Be?
(d) What are the implications for an EUV optical stepper (lithographic optical reduction print camera) with seven multilayer coated surfaces?
(e) What role might the emission spectrum of the source play in the apparent advantage of Mo/Be?
(f) What disadvantages might there be?

10.7 You are interested in using polarimetry to probe the magnetic properties of iron containing materials with polarized radiation near the iron L-edge.
(a) What photon energy range would be appropriate? You plan to use Brewster's angle optics to polarize the incident radiation and to measure Faraday rotation of the polarization vector as it traverses the material while exposed to an applied magnetic field.
(b) Describe the parameters of a W/C multilayer mirror to achieve the desired polarization, including the d-spacing, Γ, N, and σ.
(c) How can a polarimeter be constructed? Refer to the paper of J.B. Kortright *et al* in Phys. Rev. B 51, 10240 (1995).

10.8 An x-ray fluorescence nanoprobe uses multilayer coated Kirkpatrick-Baez (KB) optics to focus hard x-rays to a 30 nm diameter spot size to study elemental compositions of samples for materials science, environmental sciences, and parts-per-billion (ppb) impurity analyses in which fluorescent emissions are detected simultaneously from many elements. This is done with thick samples, in air, and in the presence of higher Z materials if necessary.
(a) Plot the reflectivity of a single W/C multilayer mirror from zero to 12 keV photon energy, for d = 4.0 nm, Γ = 0.40, N = 50 layer pairs, and a glancing incidence angle of 1.2°. Assume σ = 0.1 nm rms.
(b) What is the peak wavelength passed by the multilayer coating and what is the relative spectral bandpass?
(c) How does this relative spectral bandwidth compare to 1/N?
(d) For what elements could multi-keV fluorescence emission from K-shells be observed?
(e) From L-shells of which elements?
(f) What numerical aperture is required for the KB optics?

10.9 (a) Compute diffraction efficiencies to orders m = 1 to 5 for an opaque transmission grating with line to space ratios of 1:2, that is, where the opaque line widths are $d/3$ and the spaces are $2d/3$. (b) Compare this with the diffraction efficiencies of a symmetric (1:1) grating. (c) What does this suggest about the appearance of even order diffraction from gratings and zone plates?

10.10 Following the nomenclature in Figure 10.21, draw a right triangle containing the on-axis focal length f as one leg, the radial distance to the outer zone r_n as the second leg, and hypotenuse $f + n\lambda/2$, adding a half-wavelength to the hypotenuse from the edge of each added zone. (a) Confirm, using the Pythagorean theorem, that zone plate radii are given by

$$r_n^2 = n\lambda f + \frac{n^2\lambda^2}{4}$$

(b) Show that for a lens of small numerical aperture, NA $\ll$ 1, the $n^2\lambda^2/4$ term can be neglected, yielding the simpler expression for zone plate radii

$$r_n \cong \sqrt{n\lambda f}$$

(c) Show that in this limit sequential zones (and thus all zones) have equal area (hint: form the difference $\left(r_n^2 - r_{n-1}^2\right)$. (d) Why are equal zone areas important to the focusing and imaging properties of a zone plate lens?

10.11 Give the specifications for a zone plate lens designed to achieve a spatial resolution of 25 nm with uniform spherical wave illumination ($R \rightarrow \infty$) at a wavelength of 3.1 nm. Assume the radiation source has a relative spectral bandwidth $\Delta\lambda/\lambda = 10^{-3}$. Design the lens to have the largest possible focal length. Give the outer zone width Δr, the number of zones, N, the diameter, focal length, numerical aperture, and the depth of focus. How might depth of focus, or depth of field (DOF), affect a high resolution imaging experiment? What might this require as a 'fix'?

10.12 In Section 10.6.2, calculation of diffraction by a zone plate lens shows that the focal plane intensity is characterized by an Airy pattern, Eq. 10.40), with a first radial null at $r_{null} = 0.610\lambda / NA$, Eq.(10.41, where for x-rays NA = sinθ and where θ is measured from the axis to the edge of the lens. Indeed, this formula holds for focusing of uniform plane waves by all aberration free lenses, where diffraction from the limiting aperture sets the focal region intensity pattern, what we know as the Airy Pattern. Thus the intensity profile for all 'perfect lenses' is set by just the wavelength and numerical aperture, as described by Eq.(10.41). Of additional interest is the FWHM of the Airy pattern, 1.029 Δr, as described in the footnote on p.484, the nominal width of the focal region intensity pattern. Noting that for a zoneplate the NA is given by NA = λ/2Δr, show that in general, for perfect lenses of any kind, the FWHM of an Airy intensity pattern given by $D_{FWHM} = 0.515\lambda / NA$.

10.13 You are designing a soft x-ray microscope for imaging thick hydrated biological samples at high spatial resolution. Your goal is to achieve a spatial resolution ten times better than that of a scanning confocal visible light microscope operating at 486 nm wavelength with a 1.4 NA (NA= n sin θ) oil immersion objective lens. You choose a soft x-ray wavelength that will provide the greatest contrast by absorption between the organic material and water. (a) What wavelength will you use? (b) What outer zone width will you require for the lens in a scanning microscope? (c) What will be the zone plate lens's numerical aperture (NA)? (d) How might relative spectral bandwidth of the source affect performance? (f) Describe the proper illumination for the scanning microscope, and also for a full field microscope. How might the illumination characteristics affect achievable spatial resolution? (g) Are equal outer zone widths required for both microscopes to achieve the same spatial resolution?

10.14 (a) Discuss potential aberrations in a zone plate lens. (b) How would each affect imaging properties? (c) What measurements would reveal these aberrations? (d) How might each be avoided in fabrication or mounting of the zone plate?

10.15 For a scanning zone plate microscope what experimental features of the beamline are important for achieving near-theoretical ("diffraction limited") performance? What is the penalty for not strictly implementing this?

10.16 You are designing the pinhole illumination for a scanning zone plate microscope. To obtain near diffraction limited resolution you choose a small pinhole so as to overfill the zone plate. If your choice of wavelength and pinhole diameter lead to an Airy pattern with first null diameter equal to twice the zone plate diameter, (a) what will be the intensity variation across the zone plate, and (b) what fraction of the radiation through the pinhole would be captured by the lens? (c) How does this intensity variation affect microscope performance? Express your answer as the ratio of focal spot FWHM to that of the ideal Airy pattern obtained with uniform illumination of the same lens. (d) Extend your analyses to situations where the illuminating Airy pattern null diameter ranges from one to four times the zone plate diameter. Graph intensity variation (min/max), ratio of focal spot diameters (FWHM), and the fractional radiation collection by the lens, as a function of the illumination parameter (Airy null to zone plate diameter).

10.17 You are designing a scanning photoemission microscope for studying surfaces containing iron, cobalt, and nickel. The studies will require observations of the respective L-edge structure with a relative spectral resolution of 3×10^{-4}, and a spatial resolution of 20 nm. You wish to have a long focal length zone plate lens so as to provide the greatest working distance (sample to lens separation) and thus least complex photoelectron collection. (a) What specifications will you set for the zone plate lens? Give all parameters. (b) What type of experiments could be performed with this microscope?

10.18 (a) Determine the efficiency to first order of 100 nm thick gold, nickel, and germanium phase zone plates, at a photon energy of 500 eV, using the Kirz formula [J. Kirz, *JOSA* **64**, 301 (1974)]

$$\eta_m = \frac{1}{m^2\pi^2}\left(1 + e^{-2\varphi\beta/\delta} - 2e^{-\varphi\beta/\delta}\cos\theta\right)$$

where η_m is the efficiency to odd order m, $m = \pm 1, \pm 3, \ldots$ for the above formula, δ and β are the real and imaginary components of refractive index at wavelength λ, and $\varphi = 2\pi t\delta/\lambda$ is the phase shift for material zones of thickness t. (b) What are the efficiencies for these three materials at a photon energy of 800 eV? For δ and β values of nickel and gold consult Appendix C. For germanium and other materials of interest to you, consult the website www.cxro.LBL.gov/optical_constants.

10.19 A common hard x-ray crystal monochromator which provides wavelength tunable, high spectral resolution, of order 10^{-4} to 10^{-5}, with a fixed exit slit and forward collinear transmitted radiation, is known as a double crystal monochromator (DCM). (a) For silicon crystals with d=3.1355Å, what is the Bragg angle for 10 keV and 20 keV. (b) Based on the single crystal reflectivity curves in Figure 10.40, page 502, sketch the transmission throughput for a DCM at these two photon energies. (c) Draw an approximate diagram for the Si DCM at these two energies. It may be helpful to consult figure 10 in the paper by B.W. Batterman and D.H. Bilderback, "X-ray Monochromators and Mirrors", pp. 105-151 in *Handbook on Synchrotron radiation*, Vol. 3, (Elsevier Science, 1991), G. Brown and D.E. Moncton, Editors; or figures 6.10 and 6.16(a) in Chapter 6 of the text book by J. Als-Nielson and D. McMorrow, *Elements of Modern Physics* (Wiley,2011), Second Edition; also consult the textbook by Y. Shvyd'ko, *Elements of Modern X-Ray Physics* (Springer, Berlin, 2004.

10.20 (a) Explain the Borrmann Effect. (b)Why is this important for diffraction from crystals and multilayer coatings? (c) What occurs within the periodic structure as the incidence angle is varied to either side of the Bragg angle? Consult Chapter 3, Section 3.9, Figure3.18 and the accompanying text.

10.21 A Compound refractive lens (CRL) consists of a sequential set of parabolic lenses which can focus hard x-rays. These work best in the 10 keV or higher photon energy range where x-ray absorption is minimal, but are sometimes used at lower photon energies. The refractive focal length, as described in Chapter 10, Section 10.9, is $f = R/2N\delta$ where R is the local radius of curvature, N is the number of lens elements, where the refractive index is given by $n = 1 - \delta + i\beta$, and where it is assumed that to first order $\beta/\delta \ll 1$. These compound refractive lenses are widely used for alignment and focusing at hard x-ray undulator and FEL beamlines. At SLAC's Linear Coherent Light Source (LCLS), a free electron laser (FEL), there are several CRLs in use. Among these is a beryllium CRL with twenty lens elements, each of 50 μm local radius of curvature and an aperture of 250 μm. When operating at 8.2 keV, what is the focal length of this lens? When illuminated at full aperture, what is the numerical aperture (NA) for 8.2 keV? What is the theoretical focal spot size? [See problem 10.12, which describes the diffraction limited focal intensity distribution (Airy distribution) as having a FWHM diameter given by $D_{FWHM} = 0.515\lambda / NA$].

10.22 For a planoconcave lens with a a radius of curvature R = 10 cm, diameter D = 1 cm, and central axial thickness t = 2 mm,

(a) Calculate the focal length f at $\lambda = 0.124$ nm using the lens maker's equation $f = R/(n-1)$ and refractive index approximated by $n \approx 1 - \delta$.

(b) Approximate the fraction of radiation that reaches the focal region assuming uniform illumination of the lens. For a discussion of the lens maker's equation consult the text by Hecht, Chapter 10, reference 62.